

# **CS3223 Lecture 5**

## **Query Evaluation: Projection & Join**

# Projection: $\pi_{A_1, \dots, A_m}(R)$

- ▶  $\pi_L(R)$  projects columns given by list L from relation  $R$
- ▶ Example: **select distinct age from R**

Relation R				
name	age	weight	height	$\pi_{\text{age}}(R)$
Alice	17	48	175	age
Bob	15	60	178	17
Curly	10	65	171	15
Larry	12	70	175	10
Lucy	17	45	170	12
Moe	10	55	180	

- ▶  $\pi_L^*(R)$  same as  $\pi_L(R)$  but preserves duplicates

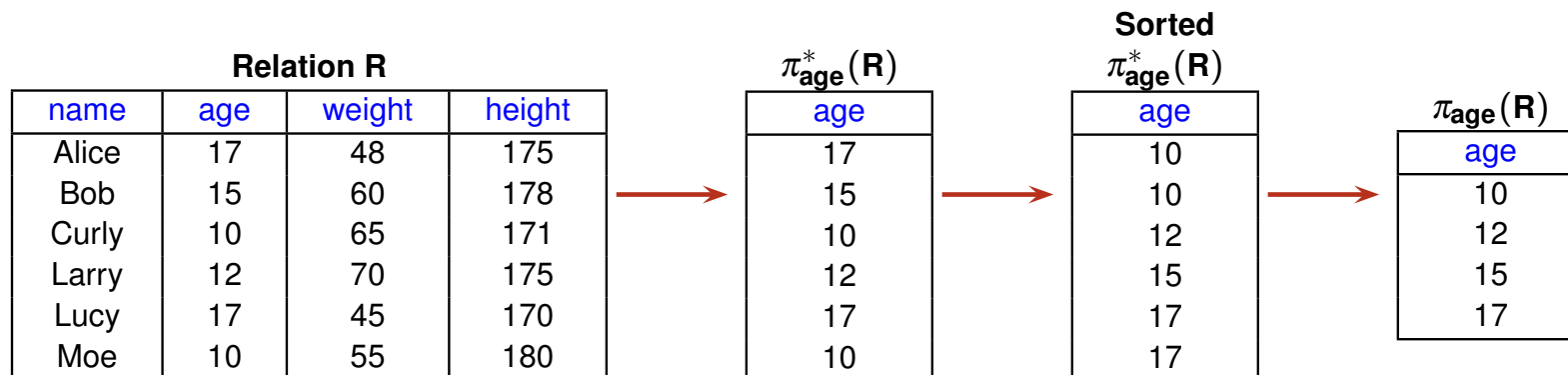
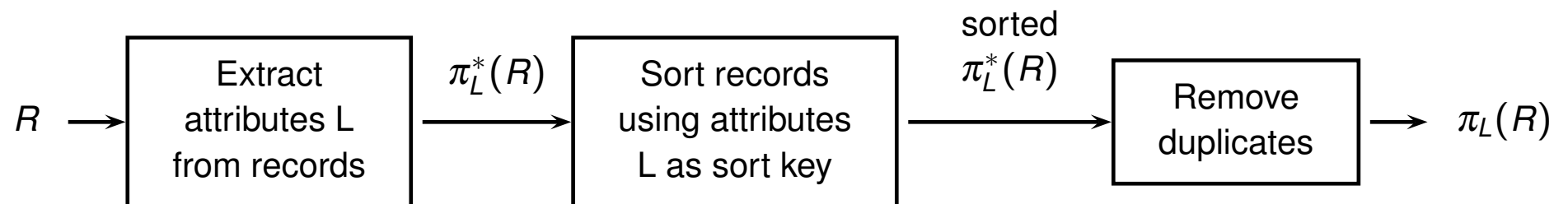
$\pi_{\text{age}}^*(R)$
age
17
15
10
12
17
10

# Projection Operation, $\pi_{A_1, \dots, A_m}(R)$

- ▶ Projection involves two tasks:
  1. remove unwanted attributes
  2. eliminate any duplicate tuples produced
- ▶ Two approaches:
  - ▶ Projection based on sorting
  - ▶ Projection based on hashing

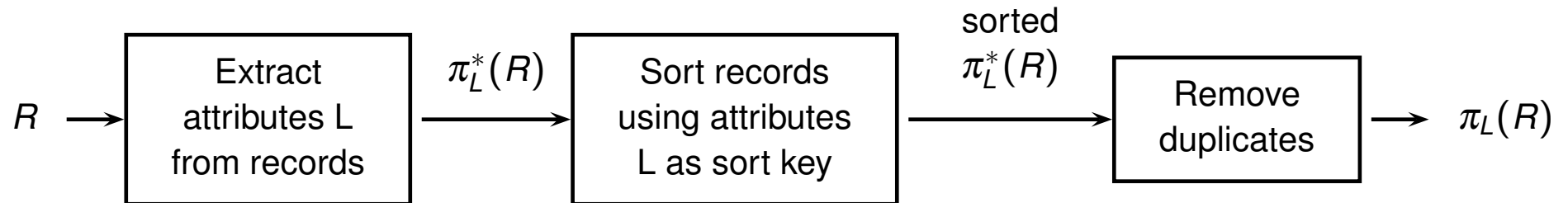
# Sort-based Approach

Consider  $\pi_L(R)$  where  $L$  denote some sequence of attributes of  $R$



# Sort-based Approach

Consider  $\pi_L(R)$  where  $L$  denote some sequence of attributes of  $R$



## Cost Analysis

### ► Step 1:

- Cost to scan records =  $|R|$
- Cost to output temporary result =  $|\pi_L^*(R)|$

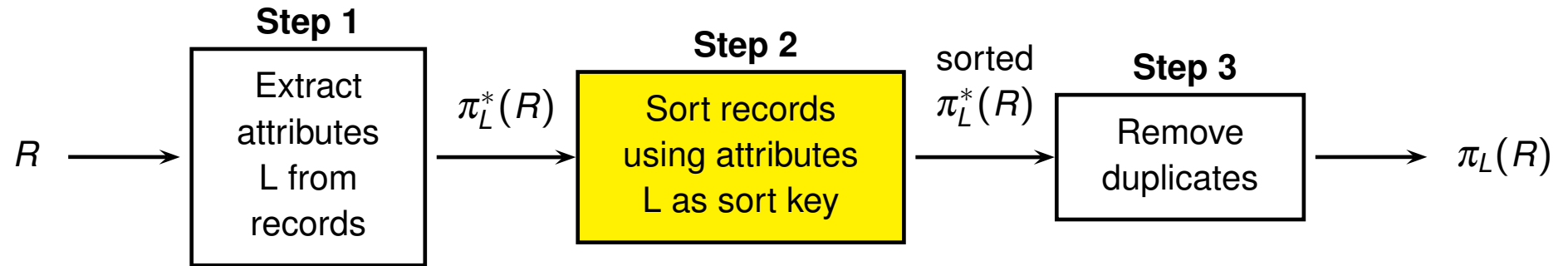
### ► Step 2:

- Cost to sort records =  $2|\pi_L^*(R)| (\log_m(N_0) + 1)$
- $N_0$  = number of initial sorted runs,  $m$  = merge factor

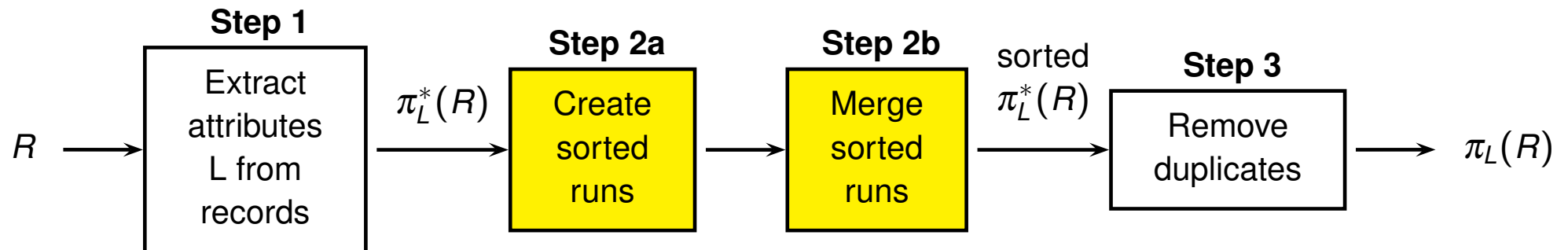
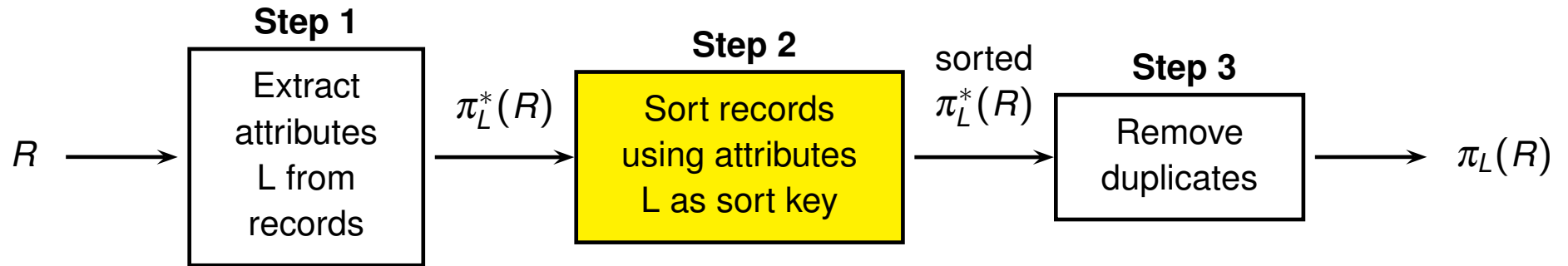
### ► Step 3:

- Cost to scan records =  $|\pi_L^*(R)|$

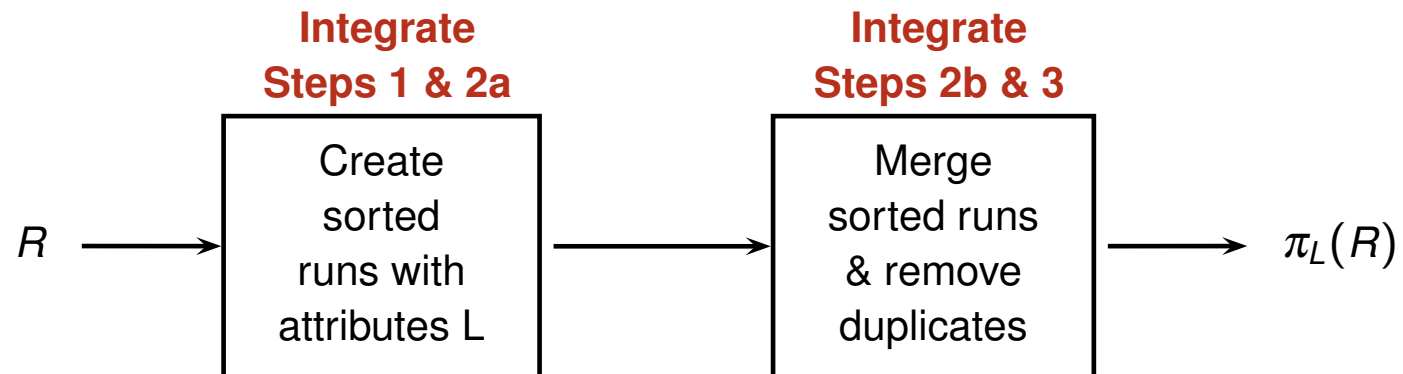
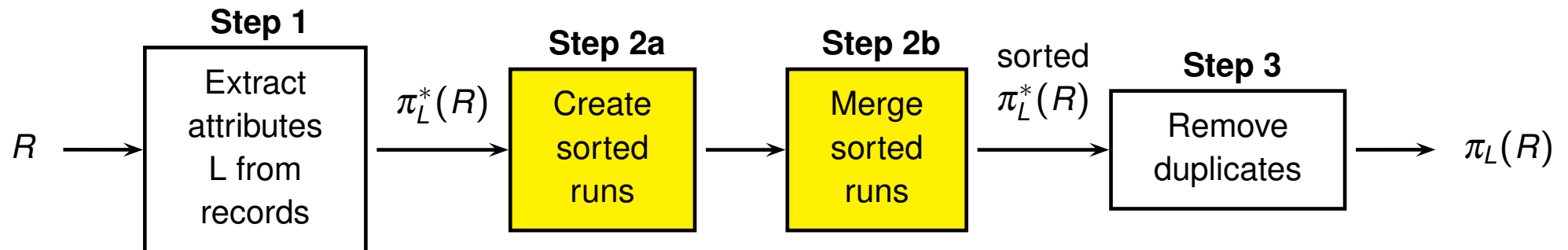
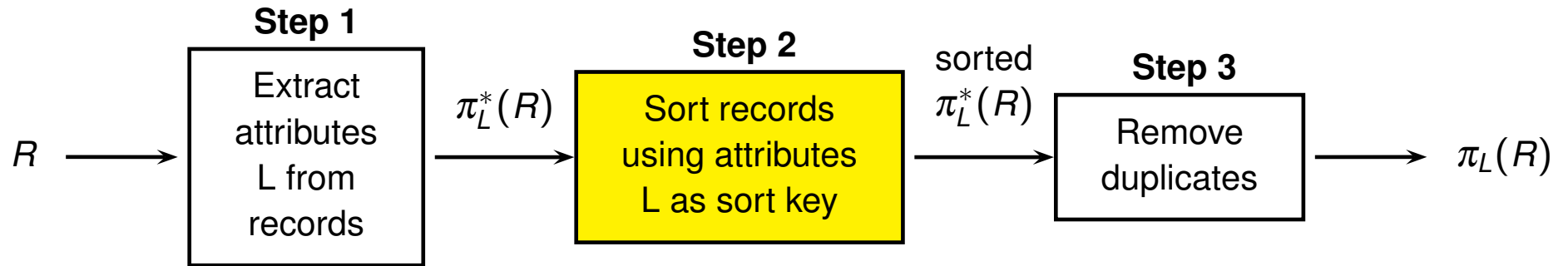
# Optimized Sort-based Approach



# Optimized Sort-based Approach



# Optimized Sort-based Approach





# Hash-based Approach

- ▶ Consider  $\pi_L(R)$
- ▶ Build a main-memory hash table to detect & remove duplicates

01. initialize an empty hash table  $T$
02. for each tuple  $t$  in  $R$  do
03.     apply hash function  $h$  on  $\pi_L(t)$
04.     let  $t$  be hashed to bucket  $B_i$  in  $T$
05.     if ( $\pi_L(t)$  is not in  $B_i$ ) then
06.         insert  $\pi_L(t)$  into  $B_i$
07. return all entries in  $T$

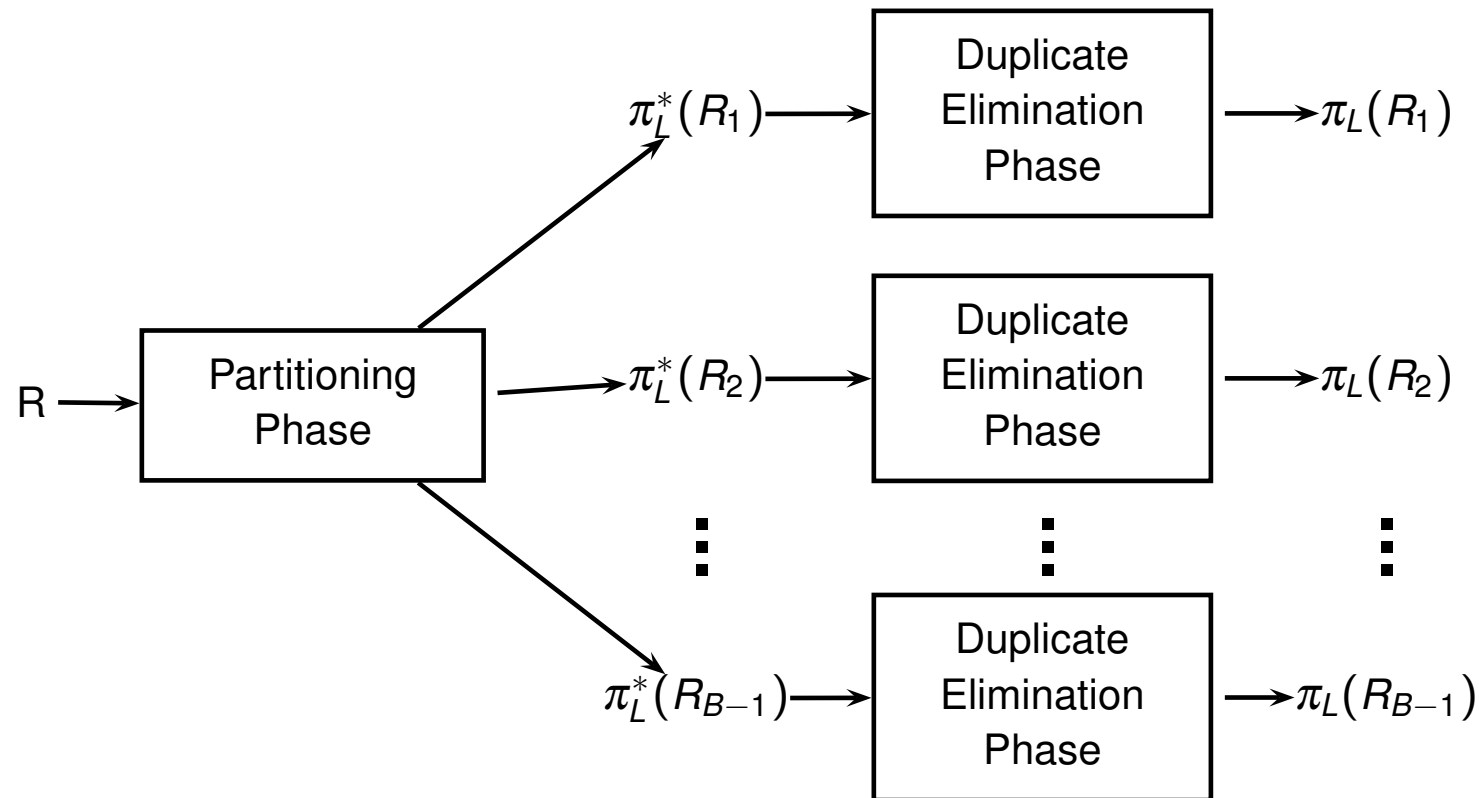
# Hash-based Approach

Consists of two phases:

1. **Partitioning phase**: partitions  $R$  into  $R_1, R_2, \dots, R_{B-1}$ 
  - ▶ Hash on  $\pi_L(t)$  for each tuple  $t \in R$
  - ▶  $R = R_1 \cup R_2 \cup \dots \cup R_{B-1}$
  - ▶  $\pi_L^*(R_i) \cap \pi_L^*(R_j) = \emptyset$  for each pair  $R_i$  &  $R_j, i \neq j$
2. **Duplicate elimination phase**: eliminates duplicates from each  $\pi_L^*(R_i)$

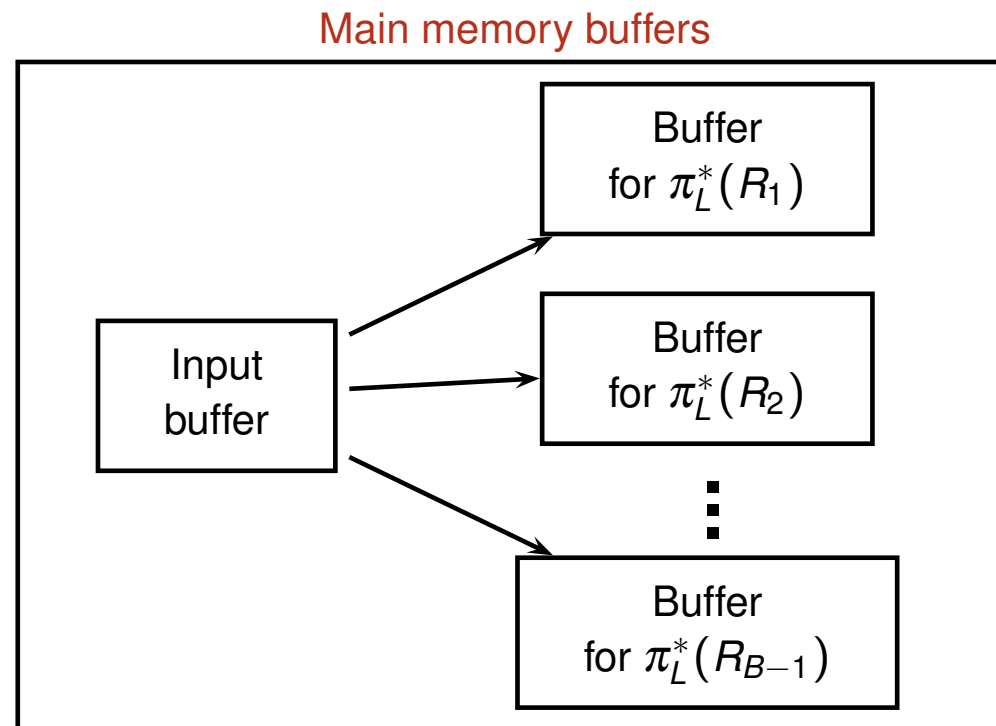
$$\pi_L(R) = \text{duplicate-free union of } \pi_L(R_1), \pi_L(R_2), \dots, \pi_L(R_{B-1})$$

# Hash-based Approach (cont.)



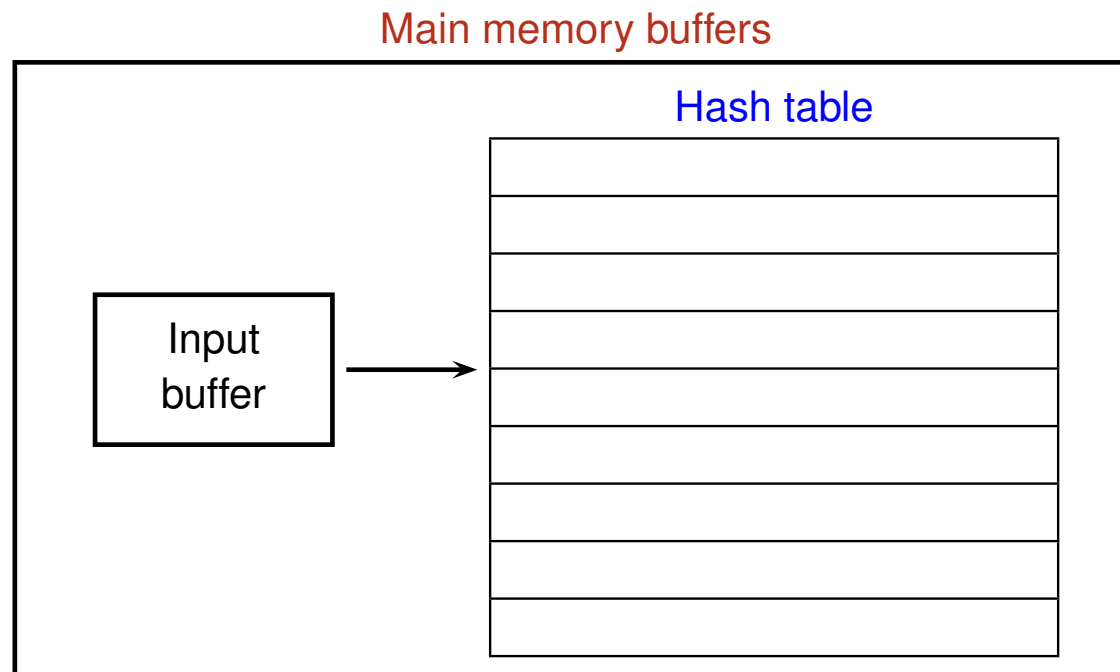
# Partitioning Phase

- ▶ Use one buffer for input &  $(B - 1)$  buffers for output
- ▶ Read  $R$  one page at a time into input buffer
- ▶ For each tuple  $t$  in input buffer,
  - ▶ project out unwanted attributes from  $t$  to form  $t'$
  - ▶ apply a hash function  $h$  on  $t'$  to distribute  $t'$  into one output buffer
  - ▶ flush output buffer to disk whenever buffer is full

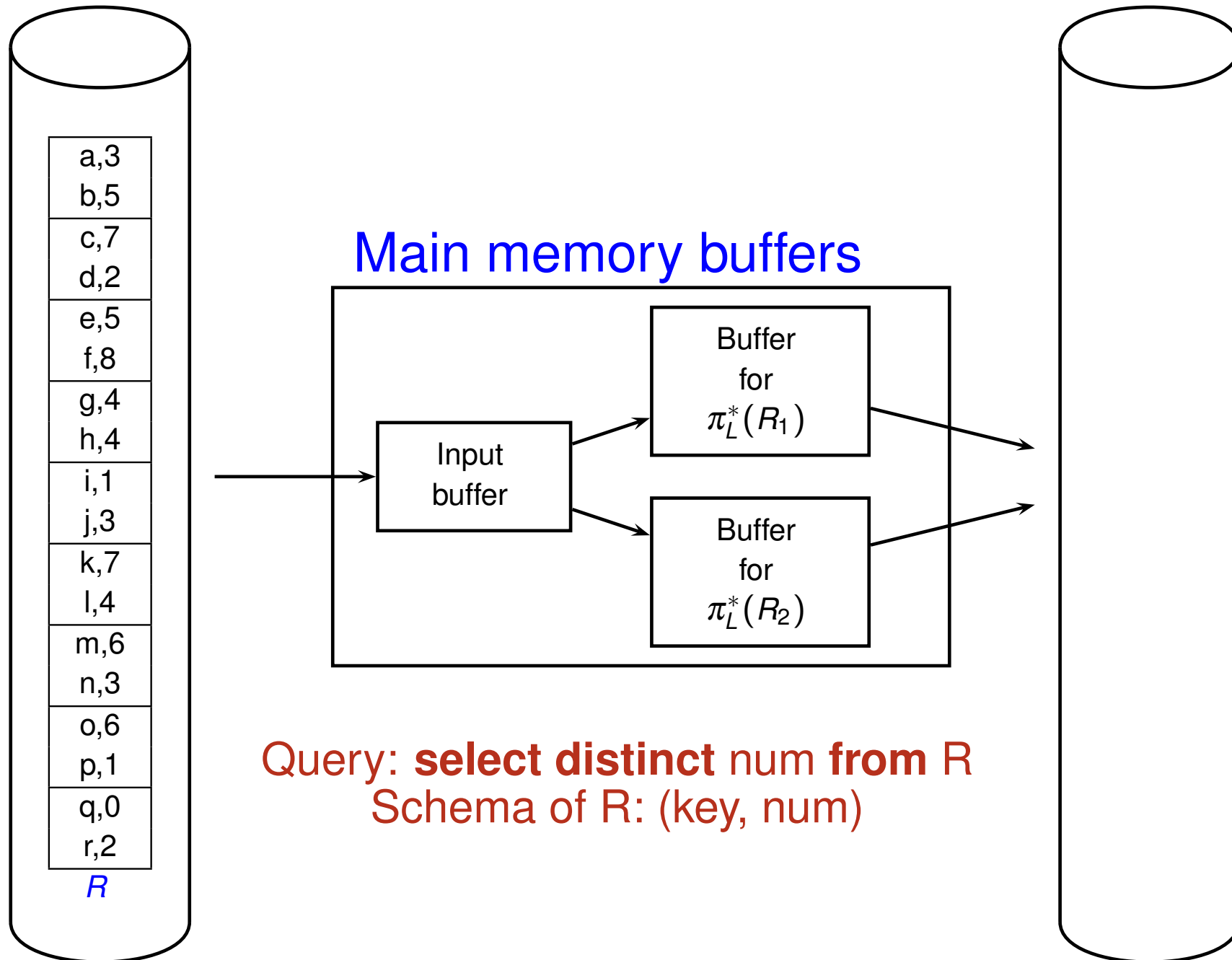


# Duplicate Elimination Phase

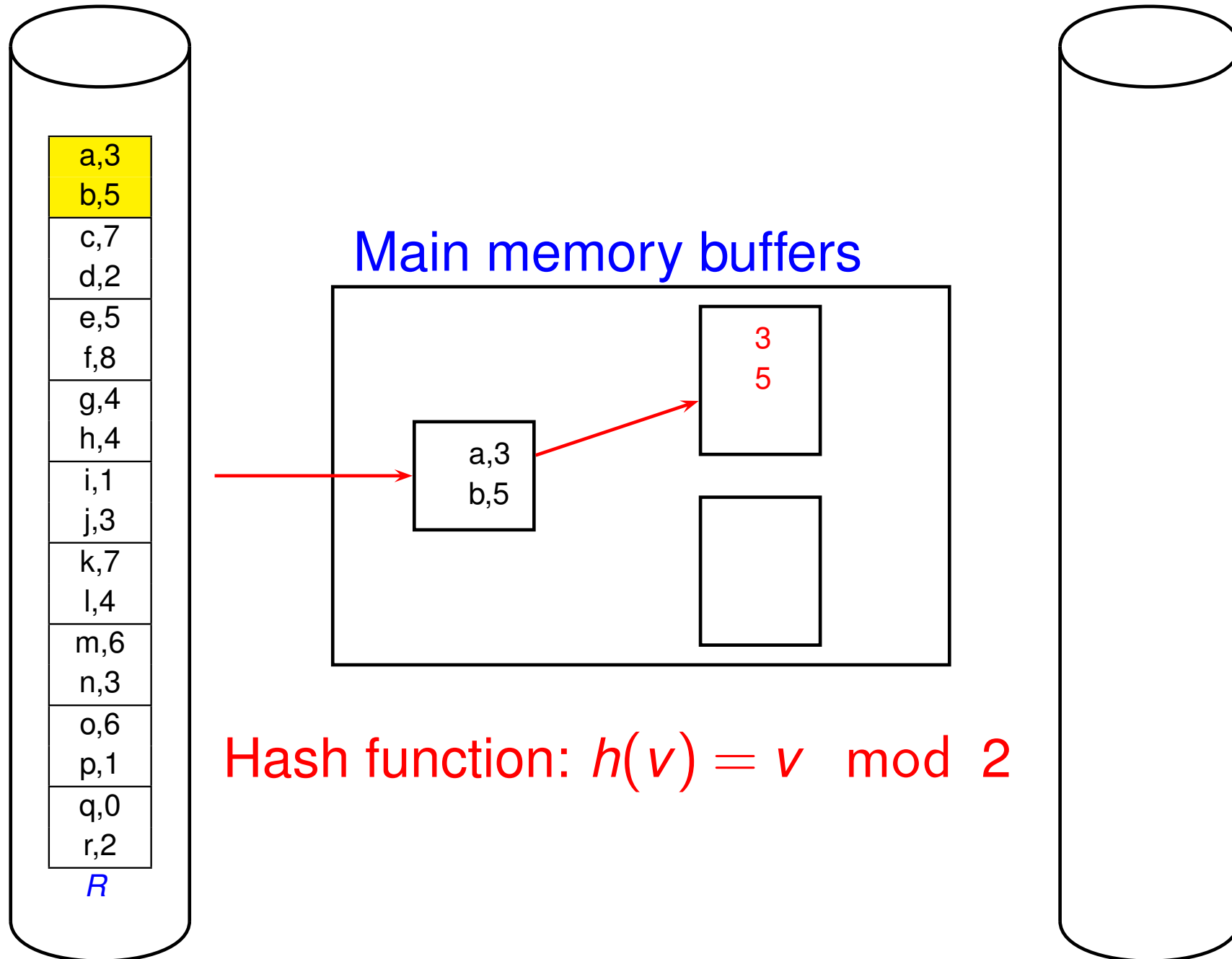
- ▶ For each partition  $R_i$ ,
  - ▶ Initialize an in-memory hash table
  - ▶ Read  $\pi_L^*(R_i)$  one page at a time; for each tuple  $t$  read,
    - Hash  $t$  into bucket  $B_j$  with hash function  $h'$  ( $h' \neq h$ )
    - Insert  $t$  into  $B_j$  if  $t \notin B_j$
  - ▶ Write out tuples in hash table to results



