

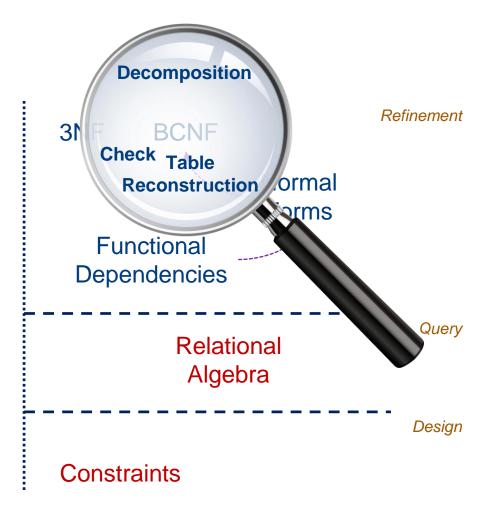
CS2102 Database Systems

Lecture 12 – Third Normal Form

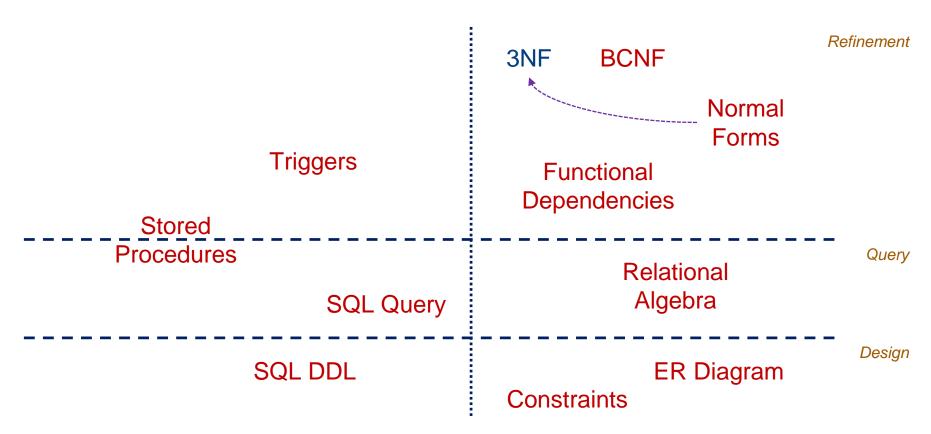
Roadmap

Previously

- BCNF Definition:
 - Every non-trivial & decomposed FD has superkey as LHS
- BCNF Check:
 - Find violation: "more but not all"
- **■** BCNF Decomposition:
 - For violation $A \rightarrow A^+$
 - \blacksquare R1(A^+)
 - $\blacksquare R2(A \cup \{R A^+\})$
 - Repeat until all tables are in BCNF
- BCNF Properties:
 - ✓ No update or deletion anomalies
 - Small redundancies
 - ✓ Original table can be reconstructed.
 - Dependencies may not be preserved in the decomposed table



Roadmap



Roadmap

- We will do this step by step
 - Dependency Preservation (why we need 3NF)



Define 3NF



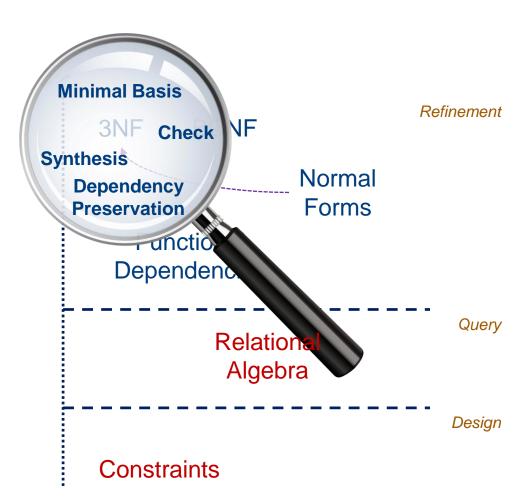
Check 3NF



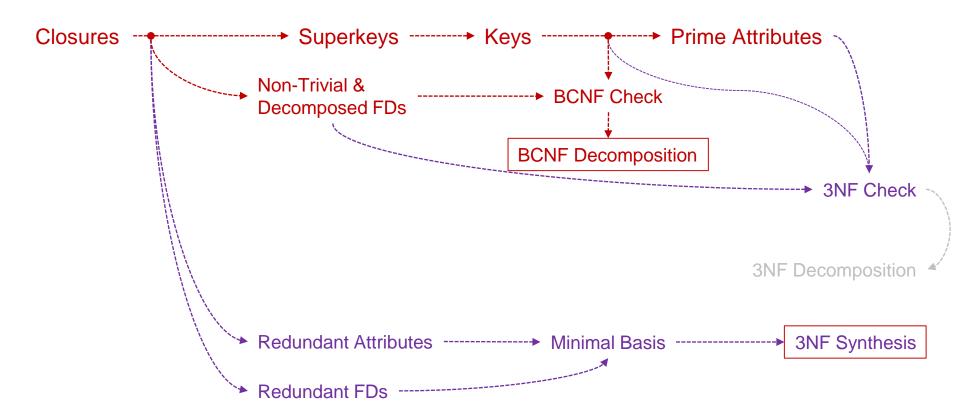
Minimal Basis



3NF Synthesis



Algorithm Roadmap

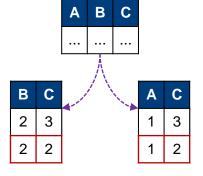


or why we need 3NF



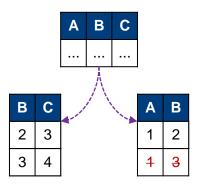
Motivating Example

- Table R(A, B, C) with AB \rightarrow C and C \rightarrow B
- \blacksquare {C} \rightarrow {B, C} violates BCNF of R(A, B, C)
 - R1(B, C) (non-trivial & decomposed FD on R1: $C \rightarrow B$)
 - \blacksquare R2(A, C) (non-trivial & decomposed FD on R2: nothing)



- Where is $AB \rightarrow C$??
 - Cannot even be derived from FDs on R1 and R2
 - Totally "lost"
- This is why we say that a BCNF decomposition may not always preserve all FDs (non dependency preserving decomposition)
 - Dilemma!
 - either the table has anomalies OR does not preserve constraints!

