CS2102 Tutorial 9: BCNF

Wk 12, Sem 2, 2021/22

1. For each of the following schema decomposition, determine whether or not it is a lossless-join decomposition.

   (a) Schema $R(A, B, C, D)$ with $F = \{A \rightarrow BCD, C \rightarrow D\}$ and decomposition $\{R1(A, B, C), R2(C, D)\}$

   (b) Schema $R(A, B, C, D)$ with $F = \{A \rightarrow BCD, C \rightarrow D\}$ and decomposition $\{R1(A, C), R2(A, B, D)\}$

   (c) Schema $R(A, B, C, D, E)$ with $F = \{AB \rightarrow C, AC \rightarrow D, E \rightarrow ABCD\}$ and decomposition $\{R1(A, B, C), R2(A, B, E), R3(A, C, D)\}$

Solution:

(a) Since $ABC \cap CD = C$ and C is a superkey of $R2$ due to $C \rightarrow D$, the decomposition is lossless.

```
{a} -> {b,c,d}   [Given]
{c} -> {d}       [Given]
compute superkeys of {a,b,c}
compute superkeys of {c,d}
```

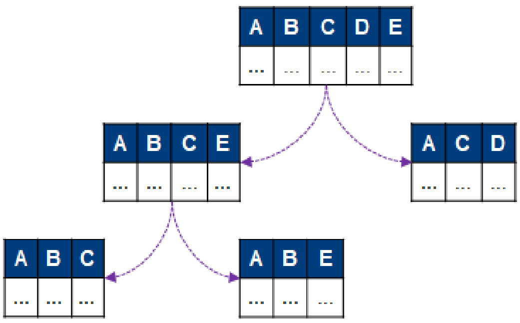
(b) Since $AC \cap ABD = A$ and A is a superkey of $R1$ due to $A \rightarrow BCD$, the decomposition is lossless.

```
{a} -> {b,c,d}   [Given]
{c} -> {d}       [Given]
compute superkeys of {a,c}
compute superkeys of {a,b,d}
```

(c) A decomposition is lossless-join if there exists a *sequence* of binary lossless-join decomposition that generates that decomposition. It turns out, for this case, there is at least one such decomposition:

   (a) Decompose $R(A, B, C, D, E)$ into $R3(A, C, D)$ and $R_t(A, B, C, E)$.
      - $ACD \cap ABCE = AC$ and $AC \rightarrow D$.

   (b) Decompose $R_t(A, B, C, E)$ into $R1(A, B, C)$ and $R2(A, B, E)$.
      - $ABC \cap ABE = AB$ and $AB \rightarrow C$.



Here, R_t is a temporary table to be decomposed later on. Note that this may not be the only steps to decomposition that produces a lossless-join decomposition.

```
{a,b} -> {c}            [Given]
{a,c} -> {d}            [Given]
{e}   -> {a,b,c,d}      [Given]
compute superkeys of {a,b,c}
compute superkeys of {a,b,e}
compute superkeys of {a,c,d}
```

2. Consider the schema $R(A, B, C, D, E)$ with $F = \{A \rightarrow E, AB \rightarrow D, CD \rightarrow AE, E \rightarrow B, E \rightarrow D\}$. Let $\delta = \{R1(B, D, E), R2(A, C, E)\}$ be a decomposition of R.

   (a) Is R in BCNF? Explain.

   (b) Is δ a lossless-join decomposition? Explain.

   (c) Is δ in BCNF? Explain.

   (d) If δ is not in BCNF, find a BCNF decomposition of R.

Solution:

(a) R is not in BCNF because $A \rightarrow E$ violates BCNF:

- $\{A\}^+ = \{ABDE\}$
- A is not superkey of R

(b) The decomposition is lossless-join because $R1 \cap R2 = \{E\}$ and $E \rightarrow BDE$.

(c) Note that $\{A\}^+ = \{ABDE\}$. Therefore, on $R2$, we have $\{A\}^+ = \{AE\}$. This indicates that $R2$ is not in BCNF.

(d) Since $R2$ is not in BCNF, we need to find a BCNF decomposition:

   (a) $\{E\}^+ = \{BDE\}$, so $E \rightarrow BD$ violates BCNF of R.

   (b) Decompose R into $R1(B, D, E)$ and $R2(A, C, E)$.

   (c) $R1$ is in BCNF because the only non-trivial functional dependency is $E \rightarrow BD$ and E is the key of $R1$.

   (d) $\{A\}^+ = \{ABDE\}$, when projected to $R2$ we have $\{A\}^+ = \{AE\}$ on $R2$, so $A \rightarrow E$ violates BCNF of $R2$.

   (e) We decompose $R2$ into $R3(A, E)$ and $R4(A, C)$.

   (f) Both $R3$ and $R4$ are in BCNF.

   The BCNF decomposition is: $\{R1(B, D, E), R3(A, E), R4(A, C)\}$.

```
{a}   -> {e}    [Given]
{a,b} -> {d}    [Given]
{c,d} -> {a,e}  [Given]
{e}   -> {b}    [Given]
{e}   -> {d}    [Given]
compute closure of {a}
compute closure of {a,b}
compute closure of {c,d}
compute closure of {e}
compute superkeys of {b,d,e}
compute superkeys of {a,c,e}
```

3. Consider the schema $R(A, B, C, D, E)$ with $F = \{AB \rightarrow CDE, AC \rightarrow BDE, B \rightarrow C, C \rightarrow B, C \rightarrow D, B \rightarrow E\}$. Find a BCNF decomposition of R.

Solution: First, note that R is not in BCNF. If it is, then our job is actually done.

1. $\{B\}^+ = \{BCDE\}$, so $B \rightarrow CDE$ violates BCNF of R.
2. Decompose R into $R1(B, C, D, E)$ and $R2(B, A)$.
3. $R1$ is in BCNF:
   - $\{B\}^+ = \{BCDE\}$, B is a key of $R1$.
   - $\{C\}^+ = \{BCDE\}$, C is a key of $R1$.
   - $\{D\}^+ = \{D\}$, this is trivial.
   - $\{E\}^+ = \{E\}$, this is trivial.
   - $\{DE\}^+ = \{DE\}$, this is trivial.
   - Any other sets of attributes are superkeys since they are superset of either $\{B\}$ or $\{C\}$.
4. $R2$ is in BCNF trivially.

Therefore, $\{R1(B, C, D, E), R2(A, B)\}$ is a BCNF decomposition of R.

```
{a,b} -> {c,d,e}        [Given]
{a,c} -> {b,d,e}        [Given]
{b}   -> {c}            [Given]
{c}   -> {b}            [Given]
{c}   -> {d}            [Given]
{b}   -> {e}            [Given]
compute closure of {b}
```