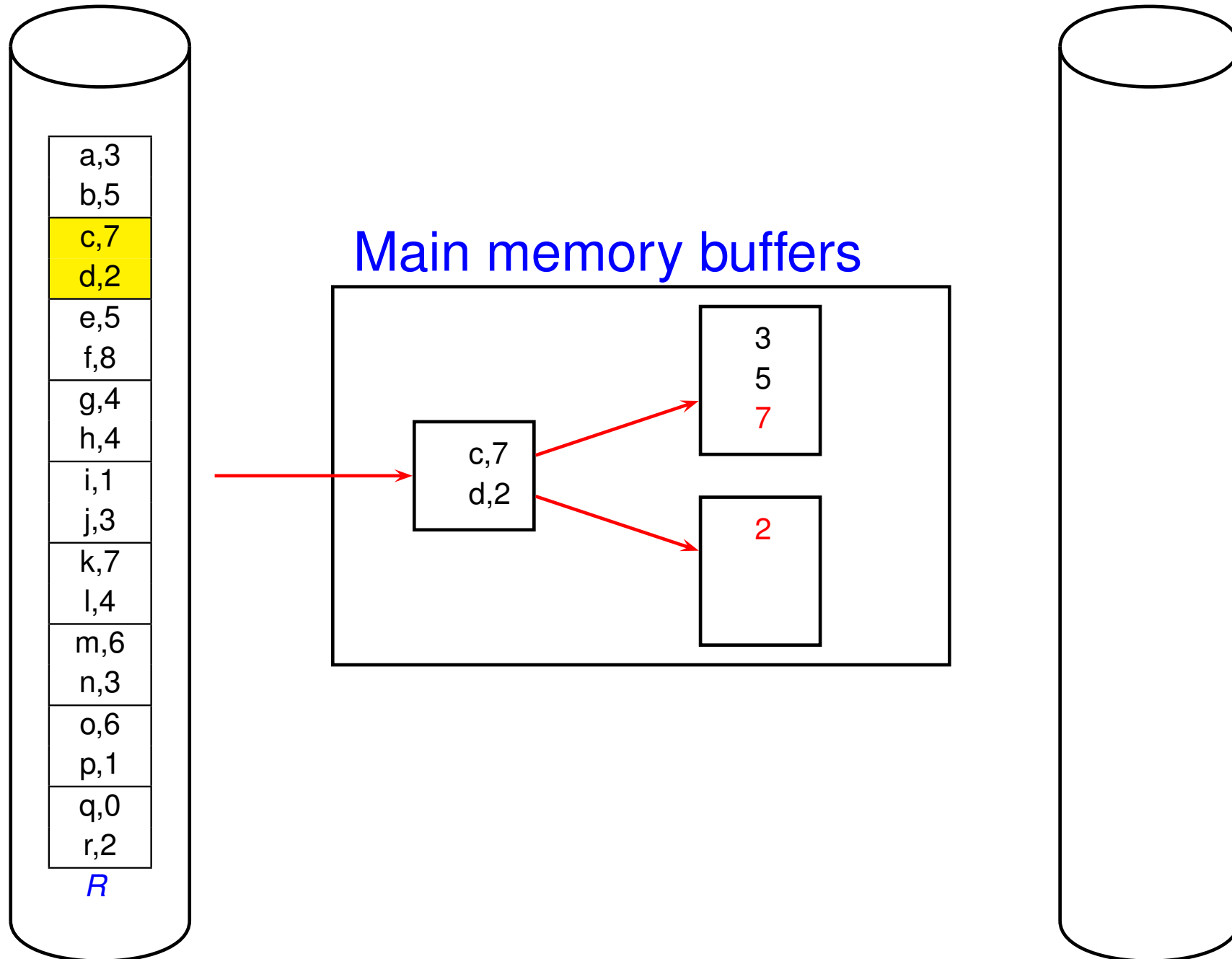
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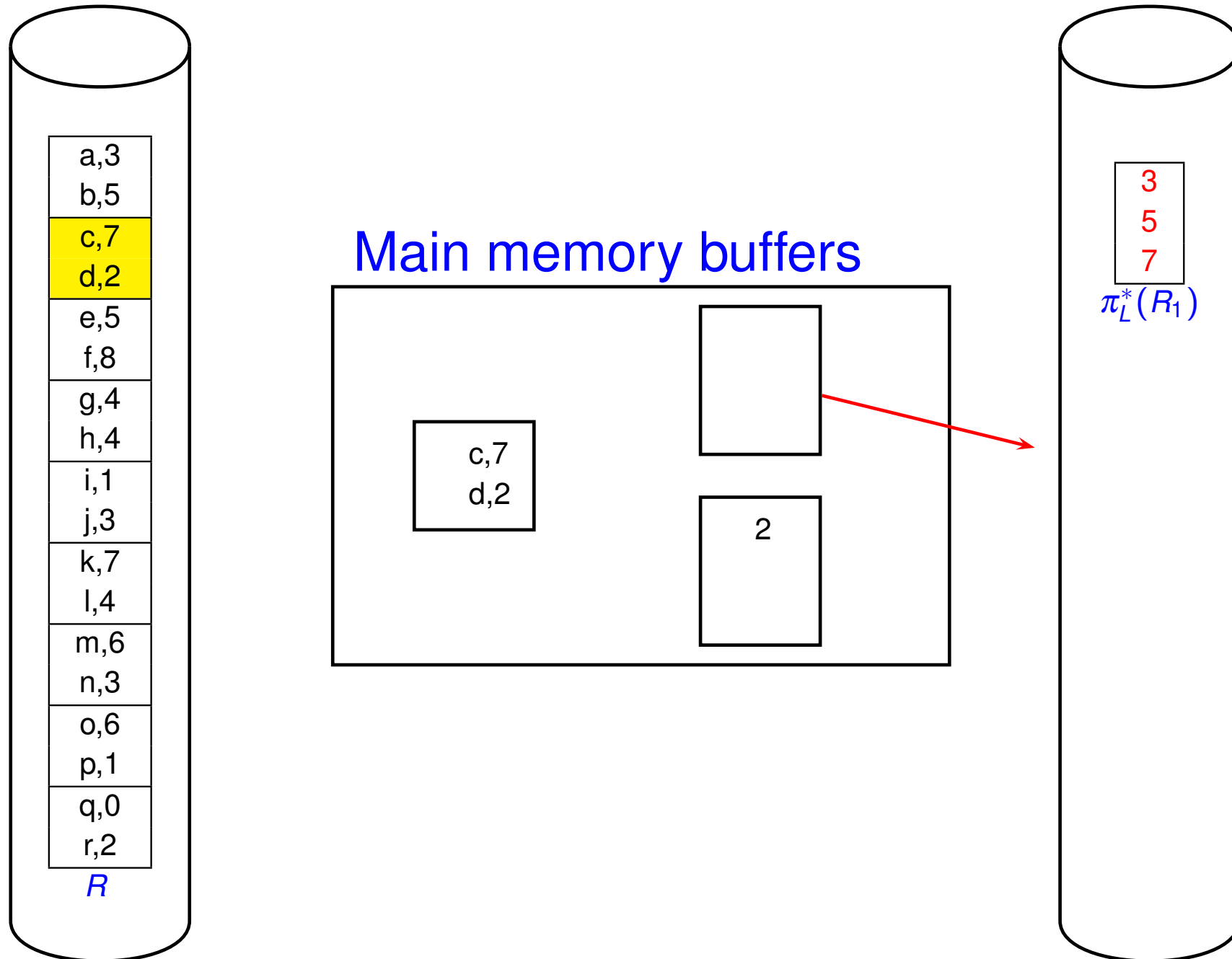


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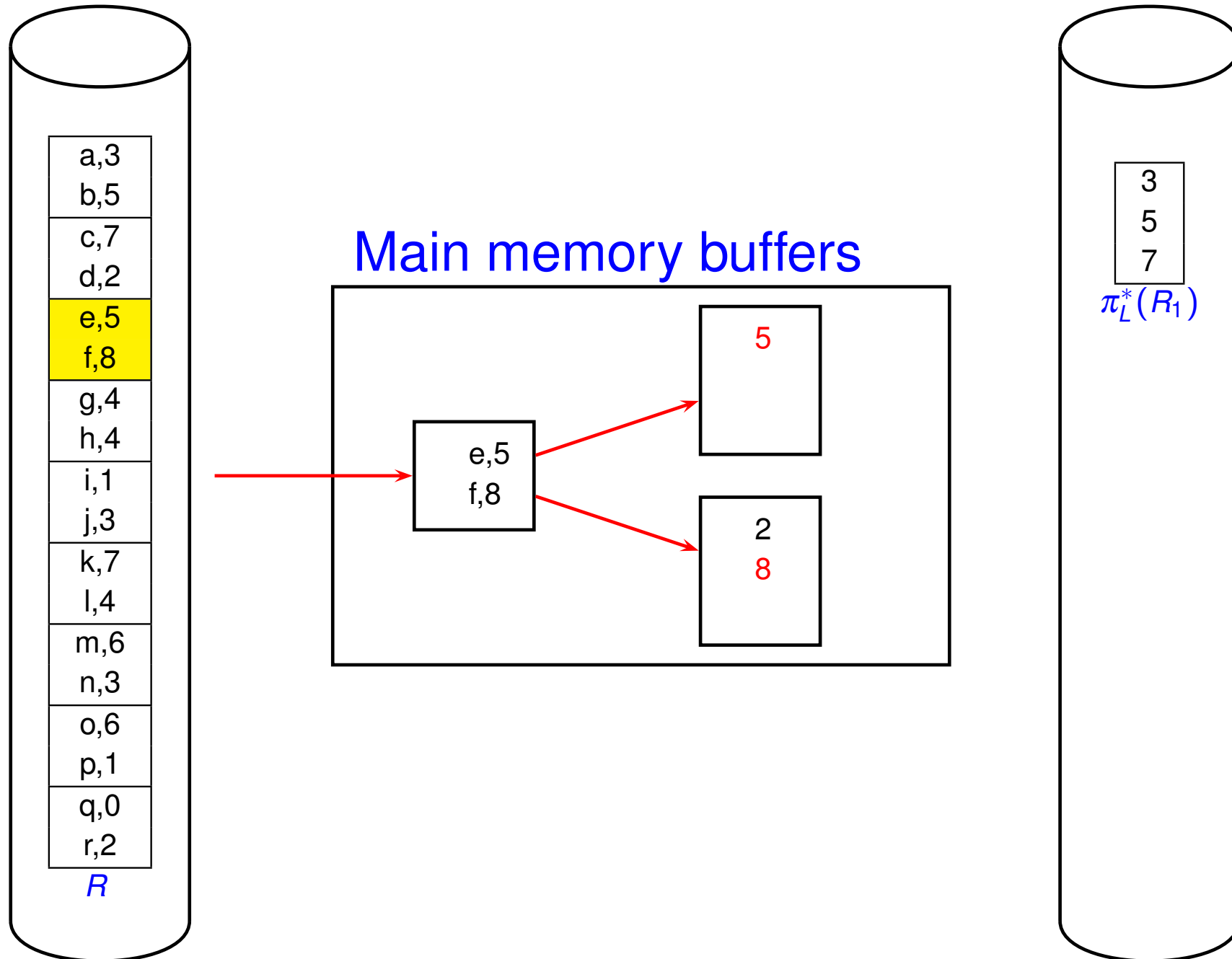




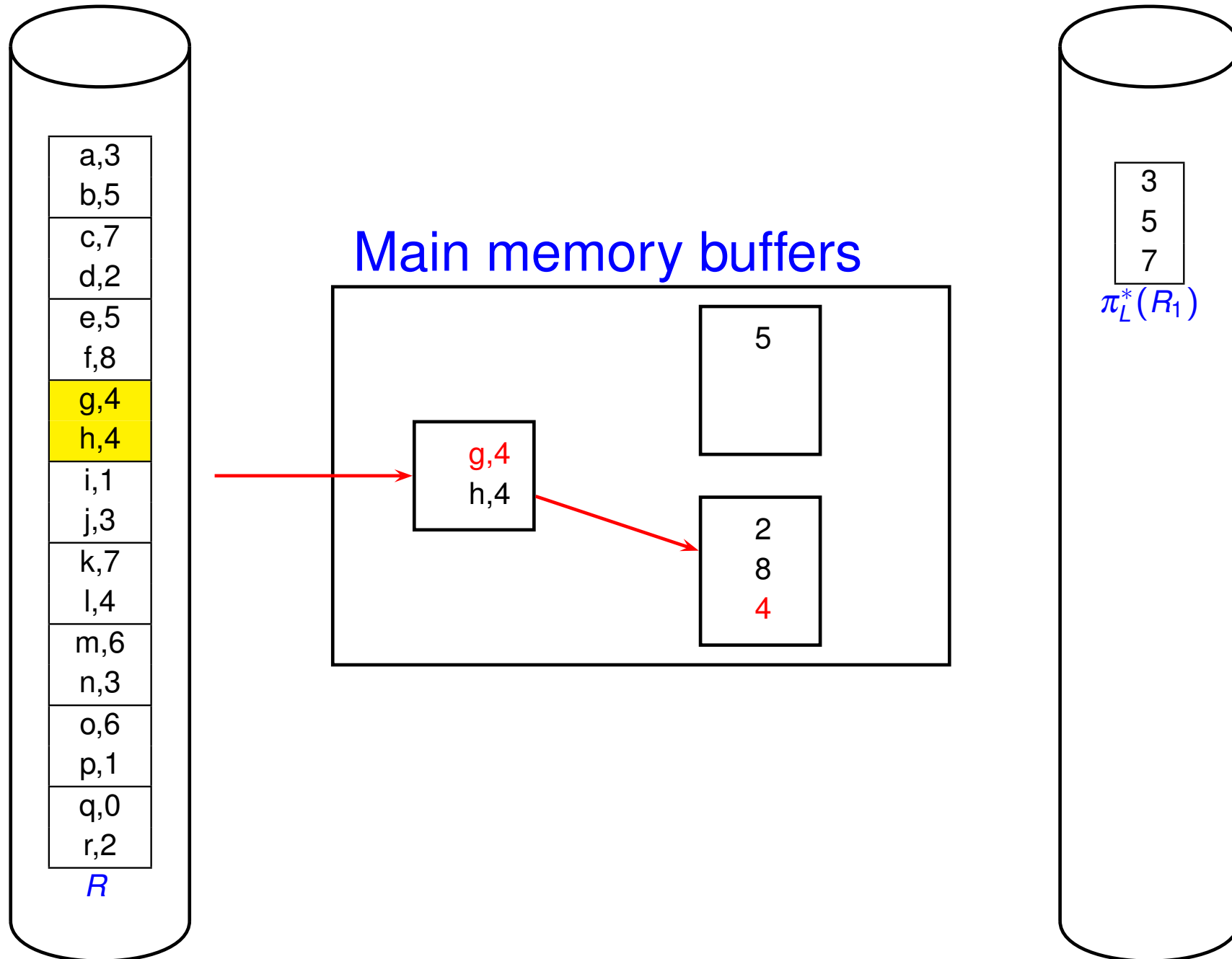
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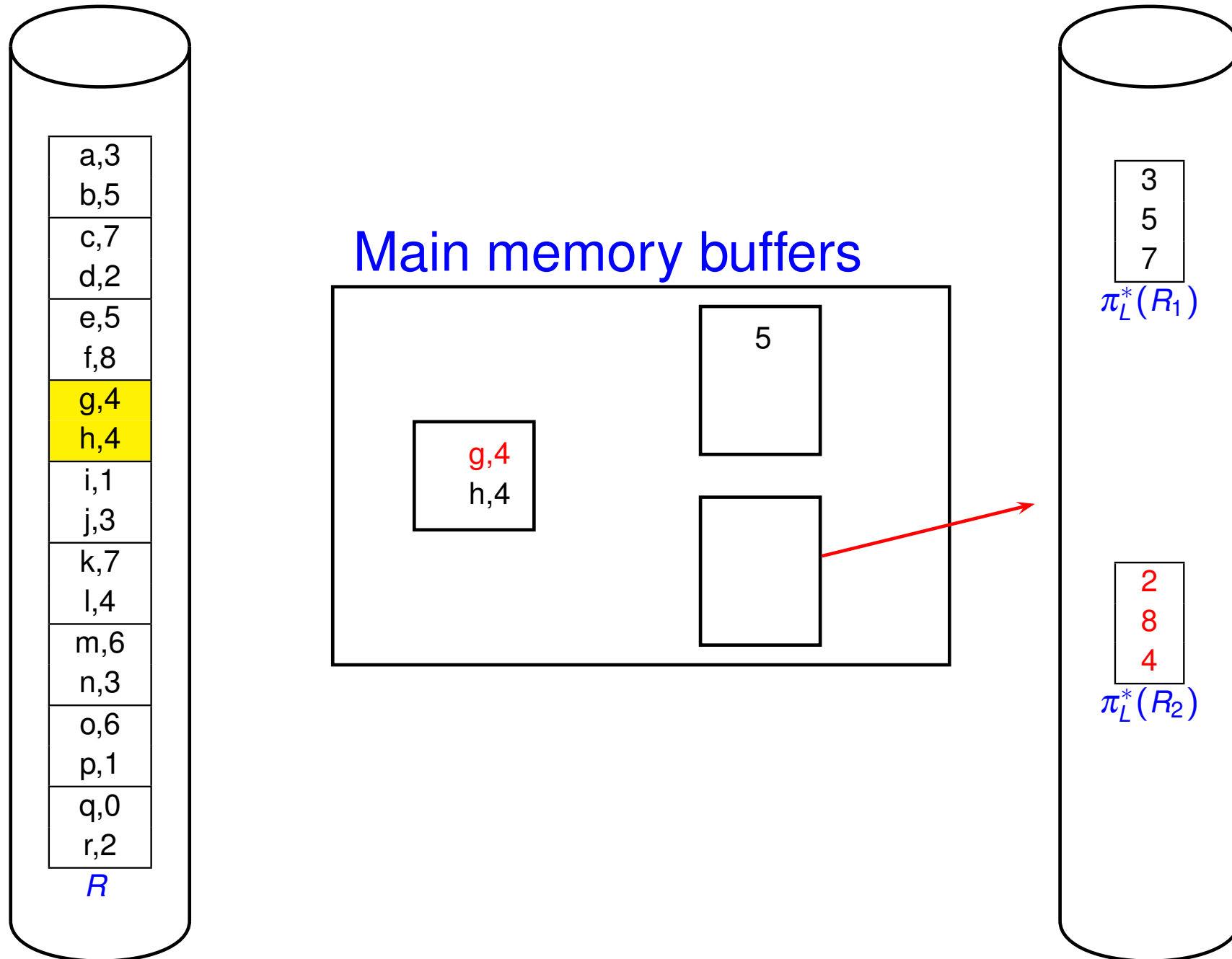
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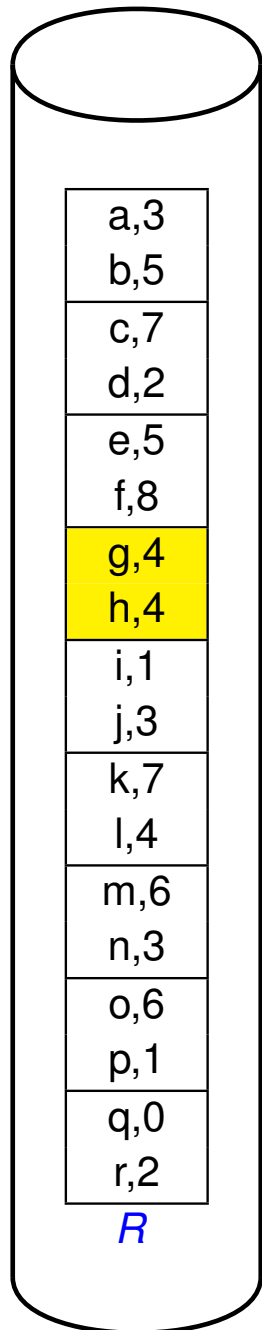
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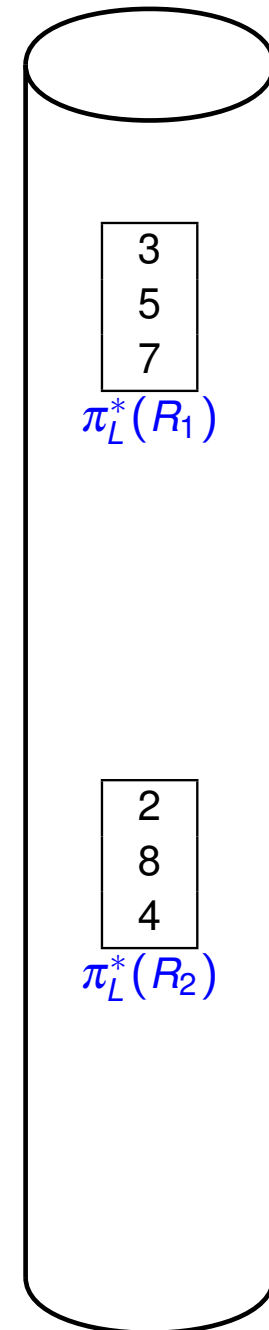
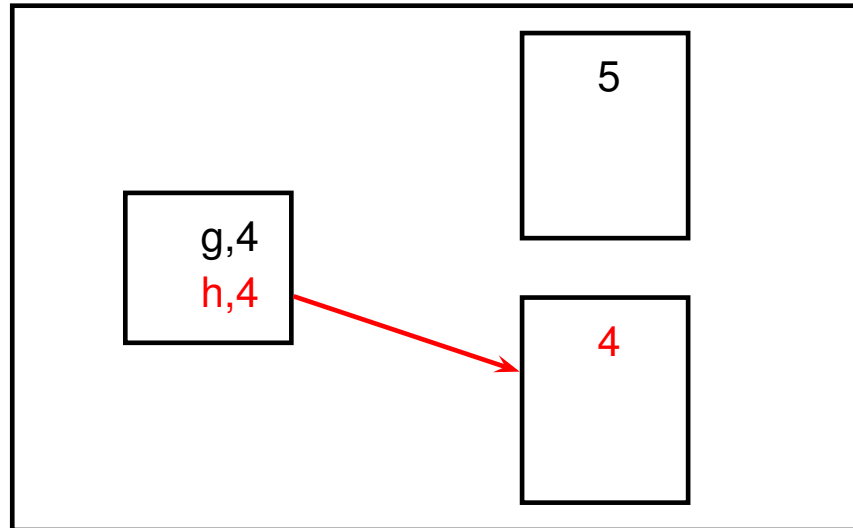
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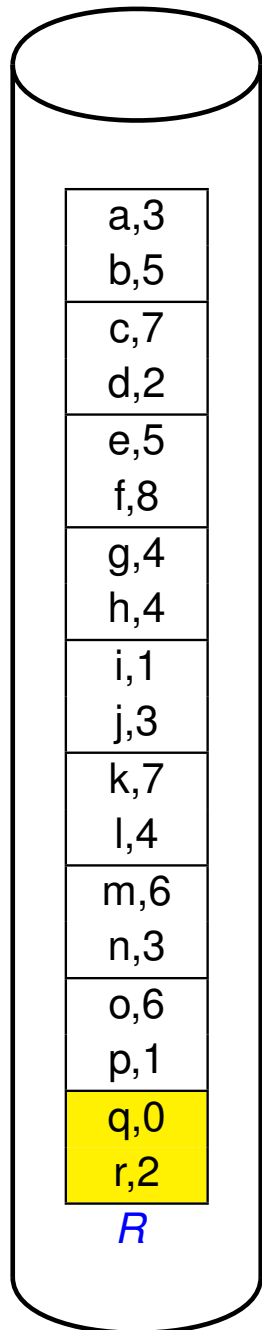
# Partitioning Phase



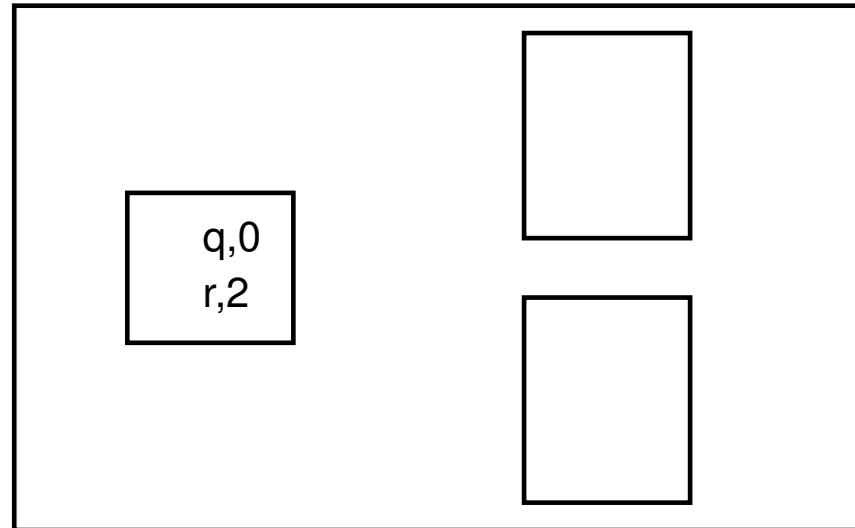
## Main memory buffers



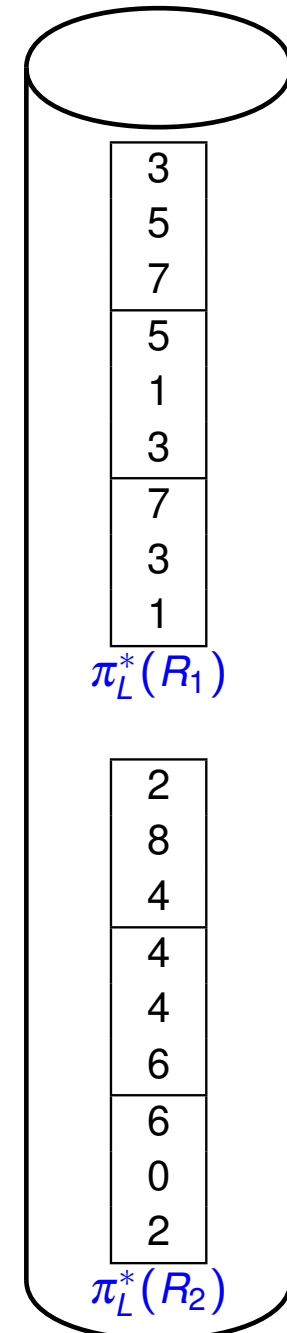
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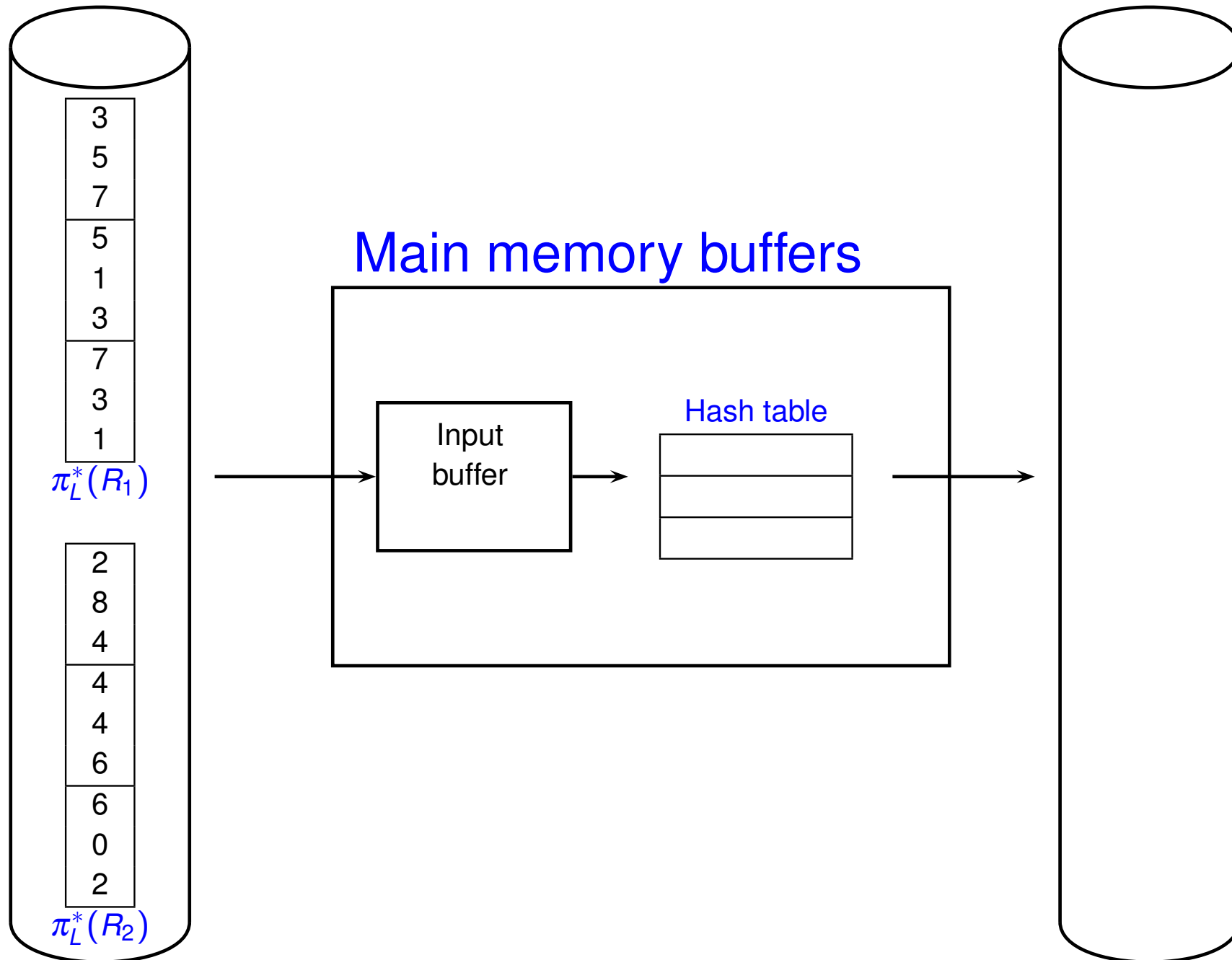
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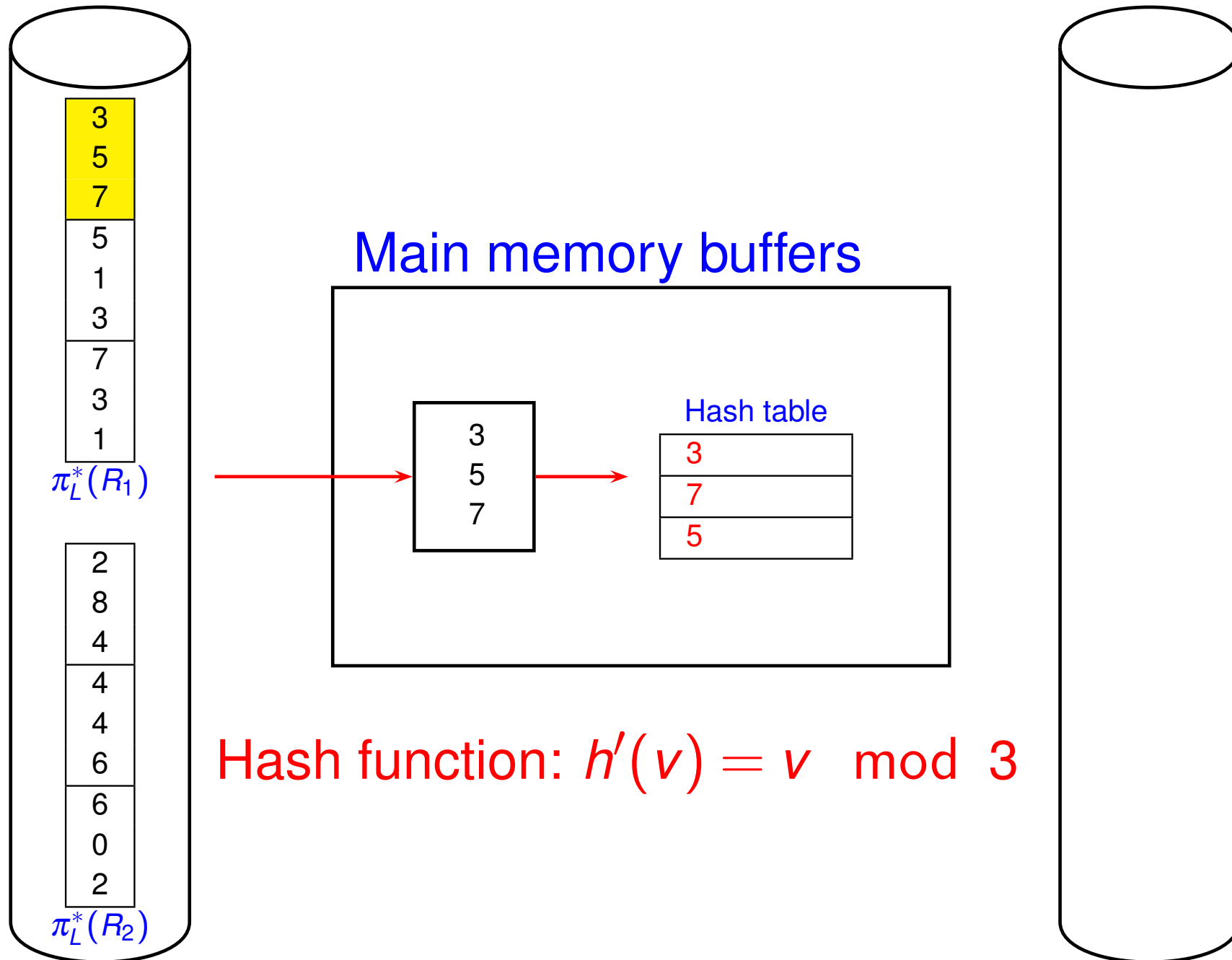
Eventually,  $R$  is partitioned  
into  $R_1$  &  $R_2$