- "What We Want" Example
 - Table R(A, B, C) with A \rightarrow B, B \rightarrow C and A \rightarrow C
 - $\{B\} \rightarrow \{B, C\} \text{ violates BCNF of } R(A, B, C)$
 - R1(B, C) (non-trivial & decomposed FD on R1: $B \rightarrow C$)
 - R2(A, B) (non-trivial & decomposed FD on R2: $A \rightarrow B$)



- Where is $A \rightarrow C$??
 - Can be derived from FDs on R1 and R2
 - Compute $\{A\}^+$ w.r.t. $\{B \rightarrow C, A \rightarrow B\} = \{A, B, C\}$
 - So this FD is *preserved* even when it spans across multiple tables
- No valid insertion into R1 or R2 can violates this functional dependencies
 - Gives rise to the notion of FD equivalence

Functional Dependency Equivalence

- Let F1 and F2 be sets of FDs
- We say that F1 is equivalent to F2 (i.e., $F1 \equiv F2$) if and only if
 - Every FD in F1 can be derived from F2 (i.e., F2 ⊢ F1)
 - Every FD in F2 can be derived from F1 (i.e., F1 + F2)
 - They can look different, but they carry the same information
- In the context of decomposition, we can let F2 be the FD from the decomposed table
 - How do we get this FD from decomposed table?
 - Union of projection

Functional Dependency Equivalence

Example

- Example: R(A, B, C, D, E)
 F1 = {A → B, AB → C, D → AC, D → E}
 F2 = {A → BC, D → AE}
 ❖ Show F1 ≡ F2
- Prove that F1 can be derived from F2
 - \blacksquare A \to B and D \to E can be derived easily using decomposition rule
 - $\{AB\}^+ = \{ABC\} \text{ so } AB \rightarrow C \text{ is implied by } F2$
 - $\{D\}^+ = \{ABCDE\} \text{ so } D \rightarrow AC \text{ is implied by } F2$

∴ F1 can be derived from F2

Functional Dependency Equivalence

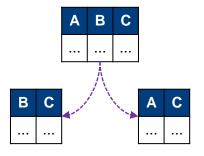
Example

- Example: R(A, B, C, D, E)
 F1 = {A → B, AB → C, D → AC, D → E}
 F2 = {A → BC, D → AE}
 ❖ Show F1 ≡ F2
- 2. Prove that F2 can be derived from F1
 - $\{A\}^+ = \{ABC\} \text{ so } A \rightarrow BC \text{ is implied by } F1$
 - $\{D\}^+ = \{ABCDE\} \text{ so } D \rightarrow AE \text{ is implied by } F1$

∴ F2 can be derived from F1

Motivating Example

- Table R(A, B, C) with AB \rightarrow C and C \rightarrow B
- $\{C\} \rightarrow \{B, C\} \text{ violates BCNF of } R(A, B, C)$
 - R1(B, C) (non-trivial & decomposed FD on R1: $C \rightarrow B$)
 - R2(A, C) (non-trivial & decomposed FD on R2: nothing)



■ F1 = {AB
$$\rightarrow$$
 C, C \rightarrow B}
■ FR1 = {C \rightarrow B}, FR2 = {}

-
$$F2 = \{C \rightarrow B\}$$

- ♦ We can show that F1 ≠ F2
 - Can we still enforce AB → C on the decomposed table?
 - YES! But not easy, need to use trigger!

Third Normal Form



- A table R is in 3NF if every non-trivial and decomposed FD either:
 - its left hand side is a superkey
 - the right hand side is a prime attributes (appears in a key)
- **Example:** R(A, B, C) with $C \rightarrow B$, $AC \rightarrow B$ and $AB \rightarrow C$
 - Key: {AB} and {AC}
 - Prime Attributes: {ABC}
 - Non-trivial and decomposed FDs on R:
 - $C \rightarrow B \qquad AB \rightarrow C \qquad AC \rightarrow B$
 - \circ B is PA \circ AB is SK \circ AC is SK
 - ∴ R satisfies 3NF

- A table R is in 3NF if every non-trivial and decomposed FD either:
 - its left hand side is a superkey
 - the right hand side is a prime attributes (appears in a key)
- **Example:** R(A, B, C) with $A \rightarrow B, B \rightarrow C, AC \rightarrow B$ and $AB \rightarrow C$
 - Key: {A}
 - Prime Attributes: {A}
 - Non-trivial and decomposed FDs on R:
 - $A \rightarrow B$ $B \rightarrow C$ \circ A is SK \circ B is NOT SK \circ C is NOT PA
 - ∴ R violates 3NF

3NF vs BCNF

- BCNF: for any non-trivial and decomposed FD
 - The left hand side is a superkey

"Every attribute must depend ONLY on superkeys!" NO EXCEPTION



- 3NF: for any non-trivial and decomposed FD
 - The left hand side is a superkey
 - OR, the right hand side is a prime attribute

"Exceptions can be made for prime attributes"



3NF vs BCNF

- BCNF: for any non-trivial and decomposed FD
 - The left hand side is a superkey

- 3NF: for any non-trivial and decomposed FD
 - The left hand side is a superkey
 - OR, the right hand side is a prime attribute

- 3NF is more "lenient" than BCNF
 - Therefore
 - Satisfying BCNF ⇒ satisfying 3NF but not necessarily vice versa
 - Violating 3NF ⇒ violating BCNF but not necessarily vice versa

Checking 3NF

- A table R is in NOT 3NF if every non-trivial and decomposed FD both:
 - its left hand side is NOT a superkey
 - the right hand side is NOT a prime attributes (does not appear in any key)

By Counterexample:

- Consider all non-trivial and decomposed FDs of R
- 2. For each non-trivial and decomposed FDs of R, check that
- More but NOT All $\{S\} \subset \{S\}^+ \subset R$

- a. the left hand side is superkey
- b. the right hand side is in prime attributes
- If not one of them, then we have a counterexample
- 3. If no counterexample found, then R satisfies 3NF

Checking 3NF

- A table R is in NOT 3NF if every non-trivial and decomposed FD both:
 - its left hand side is NOT a superkey
 - the right hand side is NOT a prime attributes (does not appear in any key)

By Counterexample:

- Consider all subset of attributes of R
- 2. Compute the closure of each subset
- 3. Consider only the subset that are not superkey (closure $\neq R$)
- 4. Remove the "trivial" attributes
- 5. We have a counterexample, if
 - a. The resulting set is non-empty, **and**
 - b. There is an attribute in the right hand side that is not in the left hand side and not a prime attribute

- **Example:** R(A, B, C, D) with $AB \rightarrow C, C \rightarrow D$ and $D \rightarrow A$
 - Consider all subset of attributes of R
 - 2. Compute the closure of each subset
 - 3. Consider only the subset that are not superkey (closure $\neq R$)
 - 4. Remove the "trivial" attributes
 - 5. We have a counterexample, if
 - a. The resulting set is non-empty, and
 - b. There is an attribute in the right hand side that is not in the left hand side and not a
 prime attribute
- Keys = {AB}, {BC}, {BD} Prime attributes = {ABCD}

$${A}^{+} = {A}$$
 ${B}^{+} = {B}$ ${C}^{+} = {ACD}$ ${D}^{+} = {AD}$
 ${AB}^{+} = {ABCD}$ ${AC}^{+} = {ACD}$ ${AD}^{+} = {AD}$
 ${BC}^{+} = {ABCD}$ ${BD}^{+} = {ABCD}$ ${CD}^{+} = {ACD}$
 ${ABC}^{+} = {ABCD}$ ${ABD}^{+} = {ABCD}$ ${ACD}^{+} = {ACD}$



