CS3230: Assignment for Week 1

Due: Sunday, 23rd Jan 2022, 11:59 pm SGT.

Please upload PDFs containing your solutions (hand-written & scanned, or typed) by 23rd Jan, 11:59 pm to `Assignments/Assignment1/Submissions`. Name the file **Assignment1_SID.pdf**, where SID should be replaced by your student ID.

You may discuss the problems with your classmates or read material online, but you should write up your solutions on your own. Please note the names of your collaborators or online sources in your submission; failure to do so would be considered plagiarism.

1. (7 points, graded for correctness) Determine, with proof, the minimum number k with the following property:

   There exists a comparison-based algorithm that can decide using at most k comparisons whether any given 10-element array of integers contains only equal numbers.

   (Note: With each comparison, the algorithm can find out whether two chosen elements x and y satisfy $x < y$, $x > y$, or $x = y$.)

2. (1 point) Prove that there exists a comparison-based sorting algorithm that can sort any 4-element array of distinct integers using at most 5 comparisons.

3. (1 point) For Christmas, your friend gave you 16 gold coins, but told you that exactly 2 of them are fake. You know that all real coins have equal weight, all fake coins have equal weight, and the weight of a fake coin is less than the weight of a real coin. You have a weighing scale where you can put some coins on each side and find out which of the two sides is heavier (or if the two sides are equally heavy). You do not have any other information (e.g., which coins are fake, or the exact weight of a real/fake coin), and cannot weigh the coins with your own hands.

   Your task is to divide the 16 coins into two subsets of 8 coins each, so that each subset contains exactly 1 fake coin. Prove that you can accomplish this task using at most 3 weighings.