# Duplicate Elimination Phase

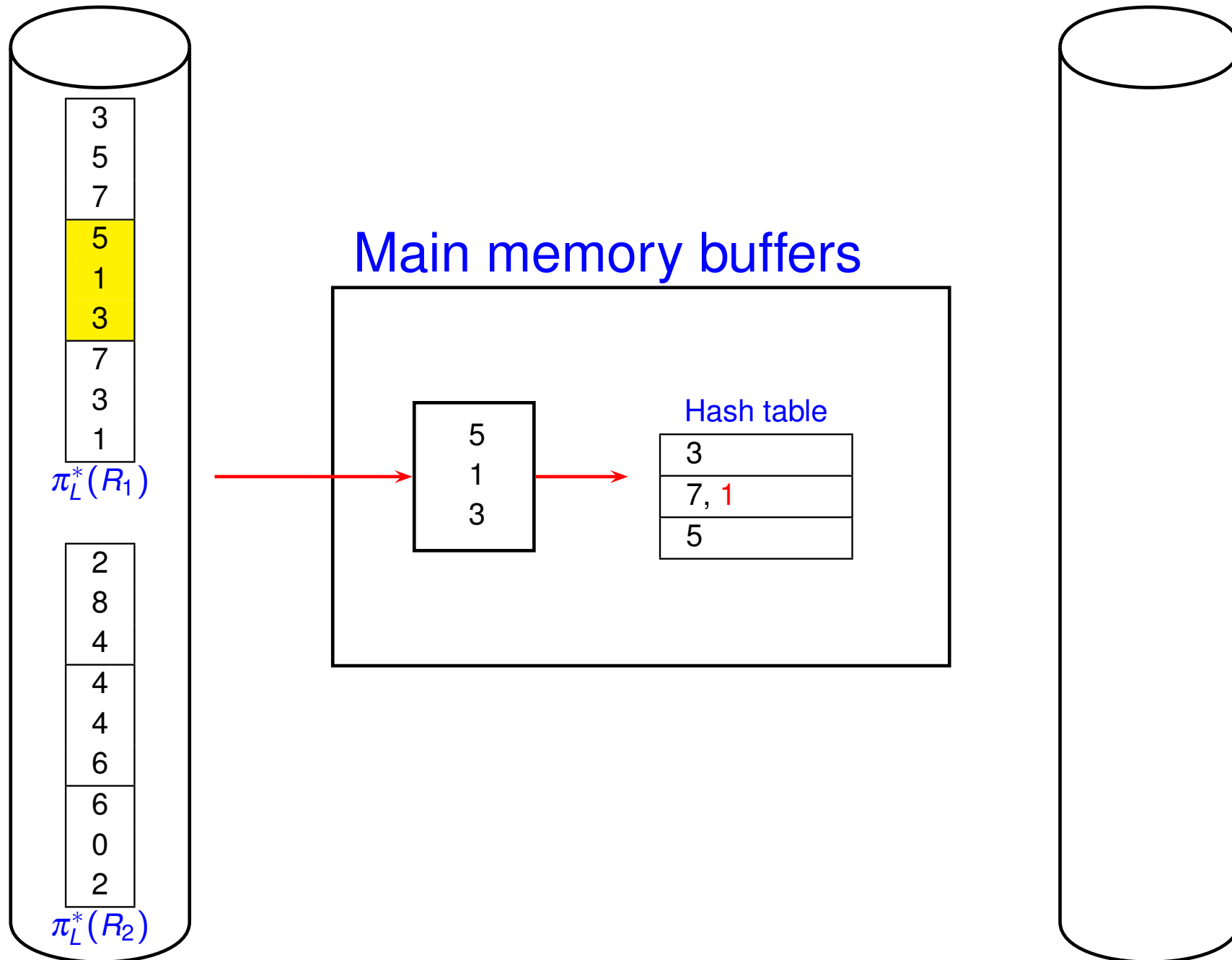


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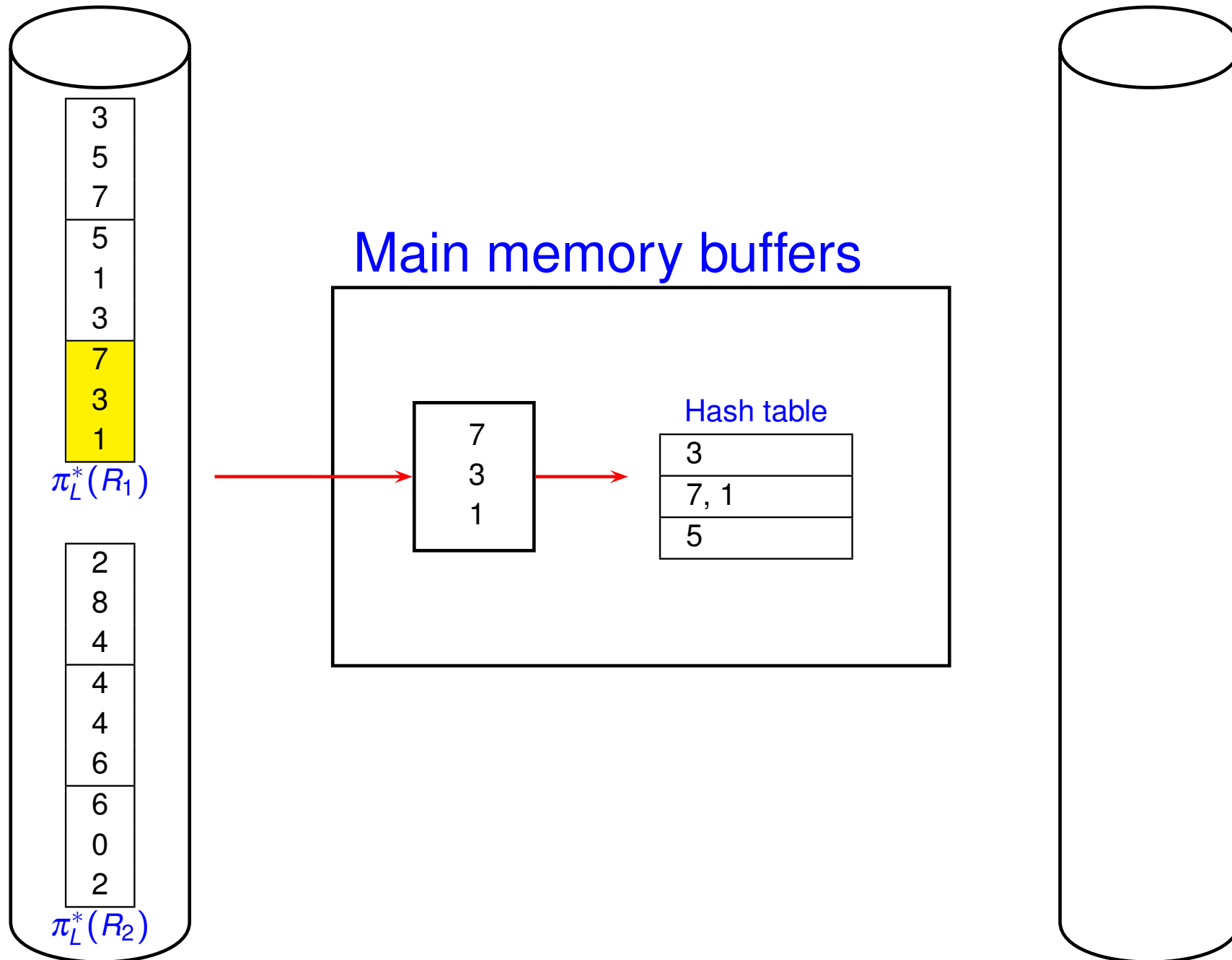




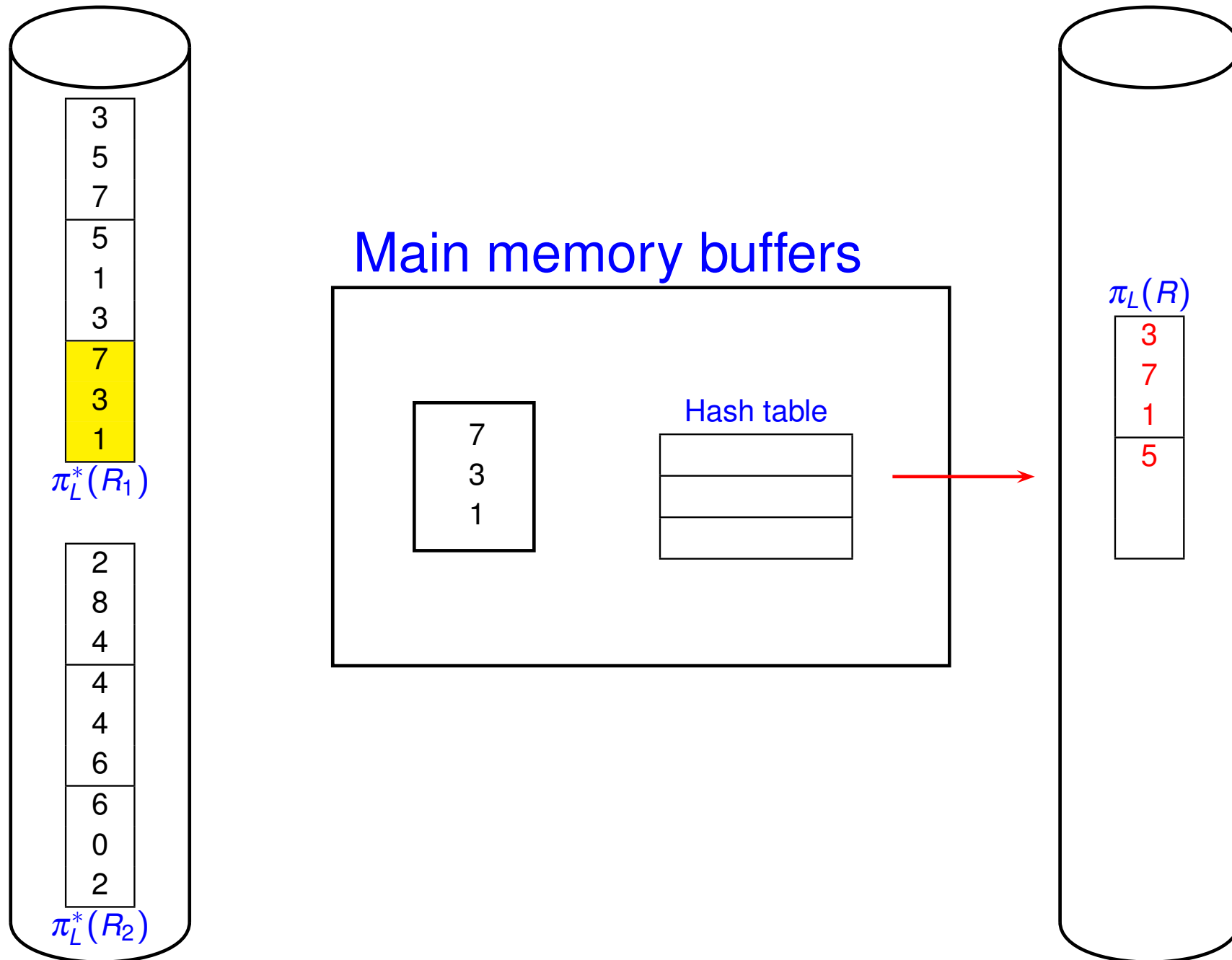
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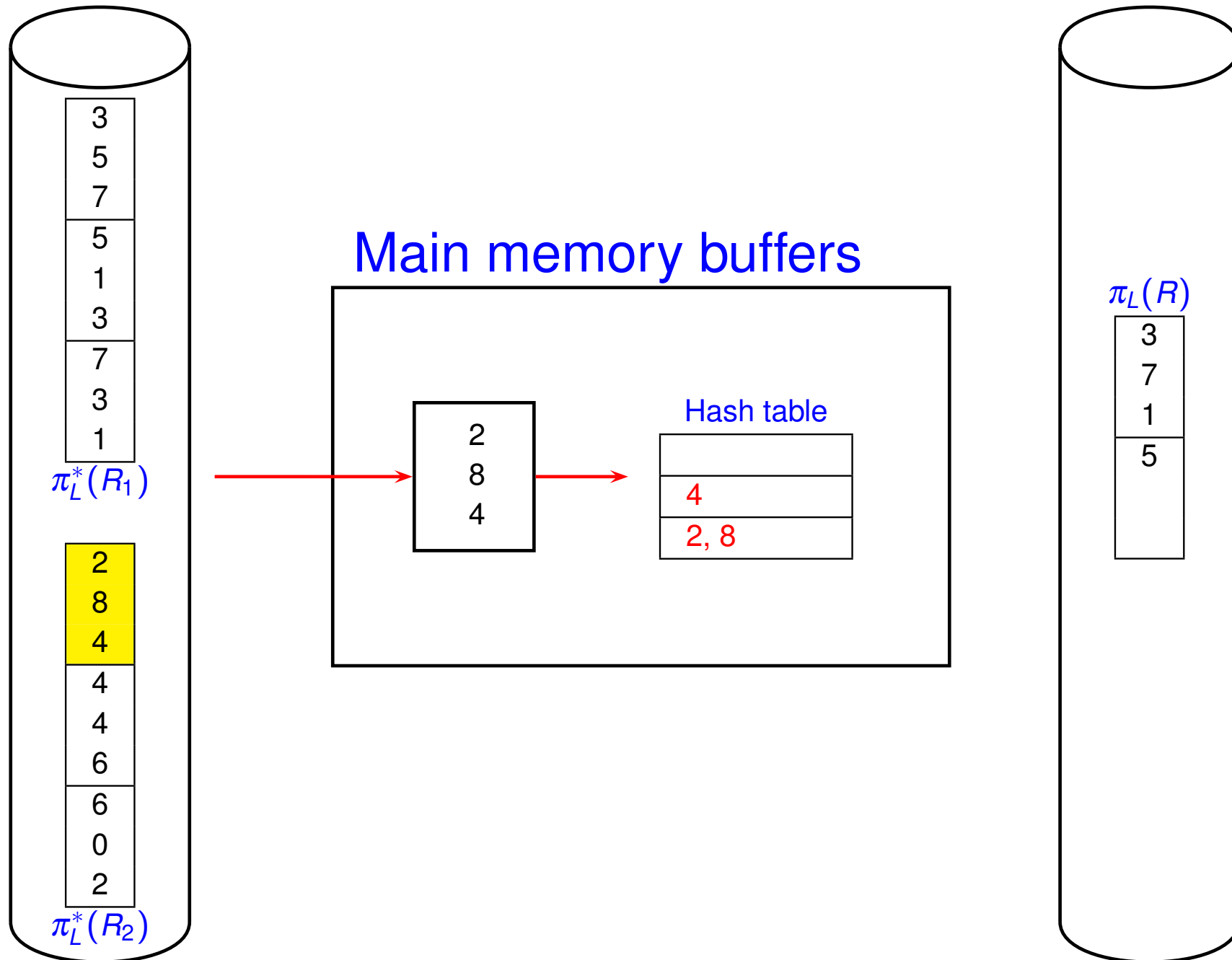
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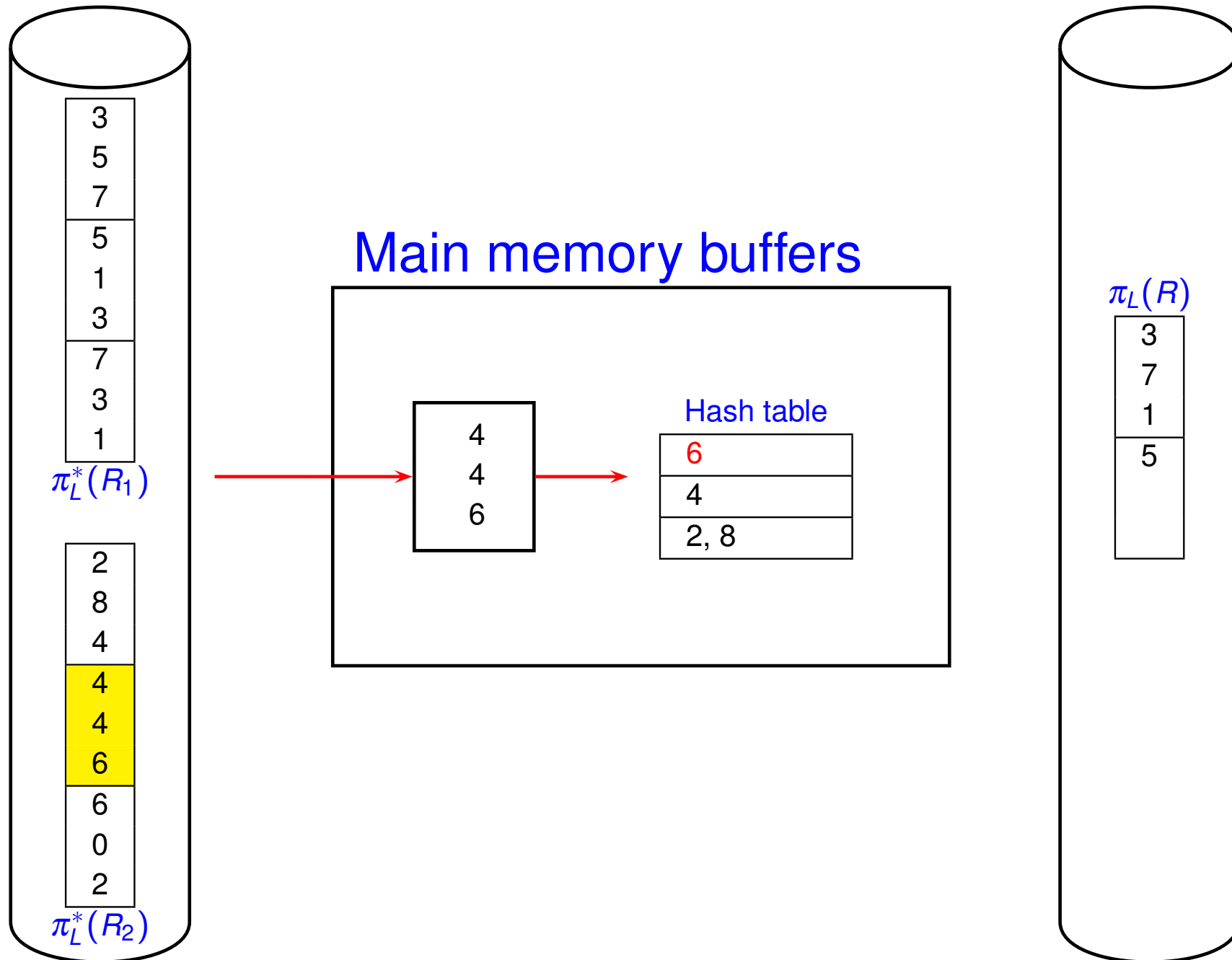
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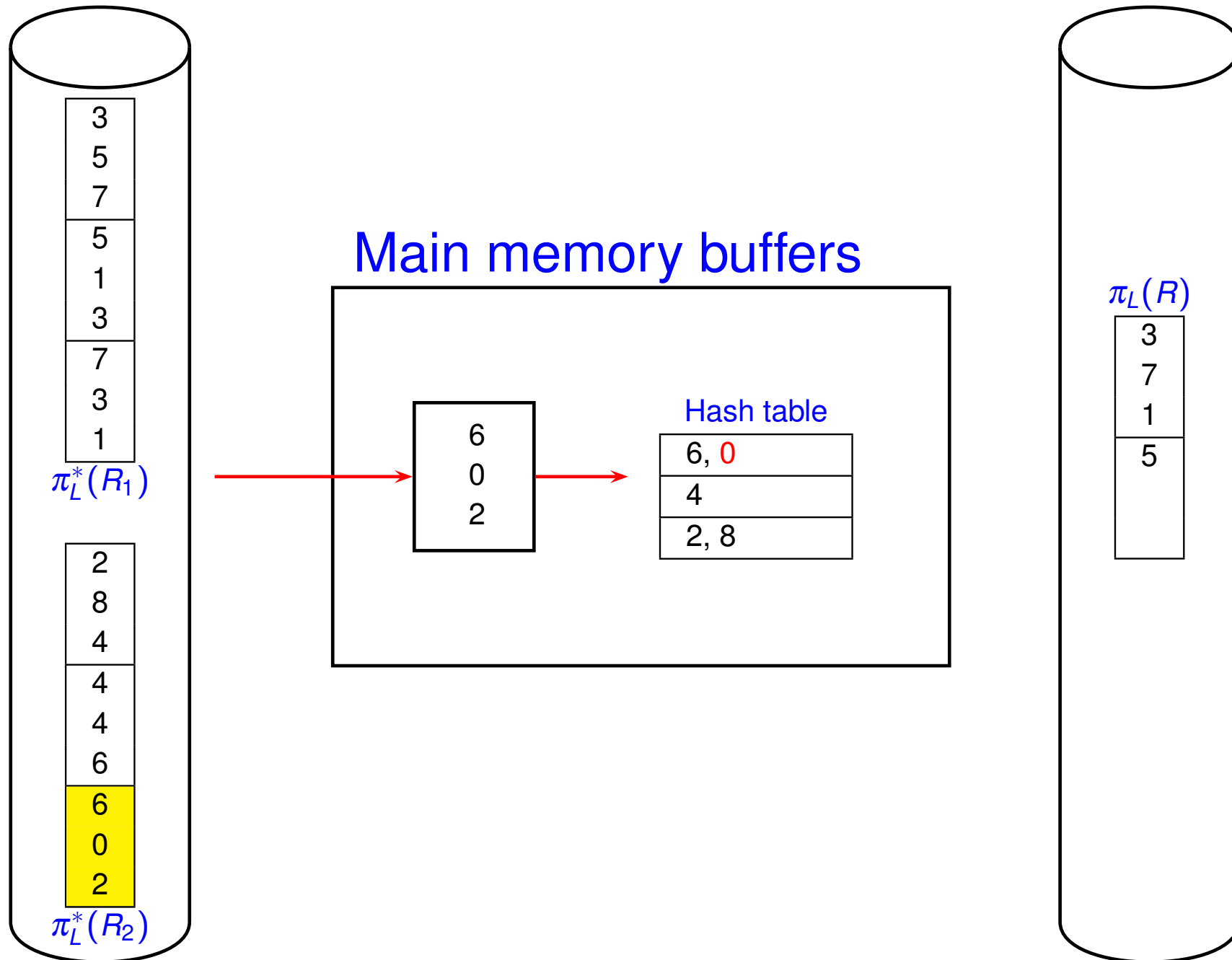
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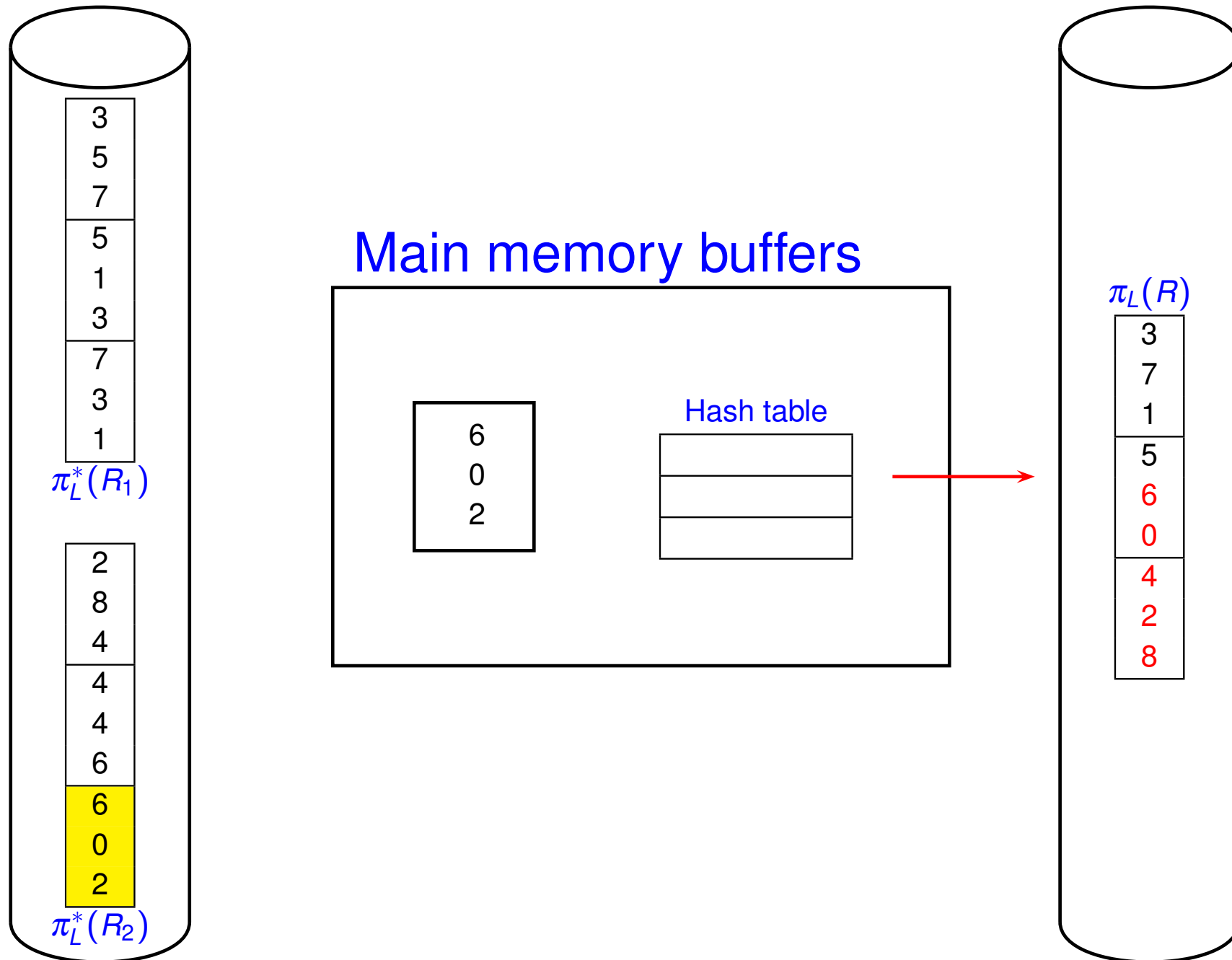
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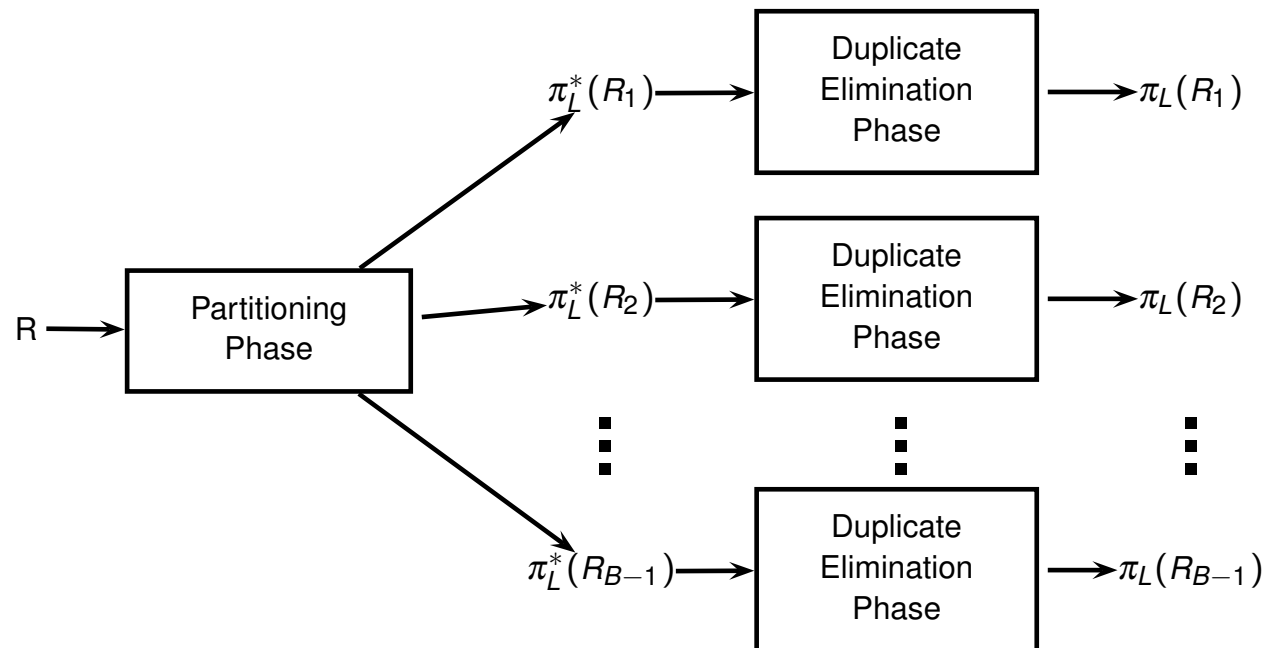


# Duplicate Elimination Phase



# Hash-based Approach: Partition Overflow

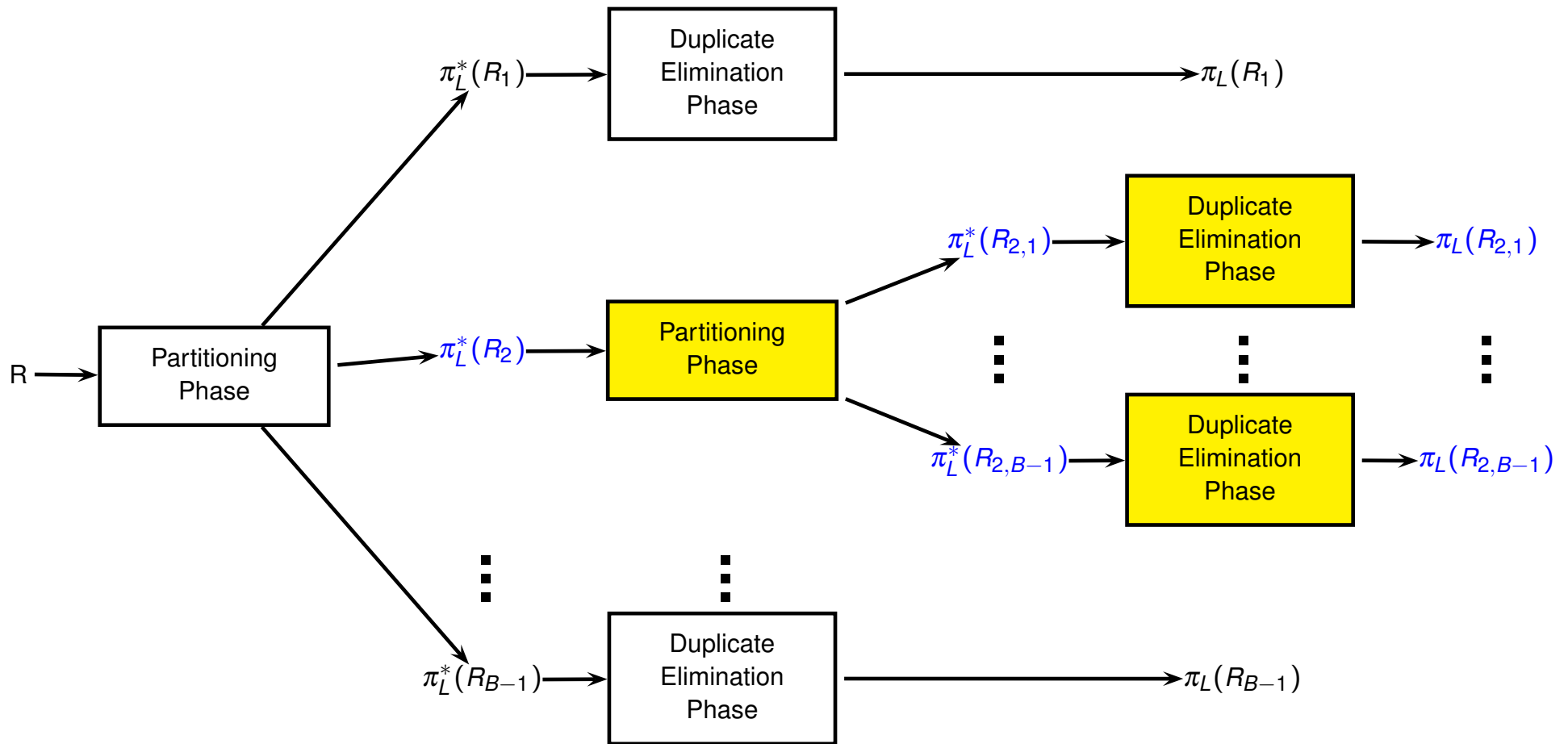
- ▶ **Partition overflow problem:** Hash table for  $\pi_L^*(R_i)$  is larger than available memory buffers
  - ▶ Recursively apply hash-based partitioning to the overflowed partition
- ▶ **Example:** Without partition overflow





# Hash-based Approach: Partition Overflow

## ► Example: Partition $R_2$ overflows



# Hash-based Approach: Analysis

- ▶ Approach is effective if  $B$  is large relative to  $|R|$
- ▶ How large should  $B$  be?
  - ▶ Assume that  $h$  distributes tuples in  $R$  uniformly
  - ▶ Each  $R_i$  has  $\frac{|\pi_L^*(R)|}{B-1}$  pages
  - ▶ Size of hash table for each  $R_i = \frac{|\pi_L^*(R)|}{B-1} \times f$ 
    - ★  $f = \text{fudge factor}$
  - ▶ Therefore, to avoid partition overflow,  $B > \frac{|\pi_L^*(R)|}{B-1} \times f$ 
    - ★ Approximately,  $B > \sqrt{f \times |\pi_L^*(R)|}$
- ▶ Analysis: Assume there's no partition overflow
  - ▶ Cost of partitioning phase:  $|R| + |\pi_L^*(R)|$
  - ▶ Cost of duplicate elimination phase:  $|\pi_L^*(R)|$
  - ▶ Total cost =  $|R| + 2|\pi_L^*(R)|$

# Sort-based vs Hash-based

## ► Hash-based

$$\text{Cost} = \underbrace{|R| + |\pi_L^*(R)|}_{\text{partitioning phase}} + \underbrace{|\pi_L^*(R)|}_{\text{duplicate elimination phase}}$$

## ► Sort-based

- Output is sorted
- Good if there are many duplicates or if distribution of hashed values are non-uniform
- If  $B > \sqrt{|\pi_L^*(R)|}$ ,
  - ★ Number of initial sorted runs  $N_0 = \lceil \frac{|R|}{B} \rceil \approx \sqrt{|\pi_L^*(R)|}$
  - ★ Number of merging passes =  $\log_{B-1}(N_0) \approx 1$
  - ★ Sort-based approach requires 2 passes for sorting
  - ★ Both hash-based & sort-based methods have same I/O cost

# Projection Operation: Using Indexes

- ▶ If there is an index whose search key contains all the wanted attributes,
  - ▶ replace table scan with index scan!
- ▶ If index is ordered (e.g.,  $B^+$ -tree) whose search key includes wanted attributes as a prefix,
  - ▶ scan data entries in order
  - ▶ compare adjacent data entries for duplicates
  - ▶ **Example:**
    - ★ Use  $B^+$ -tree index on R with key (A, B) to evaluate query  $\pi_A(R)$

# Join: $R \bowtie_{\theta} S$

## ► Database Schema:

- Employee (eid, ename, city, did)
- Department (did, dname, city, managerId)

## ► **Example 1:** Find (eid, managerId) pairs where managerId is the manager of eid

$\pi_{\text{eid,managerId}} ( \text{Employee} \bowtie_{\theta} \text{Department} )$   
where  $\theta$ : Employee.did = Department.did

## ► **Example 2:** Find (eid, did) pairs where eid and did are co-located in the same city

$\pi_{\text{eid,did}} ( \text{Employee} \bowtie_{\theta} \text{Department} )$   
where  $\theta$ : Employee.city = Department.city

# Join Algorithms

## 1. Iteration-based

- ▶ block nested loop

## 2. Index-based

- ▶ index nested loop

## 3. Partition-based

- ▶ sort-merge join
- ▶ hash join

# Join Algorithms (cont.)

- ▶ Things to consider when choosing an algorithm:
  - ▶ Types of join predicates:
    - ★ equality predicates (e.g.  $R.A_i = S.B_j$ )
    - ★ inequality predicates: (e.g.,  $R.A_i < S.B_j$ )
  - ▶ Sizes of join operands
  - ▶ Available buffer space
  - ▶ Available access methods
- ▶ Given a join  $R \bowtie_{\theta} S$ 
  - ▶  $R$  is referred to as the **outer relation**
  - ▶  $S$  is referred to as the **inner relation**

# Tuple-based Nested Loop Join

## ► Tuple-based Algorithm:

for each tuple  $r \in R$  do  
  for each tuple  $s \in S$  do  
    if ( $r$  matches  $s$ ) then  
      output  $(r, s)$  to result

5, 10
2, 10
13, 7

R

4, 10
5, 10

S



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etc ...

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etc ...