- A table R is in NOT 3NF if every non-trivial and decomposed FD both:
 - its left hand side is NOT a superkey
 - the right hand side is NOT a prime attributes (does not appear in any key)
- **Exercise:** R(A, B, C, D) with $B \rightarrow C$ and $B \rightarrow D$

■ Keys = {AB} Prime attributes = {AB}

$${A}^{+}$$
 = ${B}^{+}$ = ${C}^{+}$ = ${D}^{+}$ = ${AC}^{+}$ = ${AD}^{+}$ = ${CD}^{+}$ = ${ABC}^{+}$ = ${ABD}^{+}$ = ${ACD}^{+}$ = ${ACD}^{+}$ = ${ACD}^{+}$ =



- A table R is in **NOT** 3NF if every non-trivial and decomposed FD both:
 - its left hand side is NOT a superkey
 - the right hand side is NOT a prime attributes (does not appear in any key)
- **Exercise:** R(A, B, C, D) with $A \rightarrow B, B \rightarrow C, C \rightarrow D$ and $D \rightarrow A$

■ Keys =
$$\{A\}$$
, $\{B\}$, $\{C\}$, $\{D\}$ Prime attributes = $\{ABCD\}$

$${A}^{+} = {B}^{+} = {C}^{+} = {D}^{+} = {AB}^{+} = {AC}^{+} = {AD}^{+} = {AD}^{+} = {ABC}^{+} = {ABD}^{+} = {ACD}^{+} = {BCD}^{+} = {ACD}^{+} = {BCD}^{+} = {ACD}^{+} = {ACD$$

Yes

- A table R is in **NOT** 3NF if every non-trivial and decomposed FD both:
 - its left hand side is NOT a superkey
 - the right hand side is NOT a prime attributes (does not appear in any key)
- **Exercise:** R(A, B, C, D) with AB \rightarrow D, BD \rightarrow C, CD \rightarrow A and AC \rightarrow B
 - Keys = {AB}, {AC}, {BD}, {CD} Prime attributes = {ABCD}

$${A}^{+} = {B}^{+} = {C}^{+} = {D}^{+} = {AB}^{+} = {AC}^{+} = {AD}^{+} = {CD}^{+} = {ABC}^{+} = {ABD}^{+} = {ACD}^{+} = {BCD}^{+} = {ACD}^{+} = {ACD$$

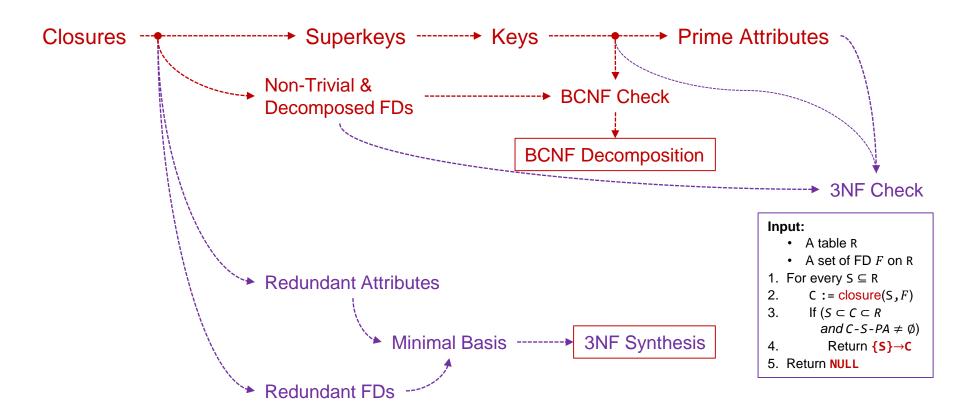


- A table R is in **NOT** 3NF if every non-trivial and decomposed FD both:
 - its left hand side is NOT a superkey
 - the right hand side is NOT a prime attributes (does not appear in any key)
- **Exercise:** R(A, B, C, D, E) with AB \rightarrow C, C \rightarrow E, E \rightarrow A and E \rightarrow D
 - Keys = {AB} , {BC} , {BE}
 Prime attributes = {ABCE}



- A table R is in **NOT** 3NF if every non-trivial and decomposed FD both:
 - its left hand side is NOT a superkey
 - the right hand side is NOT a prime attributes (does not appear in any key)
- **Exercise:** R(A, B, C, D, E) with AB \rightarrow C, DE \rightarrow C and B \rightarrow E
 - Keys = {ABD} Prime attributes = {ABD}

Algorithm Roadmap



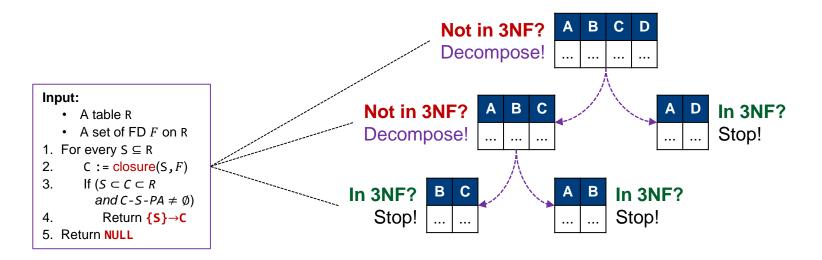
3NF Decomposition



3NF Decomposition

Normalization

- Same Idea as BCNF Decomposition:
 - Same potential problem of non dependency preserving decomposition
 - Is there a better idea?

