1. For each of the following schema decomposition, determine whether or not it is a dependency-preserving decomposition.

   (a) Schema $R(A, B, C, D)$ with $F = \{A \rightarrow BCD, C \rightarrow D\}$ and decomposition $\{R1(A, B, C), R2(C, D)\}$

   (b) Schema $R(A, B, C, D)$ with $F = \{A \rightarrow BCD, C \rightarrow D\}$ and decomposition $\{R1(A, C), R2(A, B, D)\}$

   (c) Schema $R(A, B, C, D, E)$ with $F = \{AB \rightarrow C, AC \rightarrow D, E \rightarrow ABCD\}$ and decomposition $\{R1(A, B, C), R2(A, B, E), R3(A, C, D)\}$

Solution:

(a) Yes.

- Closures on $R1$: $\{A \rightarrow BC\}$
  - $\{A\}^+ = \{ABCD\}$
  - $\{B\}^+ = \{B\}$
  - $\{C\}^+ = \{C\}$
- Closures on $R2$: $\{C \rightarrow D\}$
  - $\{C\}^+ = \{CD\}$
  - $\{D\}^+ = \{D\}$
- Union of closure: $\{A \rightarrow BC, C \rightarrow D\}$
- Derive $F = \{A \rightarrow BCD, C \rightarrow D\}$
  - $\{A\}^+ = \{ABCD\} \Rightarrow A \rightarrow BCD$
  - $\{C\}^+ = \{CD\} \Rightarrow C \rightarrow D$

All functional dependencies in F is preserved.

(b) No.

- Closures on $R1$: $\{A \rightarrow C\}$
  - $\{A\}^+ = \{ABCD\}$
  - $\{C\}^+ = \{CD\}$
- Closures on $R2$: $\{A \rightarrow BD\}$
  - $\{A\}^+ = \{ABCD\}$
  - $\{B\}^+ = \{B\}$
  - $\{D\}^+ = \{D\}$
- Union of closure: $\{A \rightarrow C, A \rightarrow BD\}$
- Derive $F = \{A \rightarrow BCD, C \rightarrow D\}$
  - $\{A\}^+ = \{ABCD\} \Rightarrow A \rightarrow BCD$
  - $\{C\}^+ = \{C\} \Rightarrow C \nrightarrow D$

$C \rightarrow D$ is not preserved.

(c) Yes.

- Closures on $R1$: $\{AB \rightarrow C\}$
  - $\{A\}^+ = \{A\}$
  - $\{B\}^+ = \{B\}$
  - $\{C\}^+ = \{C\}$
  - $\{AB\}^+ = \{ABCD\}$
  - $\{AC\}^+ = \{ACD\}$
  - $\{BC\}^+ = \{BC\}$
- Closures on $R2$: $\{E \rightarrow AB\}$
  - $\{A\}^+ = \{A\}$
  - $\{B\}^+ = \{B\}$
  - $\{E\}^+ = \{ABCDE\}$
  - $\{AB\}^+ = \{ABCD\}$
- Closures on $R3$: $\{AC \rightarrow D\}$
  - $\{A\}^+ = \{A\}$
  - $\{C\}^+ = \{C\}$
  - $\{D\}^+ = \{D\}$
  - $\{AC\}^+ = \{ACD\}$
  - $\{AD\}^+ = \{AD\}$
  - $\{CD\}^+ = \{CD\}$
- Union of closure: $\{AB \rightarrow C, E \rightarrow AB, AC \rightarrow D\}$
- Derive $F = \{AB \rightarrow C, AC \rightarrow D, E \rightarrow ABCD\}$
  - $\{AB\}^+ = \{ABCD\} \Rightarrow AB \rightarrow C$
  - $\{E\}^+ = \{ABCDE\} \Rightarrow E \rightarrow AB$
  - $\{AC\}^+ = \{ACD\} \Rightarrow AC \rightarrow D$

All functional dependencies in F is preserved.