## ► I/O Cost Analysis:

$$\underbrace{|R|}_{\text{scan R}} + \underbrace{||R|| \times |S|}_{\text{scan S}}$$

# Page-based Nested Loop Join

## ► Page-based Algorithm:

```
for each page  $P_R$  of  $R$  do
  for each page  $P_S$  of  $S$  do
    for each tuple  $r \in P_R$  do
      for each tuple  $s \in P_S$  do
        if ( $r$  matches  $s$ ) then
          output ( $r, s$ ) to result
```

5, 10
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13, 7

R

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  for each page  $P_S$  of  $S$  do  
    for each tuple  $r \in P_R$  do  
      for each tuple  $s \in P_S$  do  
        if ( $r$  matches  $s$ ) then  
          output  $(r, s)$  to result

5, 10
2, 10
13, 7

R

4, 10
5, 10

S

# Page-based Nested Loop Join

## ► Page-based Algorithm:

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etc ...

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for each page  $P_R$  of  $R$  do  
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5, 10	
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etc ...

## ► I/O Cost Analysis:

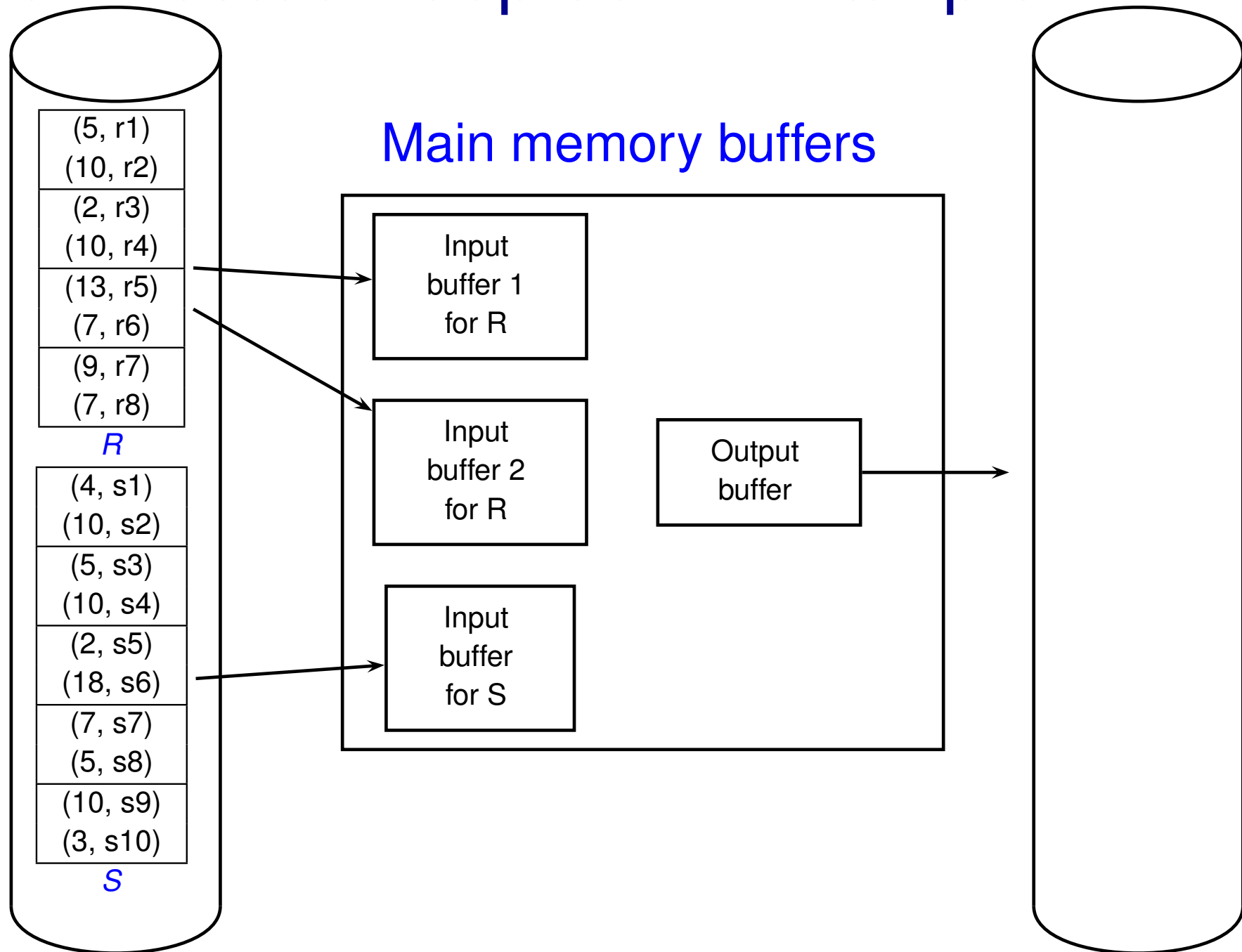
$$\underbrace{|R|}_{\text{scan R}} + \underbrace{|R| \times |S|}_{\text{scan S}}$$

# Block Nested Loop Join

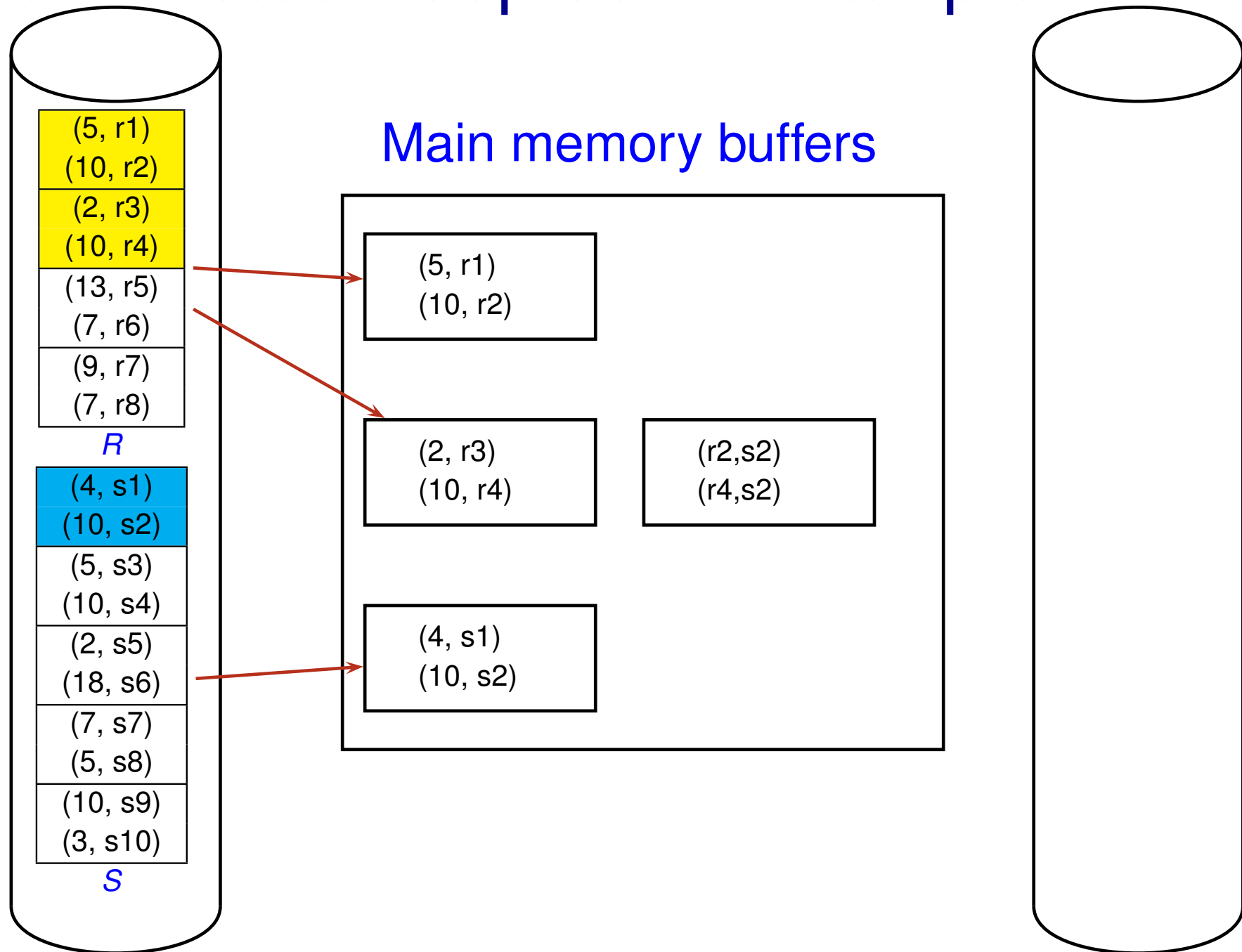
- ▶ **Motivation:** How to better exploit buffer space to minimize number of I/Os?
- ▶ Assume  $|R| \leq |S|$
- ▶ **Buffer space allocation:** Allocate one page for  $S$ , one page for output & remaining pages for  $R$
- ▶ **Algorithm (using  $B$  buffer pages):**
  - while (scan of  $R$  is not done) do
    - read next  $(B - 2)$  pages of  $R$  into buffer
    - for each page  $P_S$  of  $S$  do
      - read  $P_S$  into buffer
      - for each tuple  $r$  of  $R$  in buffer and each tuple  $s \in P_S$  do
        - if ( $r$  matches  $s$ ) then output  $(r, s)$  to result
- ▶ **I/O Cost:**  $|R| + (\lceil \frac{|R|}{B-2} \rceil \times |S|)$



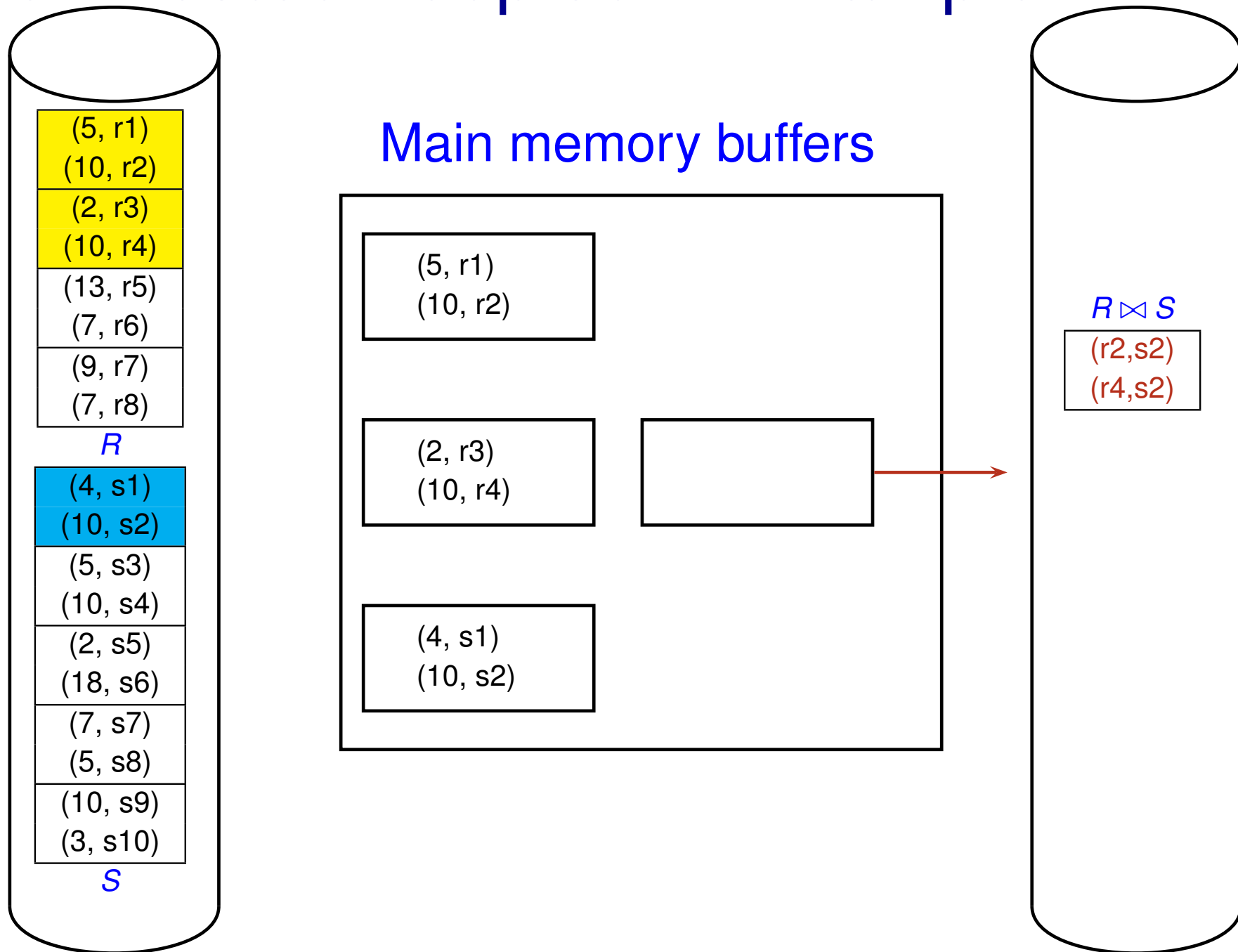
# Block Nested Loop Join: Example



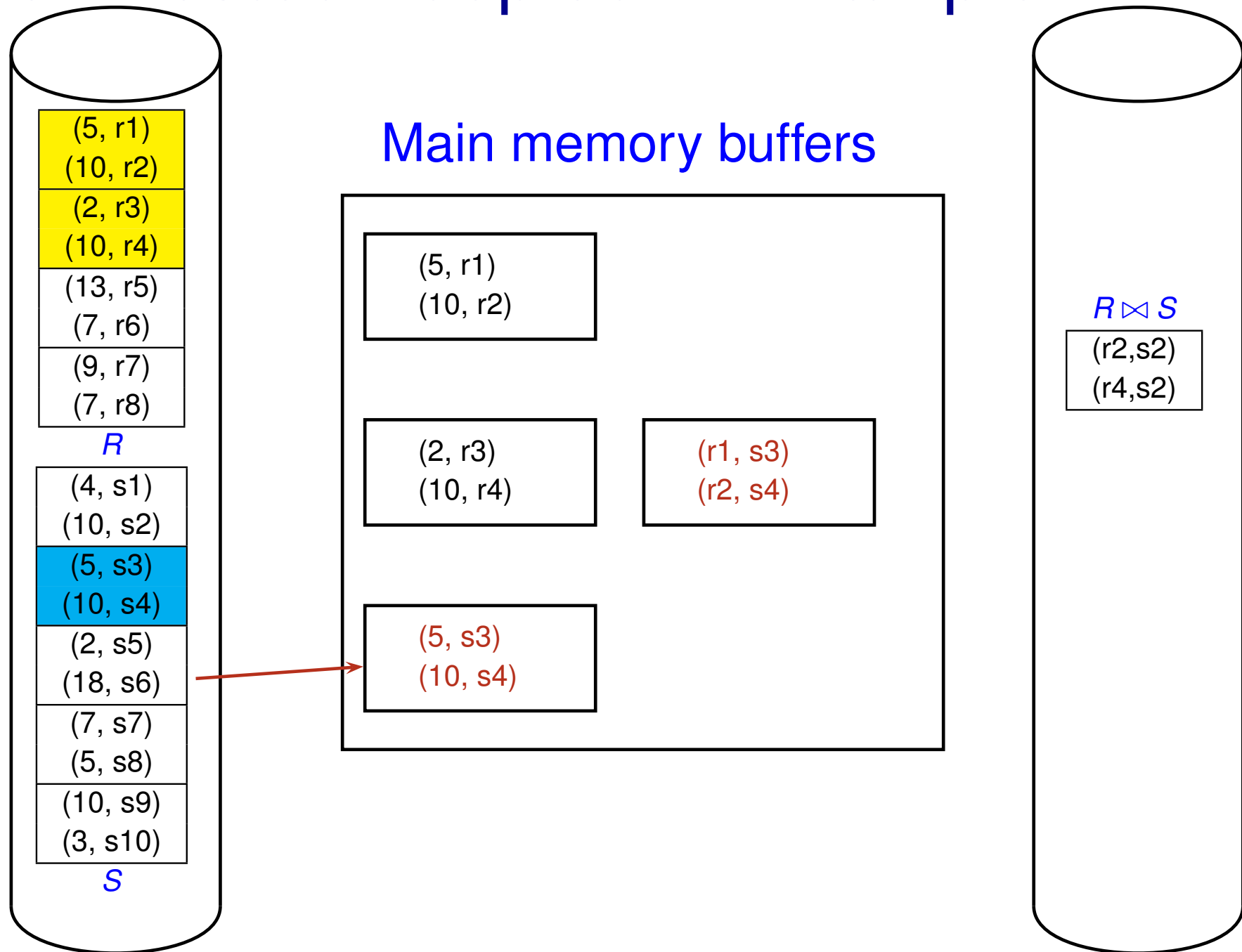
# Block Nested Loop Join: Example



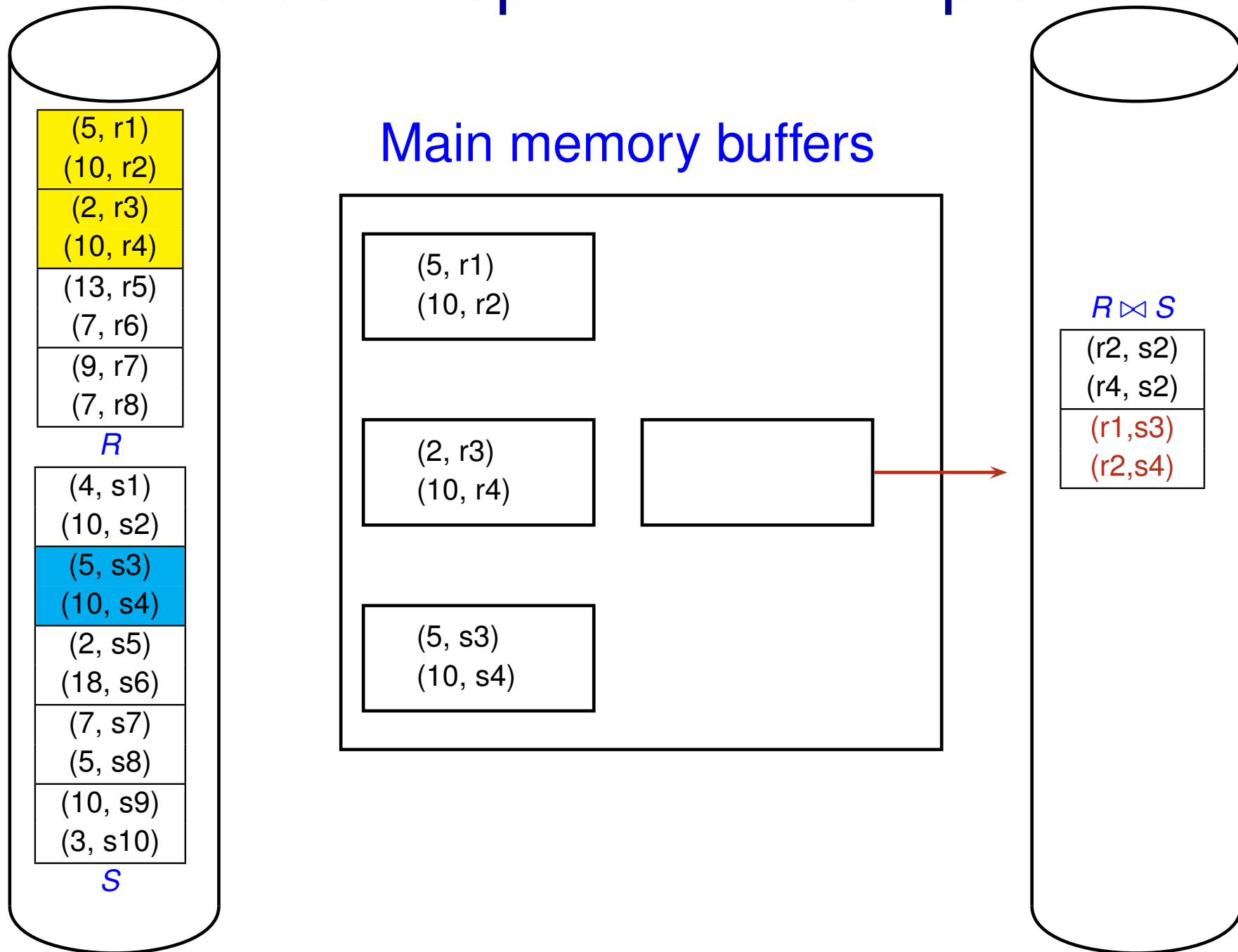
# Block Nested Loop Join: Example



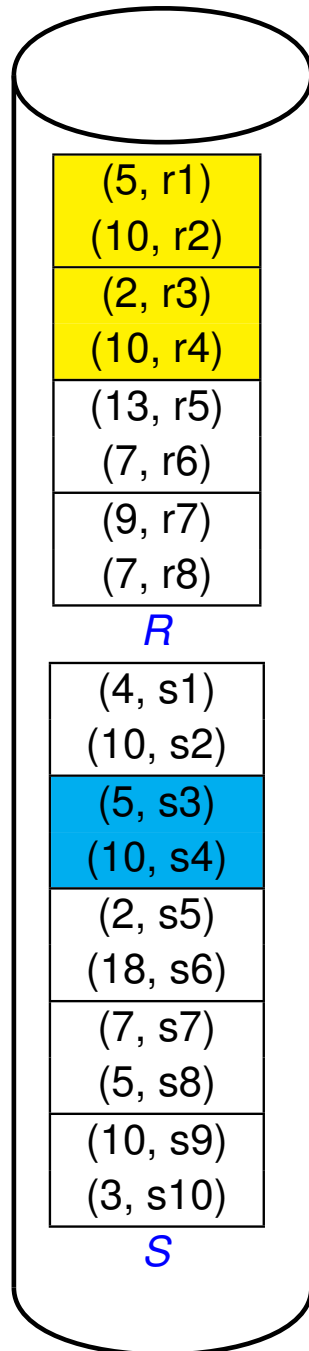
# Block Nested Loop Join: Example



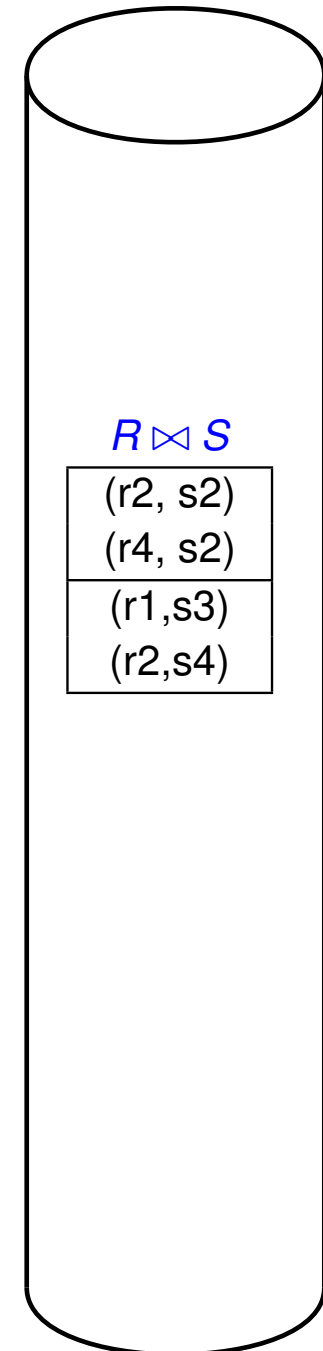
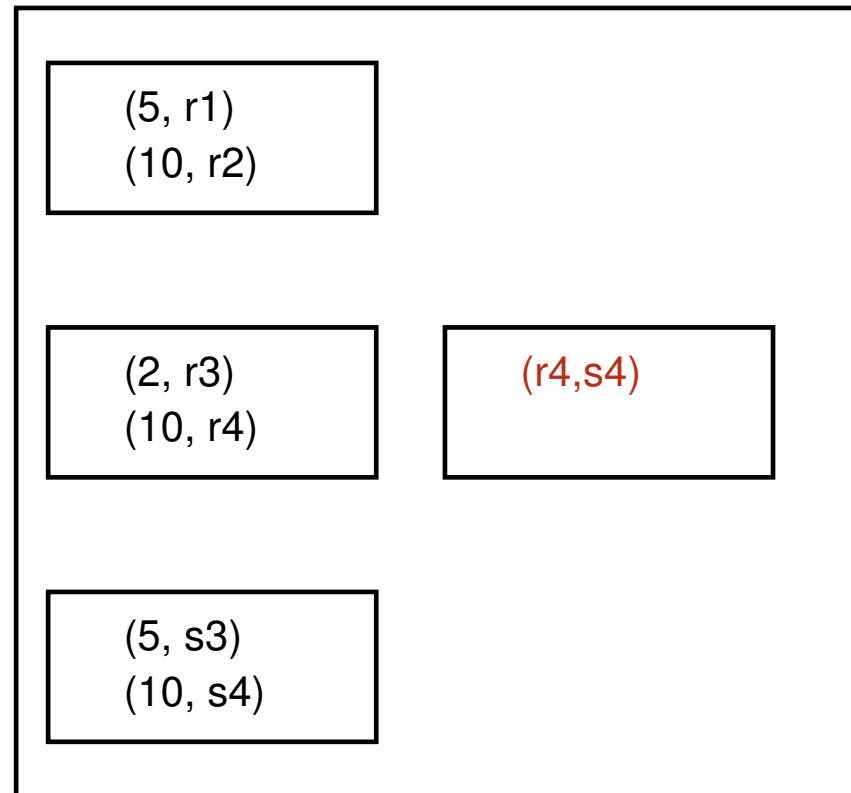
# Block Nested Loop Join: Example