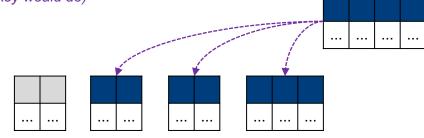


on our path to 3NF synthesis



A Sneak Peek

- 3NF Synthesis
 - Input: Table R and a set of FDs F
 - 1. Derive minimal basis of F
 - 2. From the minimal basis, combine the FDs for which the left hand sides are the same (union rule, producing what's called as canonical cover)
 - 3. Create a table for each FDs remained in the minimal basis after union
 - 4. If none of the tables contains a key of the original table R, create a table that contains a key of R (any key would do)



Definition

- Given a set F of FDs, the minimal basis of F is a *simplified* version of F
 - Also called minimal cover

(let's call this F_b)

- How simplified?
 - Four conditions
 - 1. Every FD in F_h can be derived from F and vice versa
 - 2. Every FD in F_h are non-trivial and decomposed FD
 - 3. For each FD in F_b, none of the attributes on the left hand side is redundant (no redundant attributes)
 - 4. No FD in F_h are redundant (no redundant FD)
- Redundant means that we can remove them (attributes or FDs) without affecting the original FD (i.e., still equivalent)

- Given a set F of FDs, the minimal basis of F is a *simplified* version of F
 - Also called minimal cover (let's call this F_b)
- How simplified?
 - Four conditions
 - 1. Every FD in F_b can be derived from F and vice versa

```
- F = {A \rightarrow BD, AB \rightarrow C, C \rightarrow D, BC \rightarrow D}

\checkmark F1 = {A \rightarrow B, A \rightarrow C, C \rightarrow D}

\times F2 = {A \rightarrow D, A \rightarrow C, C \rightarrow D}

\times F3 = {A \rightarrow B, A \rightarrow C, C \rightarrow D, D \rightarrow C}
```

- Given a set F of FDs, the minimal basis of F is a *simplified* version of F
 - Also called minimal cover (let's call this F_h)
- How simplified?
 - Four conditions
 - 2. Every FD in F_b are non-trivial and decomposed FD

```
- F = {A \rightarrow BD, AB \rightarrow C, C \rightarrow D, BC \rightarrow D}

\checkmark F1 = {A \rightarrow B, A \rightarrow C, C \rightarrow D}

\times F2 = {A \rightarrow BD, A \rightarrow C, C \rightarrow D}

\checkmark F3 = {A \rightarrow B, A \rightarrow C, C \rightarrow D, BC \rightarrow D}
```

- Given a set F of FDs, the minimal basis of F is a simplified version of F
 - Also called minimal cover (let's call this F_h)
- How simplified?
 - Four conditions
 - 3. For each FD in F_b, none of the attributes on the left hand side is redundant (no redundant attributes)
 - $F = \{A \rightarrow BD, AB \rightarrow C, C \rightarrow D, BC \rightarrow D\}$
 - Consider AB → C, if we remove B from left hand side, we have A → C
 - We can derive A \rightarrow C from F since {A}⁺ = {ABCD} (i.e., A \rightarrow C is "hidden" in F)
 - So we can add A → C without adding extraneous constraints
 - But once we add A → C then AB → C is redundant!
 - Effectively, we found that B is redundant in AB → C
 - \therefore AB \rightarrow C should not be in minimal basis

- Given a set F of FDs, the minimal basis of F is a simplified version of F
 - Also called minimal cover (let's call this F_b)
- How simplified?
 - Four conditions
 - 4. No FD in F_h are redundant (no redundant FD)
 - $F = \{A \rightarrow BD, AB \rightarrow C, C \rightarrow D, BC \rightarrow D\}$
 - Consider BC → D
 - We can derive it from $C \rightarrow D$
 - So we can remove it without removing any important information
 - ∴ BC → D should not be in minimal basis

Conditions

- 1. $F_b \equiv F$
- Non-trivial and decomposed
- No redundant attributes
- 4. No redundant FD

Example

- \blacksquare F = {A \rightarrow B, B \rightarrow C, A \rightarrow C} F_h = {A \rightarrow B, B \rightarrow C}
- Is F_b a minimal basis for F?
 - 1. A \rightarrow C in F can be derived from F_h
 - F_h is F by removal of A \rightarrow C
 - All FDs in F_h are non-trivial and decomposed
 - 3. For any FD in F_b, if we remove an attribute from left hand side, then the FD cannot be derived from F (in fact, they have no left hand side!)
 - 4. If any FD in F_h is removed, then some FD in F cannot be derived

∴ F_h is a minimal basis for F

Conditions

- 1. $F_b \equiv F$
- 2. Non-trivial and decomposed
- No redundant attributes
- 4. No redundant FD

Example

- $F = \{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$ $F_b = \{A \rightarrow B, AB \rightarrow C\}$
- Is F_h a minimal basis for F?
 - 1. A \rightarrow C in F can **NOT** be derived from F_h

∴ F_h is **NOT** a minimal basis for F

Conditions

- 1. $F_b \equiv F$
- Non-trivial and decomposed
- No redundant attributes
- 4. No redundant FD

Example

- $F = \{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$ $F_b = \{A \rightarrow BC, A \rightarrow C\}$
- Is F_b a minimal basis for F?
 - 1. B \rightarrow C in F can **NOT** be derived from F_h
 - 2. Also...
 - A → BC is **NOT** a decomposed FD
 - \therefore F_b is **NOT** a minimal basis for F

Conditions

- $F_b \equiv F$
- Non-trivial and decomposed
- No redundant attributes
- No redundant FD

Example

$$\blacksquare$$
 F = {A \rightarrow B, A \rightarrow C, C \rightarrow B} F_b = {A \rightarrow B, AB \rightarrow C, C \rightarrow B}

$$F_h = \{A \rightarrow B, AB \rightarrow C, C \rightarrow B\}$$

- Is F_b a minimal basis for F?
 - 1.

 - B in AB \rightarrow C in can be removed in Fb

(redundant attribute)

$$- \{A\}^+ = \{ABC\}$$

∴ F_h is **NOT** a minimal basis for F

Conditions

- $F_b \equiv F$
- Non-trivial and decomposed
- No redundant attributes
- No redundant FD

Example

$$\blacksquare$$
 F = {A \rightarrow B, B \rightarrow C, A \rightarrow C} F_b = {A \rightarrow B, B \rightarrow C, A \rightarrow C}

$$F_h = \{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$$

- Is F_b a minimal basis for F?
 - 1. ✓

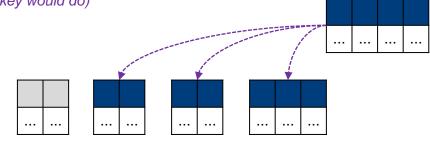
 - 3. ✓
 - 4. A \rightarrow C can be removed!

(redundant FD)

∴ F_h is **NOT** a minimal basis for F

A Reminder on Why We Need Minimal Basis

- 3NF Synthesis
 - Input: Table R and a set of FDs F
 - 1. Derive *minimal basis* of F
 - 2. From the *minimal basis*, combine the FDs for which the left hand sides are the same (union rule, producing what's called as canonical cover)
 - 3. Create a table for each FDs remained in the *minimal basis* after union
 - 4. If none of the tables contains a key of the original table R, create a table that contains a key of R (any key would do)



Conditions

- 1. $F_b \equiv F$
- Non-trivial and decomposed
- No redundant attributes
- 4. No redundant FD

Algorithm

- 2. Transform the FDs so that it is non-trivial and decomposed (maintain $F_b \equiv F$)
- 3. Remove redundant attributes on the left hand sides of each FDs (maintain $F_b \equiv F$)
- 4. Remove redundant FDs (maintain $F_b \equiv F$)
- Clearly, at the end, we have satisfied all conditions

(as long as we maintain $F_b \equiv F$)

But how do we perform step 3?

