CS3230 Finals Q4 A0214561M

4. a) fabulous $\sum_{i \in S} a_i \geq \frac{A}{2}$ $\sum_{i \in S} b_i \geq \frac{B}{2}$ $S \subseteq \{1, 2, .. n\}$

FABULOUS HALF $\Rightarrow$ fabulous set of size $\frac{n}{2}$

$\updownarrow$

(1) FAB HALF is NP

A certificate for a YES instance is a fabulous set of size $\frac{n}{2}$

A verifier can check in polynomial time that sum of $\frac{n}{2}$ front sides $\geq \frac{A}{2}$

and sum of $\frac{n}{2}$ back sides $\geq \frac{B}{2}$

(2) FABHALF is NP Hard

PARTITION EQUAL $\leq_p$ FABHALF

Reduction: Given $x_1 .. x_n$ as input to PARTEQUAL

let $a_i = x_i$ and $b_i = \frac{2S}{n} - a_i$ be front and back of each sheet

with $S = \sum_{i=1} x_i = \sum_{i=1}^{n} a_i$ $B = \sum_{i=1}^{n} b_i = 2S - S = S$

reduction in polynomial time

YES instance of PARTEQUAL $\Rightarrow$ subset T of $\frac{n}{2}$ integers which sum to $\frac{S}{2}$

corresponding $\sum_{i \in T} a_i = \frac{S}{2} \Rightarrow \sum_{i \in T} a_i \geq \frac{A}{2}$ $A = S$

$\sum_{i \in T} b_i = \frac{2S}{n} \cdot \frac{n}{2} + \sum_{i \in T} x_i = S - \frac{S}{2} = \frac{S}{2}$

$\Rightarrow \sum b_i \geq \frac{B}{2}$ $\therefore$ YES instance of FABHALF

YES instance of FABHALF $\Rightarrow$ $\sum_{i \in T} a_i \geq \frac{A}{2} = \frac{S}{2}$

since $b_i = \frac{2S}{n} - x_i$ $\sum_{i \in T} b_i = S - \sum_{i \in T} a_i$

$\leq \frac{S}{2} \leq \frac{B}{2}$

since $\sum b_i \geq \frac{B}{2} \Rightarrow \sum a_i = \sum b_i = \frac{S}{2}$

partition x_i using a_i and rest not including a_i $\Rightarrow$ YES instance of PARTEQUAL

b) knapsack with $W = \frac{n}{2}$ $V = \frac{S}{2}$ weights = 1

$O(nAB)$

c)

Algorithm in part b is not polynomial in size of input

pseudo polynomial

hence FAIRHALF cannot be solved in polynomial time

using pseudopolynomal time solution for p is

have not prove P = NP