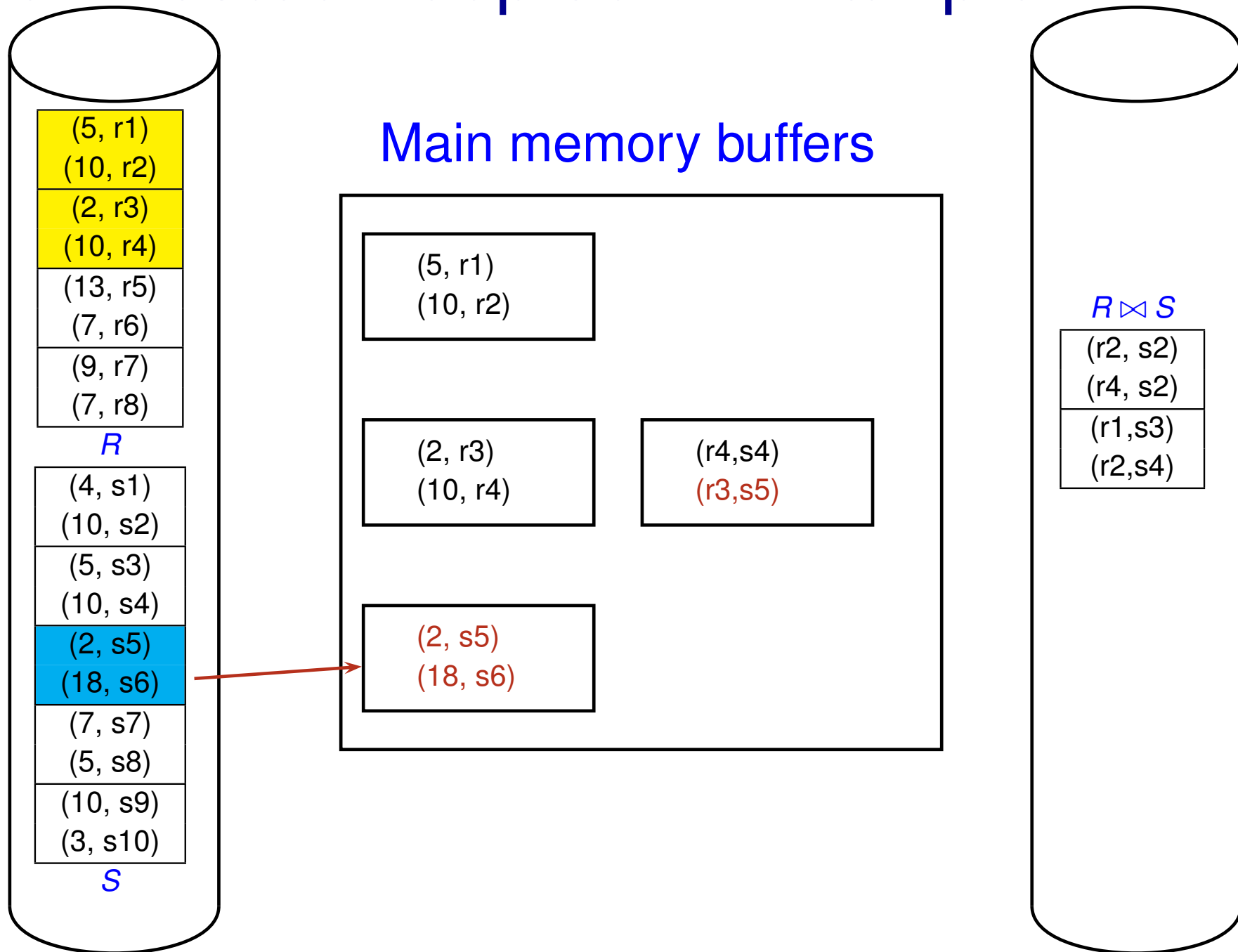
# Block Nested Loop Join: Example



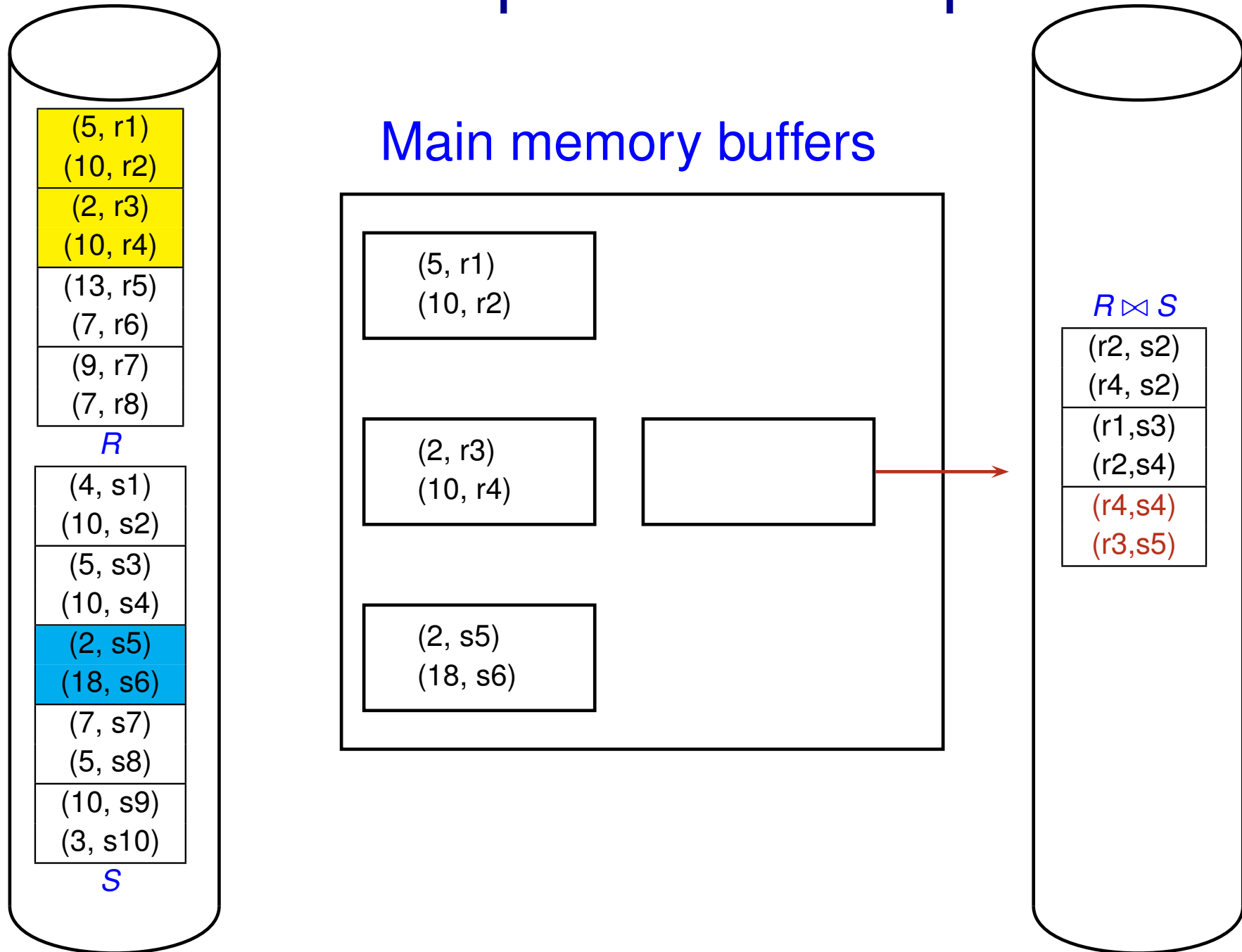
## Main memory buffers



# Block Nested Loop Join: Example

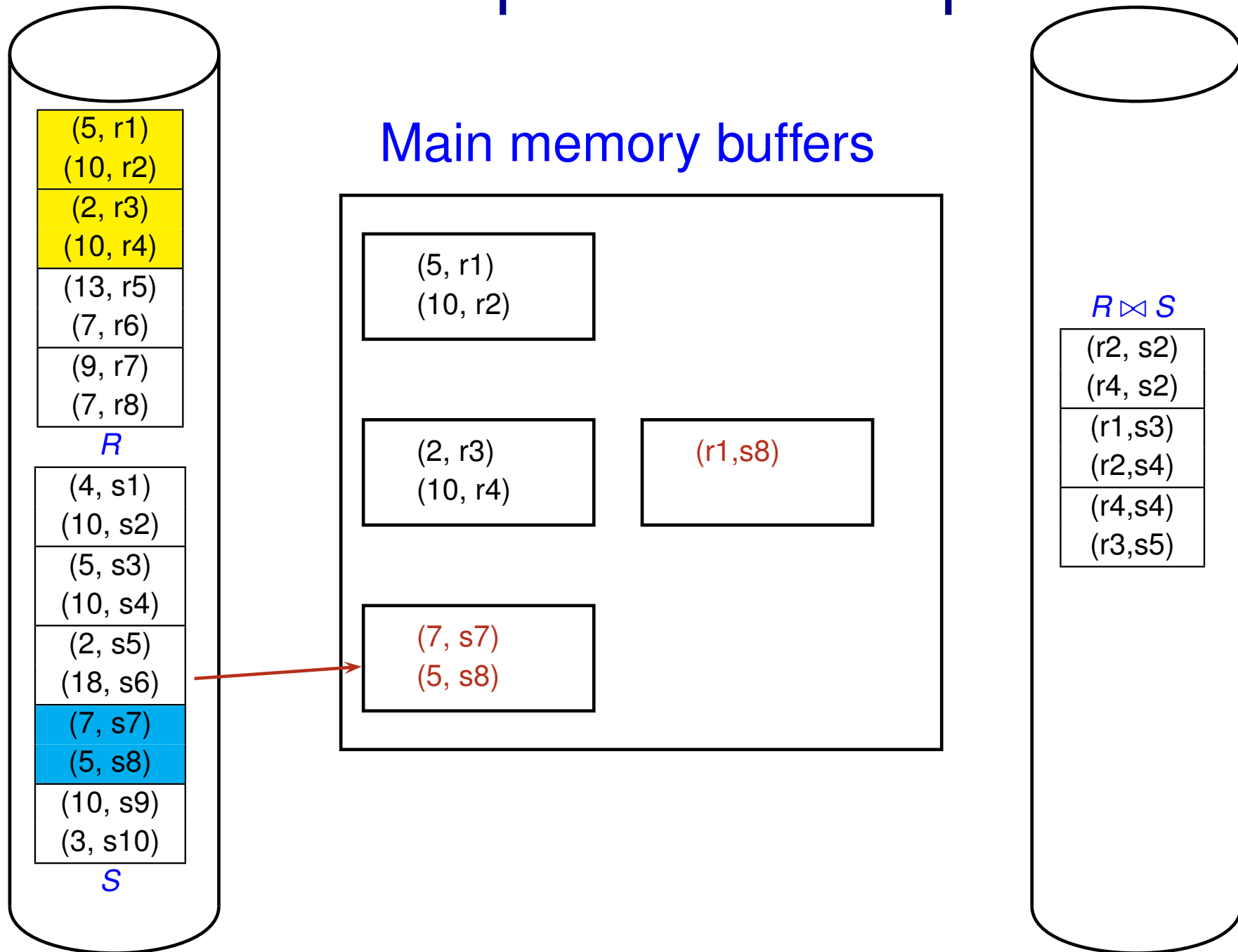


# Block Nested Loop Join: Example

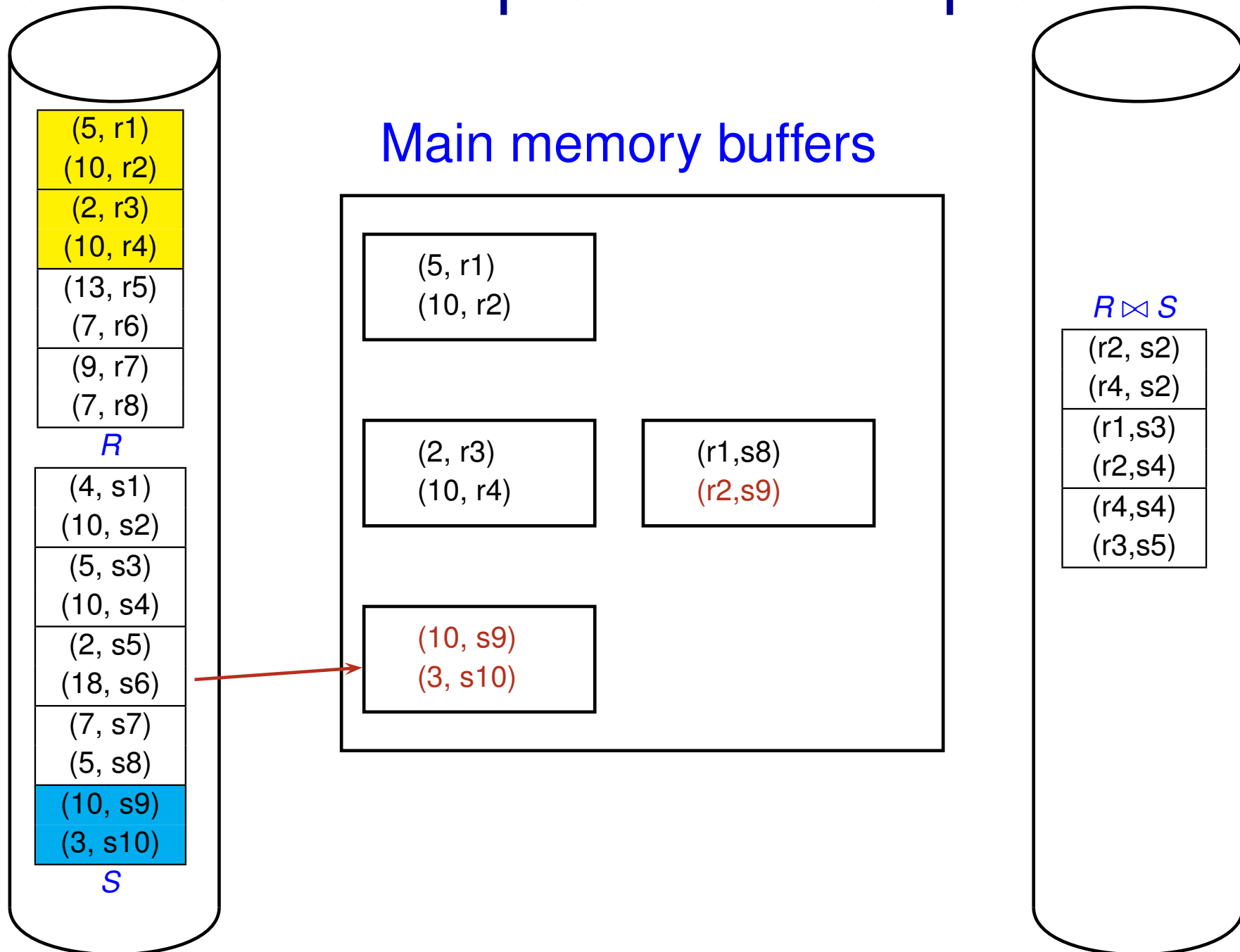




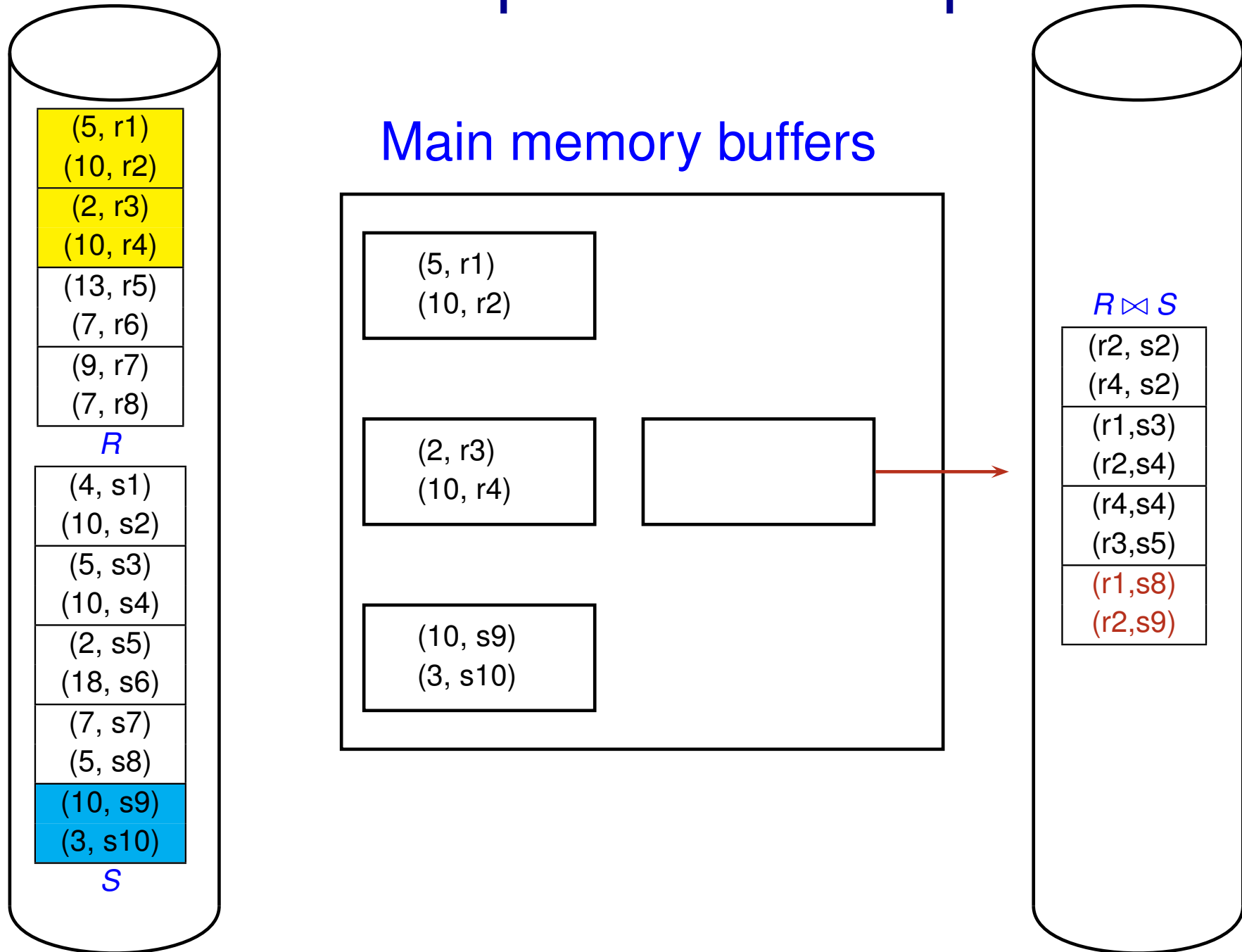
# Block Nested Loop Join: Example



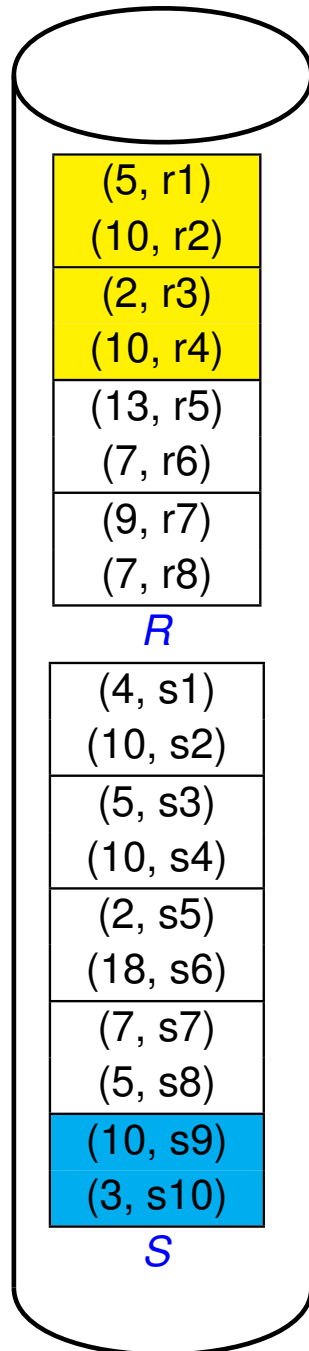
# Block Nested Loop Join: Example



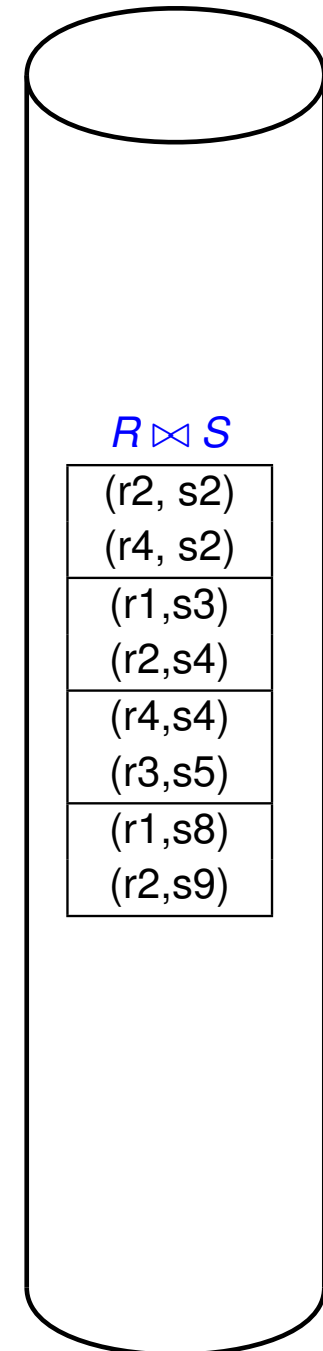
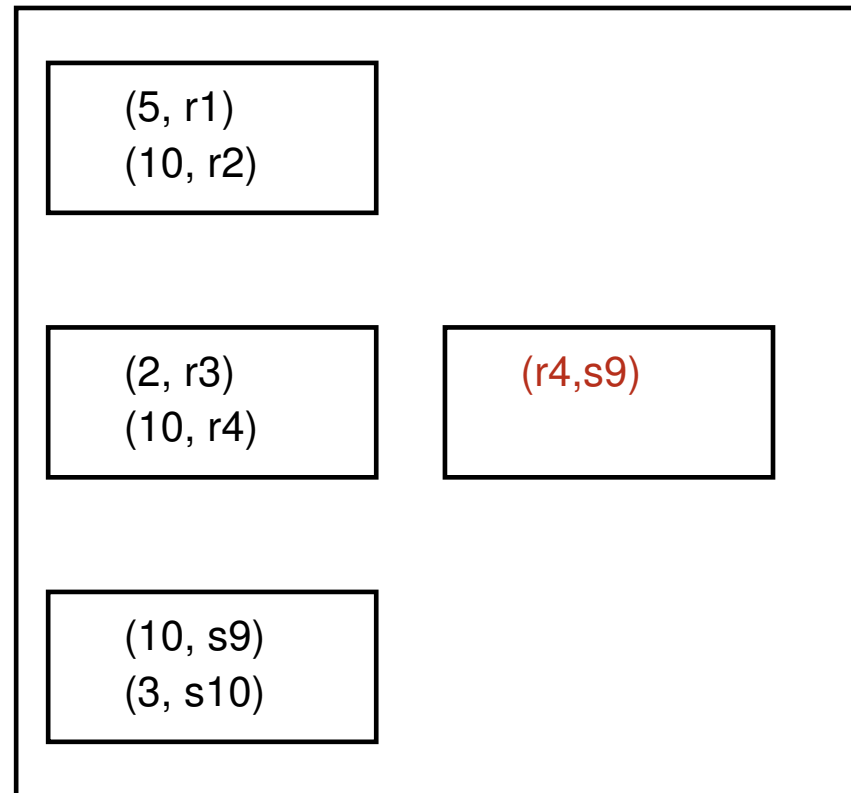
# Block Nested Loop Join: Example



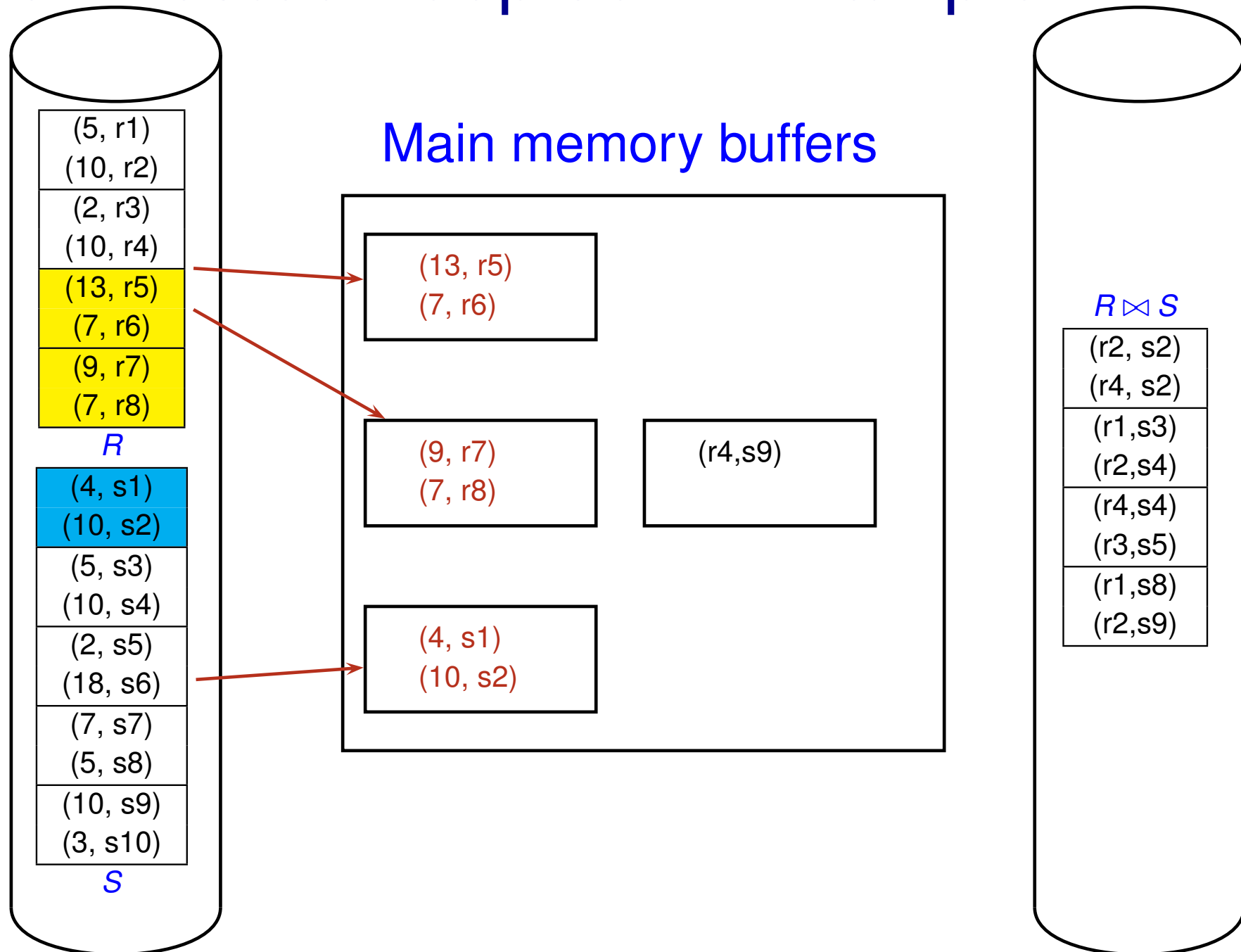
# Block Nested Loop Join: Example



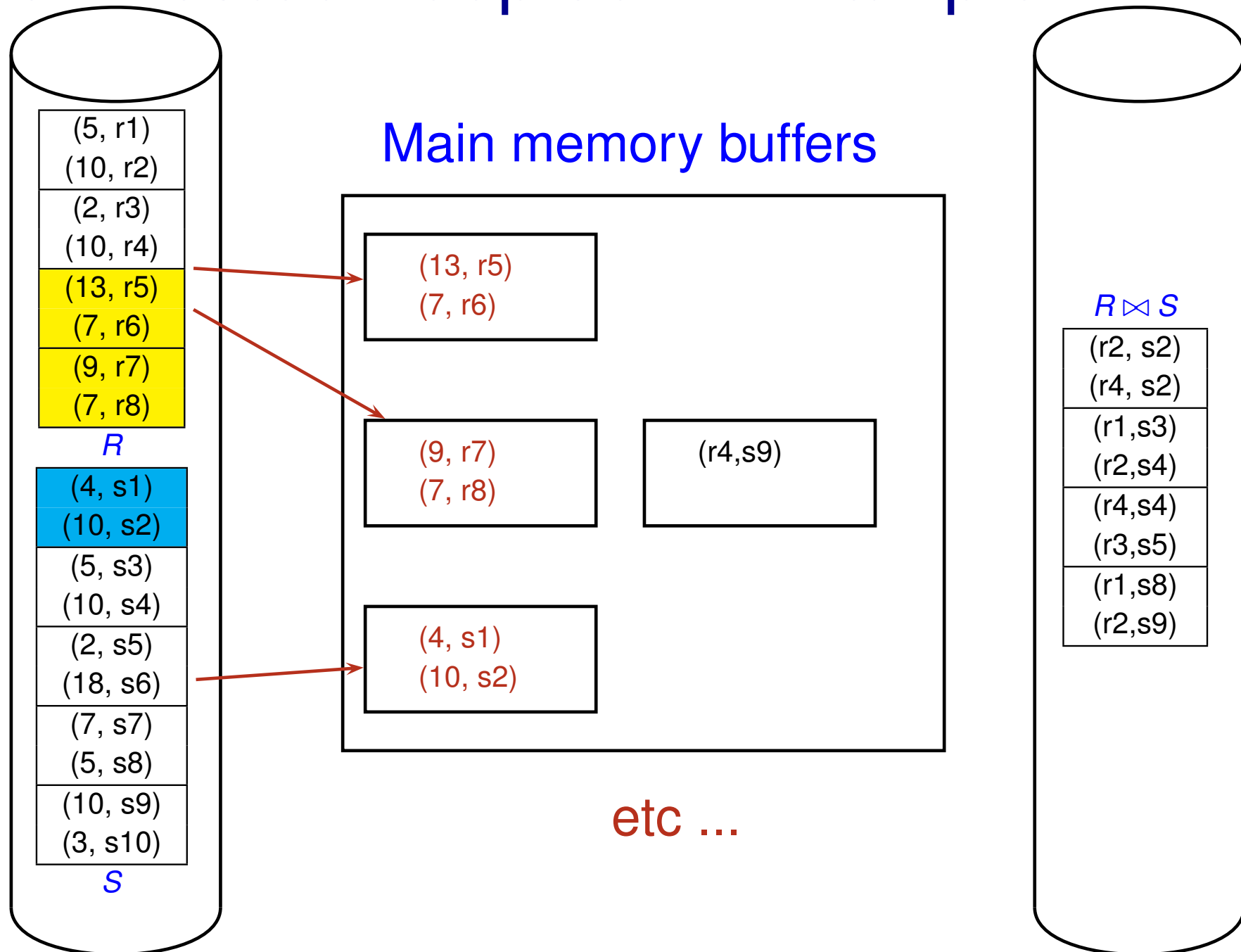
## Main memory buffers



# Block Nested Loop Join: Example

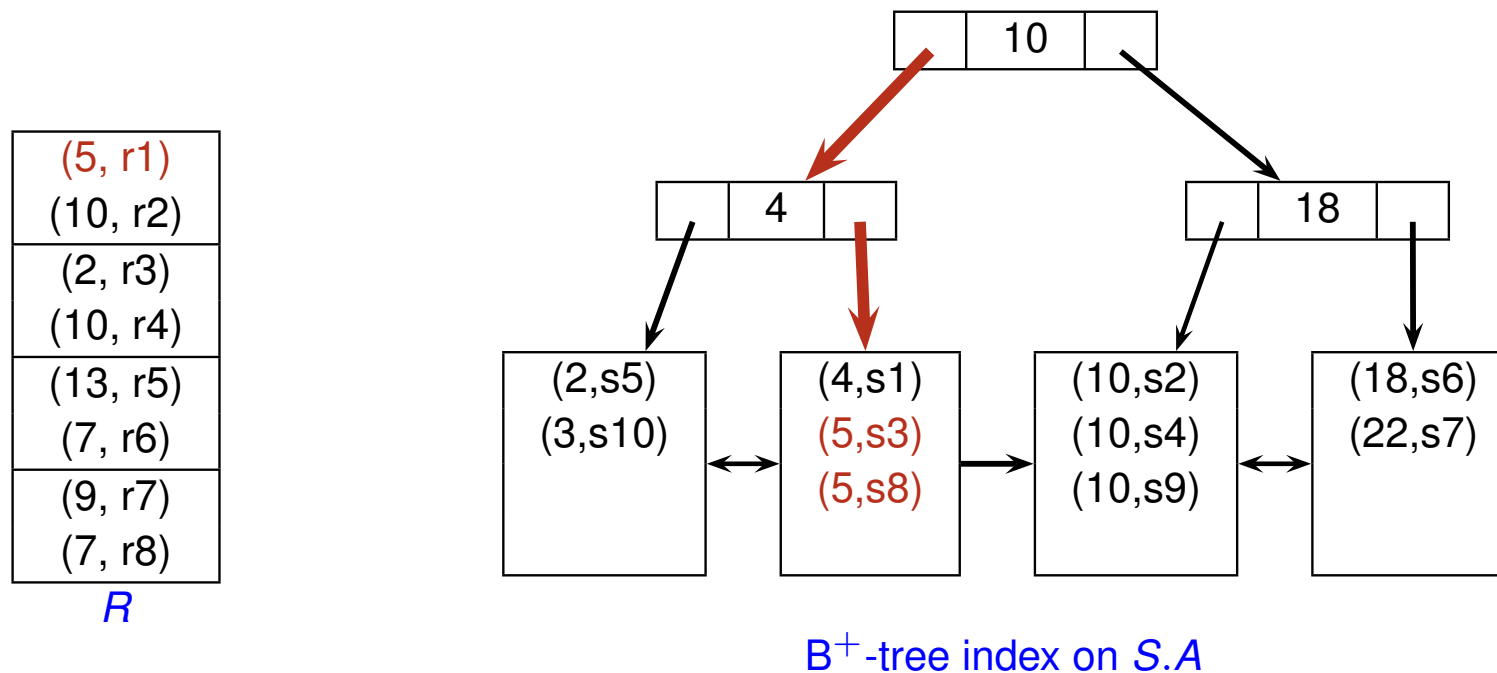


# Block Nested Loop Join: Example



# Index Nested Loop Join

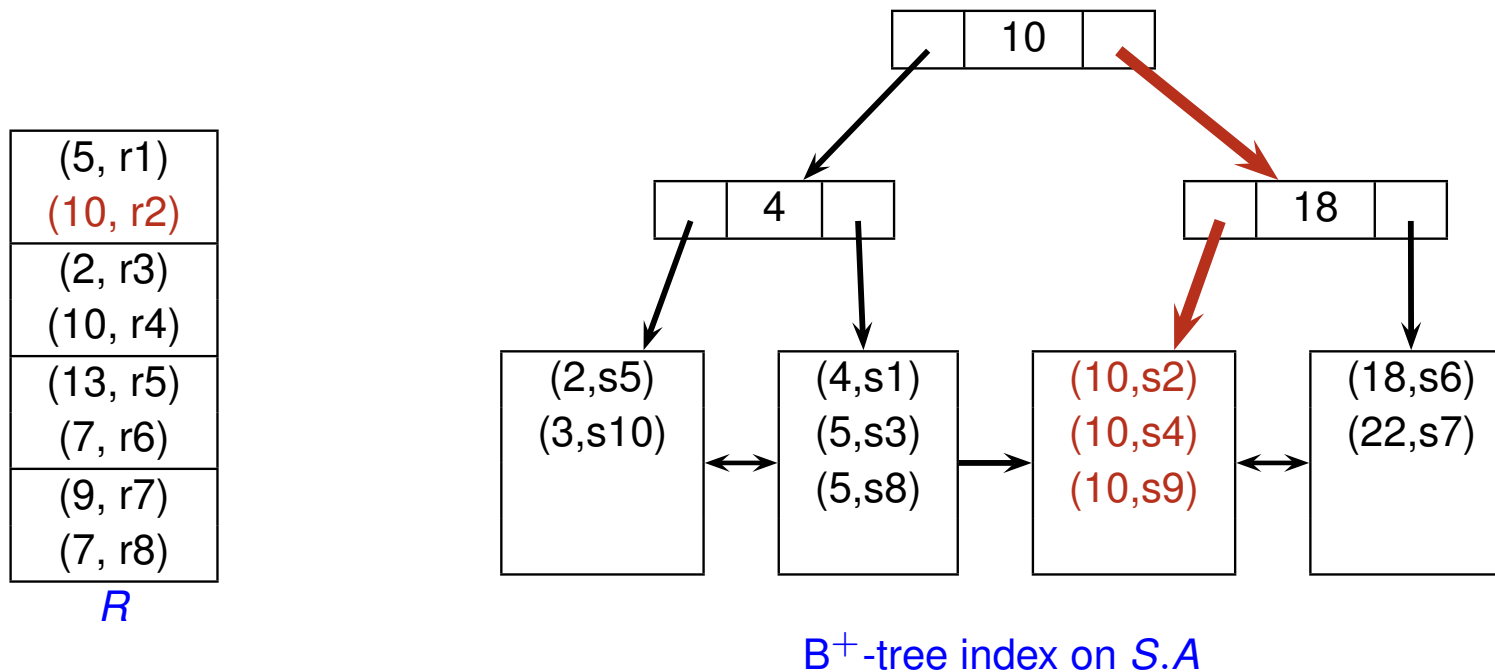
- ▶ Consider  $R(A, B) \bowtie_A S(A, C)$
- ▶ Assume that there's a  $B^+$ -tree index on  $S.A$



First, join  $(5, r1) \in R$  with matching tuples in  $S$

# Index Nested Loop Join

- ▶ Consider  $R(A, B) \bowtie_A S(A, C)$
- ▶ Assume that there's a  $B^+$ -tree index on  $S.A$



Next, join  $(10, r2) \in R$  with matching tuples in  $S$ , and so on ...