Conditions

- 1. $F_h \equiv F$
- Non-trivial and decomposed
- No redundant attributes
- 4. No redundant FD

Algorithm: Remove Redundant Attribute

2. Transform the FDs so that it is non-trivial and decomposed

- (maintain $F_b = F$
- 3. Remove redundant attributes on the left hand sides of each FDs
- (maintain $F_b \equiv F$)

4. Remove redundant FDs

 $maintain F_b \equiv F$

■ Consider an FD $\{A\}$ → B in F for any A and B

(in here, {A} is a set of attributes)

- 1. Consider an attribute C in {A}
 - Compute {A-C}+ using F

- (in here, {A-C} means we remove C from {A})
- If $B \in \{A-C\}^+$, then we can remove C

- (because $\{A-C\}\rightarrow B$)
- Repeat step 1 but with {A} → B changed with {A-C} → B
- Do this for all FDs

Conditions

- 1. $F_b \equiv F$
- Non-trivial and decomposed
- No redundant attributes
- 4. No redundant FD

Algorithm: Remove Redundant Attribute

- 2. Transform the FDs so that it is non-trivial and decomposed
- 3. Remove redundant attributes on the left hand sides of each FDs
- 4. Remove redundant FDs

$(maintain F_h \equiv F)$

(maintain $F_h \equiv F$)

 $maintain F_b \equiv F$

■ Example:

- $F = \{A \rightarrow B, A \rightarrow D, AB \rightarrow C, C \rightarrow D, BC \rightarrow D\}$
 - Consider $AB \rightarrow C$

$$\circ \quad \{A\}^+ = \{A,B,C,D\}$$

- B can be removed from AB \rightarrow C
- Repeat with

$$\circ$$
 F = {A \rightarrow B, A \rightarrow D, A \rightarrow C, C \rightarrow D, BC \rightarrow D}

- For {A} → B in F
- 1. Consider an attribute C in {A}
 - Compute {A-C}+ using F
 - If B in {A-C}+, replace {A} → B
 with {A-C} → B then repeat
- 2. Do this for all FDs

Conditions

- 1. $F_h \equiv F$
- Non-trivial and decomposed
- No redundant attributes
- 4. No redundant FD

Algorithm: Remove Redundant Attribute

- 2. Transform the FDs so that it is non-trivial and decomposed
- 3. Remove redundant attributes on the left hand sides of each FDs
- 4. Remove redundant FDs

$(maintain F_b \equiv F)$

(maintain $F_h \equiv F$)

 $maintain F_b \equiv F$

■ Example:

- $F = \{A \rightarrow B, A \rightarrow D, A \rightarrow C, C \rightarrow D, BC \rightarrow D\}$
 - Consider $BC \rightarrow D$

$$\circ \quad \{B\}^+ = \{B\}$$

■ C can **NOT** be removed from BC \rightarrow D

$$\circ \quad \{C\}^+ = \{CD\}$$

- B can be removed from BC → D
- Repeat with

$$\circ \quad \mathsf{F} = \{\mathsf{A} \to \mathsf{B}, \; \mathsf{A} \to \mathsf{D}, \; \mathsf{A} \to \mathsf{C}, \; \mathsf{C} \to \mathsf{D}, \; \mathsf{C} \to \mathsf{D}\}$$

- For {A} → B in F
- 1. Consider an attribute C in {A}
 - Compute {A-C}+ using F
 - If B in {A-C}+, replace {A} → B
 with {A-C} → B then repeat
- 2. Do this for all FDs

Conditions

- 1. $F_h \equiv F$
- Non-trivial and decomposed
- No redundant attributes
- 4. No redundant FD

Algorithm: Remove Redundant Attribute

- 2. Transform the FDs so that it is non-trivial and decomposed
- 3. Remove redundant attributes on the left hand sides of each FDs
- 4. Remove redundant FDs

- $(maintain F_b \equiv F)$
- (maintain $F_b \equiv F$)
- $maintain F_b \equiv F$

■ Example:

- F = {A → B, A → D, A → C, C → D, C → D}
 Nothing else to remove
- Question:
 - Is this always gives us a unique solution?

Yes

- For {A} → B in F
- 1. Consider an attribute C in {A}
 - Compute {A-C}+ using F
 - If B in {A-C}+, replace {A} → B
 with {A-C} → B then repeat
- 2. Do this for all FDs

Conditions

- 1. $F_h \equiv F$
- Non-trivial and decomposed
- No redundant attributes
- 4. No redundant FD

Algorithm: Remove Redundant Attribute

- 2. Transform the FDs so that it is non-trivial and decomposed
- 3. Remove redundant attributes on the left hand sides of each FDs
- 4. Remove redundant FDs

$(maintain F_h \equiv F)$

(maintain $F_h \equiv F$)

 $maintain F_b \equiv F$

■ Example:

- \blacksquare F = {A \rightarrow B, B \rightarrow A, AB \rightarrow C}
 - Consider $AB \rightarrow C$

$$\circ \quad \{A\}^+ = \{A,B,C\}$$

- B can be removed from AB \rightarrow C
- We could have started with

$$\circ$$
 {B}⁺ = {A,B,C}

- \blacksquare A can be removed from AB \rightarrow C
- BUT, we cannot remove both!
 - So two possible solutions

- For {A} → B in F
- 1. Consider an attribute C in {A}
 - Compute {A-C}+ using F
 - If B in {A-C}+, replace {A} → B
 with {A-C} → B then repeat
- 2. Do this for all FDs

Conditions

- 1. $F_b \equiv F$
- Non-trivial and decomposed
- No redundant attributes
- 4. No redundant FD

Algorithm

2. Transform the FDs so that it is non-trivial and decomposed

- (maintain $F_b \equiv F$)
- 3. Remove redundant attributes on the left hand sides of each FDs
- (maintain $F_h \equiv F$)

Remove redundant FDs

(maintain $F_b \equiv F$)

Clearly, at the end, we have satisfied all conditions

(as long as we maintain $F_b \equiv F$)

> But how do we perform **step 4**?

