

1. a) superkey is a subset of attributes that uniquely identifies a tuple in a relation

A	X	0
B	X	0, 1
C	X	2
D	X	0
AB	X	00
AC	✓	no duplicates
AD	✓	no duplicates

BC	X	12
BD	X	10
CD	X	20
ABC	✓	no duplicates
ABD	✓	no duplicates
ACD	✓	no duplicates
BCD	X	120
ABCD	✓	no duplicates

supersets of superkeys/keys are also superkeys

superkeys of R are $\{A, C\}$, $\{A, D\}$, $\{A, B, C\}$
 $\{A, B, D\}$, $\{A, C, D\}$, $\{A, B, C, D\}$

b) keys are superkeys that are minimal
 candidate keys are the set of all keys for a relation

if $\{A, C\}$ is a superkey, any superkey which is a superset of $\{A, C\}$ cannot be minimal, hence cannot be a candidate key

only $\{A, C\}$, $\{A, D\}$, $\{A, B, D\}$ remain

since $\{A, C\}$ is a superkey and is also minimal, it must be a candidate key

if $\{A, D\}$ is a candidate key, then $\{A, B, D\}$ cannot

2. A foreign key is a set of attributes that refer to the primary key of another relation

each foreign key in referencing relation must

(1) appear as primary key in referenced relation OR

(2) null value

S. \rightarrow R.A

W	✓	
X	X	4 does not map to R.A
Y	✓	
Z	✓	null ok

W, Y, Z are possible foreign keys

3. a) Equivalent

attribute A available for select and project in both queries
order of operation does not impact result

b) Not Equivalent

Q₂ is invalid as attribute C is no longer available after projection
selection condition refers to non-existent attribute

c) Equivalent

both queries have $|S| \times |T|$ entries and columns D, Y

d) Equivalent

projection reorders both columns to be the same

e) Equivalent

cross product in relational algebra is associative
order does not affect result

f) Equivalent

both projection and union removes duplicates

g) Not equivalent

eg A tuple (a, c) in R and (a, d) in S

Q_1 : (a, c) still remains after $R - S$ hence (a) in $\pi_A(R - S)$

Q_2 : $\pi_A(R) \Rightarrow \{(a)\}$
 $\pi_A(S) \Rightarrow \{(a)\}$ } set difference = \emptyset

4. a) Find all pizzas Alice likes but not liked by Bob

set difference between set of pizzas Alice likes and set of pizzas Bob likes

$$\pi_{\text{pizza}} (\sigma_{\text{name} = \text{'Alice'}} (\text{Likes})) - \pi_{\text{pizza}} (\sigma_{\text{name} = \text{'Bob'}} (\text{Likes}))$$

b) Find all customer-restaurant pairs (C, R) where C and R in the same area
 C likes some pizza sold by R

① To find (C, R) pairs in the same area

$$Q_1 = \text{Customers} \bowtie \text{Restaurants}$$

(natural join over common
area attribute)

② To find pizzas C likes

$$Q_2 = Q_1 \bowtie \text{Likes}$$

(natural join over
cname)

③ To find pizzas sold by R liked by C

$$Q_3 = Q_2 \bowtie \text{sells}$$

(natural join over
pname and pizza)

④ project the (C, R) pairs

$$\pi_{\text{cname, rname}} (Q_3)$$

$$\pi_{\text{cname, rname}} (\text{Customers} \bowtie \text{Restaurants} \bowtie \text{Likes} \bowtie \text{sells})$$

equivalent to

$$\left(\pi_{c1, r1} \left(\sigma_{(a1=a2) \wedge (c1=c2) \wedge (r1=r2) \wedge (p1=p2)} \left(\begin{array}{l} P(c1, a1) (\text{Customers}) \\ \times \\ P(r1, a2) (\text{Restaurants}) \\ \times \\ P(c2, p1) (\text{Likes}) \\ \times \\ P(r2, p2, price) (\text{sells}) \end{array} \right) \right) \right)$$

c) Find pizzas each customer dislikes

if $(cname, pizza)$ not in Likes, customer dislikes

dislikes is the set difference between all customer-pizza pairs and Likes

$$\boxed{\pi_{cname, pizza}(\text{Customers} \times \text{Pizza}) - \text{Likes}}$$

d) Find all customer pairs (c_1, c_2) such that c_1 likes some pizza that c_2 dislikes

natural join on pizza attribute of Likes and Dislikes

$$\boxed{\pi_{cname, cname2}(\text{Likes} \bowtie \rho_{cname2 \leftarrow cname}(\text{Dislikes}))}$$

e) Find customer pairs (c_1, c_2) such that $c_1 < c_2$ and they like the same pizzas

customer pairs who like at least 1 pizza + ordered

$$Q_1 = \pi_{cname, cname2}(\sigma_{cname < cname2}(\text{Likes} \times \rho_{cname2 \leftarrow cname}(\text{Likes})))$$

set difference with LikeDislike to remove pair who do not like same pizza

$$Q_2 = \text{LikeDislike} \cup \pi_{cname2, cname}(\text{LikeDislike})$$

$$Q_1 - Q_2$$

f) Find most expensive for each restaurant

Max

$$Q_1 = \pi_{name, price} (sell)$$

\Rightarrow there exists a price that is higher \Rightarrow not max

$$Q_2 = \pi_{name, price} \left((name = name_2) \wedge (price < price_2) \left(Q_1 \times P_{(name_2, price_2)} Q_1 \right) \right)$$

$$Q_1 - Q_2$$

set difference between all (restaurant-price) pairs and prices which are not max for each restaurant

g) pizzas sold and their areas

$$Q_1 = sell \bowtie Restaurant$$

natural join over name

$$Q_2 = \pi_{name, pizza} (curtleson \bowtie Q_1)$$

natural join over area
include dangling curtains
where no pizza sold in their areas

5.

- R_1 : All pizzas Maggie likes
- R_2 : All (restaurant \times pizza which Maggie likes) pairs
- R_3 : Restaurants that do not sell pizzas Maggie likes
- R_4 : Restaurants that sell pizzas Maggie likes
- R_5 : All pizzas Ralph likes
- R_6 : Restaurants that sell pizzas Ralph likes
- R_7 : Restaurants that sell pizzas Maggie likes but not Ralph