# Index Nested Loop Join

- ▶ **Precondition:** there is an index on the join attribute(s) of inner relation
- ▶ **Idea:**
  - for each tuple  $r \in R$  do
  - use  $r$  to probe  $S$ 's index to find matching tuples

## ▶ **Analysis:**

- ▶ Let  $R.A_i = S.B_j$  be the join condition
- ▶ Uniform distribution assumption:  
each  $R$ -tuple joins with  $\lceil \frac{||S||}{||\pi_{B_j}(S)||} \rceil$  number of  $S$ -tuples

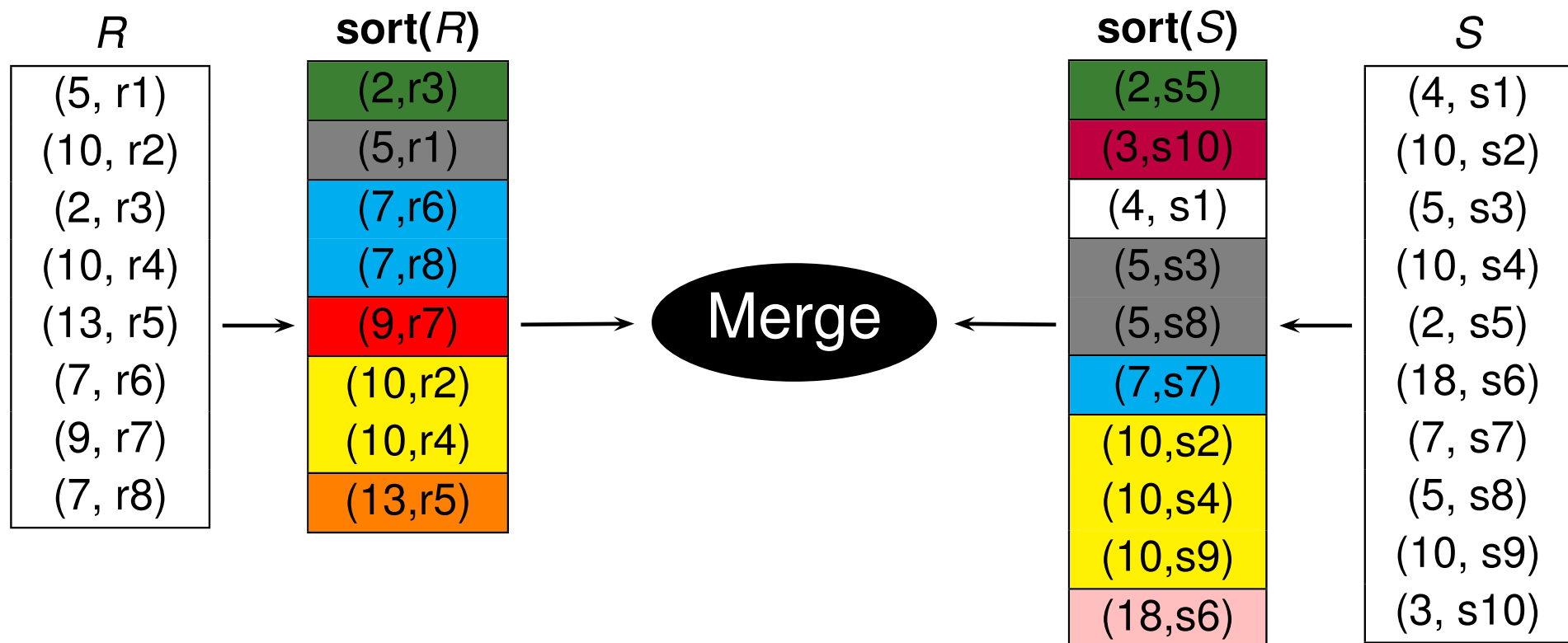
- ▶ For a format-1  $B^+$ -tree index on  $S$ ,

$$\star \text{ I/O Cost} = \underbrace{|R|}_{\text{scan } R} + \underbrace{||R|| \times J}_{\text{join each } R\text{-tuple with } S}$$

$$\star J = \underbrace{\log_F\left(\lceil \frac{||S||}{b_d} \rceil\right)}_{\text{search index's internal nodes}} + \underbrace{\left\lceil \frac{||S||}{b_d ||\pi_{B_j}(S)||} \right\rceil}_{\text{search index's leaf nodes}}$$

# Sort-Merge Join

- ▶ **Idea**: sort both relations based on join attributes & merge them
- ▶ A sorted relation  $R$  consists of **partitions**  $R_i$  of records where  $r, r' \in R_i$  iff  $r$  and  $r'$  have the same values for the join attribute(s)



# Sort-Merge Join: Merging Phase

- ▶ Each tuple in R-partition merges with all tuples in matching S-partition
- ▶ A *pointer* is maintained for each sorted join operand
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- ▶ Search for matching partitions by advancing the pointer that is pointing to a “smaller” tuple
- ▶ Need to remember position of first tuple in matching S-partition to enable rewinding of S-pointer
- ▶ **Example:**

R: 2 5 7 10 10 13

S: 4 5 5 10 10 18 22

---

*R* ⋈ *S*:

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S:	4	5	5	10	10	18	22

---

$R \bowtie S: (5,5)(5,5)(10,10)(10,10)(10,10)(10,10)$

# Sort-Merge Join Algorithm for $R \bowtie_{R.A_i=S.B_j} S$

01. if ( $R$  is not sorted) then sort  $R$
02. if ( $S$  is not sorted) then sort  $S$
03.  $t_r$  = first tuple in  $R$
04.  $t_s$  = first tuple in  $S$
05.  $p_s$  = first tuple in  $S$  partition
06. while ( $t_r \neq null$ ) and ( $p_s \neq null$ ) do
07.     while ( $t_r.A_i < p_s.B_j$ ) do
08.          $t_r$  = next tuple in  $R$  after  $t_r$
09.     while ( $t_r.A_i > p_s.B_j$ ) do
10.          $p_s$  = next tuple in  $S$  after  $p_s$
11.      $t_s = p_s$
12.     while ( $t_r.A_i = p_s.B_j$ ) do
13.          $t_s = p_s$
14.         while ( $t_s.B_j = t_r.A_i$ ) do
15.             add ( $t_r, t_s$ ) to result
16.              $t_s$  = next tuple in  $S$  after  $t_s$
17.          $t_r$  = next tuple in  $R$  after  $t_r$
18.      $p_s = t_s$

# Sort-Merge Join: Analysis

- ▶ **I/O cost = Cost to sort  $R$  + Cost to sort  $S$  + Merging cost**
- ▶ **Cost to sort  $R = 2|R| (\log_m(N_R) + 1)$**  if using external merge sort
  - ▶  $N_R$  = number of initial sorted runs of  $R$ ,  $m$  = merge factor
- ▶ **Cost to sort  $S = 2|S| (\log_m(N_S) + 1)$**  if using external merge sort
  - ▶  $N_S$  = number of initial sorted runs of  $S$ ,  $m$  = merge factor
- ▶ If each  $S$  partition is scanned at most once during merging,
  - ▶ **Merging cost =  $|R| + |S|$**
- ▶ Worst case occurs when each tuple of  $R$  requires scanning entire  $S$ !
  - ▶ **Merging cost =  $|R| + ||R|| \times |S|$**

# Sort-Merge Join: Optimization

## ► Conventional Sort-Merge Join

- **Sort R**: create sorted runs of R; merge sorted runs of R
- **Sort S**: create sorted runs of S; merge sorted runs of S
- **Join R and S**: merge sorted R & sorted S

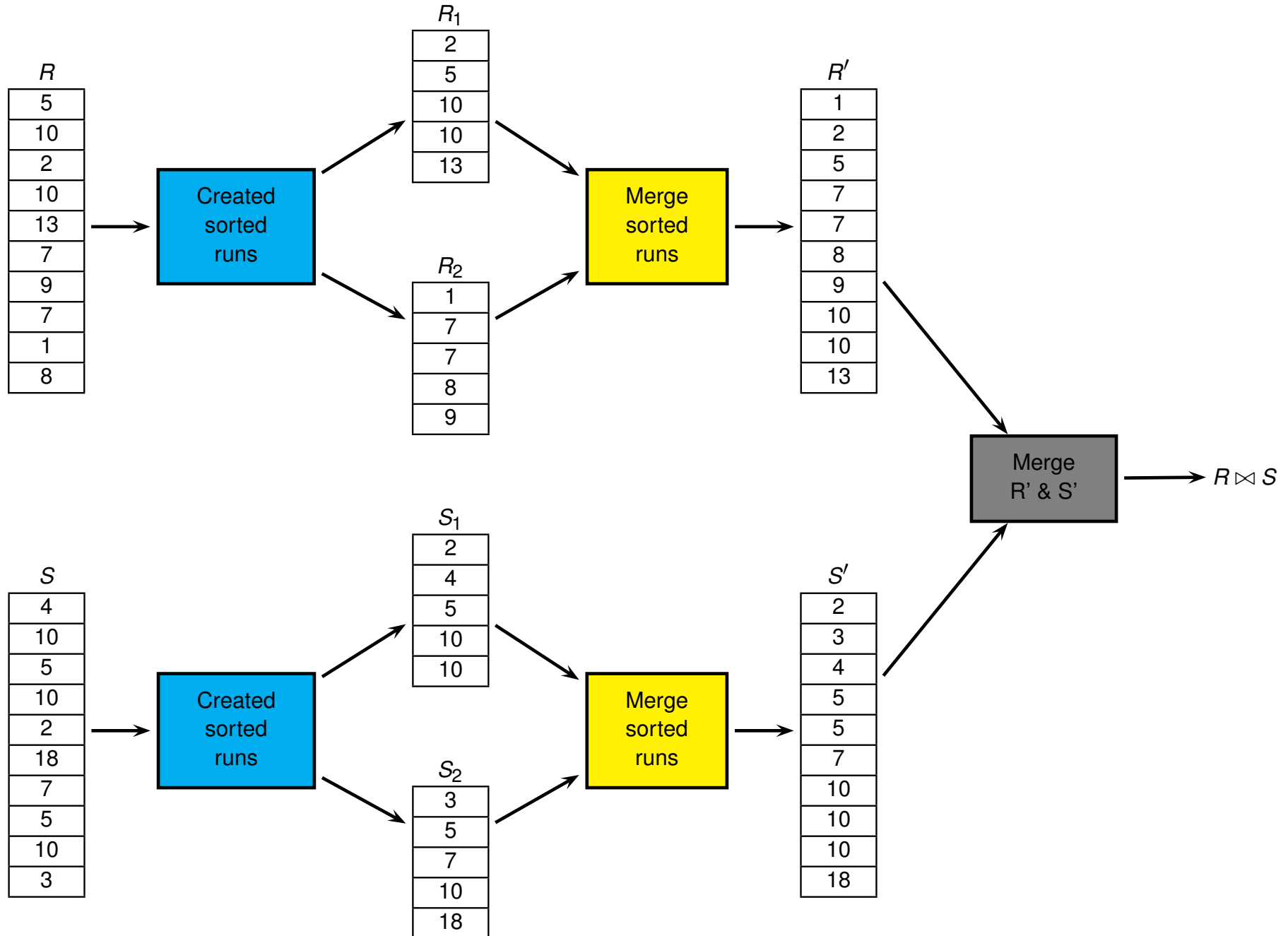
## ► Idea: Combine merge phase of sorting & merge phase of join

- It's not necessary to merge sorted runs into a single run before performing join
- If  $B > N(R, i) + N(S, j)$  for some  $i$  &  $j$ , sorting of R and S can stop
  - ★  $N(R, i)$  = total number of sorted runs of R at the end of pass  $i$  of sorting R

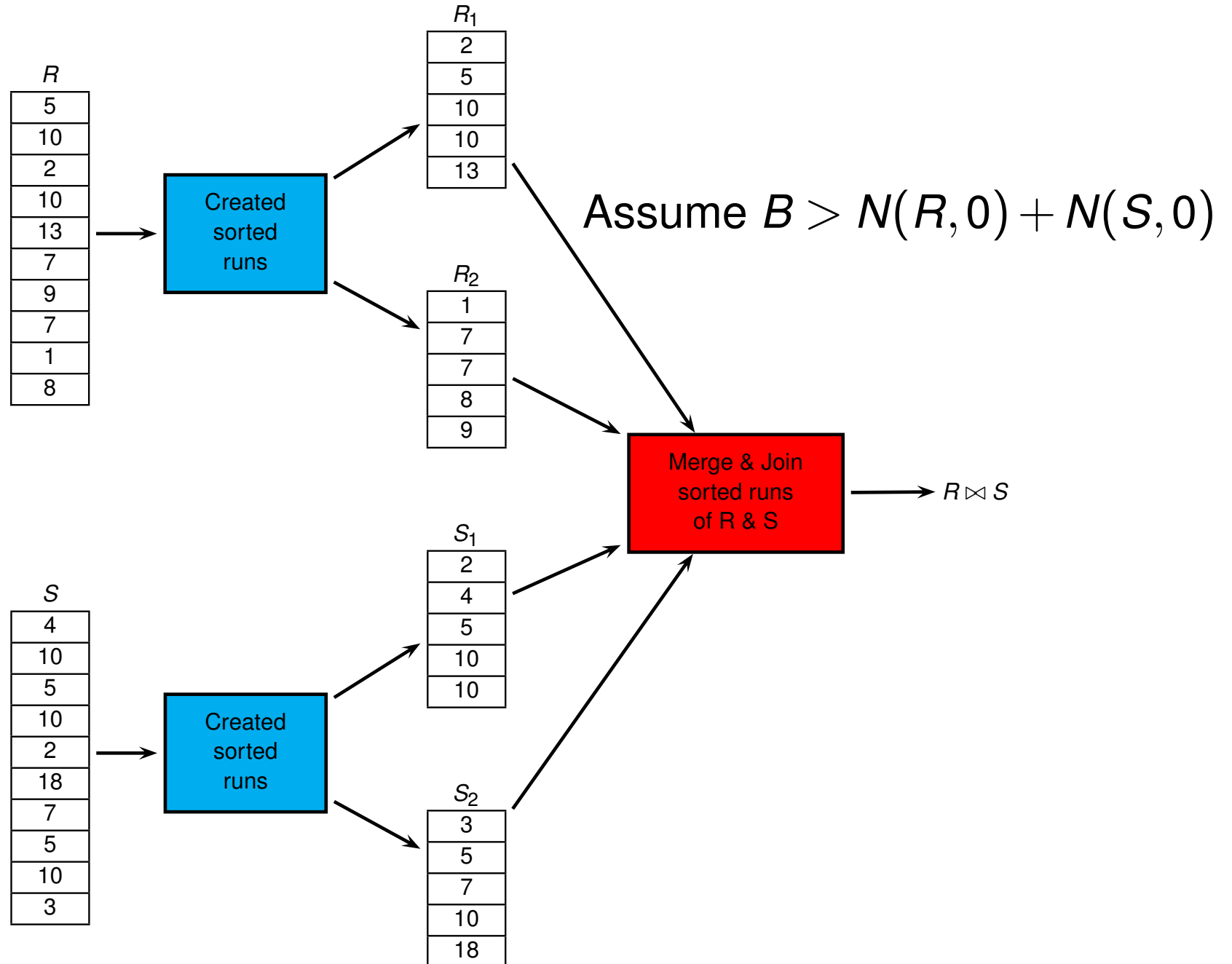
## ► Optimized Sort-Merge Join

- Create sorted runs of R; merge sorted runs of R partially
- Create sorted runs of S; merge sorted runs of S partially
- Merge remaining sorted runs of R & S and join them at the same time

# Conventional Sort-Merge Join



# Optimized Sort-Merge Join



# Optimized Sort-Merge Join: Analysis

- ▶ Assume  $|R| \leq |S|$
- ▶ If  $B > \sqrt{2|S|}$ 
  - ▶ Number of initial sorted runs of  $S < \sqrt{\frac{|S|}{2}}$
  - ▶ Total number of initial sorted runs of  $R$  and  $S < \sqrt{2|S|}$
  - ▶ One pass is sufficient to merge and join the initial sorted runs  $R$  &  $S$
  - ▶ I/O Cost =  $3 \times (|R| + |S|)$



# Hash Join, $R \bowtie_{R.A=S.B} S$

## ► Idea:

- Partition  $R$  and  $S$  into  $k$  partitions using some hash function  $h$ 
  - ★  $R = R_1 \cup R_2 \cup \dots \cup R_k$ ,  $t \in R_i$  iff  $h(t.A) = i$
  - ★  $S = S_1 \cup S_2 \cup \dots \cup S_k$ ,  $t \in S_i$  iff  $h(t.B) = i$
  - ★  $\pi_A(R_i) \cap \pi_B(S_j) = \emptyset$  for each  $R_i$  &  $S_j$ ,  $i \neq j$
- Joins corresponding pair of partitions
  - ★  $R \bowtie S = (R_1 \bowtie S_1) \cup (R_2 \bowtie S_2) \cup \dots \cup (R_k \bowtie S_k)$

## ► Algorithms:

- Grace hash join
- Hybrid hash join (not covered in lecture)

# Grace Hash Join, $R \bowtie_{R.A=S.B} S$

► Consists of three phases:

1. Partition  $R$  into  $R_1, \dots, R_k$
2. Partition  $S$  into  $S_1, \dots, S_k$
3. Probing phase: probes each  $R_i$  with  $S_i$ 
  - ★ Read  $R_i$  to build a hash table
  - ★ Read  $S_i$  to probe hash table

►  $R$  is called the **build relation** &  $S$  is called the **probe relation**

# Grace Hash Join, $R \bowtie_{R.A=S.B} S$

► Consists of three phases:

1. Partition  $R$  into  $R_1, \dots, R_k$
2. Partition  $S$  into  $S_1, \dots, S_k$
3. Probing phase: probes each  $R_i$  with  $S_i$ 
  - ★ Read  $R_i$  to build a hash table
  - ★ Read  $S_i$  to probe hash table

►  $R$  is called the **build relation** &  $S$  is called the **probe relation**

## Partitioning (building) phases

initialize a hash table  $T$  with  $k$  buckets

for each tuple  $r \in R$  do

    insert  $r$  into bucket  $h(r.A)$  of  $T$

write each bucket  $R_i$  of  $T$  to disk

initialize a hash table  $T$  with  $k$  buckets

for each tuple  $s \in S$  do

    insert  $s$  into bucket  $h(s.B)$  of  $T$

write each bucket  $S_i$  of  $T$  to disk

## Probing (matching) phase

for  $i = 1$  to  $k$  do

    initialize a hash table  $T$

    for each tuple  $r$  in partition  $R_i$  do

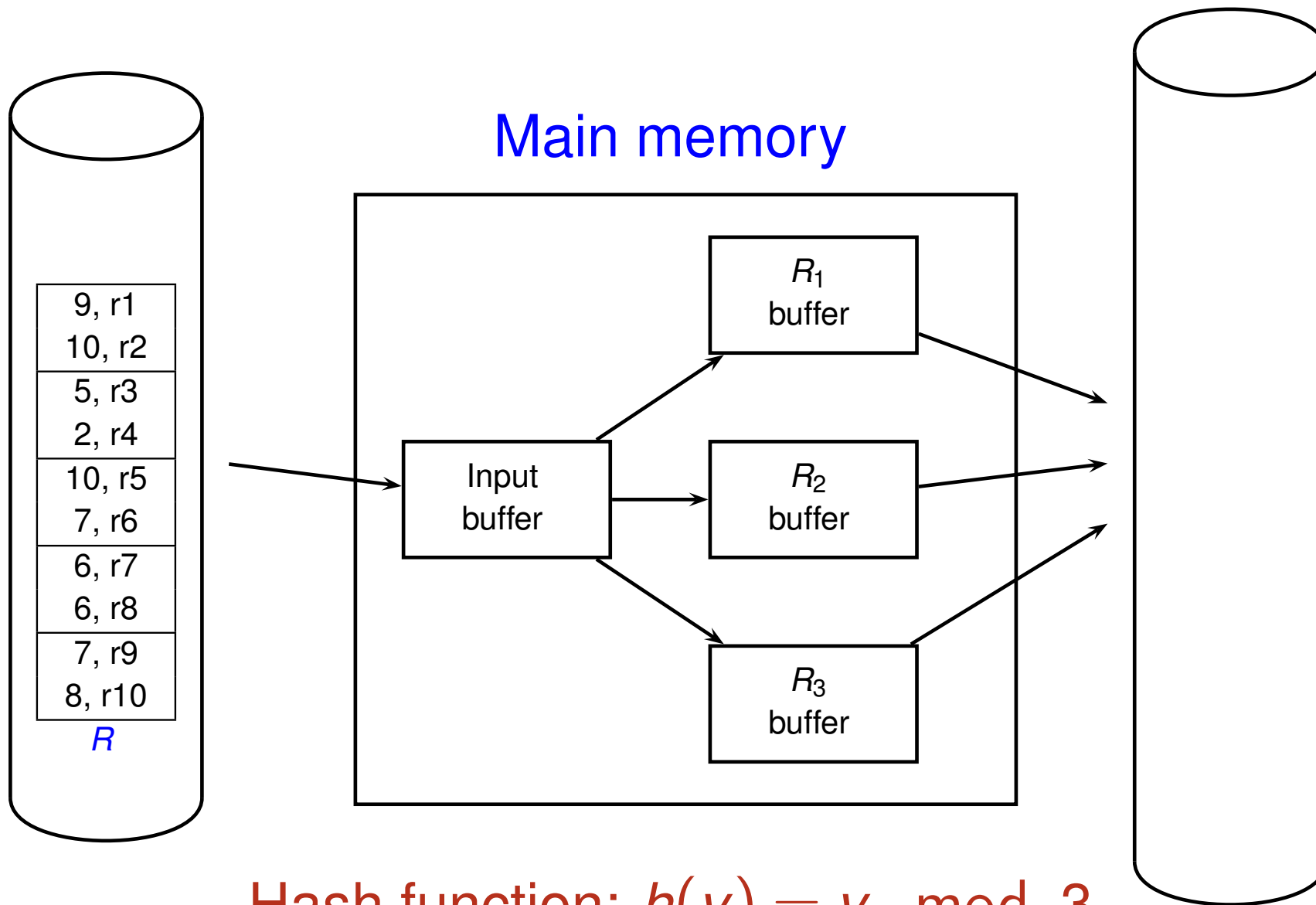
        insert  $r$  into bucket  $h'(r.A)$  of  $T$

    for each tuple  $s$  in partition  $S_i$  do

        for each tuple  $r$  in bucket  $h'(s.B)$  of  $T$  do

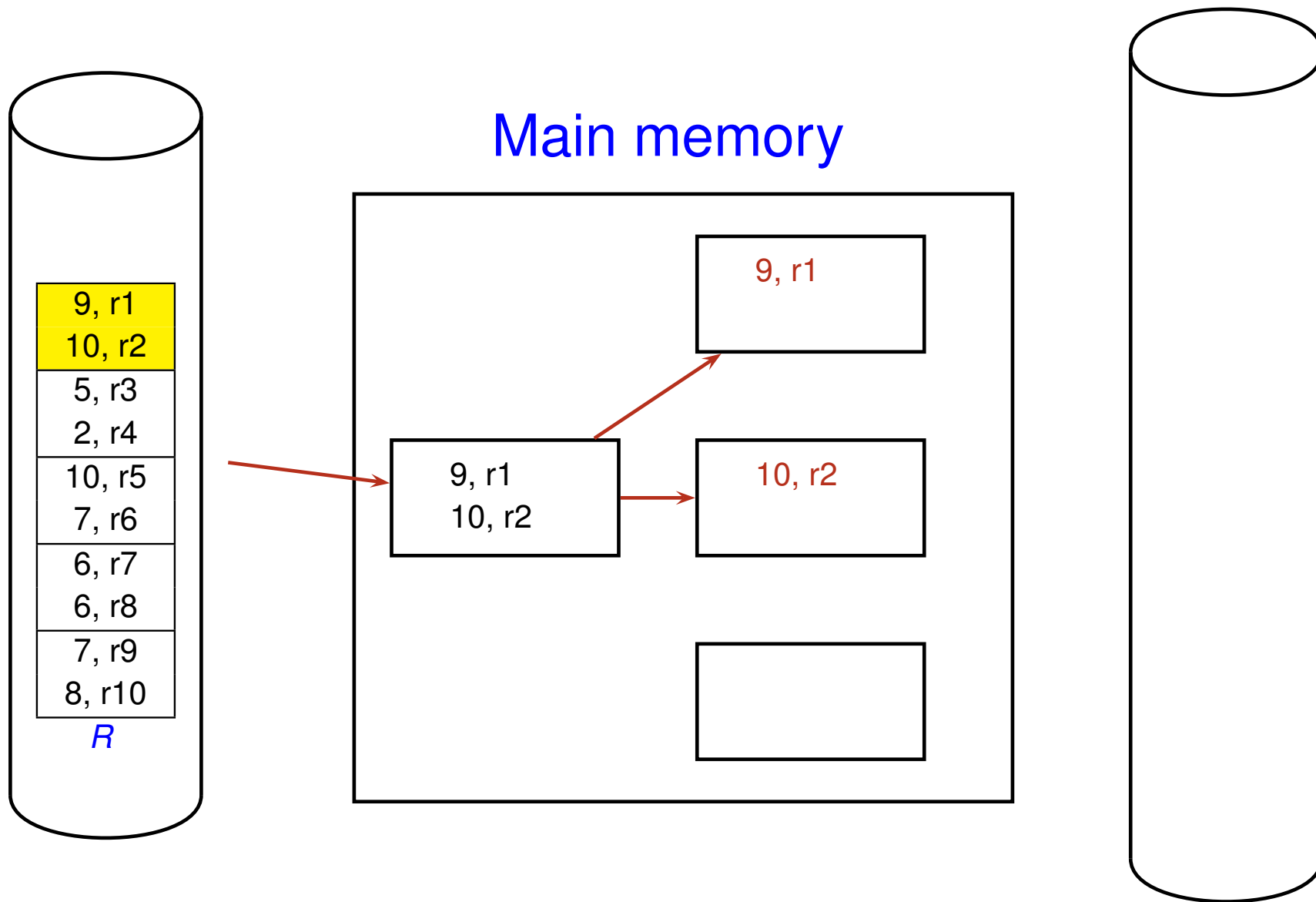
            if  $r$  and  $s$  matches then output  $(r, s)$

# Grace Hash Join: Partitioning Relation R

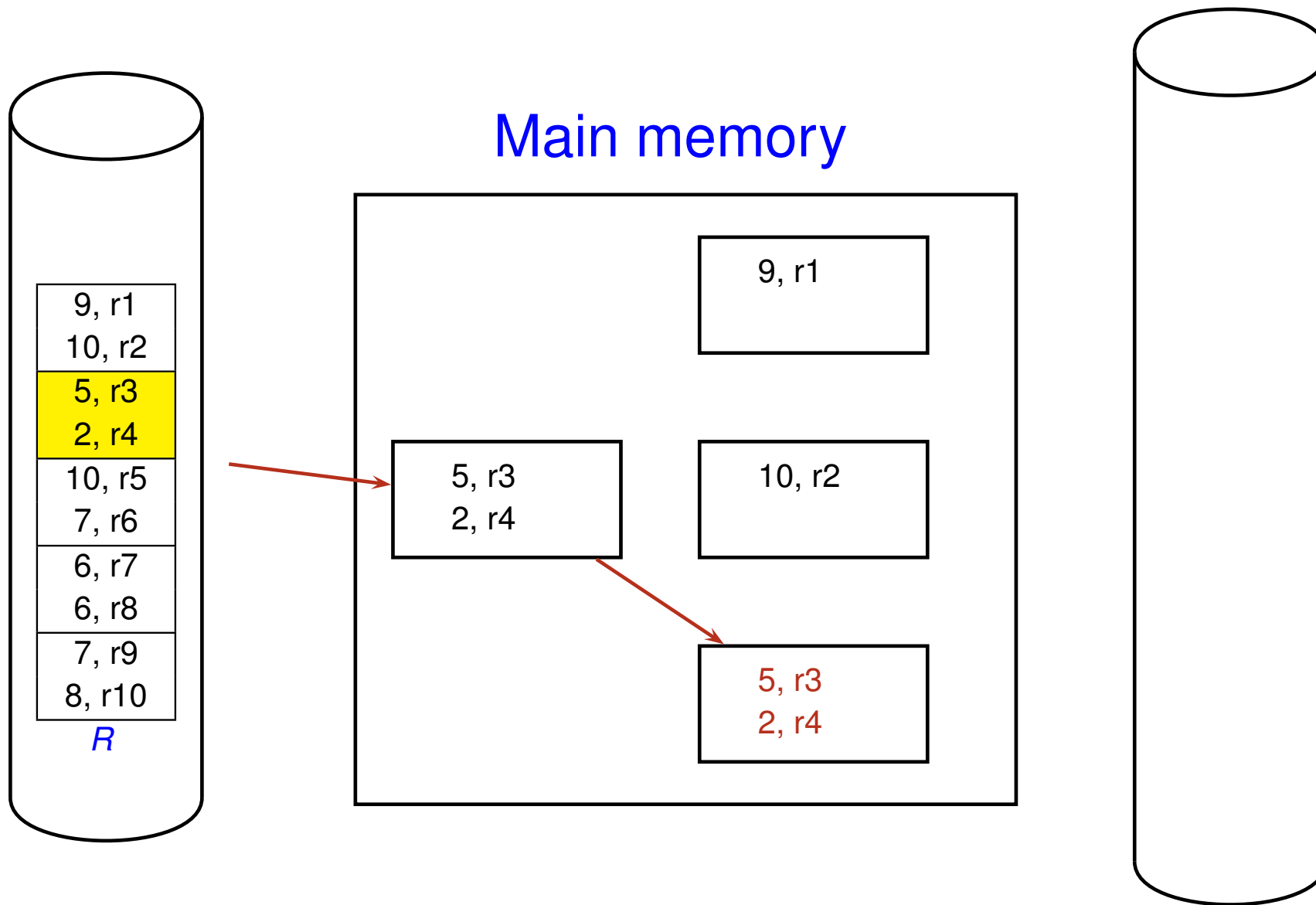


Hash function:  $h(v) = v \bmod 3$

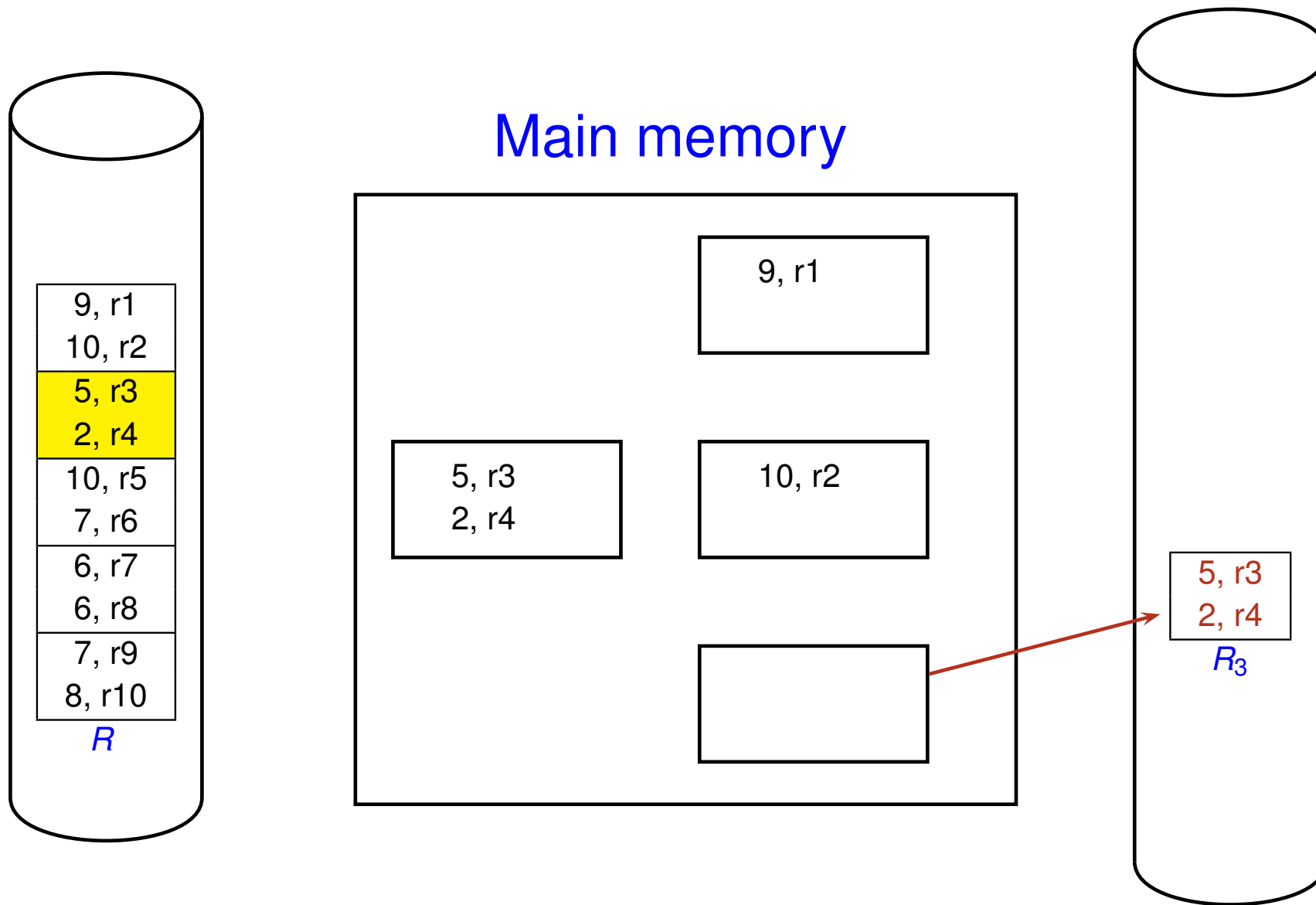
# Grace Hash Join: Partitioning Relation R



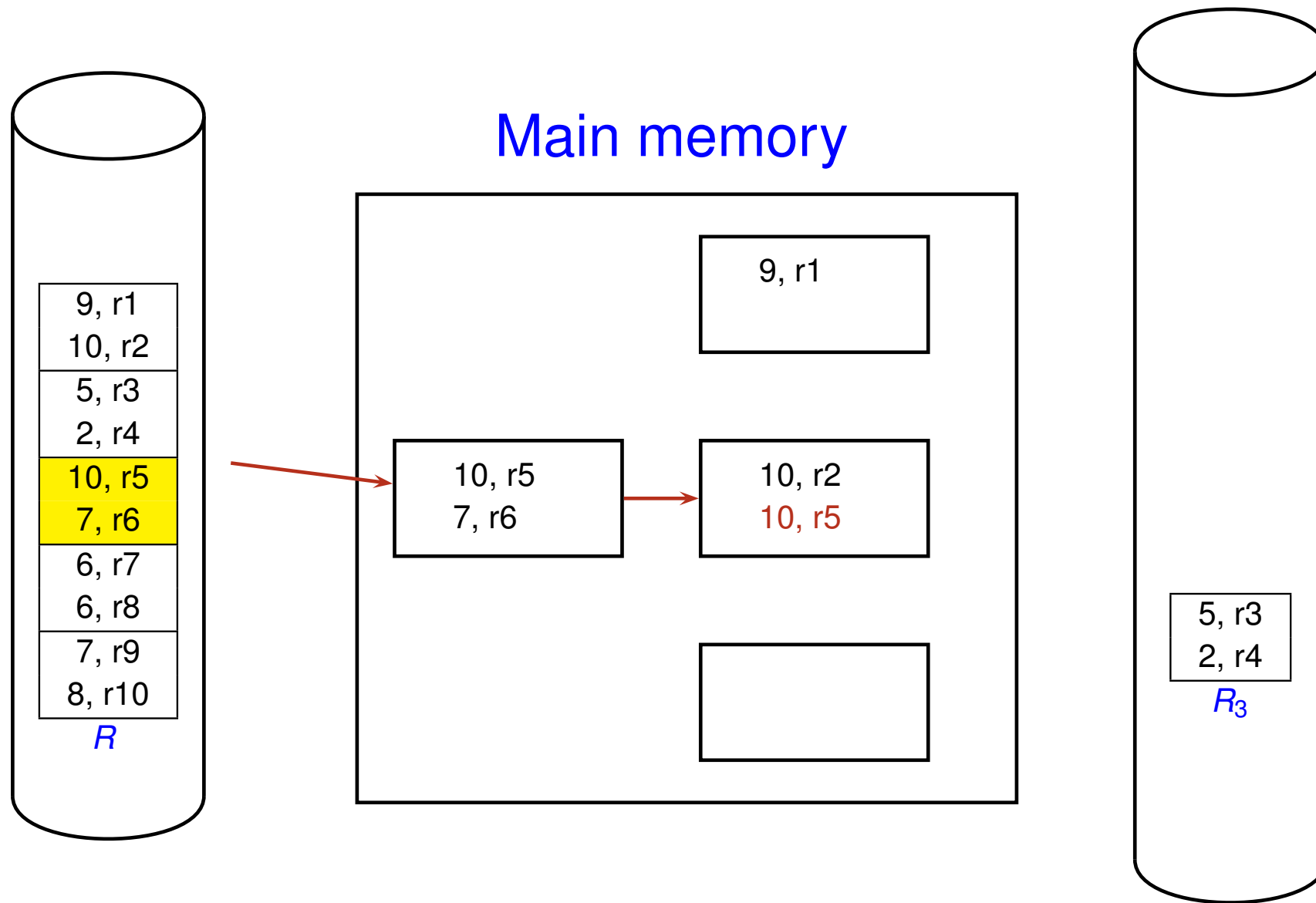
# Grace Hash Join: Partitioning Relation R