Conditions

- $F_h \equiv F$
- Non-trivial and decomposed
- No redundant attributes
- No redundant FD

Algorithm: Remove Redundant FD

- 2. Transform the FDs so that it is non-trivial and decomposed
- Remove redundant attributes on the left hand sides of each FDs
- Remove redundant FDs (maintain $F_h \equiv F$)
- Consider an FD $\{A\} \rightarrow B$ in F for any A and B

(in here, {A} is a set of attributes)

- **Observation:**
 - If we can remove $\{A\} \rightarrow B$, that means $F \{\{A\} \rightarrow B\} \equiv F$
 - The only difference is the removal of $\{A\} \rightarrow B$
 - Clearly, all FD in F $\{A\} \rightarrow B\}$ can be derived from F
 - So what we need to show is only that $\{A\} \rightarrow B$ can be derived from F - $\{\{A\} \rightarrow B\}$
 - Compute $\{A\}^+$ using $F \{\{A\} \rightarrow B\}$
 - Then check if B is in {A}+

Conditions

- $F_h \equiv F$
- Non-trivial and decomposed
- No redundant attributes
- No redundant FD

Algorithm: Remove Redundant FD

- 2. Transform the FDs so that it is non-trivial and decomposed
- Remove redundant attributes on the left hand sides of each FDs
- Remove redundant FDs

(maintain $F_h \equiv F$)

(in here, {A} is a set of attributes)

- Consider an FD $\{A\} \rightarrow B$ in F for any A and B
 - 1. Compute $\{A\}^+$ using $F \{\{A\} \rightarrow B\}$
 - If B $\in \{A\}^+$, then we can remove $\{A\} \rightarrow B$
 - (because $\{A\}\rightarrow B$ even in its absence)
 - Repeat with next FD but with $\{A\} \rightarrow B$ removed
 - Do this for all FDs

Conditions

- 1. $F_h \equiv F$
- Non-trivial and decomposed
- No redundant attributes
- 4. No redundant FD

Algorithm: Remove Redundant FD

- 2. Transform the FDs so that it is non-trivial and decomposed
- 3. Remove redundant attributes on the left hand sides of each FDs
- Remove redundant FDs

Example:

```
    F = {A → B, A → D, A → C, C → D}
    Consider A → B
    (A)+ w.r.t {A → D, A → C, C → D}
    = {A,C,D}
```

 \blacksquare A \rightarrow B can **NOT** be remove

 $(maintain F_b \equiv F)$

 $(maintain F_b \equiv F)$

(maintain $F_b \equiv F$)

- For {A} → B in F
 - 1. Compute {A}⁺ using F-{{A}→B}
 If B in {A}⁺, remove {A} → B
 then repeat with next FD
- 2. Do this for all FDs

Conditions

- 1. $F_b \equiv F$
- Non-trivial and decomposed
- No redundant attributes
- 4. No redundant FD

Algorithm: Remove Redundant FD

- 2. Transform the FDs so that it is non-trivial and decomposed
- 3. Remove redundant attributes on the left hand sides of each FDs
- 4. Remove redundant FDs

Example:

- \blacksquare F = {A \rightarrow B, A \rightarrow D, A \rightarrow C, C \rightarrow D}
 - Consider $A \rightarrow D$
 - - \blacksquare A \rightarrow D can be removed
 - Repeat with

$$\circ \quad F = \{A \rightarrow B, A \rightarrow C, C \rightarrow D\}$$

$(maintain F_n \equiv F)$

$$(maintain F_h \equiv F)$$

(maintain $F_b \equiv F$)

- For {A} → B in F
- 1. Compute $\{A\}^+$ using $F \{\{A\} \rightarrow B\}$ - If B in $\{A\}^+$, remove $\{A\} \rightarrow B$ then repeat with next FD
- 2. Do this for all FDs

Conditions

- 1. $F_b \equiv F$
- Non-trivial and decomposed
- No redundant attributes
- 4. No redundant FD

Algorithm: Remove Redundant FD

- 2. Transform the FDs so that it is non-trivial and decomposed
- 3. Remove redundant attributes on the left hand sides of each FDs
- 4. Remove redundant FDs

Example:

```
    F = {A → B, A → C, C → D}
    Consider A → C
    (A)<sup>+</sup> w.r.t {A → B, C → D}
    = {A,B}
```

■ A → C can NOT be removed

$(maintain F_b \equiv F)$

(maintain $F_b \equiv F$)

(maintain $F_b \equiv F$)

- For {A} → B in F
- 1. Compute {A}+ using F-{{A}→B}
 If B in {A}+, remove {A} → B
 then repeat with next FD
- 2. Do this for all FDs

Conditions

- 1. $F_b \equiv F$
- Non-trivial and decomposed
- No redundant attributes
- 4. No redundant FD

Algorithm: Remove Redundant FD

- 2. Transform the FDs so that it is non-trivial and decomposed
- 3. Remove redundant attributes on the left hand sides of each FDs
- Remove redundant FDs

Example:

```
    F = {A → B, A → C, C → D}
    Consider C → D
    (C)+ w.r.t {A → B, A → C}
    = {C}
```

 \blacksquare C \rightarrow D can **NOT** be removed

 $(maintain F_b \equiv F)$

(maintain $F_b \equiv F$)

(maintain $F_b \equiv F$)

Algorithm

- For {A} → B in F
- 1. Compute $\{A\}^+$ using $F \{\{A\} \rightarrow B\}$ - If B in $\{A\}^+$, remove $\{A\} \rightarrow B$

then repeat with next FD

2. Do this for all FDs

Conditions

- 1. $F_h \equiv F$
- Non-trivial and decomposed
- No redundant attributes
- 4. No redundant FD

Algorithm: Remove Redundant FD

- 2. Transform the FDs so that it is non-trivial and decomposed
- 3. Remove redundant attributes on the left hand sides of each FDs
- 4. Remove redundant FDs
- Example:
 - \blacksquare F = {A \rightarrow B, A \rightarrow C, C \rightarrow D}
 - No other FDs to consider
- Question:
 - Is this always gives us a unique solution?

```
Yes
```

- $(maintain F_b \equiv F)$
- (maintain $F_b \equiv F$)
- (maintain $F_b \equiv F$)

- For $\{A\} \rightarrow B$ in F
 - 1. Compute {A}⁺ using F-{{A}→B}
 If B in {A}⁺, remove {A} → B
 then repeat with next FD
- 2. Do this for all FDs

Conditions

- 1. $F_h \equiv F$
- Non-trivial and decomposed
- No redundant attributes
- 4. No redundant FD

Algorithm: Remove Redundant FD

- 2. Transform the FDs so that it is non-trivial and decomposed
- 3. Remove redundant attributes on the left hand sides of each FDs
- 4. Remove redundant FDs

Example:

- \blacksquare F = {A \rightarrow B, A \rightarrow C, B \rightarrow A, B \rightarrow C, C \rightarrow A, C \rightarrow B}
 - Can remove either one but not both
 - \circ A \rightarrow B
 - \circ A \rightarrow C
 - {A}+ w.r.t. {A \rightarrow B, B \rightarrow A, B \rightarrow C, C \rightarrow A, C \rightarrow B} = {A, B, C}