2. Consider the schema $R(A, B, C, D)$ with $F = \{ABC \rightarrow D, D \rightarrow A\}$.

   (a) Is R in BCNF? Explain.

   (b) Is R in 3NF? Explain.

Solution:

(a) $\{D\}^+ = \{AD\}$, so $D \rightarrow A$ violates BCNF of R.

(b) Keys of R are only ABC and BCD. All attributes are prime attributes. There will not be any violation of 3NF on R.

3. Consider the schema $R(A, B, C, D)$ with $F = \{A \rightarrow E, CD \rightarrow A, E \rightarrow B, E \rightarrow D, A \rightarrow BD\}$.

   (a) Is R in 3NF? Explain.

   (b) If R is not in 3NF, find a 3NF decomposition of R.

   (c) Is your decomposition in (b) in BCNF?

Solution:

(a) Keys of R are AC, CD and CE. Consider $E \rightarrow B$.

- E is not a superkey.
- B is not a prime attribute.

$E \rightarrow B$ violates 3NF of R.

(b) 3NF synthesis.

- Minimal basis: $\{A \rightarrow E, CD \rightarrow A, E \rightarrow B, E \rightarrow D\}$
- Canonical cover: $\{A \rightarrow E, CD \rightarrow A, E \rightarrow BD\}$ *union RHS*
- Construct table: $\{R1(A, E), R2(A, C, D), R3(B, D, E)\}$
- Check keys: at least AC and CD are contained in $R2$, so no need to add additional table.

Decomposition: $\{R1(A, E), R2(A, C, D), R3(B, D, E)\}$.

(c) Check each decomposed table one by one:

- $R1$ is in BCNF trivially.
- $R2$ is not in BCNF:
  - $\{A\}^+ = \{ABDE\}$.
  - $A \rightarrow D$ violates BCNF of $R2$.

Note that there is a possibility that using 3NF synthesis, you construct a BCNF decomposition. In which case, you are lucky since BCNF decomposition algorithm may miss this decomposition since (1) the algorithm is inherently probabilistic and (2) it can be time consuming to search all possibilities.

4. Consider the schema $R(A, B, C, D, E)$ with $F = \{AB \rightarrow CDE, AC \rightarrow BDE, B \rightarrow C, C \rightarrow B, C \rightarrow D, B \rightarrow E\}$.

   (a) Is R in 3NF? Explain.

   (b) If R is not in 3NF, find a 3NF decomposition of R.

   (c) Is your decomposition in (b) in BCNF?

Solution:

(a) Keys of R are AB and AC. Consider $C \rightarrow D$.

- C is not a superkey.
- D is not a prime attribute.

$C \rightarrow D$ violates 3NF of R.

(b) 3NF synthesis.

- Minimal basis: $\{B \rightarrow C, C \rightarrow B, C \rightarrow D, C \rightarrow E\}$
- Canonical cover: $\{B \rightarrow C, C \rightarrow BDE\}$ *union RHS*
- Construct table: $\{R1(B, C), R2(B, C, D, E)\}$
- Check keys: AB and CD are NOT contained in either $R1$ or $R2$, so need to add additional table.
  - Can add either $R3(A, B)$ or $R3(A, C)$.
  - Say we add $R3(A, B)$.

Decomposition: $\{R1(B, C), R2(B, C, D, E), R3(A, B)\}$.

(c) Check each decomposed table one by one:

- $R1$ is in BCNF trivially.
- $R2$ is in BCNF:
  - $\{B\}^+ = \{BCDE\}$.
  - $\{C\}^+ = \{BCDE\}$.
  - $\{D\}^+ = \{D\}$.
  - $\{E\}^+ = \{E\}$.
  - $\{DE\}^+ = \{DE\}$.
- $R3$ is in BCNF trivially.

This is the possibility that 3NF synthesis actually finds a BCNF decomposition.