# Grace Hash Join: Partitioning Relation R

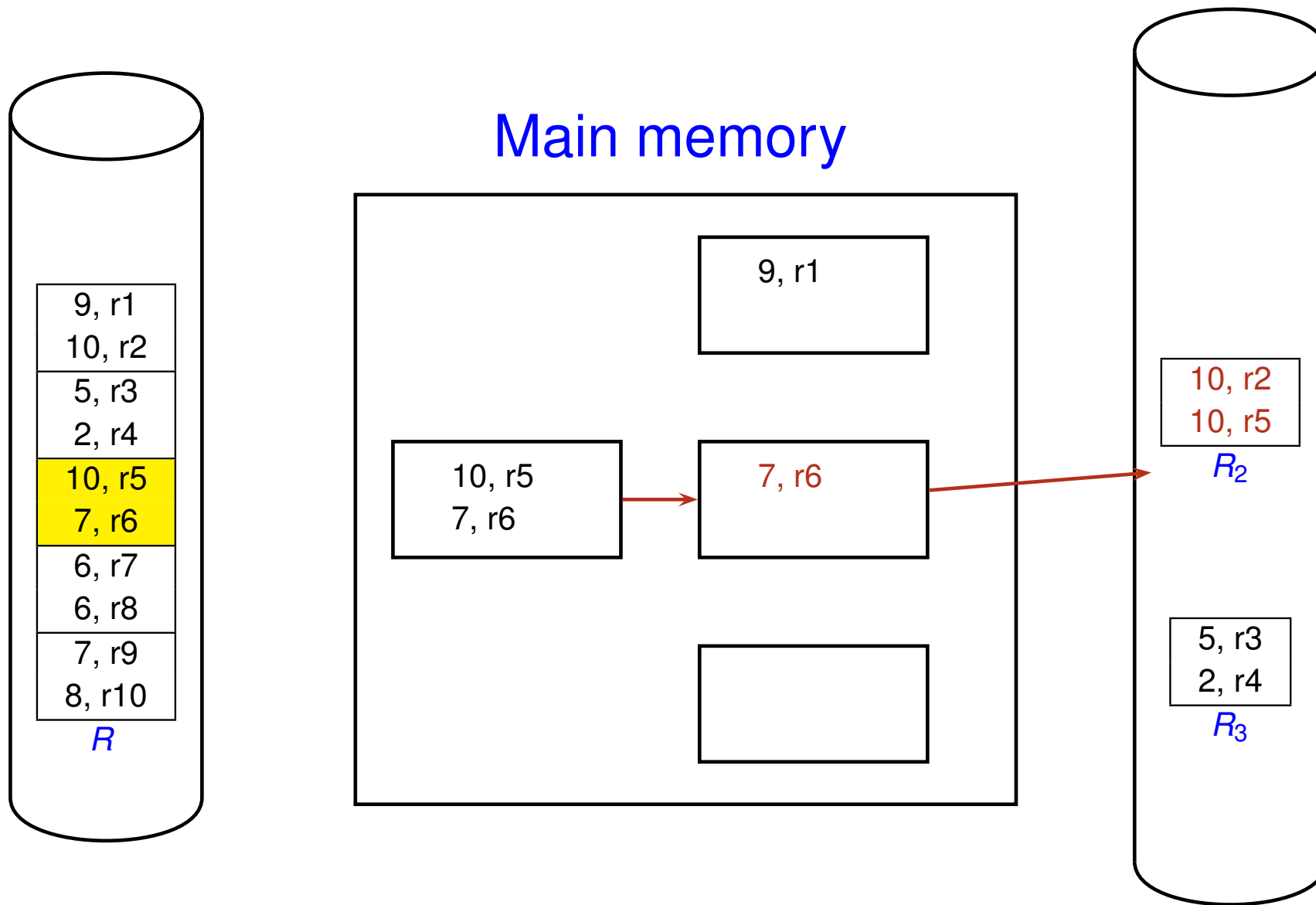


# Grace Hash Join: Partitioning Relation R

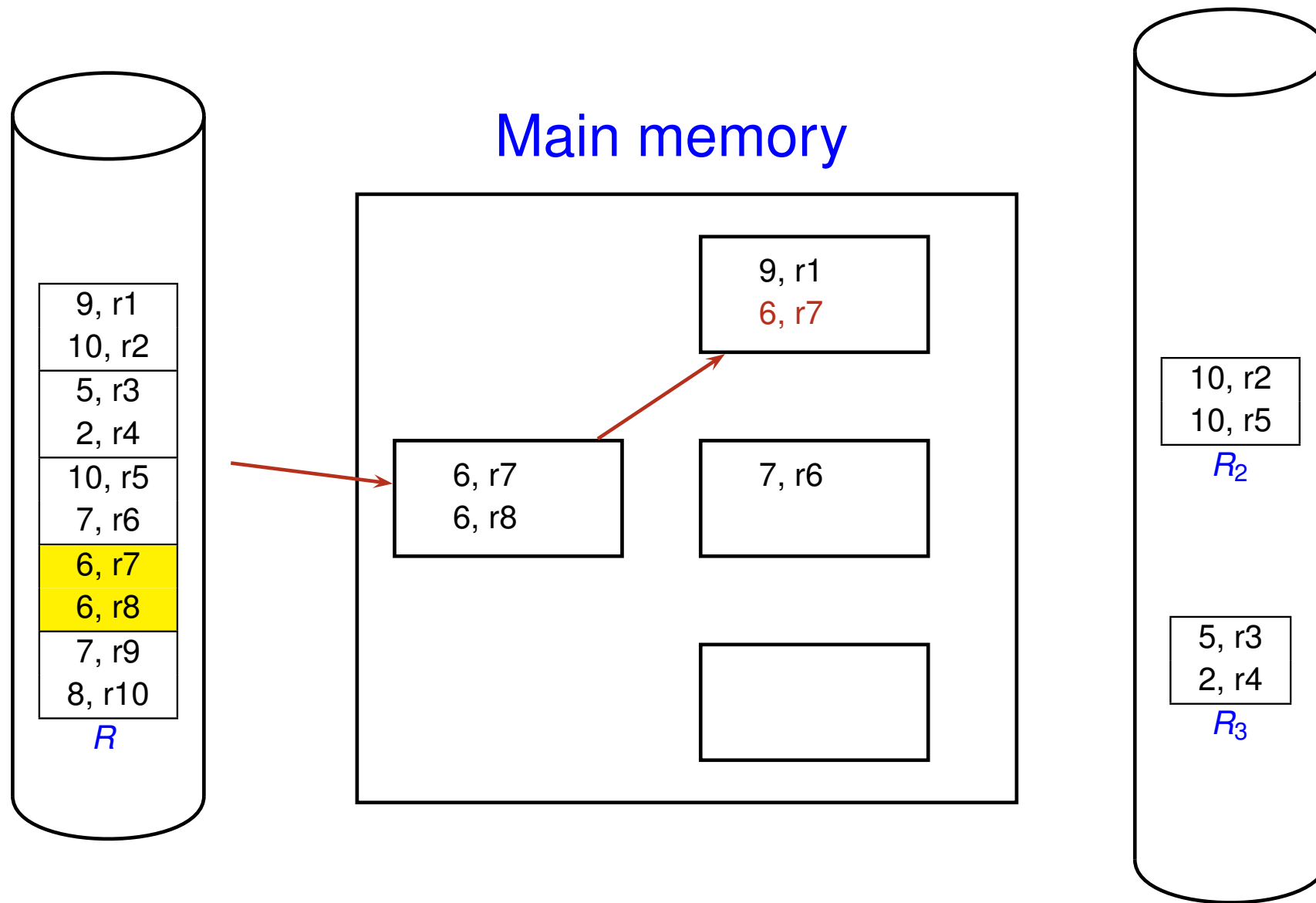




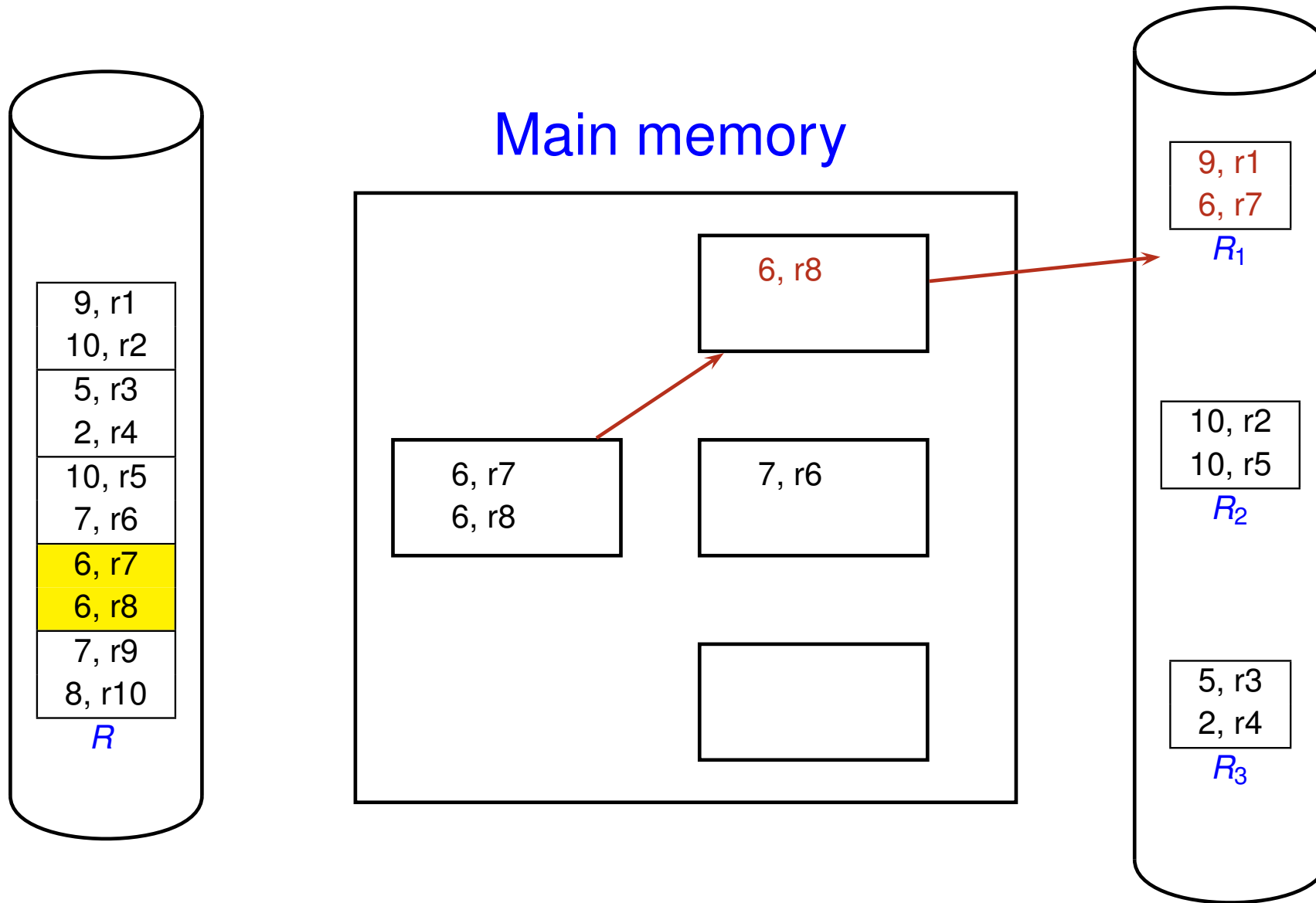
# Grace Hash Join: Partitioning Relation R



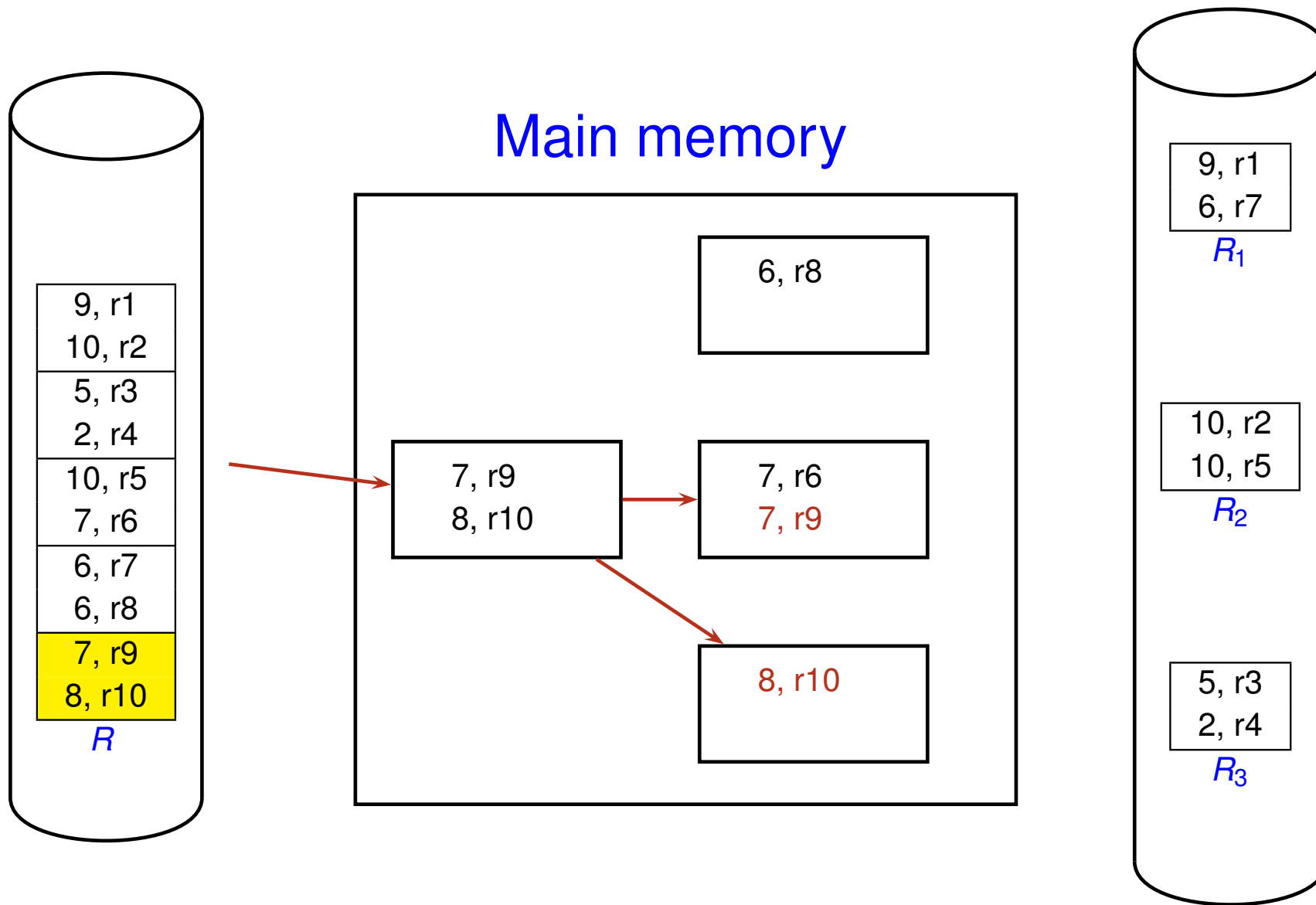
# Grace Hash Join: Partitioning Relation R



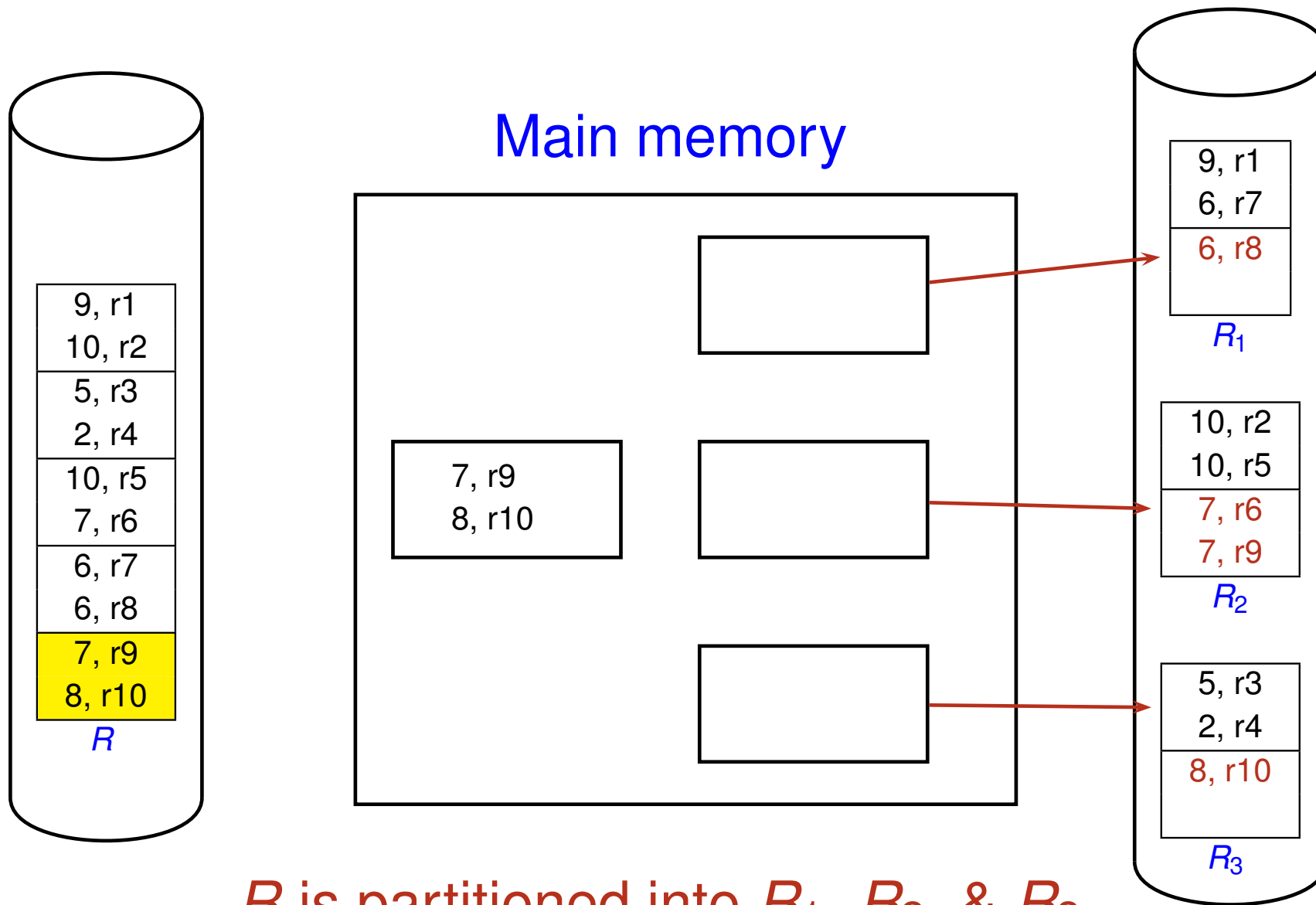
# Grace Hash Join: Partitioning Relation R



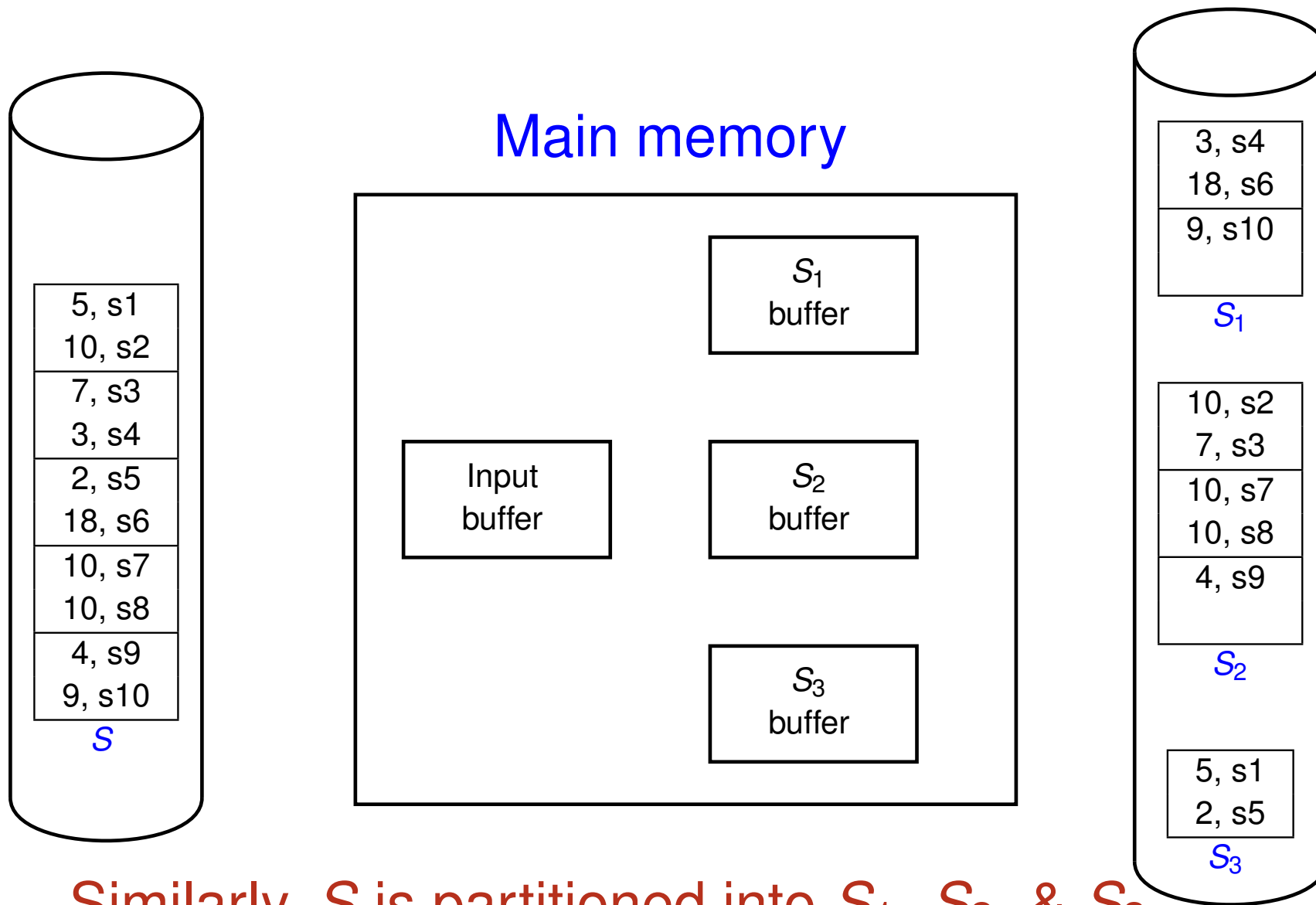
# Grace Hash Join: Partitioning Relation R