 $(maintain F_b \equiv F)$

(maintain $F_b \equiv F$)

(maintain $F_b \equiv F$)

- For {A} → B in F
- 1. Compute {A}+ using F-{{A}→B}
 - If B in {A}⁺, remove {A} → B
 then repeat with next FD
- 2. Do this for all FDs

Conditions

- $I. \quad F_h \equiv F$
- Non-trivial and decomposed
- No redundant attributes
- No redundant FD

Algorithm

2. Transform the FDs so that it is non-trivial and decomposed

- $(maintain F_b \equiv F)$
- 3. Remove redundant attributes on the left hand sides of each FDs

```
(maintain F_h \equiv F)
```

- For $\{A\} \rightarrow B$ in F
- 1. Consider an attribute C in {A}
 - Compute {A-C}+ using F
 - If B in $\{A-C\}^+$, replace $\{A\} \rightarrow B$ with $\{A-C\} \rightarrow B$ then repeat
- 2. Do this for all FDs
- Remove redundant FDs

(maintain $F_h \equiv F$)

- For {A} → B in F
 - 1. Compute $\{A\}^+$ using $F \{\{A\} \rightarrow B\}$
 - If B in $\{A\}^+$, remove $\{A\} \rightarrow B$ then repeat with next FD
 - 2. Do this for all FDs
- Clearly, at the end, we have satisfied all conditions

(as long as we maintain $F_b \equiv F$)

Conditions

(maintain $F_h \equiv F$)

(maintain $F_h \equiv F$)

- 1. $F_b \equiv F$
- 2. Non-trivial and decomposed
- No redundant attributes
- 4. No redundant FD

Example

- \blacksquare F = {BC \rightarrow DE, A \rightarrow E, D \rightarrow A, E \rightarrow B}
- 2. Transform the FDs so that it is non-trivial and decomposed
 - $F = \{BC \rightarrow D, BC \rightarrow E, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$
- 3. Remove redundant attributes on the left hand sides of each FDs
 - Two candidates
 - 1. BC \rightarrow D
 - 2. BC \rightarrow E

Conditions

- 1. $F_b \equiv F$
- 2. Non-trivial and decomposed
- No redundant attributes
- 4. No redundant FD

Example

- $F = \{BC \rightarrow D, BC \rightarrow E, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$
- 3. Remove redundant attributes on the left hand sides of each FDs

- Consider BC → D
 - $\{B\} + = \{B\}$
 - So C can NOT be removed
 - $\{C\} + = \{C\}$
 - So B can NOT be removed

Conditions

- 1. $F_b \equiv F$
- Non-trivial and decomposed
- No redundant attributes
- 4. No redundant FD

Example

- $F = \{BC \rightarrow D, BC \rightarrow E, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$
- 3. Remove redundant attributes on the left hand sides of each FDs

- Consider BC → E
 - $\{B\} + = \{B\}$
 - So C can NOT be removed
 - $\{C\} + = \{C\}$
 - So B can NOT be removed

Conditions

- 1. $F_b \equiv F$
- Non-trivial and decomposed
- No redundant attributes
- 4. No redundant FD

Example

- $F = \{BC \rightarrow D, BC \rightarrow E, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$
- 2. Transform the FDs so that it is non-trivial and decomposed
 - $F = \{BC \rightarrow D, BC \rightarrow E, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$
- 3. Remove redundant attributes on the left hand sides of each FDs
 - No redundant attributes
- 4. Remove redundant FDs
 - Need to check everything

(maintain $F_b \equiv F$)

(maintain $F_h \equiv F$)

Conditions

- 1. $F_b \equiv F$
- Non-trivial and decomposed
- No redundant attributes
- 4. No redundant FD

Example

- \blacksquare F = {BC \rightarrow D, BC \rightarrow E, A \rightarrow E, D \rightarrow A, E \rightarrow B}
- 4. Remove redundant FDs

(maintain $F_h \equiv F$)

■ Consider BC → D

-
$$\{BC\}^+$$
 w.r.t. $\{BC \rightarrow E, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$
= $\{B, C\}$

 \circ So BC \rightarrow D is **NOT** redundant

Conditions

- 1. $F_b \equiv F$
- Non-trivial and decomposed
- No redundant attributes
- 4. No redundant FD

Example

- \blacksquare F = {BC \rightarrow D, BC \rightarrow E, A \rightarrow E, D \rightarrow A, E \rightarrow B}
- 4. Remove redundant FDs

- Consider BC → E
 - {BC}+ w.r.t. {BC → D, A → E, D → A, E → B}
 = {A, B, C, D, E}
 So BC → E is redundant
- ❖ Continue with $F = \{BC \rightarrow D, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$

Conditions

- 1. $F_b \equiv F$
- 2. Non-trivial and decomposed
- No redundant attributes
- 4. No redundant FD

Example

- \blacksquare F = {BC \rightarrow D, A \rightarrow E, D \rightarrow A, E \rightarrow B}
- 4. Remove redundant FDs
 - Consider A → E

-
$$\{A\}^+$$
 w.r.t. $\{BC \rightarrow D, D \rightarrow A, E \rightarrow B\}$
= $\{A\}$

So A → E is NOT redundant

Conditions

- 1. $F_b \equiv F$
- 2. Non-trivial and decomposed
- No redundant attributes
- 4. No redundant FD

Example

- \blacksquare F = {BC \rightarrow D, A \rightarrow E, D \rightarrow A, E \rightarrow B}
- 4. Remove redundant FDs
 - Consider D → A

-
$$\{D\}^+$$
 w.r.t. $\{BC \rightarrow D, A \rightarrow E, E \rightarrow B\}$
= $\{D\}$

So D → A is NOT redundant

Conditions

- 1. $F_b \equiv F$
- 2. Non-trivial and decomposed
- No redundant attributes
- 4. No redundant FD

Example

- \blacksquare F = {BC \rightarrow D, A \rightarrow E, D \rightarrow A, E \rightarrow B}
- 4. Remove redundant FDs
 - Consider E → B
 - $\{E\}^+$ w.r.t. $\{BC \rightarrow D, A \rightarrow E, D \rightarrow A\}$ = $\{E\}$
 - \circ So E \rightarrow B is **NOT** redundant

Conditions

- 1. $F_b \equiv F$
- Non-trivial and decomposed
- No redundant attributes
- 4. No redundant FD

Example

$$\blacksquare$$
 F = {BC \rightarrow D, A \rightarrow E, D \rightarrow A, E \rightarrow B}