# Grace Hash Join: Partitioning Relation R

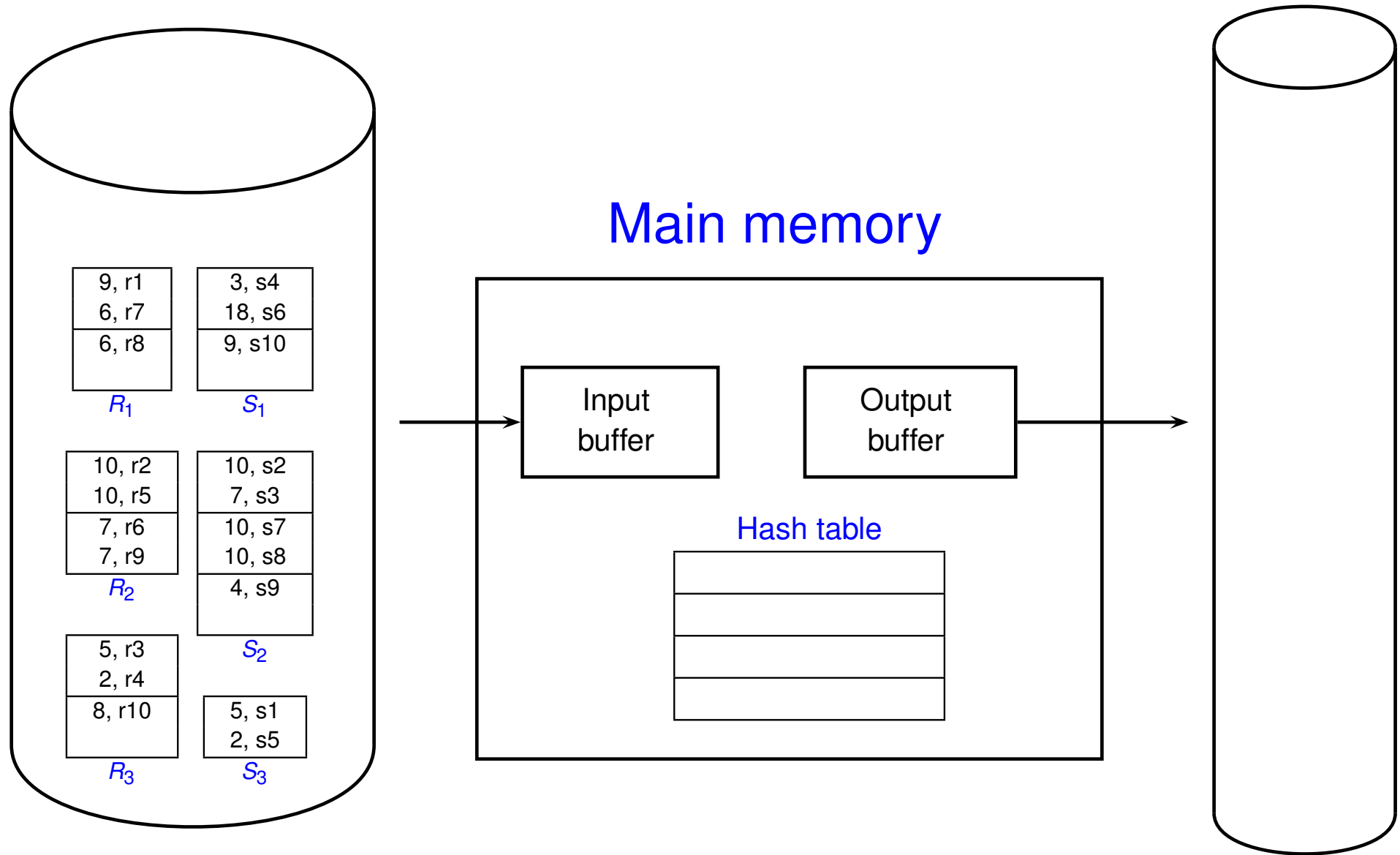


# Grace Hash Join: Partitioning Relation S

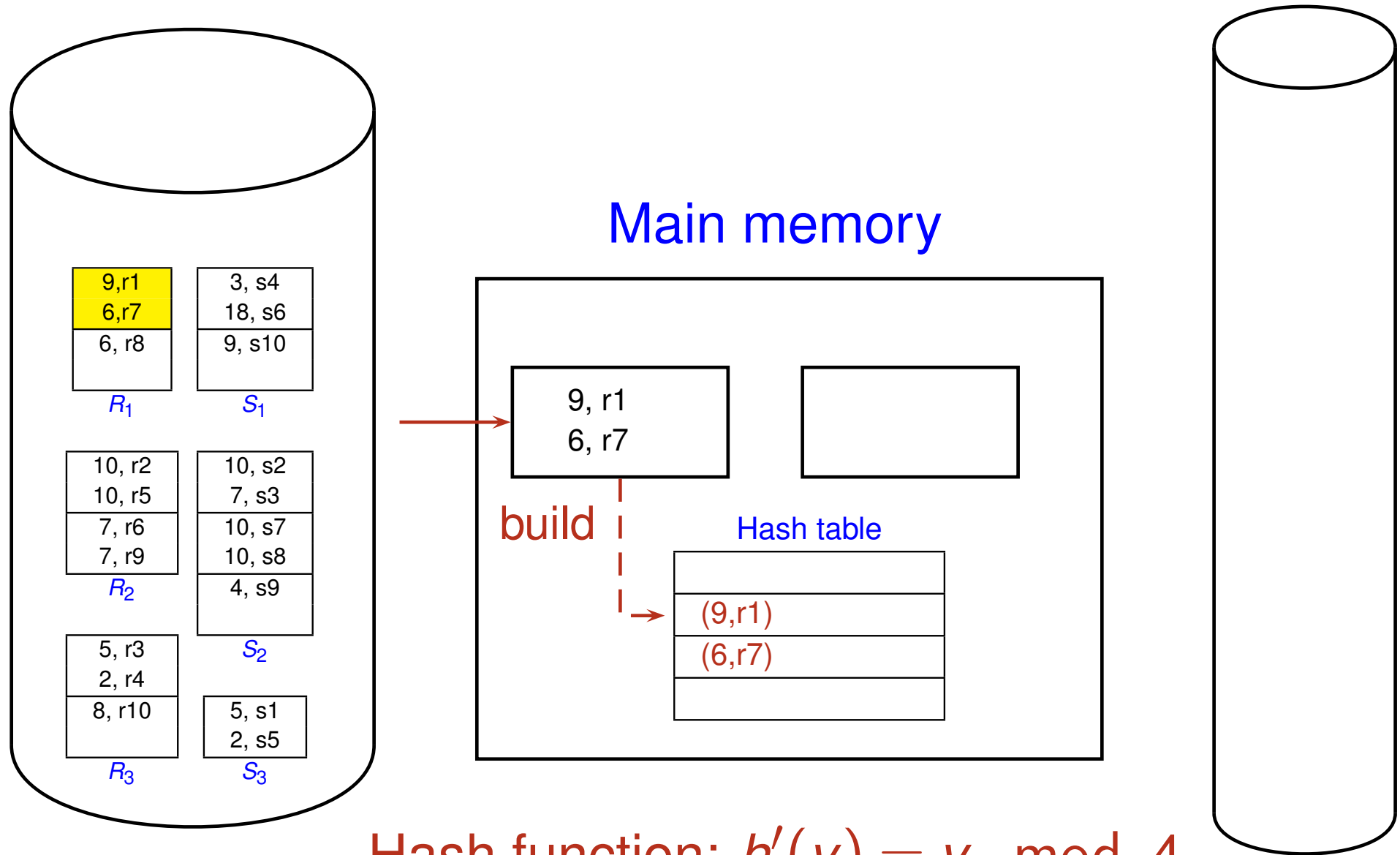


Similarly,  $S$  is partitioned into  $S_1$ ,  $S_2$ , &  $S_3$

# Grace Hash Join: Probing Phase

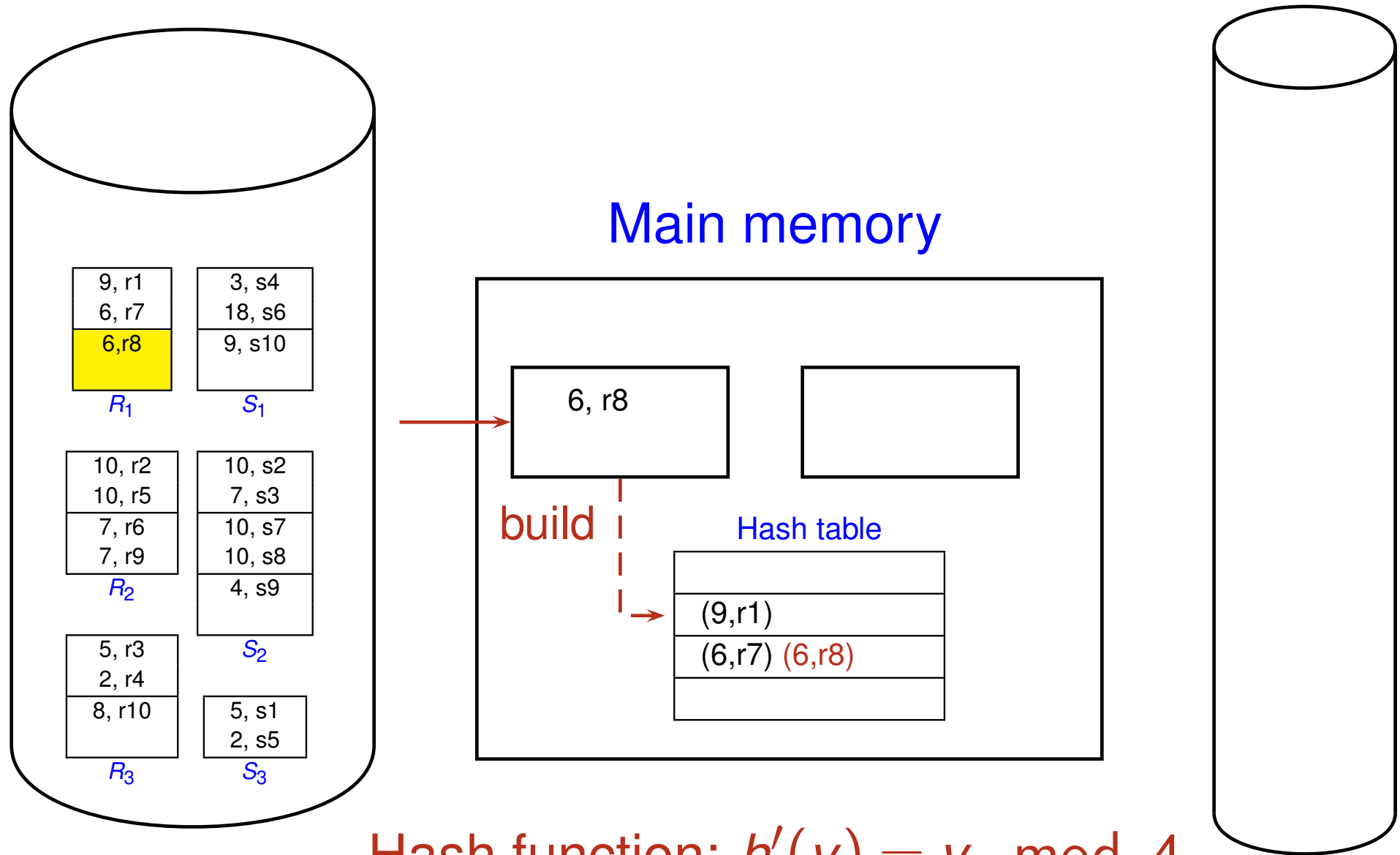


# Grace Hash Join: Probing Phase

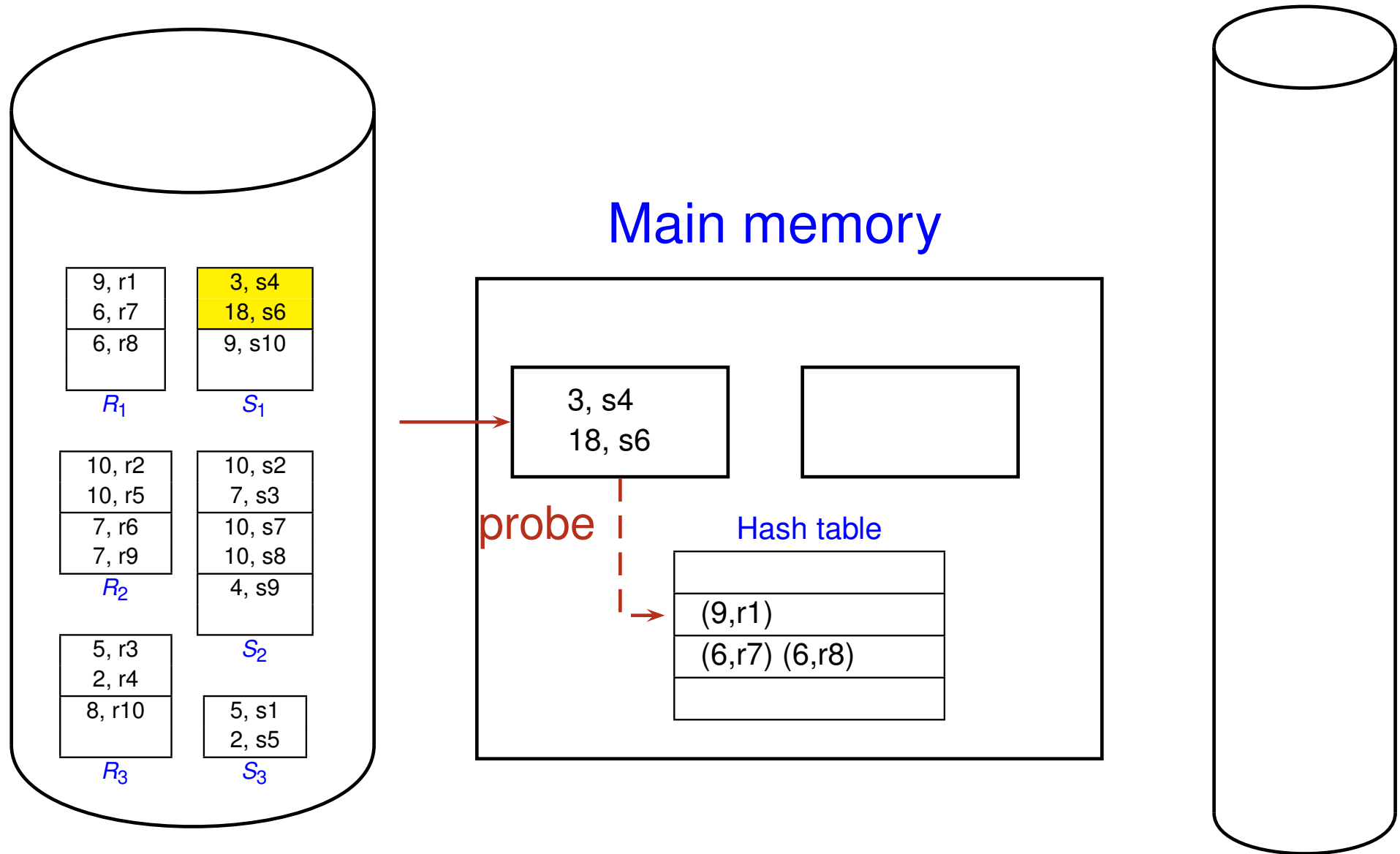




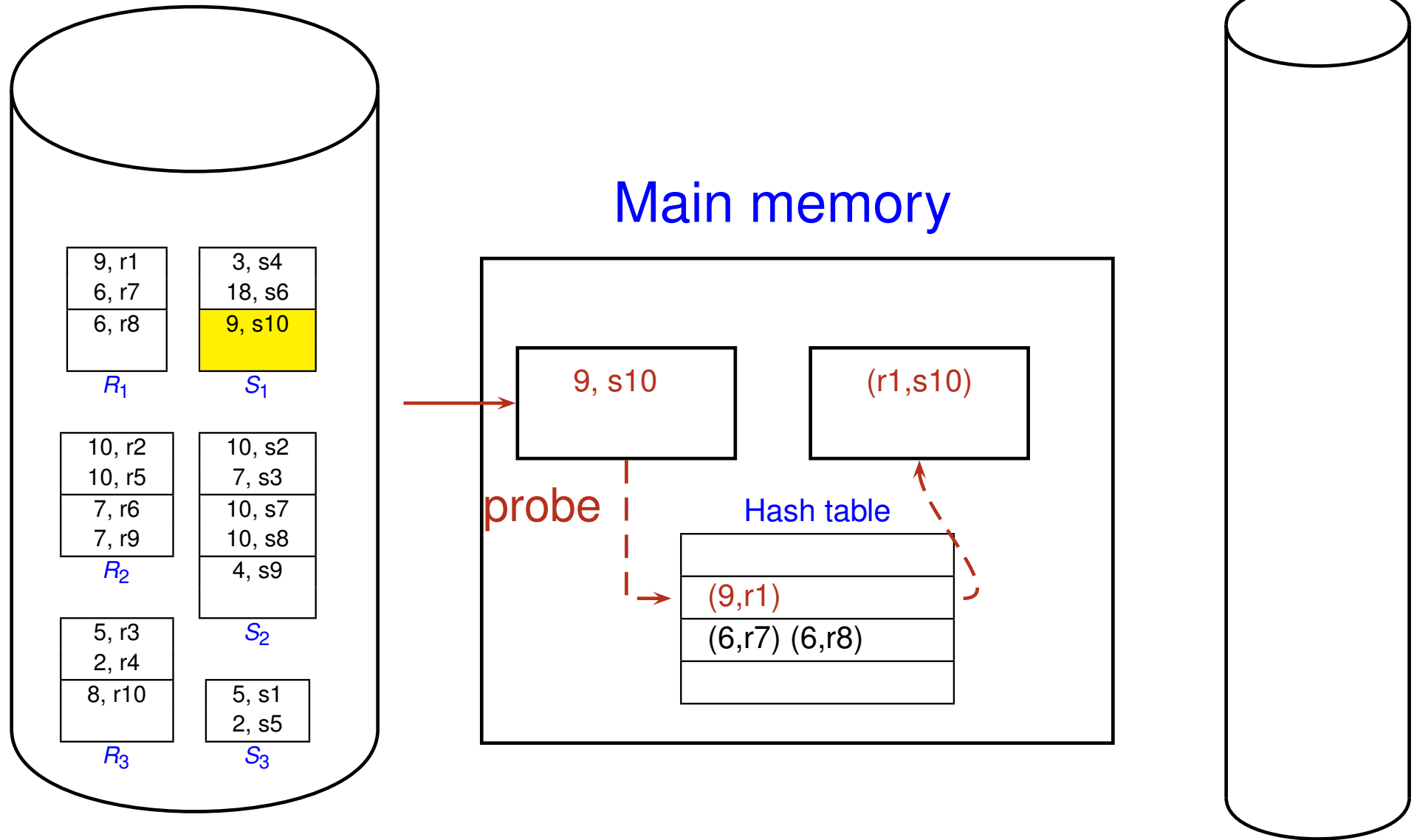
# Grace Hash Join: Probing Phase



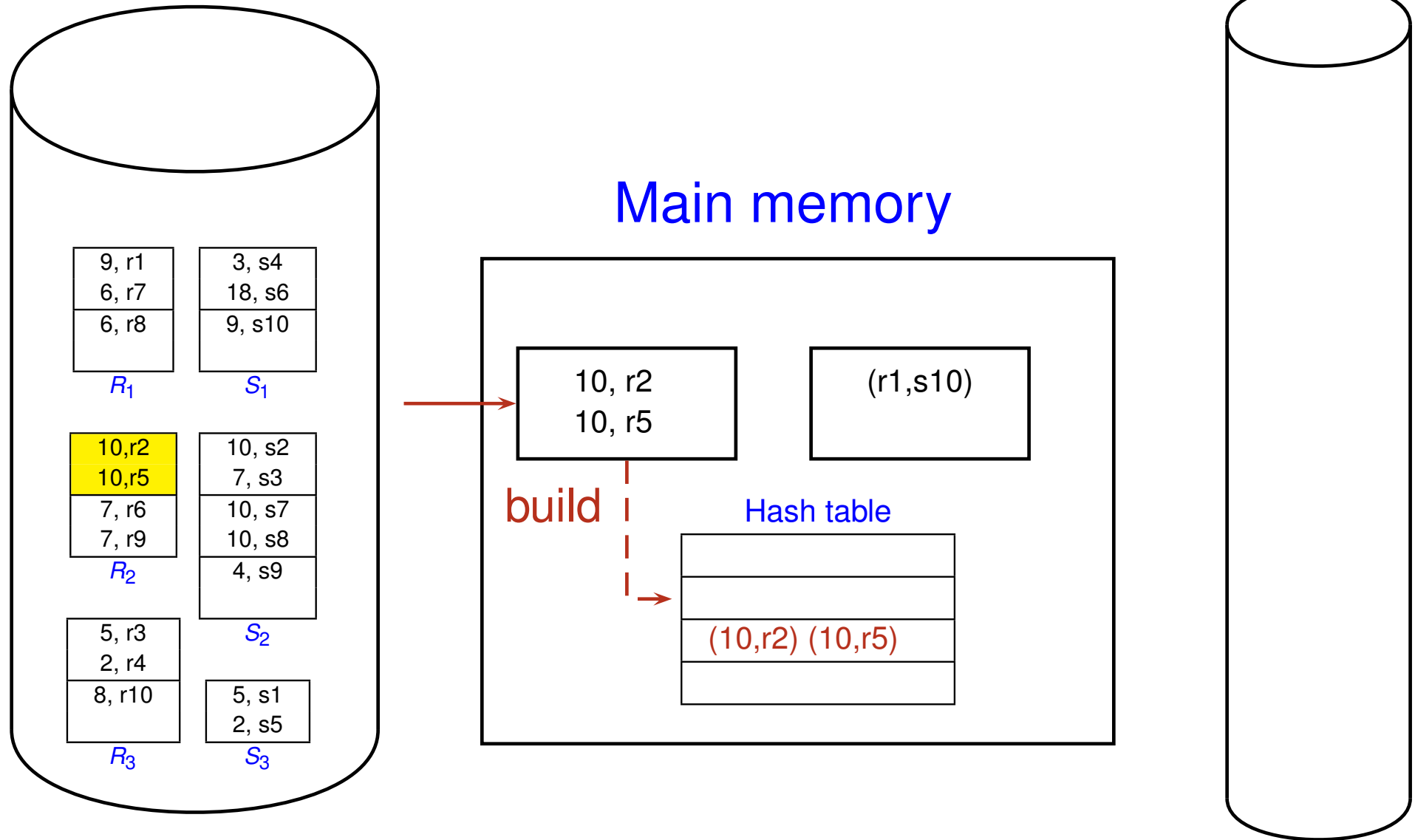
# Grace Hash Join: Probing Phase



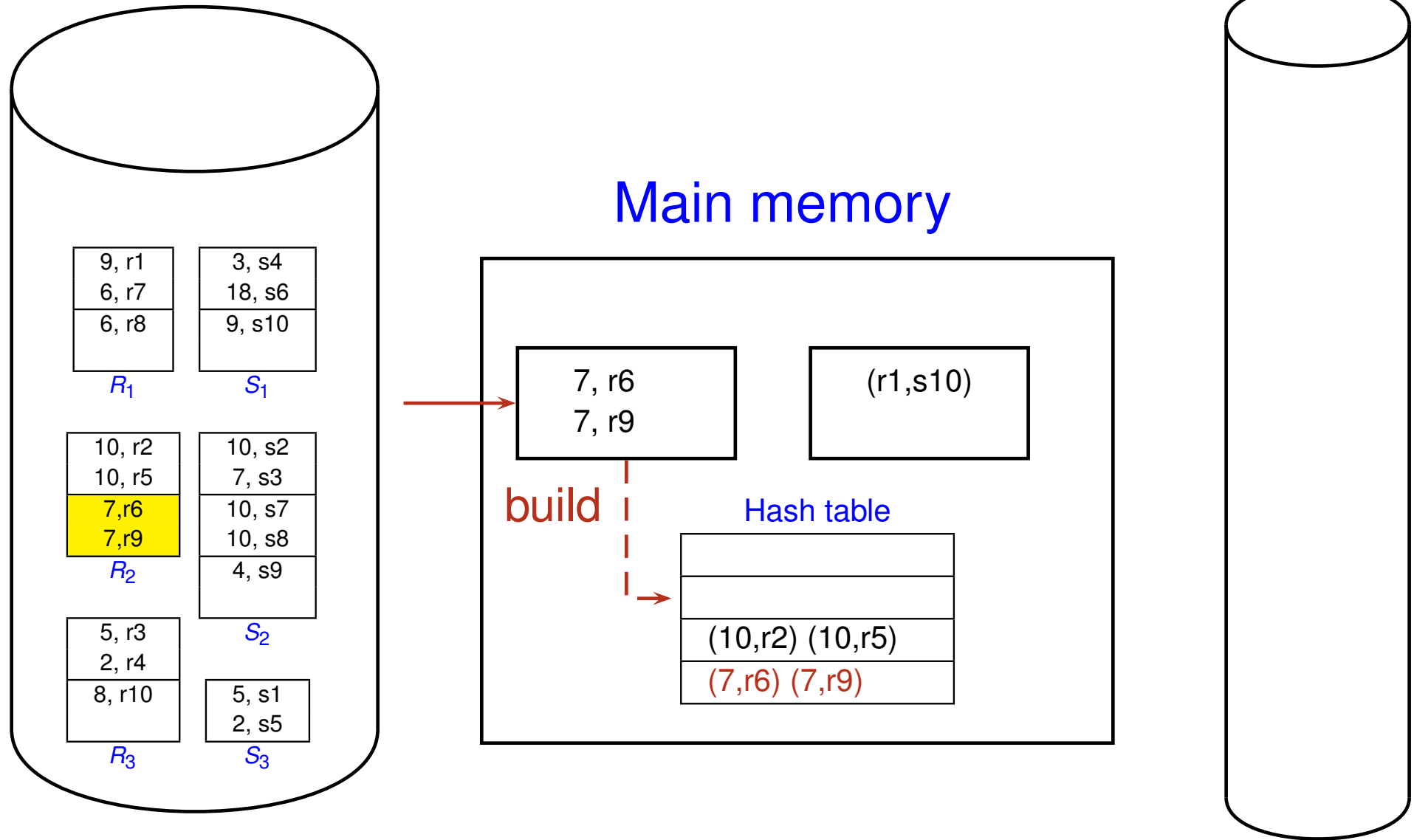
# Grace Hash Join: Probing Phase



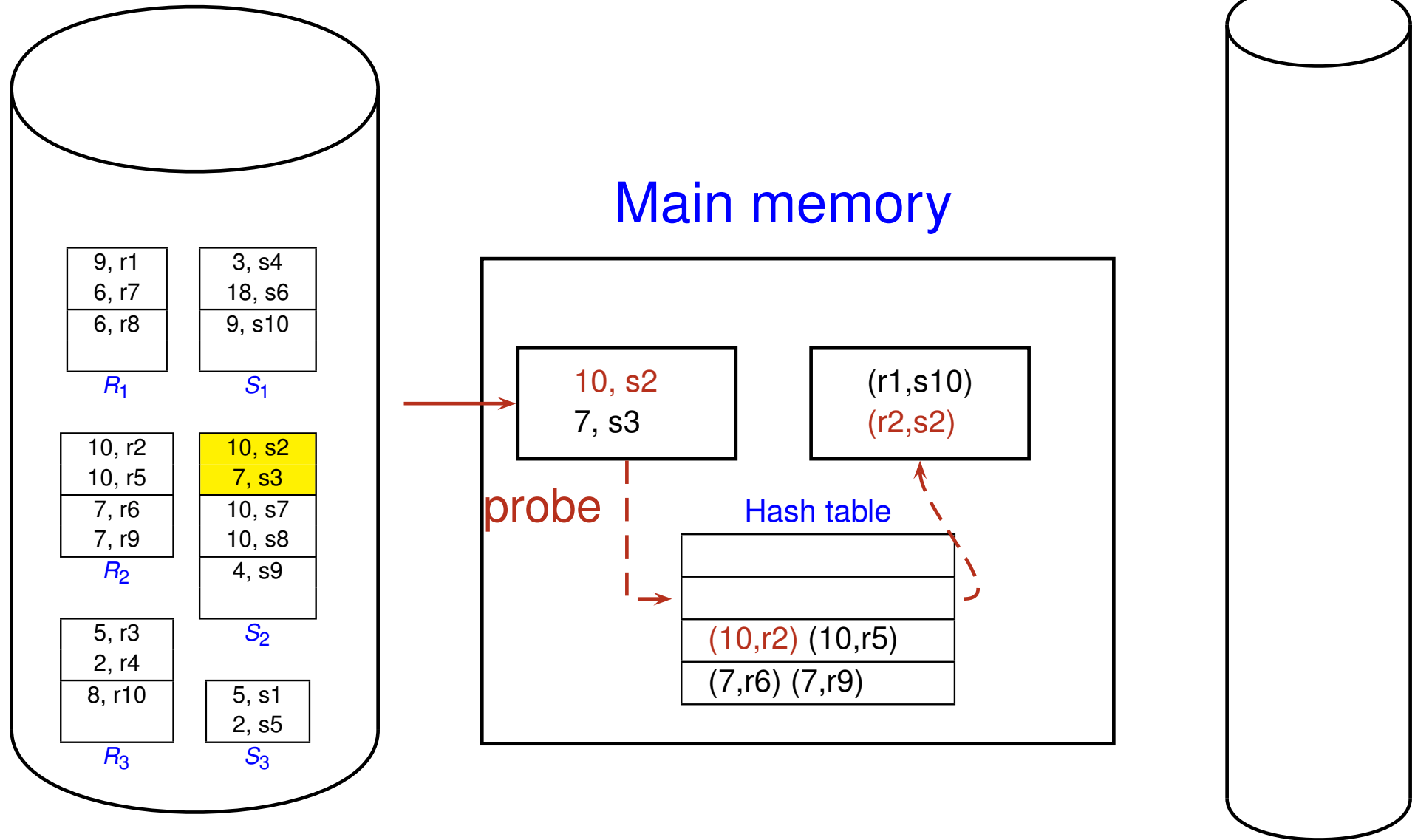
# Grace Hash Join: Probing Phase



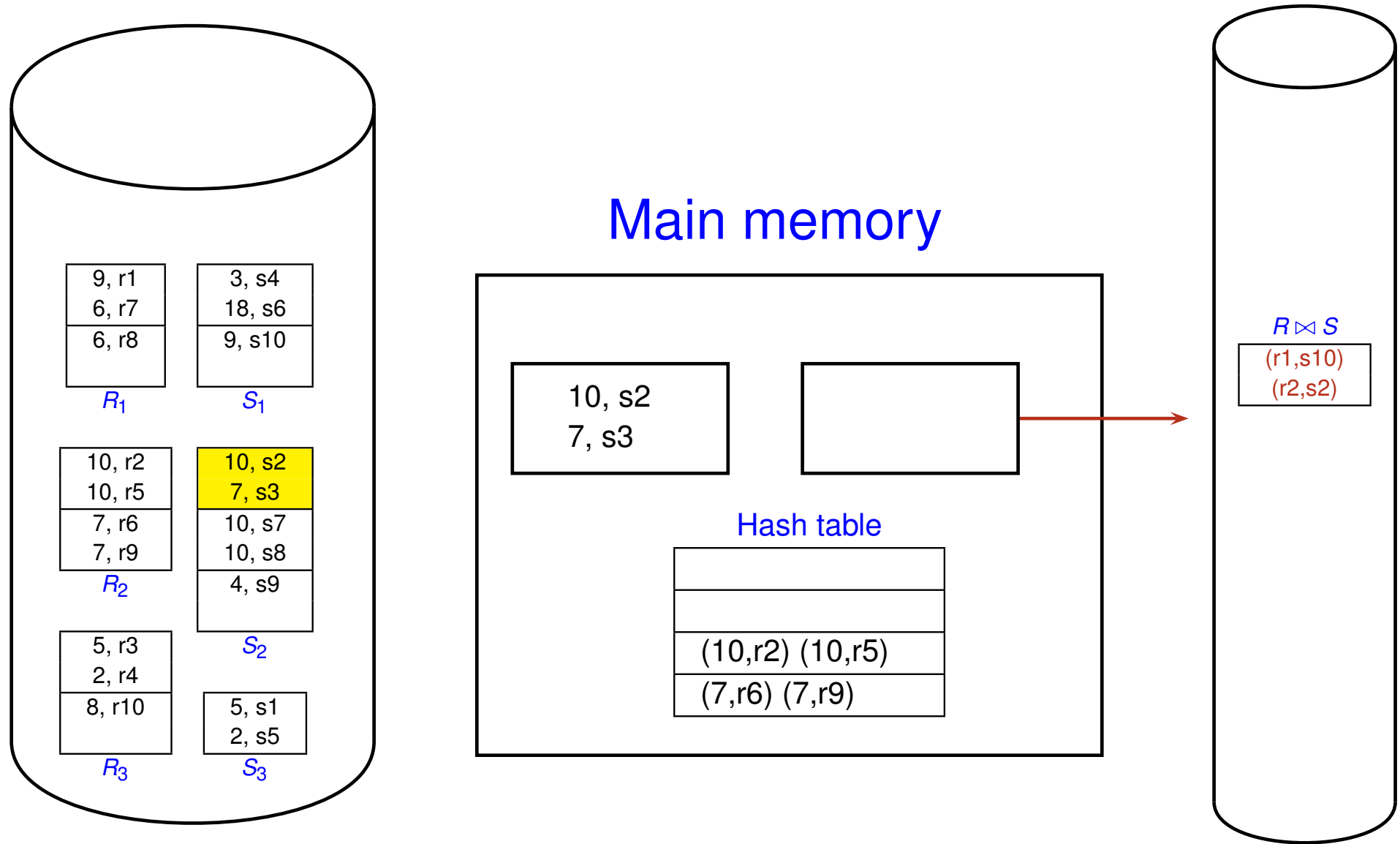
# Grace Hash Join: Probing Phase



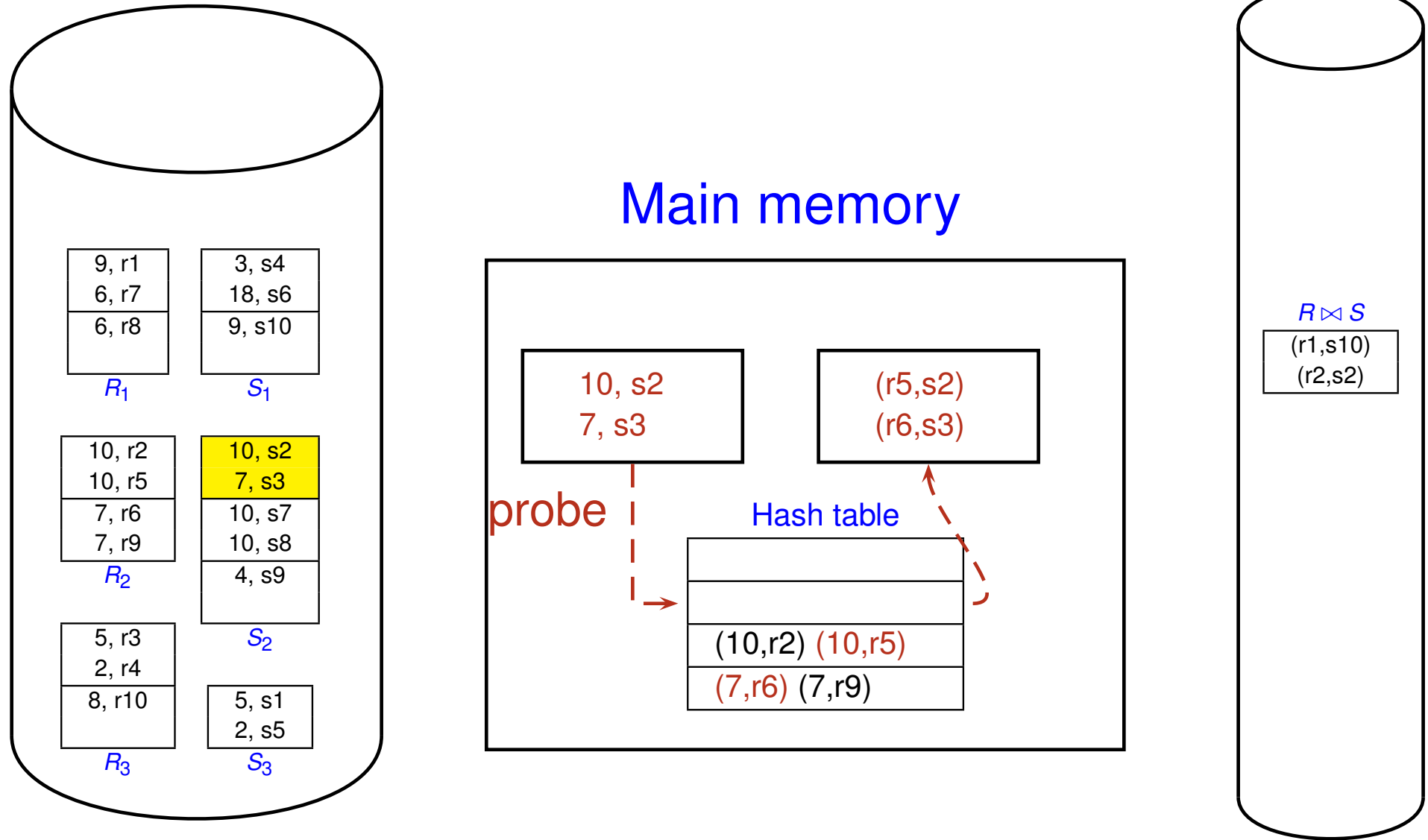
# Grace Hash Join: Probing Phase



# Grace Hash Join: Probing Phase

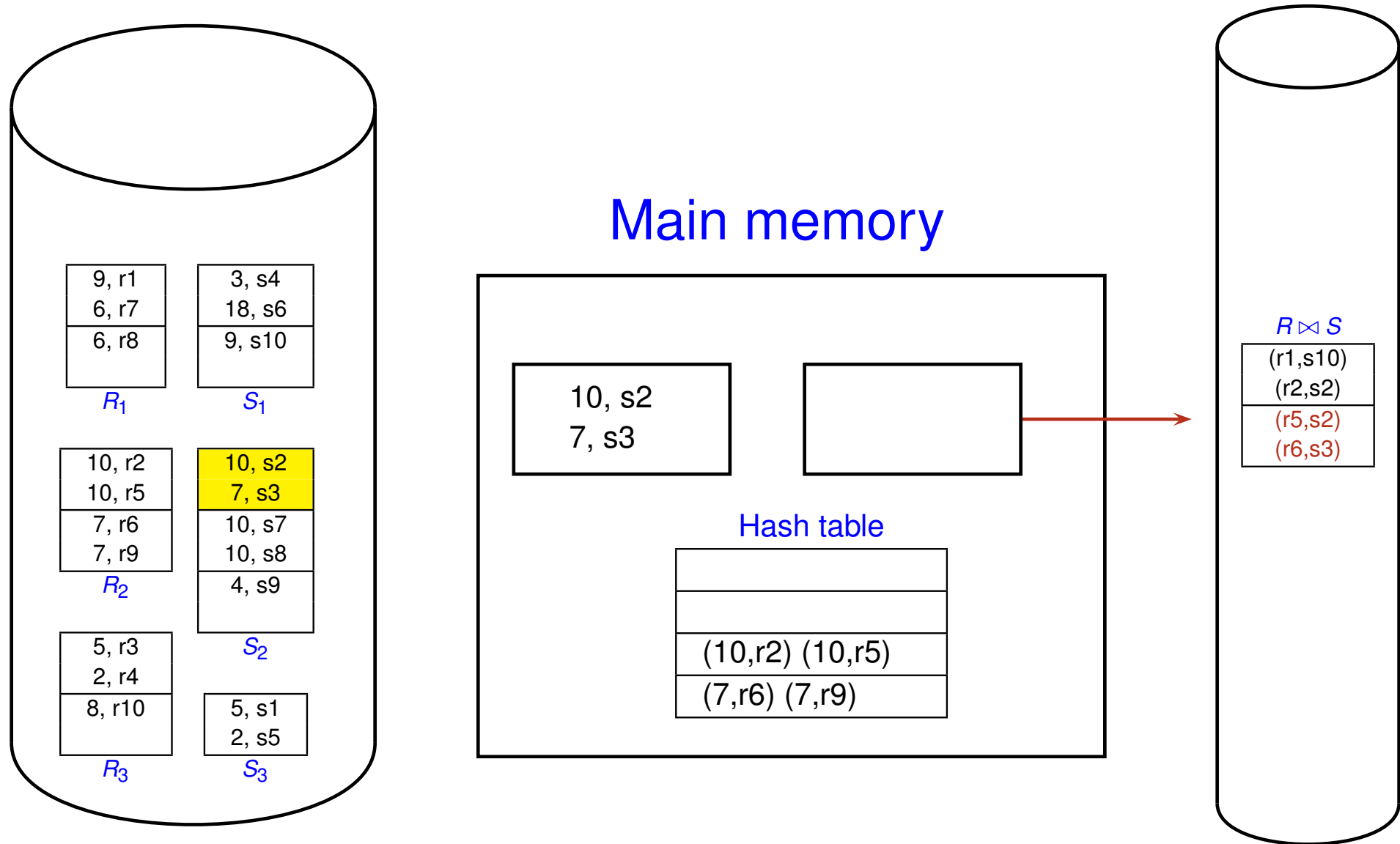


# Grace Hash Join: Probing Phase

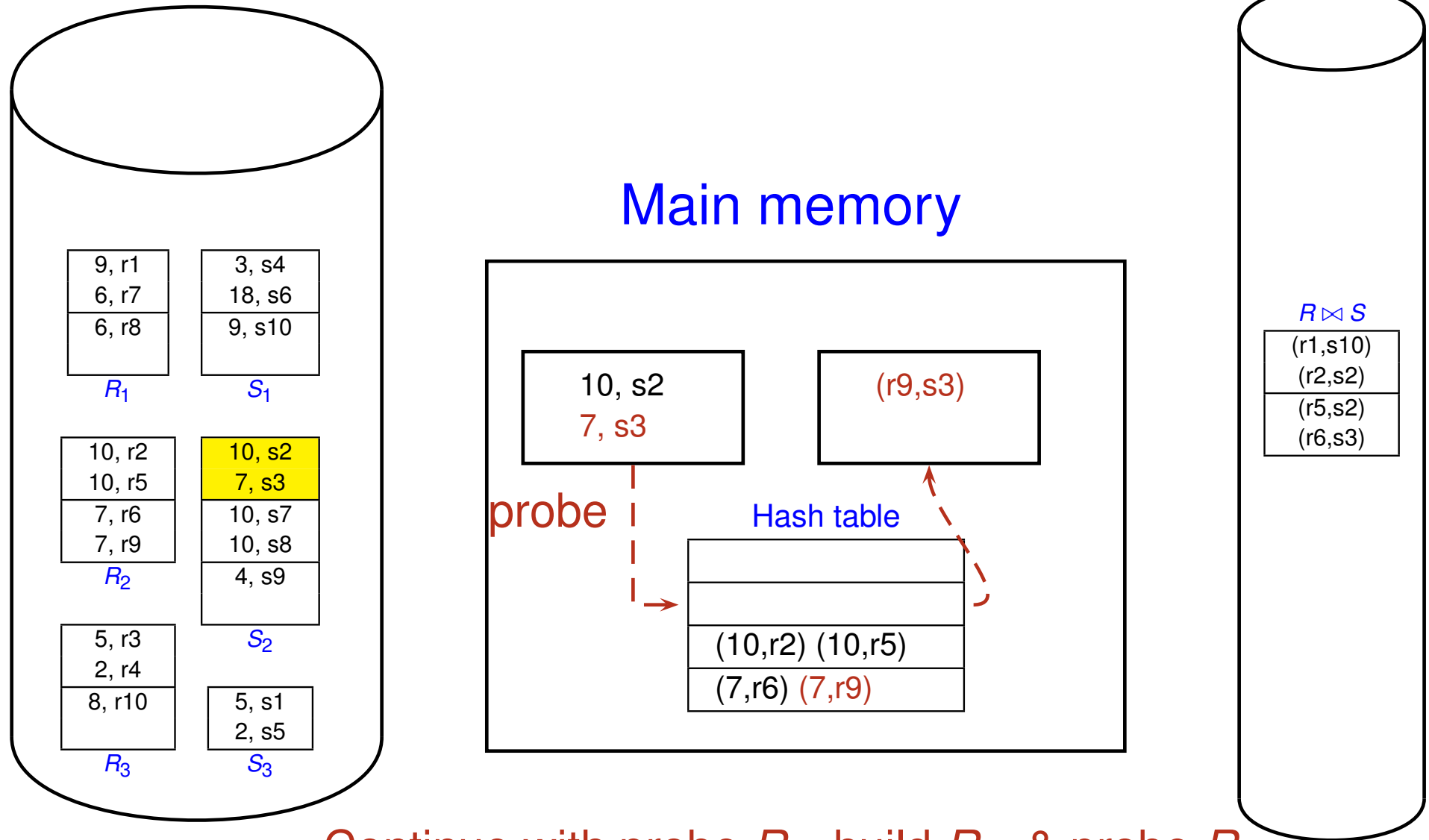




# Grace Hash Join: Probing Phase



# Grace Hash Join: Probing Phase



Continue with probe  $R_2$ , build  $R_3$ , & probe  $R_3$

# Grace Hash Join: Analysis

- ▶ To minimize size of each partition of  $R_i$ ,
  - ▶ Set  $k = B - 1$  given  $B$  buffer pages
- ▶ Assuming uniform hashing distribution,
  - ▶ size of each partition  $R_i$  is  $\frac{|R|}{B-1}$
  - ▶ size of hash table for  $R_i$  is  $\frac{f \times |R|}{B-1}$ , where  $f$  is a fudge factor
  - ▶ During probing phase,  $B > \frac{f \times |R|}{B-1} + 2$   
(with one input buffer for  $S_i$  & one output buffer)
  - ▶ Approximately,  $B > \sqrt{f \times |R|}$
- ▶ **Partition overflow problem**
  - ▶ Hash table for  $R_i$  does not fit in memory
  - ▶ Solution: recursively apply partitioning to overflow partitions
- ▶ I/O cost = Cost of partitioning phases + Cost of probing phase
  - ▶ I/O cost =  $3(|R| + |S|)$  if there's no partition overflow problem

# General Join Conditions

## ► Multiple equality-join conditions

- Example:  $(R.A = S.A) \text{ and } (R.B = S.B)$
- Algorithms:
  - ★ **Index Nested Loop Join**: use index on all or some of join attributes
  - ★ **Sort-Merge Join**: need to sort on combination of attributes
  - ★ Other algorithms essentially unchanged

## ► Inequality-join conditions

- Example:  $(R.A < S.A)$
- Algorithms:
  - ★ **Index Nested Loop Join**: requires a  $B^+$ -tree index
  - ★ **Sort-Merge Join**: not applicable
  - ★ **Hash-based Joins**: not applicable
  - ★ Other algorithms essentially unchanged