- 2. Transform the FDs so that it is non-trivial and decomposed
 - $F = \{BC \rightarrow D, BC \rightarrow E, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$
- 3. Remove redundant attributes on the left hand sides of each FDs
 - No redundant attributes
- 4. Remove redundant FDs
 - BC → E is redundant

$$\bullet$$
 F_b = {BC \rightarrow D, A \rightarrow E, D \rightarrow A, E \rightarrow B}

(maintain
$$F_b \equiv F$$
)

(maintain
$$F_h \equiv F$$
)

(maintain
$$F_b \equiv F$$
)

Conditions

- 1. $F_b \equiv F$
- Non-trivial and decomposed
- No redundant attributes
- 4. No redundant FD

Exercise

■
$$F = \{A \rightarrow C, AC \rightarrow D, AD \rightarrow B\}$$

2. Transform the FDs so that it is non-trivial and decomposed

■
$$F = \{A \rightarrow C, AC \rightarrow D, AD \rightarrow B\}$$

- 3. Remove redundant attributes on the left hand sides of each FDs
 - Two candidates

1. AC
$$\rightarrow$$
 D

(C is redundant)

2. AD
$$\rightarrow$$
 B

(D is redundant)

■
$$F = \{A \rightarrow C, A \rightarrow D, A \rightarrow B\}$$

4. Remove redundant FDs

No redundant FD

$$\blacksquare$$
 $F_b = \{A \rightarrow C, A \rightarrow D, A \rightarrow B\}$

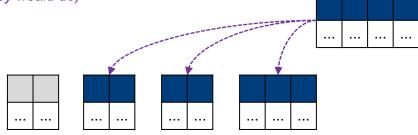
(maintain $F_h \equiv F$)

(maintain $F_h \equiv F$)

our final product



- 3NF Synthesis
 - Input: Table R and a set of FDs F
 - 1. Derive minimal basis of F
 - 2. From the minimal basis, combine the FDs for which the left hand sides are the same (union rule, producing what's called as canonical cover)
 - 3. Create a table for each FDs remained in the minimal basis after union
 - 4. If none of the tables contains a key of the original table R, create a table that contains a key of R (any key would do)



3NF Synthesis

- Derive minimal basis of F
- 2. Produce canonical cover
- 3. Create a table for each FD
- 4. Add the *key* if missing

Algorithm

- **Example:** R(A, B, C, D, E) with BC \rightarrow DE, A \rightarrow E, D \rightarrow A and E \rightarrow B
 - $F = \{BC \rightarrow DE, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$
 - Derive minimal basis of F

-
$$F_h = \{BC \rightarrow D, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$$

2. Produce canonical cover

-
$$F_C = \{BC \rightarrow D, A \rightarrow E, D \rightarrow A, E \rightarrow B\}$$

3. Create a table for each FD

4. Add the key if missing

- R1 contains a key

3NF Synthesis

- Derive minimal basis of F
- 2. Produce canonical cover
- 3. Create a table for each FD
- 4. Add the *key* if missing

Lossless Join Decomposition

- **Example:** R(A, B, C, D) with $A \rightarrow B$ and $C \rightarrow D$
 - \blacksquare F = {A \rightarrow B, C \rightarrow D}
 - 1. Derive minimal basis of F

-
$$F_h = \{A \rightarrow B, C \rightarrow D\}$$

2. Produce canonical cover

-
$$F_C = \{A \rightarrow B, C \rightarrow D\}$$

- 3. Create a table for each FD
 - R1(A, B) and R2(C, D) ---
- 4. Add the key if missing
 - Keys = $\{AC\}$
 - Add R3(A, C)

Why add Key?

R1(A, B) and R2(C, D) cannot be used to reconstruct R(A, B, C, D)

3NF Synthesis

- Derive minimal basis of F
- 2. Produce canonical cover
- 3. Create a table for each FD
- 4. Add the *key* if missing

Algorithm

Exercise: R(A, B, C, D, E) with A \rightarrow B, A \rightarrow C, B \rightarrow C, E \rightarrow C and E \rightarrow D

$$\blacksquare$$
 F = {A \rightarrow B, A \rightarrow C, B \rightarrow C, E \rightarrow C, E \rightarrow D}

Derive minimal basis of F

-
$$F_b = \{A \rightarrow B, B \rightarrow C, E \rightarrow C, E \rightarrow D\}$$

2. Produce canonical cover

-
$$F_C = \{A \rightarrow B, B \rightarrow C, E \rightarrow CD\}$$

3. Create a table for each FD

4. Add the key if missing

- Keys =
$$\{AE\}$$

- Add R4(A, E)

3NF Synthesis

- Derive minimal basis of F
- 2. Produce canonical cover
- 3. Create a table for each FD
- 4. Add the *key* if missing

Algorithm

Exercise: R(A, B, C, D, E) with A \rightarrow B, AB \rightarrow C, C \rightarrow DE, E \rightarrow C and E \rightarrow D

■
$$F = \{A \rightarrow B, AB \rightarrow C, C \rightarrow DE, E \rightarrow C, E \rightarrow D\}$$

Derive minimal basis of F

-
$$F_b = \{A \rightarrow B, A \rightarrow C, C \rightarrow D, C \rightarrow E, E \rightarrow C\}$$

2. Produce canonical cover

-
$$F_C = \{A \rightarrow BC, C \rightarrow DE, E \rightarrow C\}$$

3. Create a table for each FD

4. Add the key if missing

- Keys = $\{A\}$
- R1 contains a key

Final Thought

closing statement

Summary

- Poorly designed tables give rise to redundancy, update anomalies and deletion anomalies
- BCNF eliminates these problems
 - BCNF: for any non-trivial and decomposed FD on a table R, its left hand side is a superkey for R
 - But BCNF does not always preserve all FDs (non dependency preserving)
 - We may need to perform a join of multiple tables to check whether an FD holds
- **3NF** is slightly weaker than BCNF (has more redundancies, has update and deletion anomalies in some rare cases) but preserves all FDs
 - 3NF: for any non-trivial and decomposed FD on a table R, either its left hand side is a superkey for R, or its right hand side is a prime attribute

BCNF or 3NF or *Lower*?

- BCNF is only inferior to 3NF in the sense that sometimes it does not preserve all FDs
- Idea:
 - Go for BCNF if we can find a BCNF decomposition that preserves all FDs
 - If such decomposition cannot be found, then
 - Still go for BCNF if preserving all FDs is not important
 - Or go for 3NF otherwise
 - If we are lucky, 3NF synthesis may actually find a BCNF decomposition!
- Should we go lower than 3NF?
 - Notice how there can be many tables produced by 3NF
 - What will happen to queries?
 - We may have to perform lots of joins
 - This can slow down queries by a lot since joins are expensive
 - Even 3NF may not be suitable in that case

QUESTION?