CS3230 Midterm Q4

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4. a) 15 players ⇒ total # matches = $\binom{15}{2} = 105$

Using a knockout tournament strategy of asking (P_i beat P_j)
since every match has a winner, the person who lost would not have
been able to beat 14 other players.

WLOG, let person with smaller number win

1st round (7 queries): $P_1 > P_2$ $P_3 > P_4$ $P_5 > P_6$ $P_7 > P_8$ $P_9 > P_{10}$ $P_{11} > P_{12}$ $P_{13} > P_{14}$ P_{15}

2nd round (4 queries): $P_1 > P_3$ $P_5 > P_7$ $P_9 > P_{11}$ $P_{13} > P_{15}$

3rd round (2 queries): $P_1 > P_5$ $P_9 > P_{13}$

4th round (1 query): $P_1 > P_9$

At the end of 4 rounds = 14 queries made to rule out 14 players.

check winner with all other 10 players not faced in knockout to
determine if exists a player who beat all other 14 players

Query ($P_1 > P_4$, $P_1 > P_6$ ... 7, 8, 10, 11, 12, 13, 14, 15)

total <u>24 queries</u> in the worst case. //

b) set of players who won highest number of matches

105 matches played means 105 total wins

evasive problem....

Idea is to let every player win 7 games...

Suppose M is an algorithm that correctly determines players with highest # wins using < 105 queries.
There must be a match eg P_1 vs P_2 which was not queried.
Let A be outcome where every player won 7 games and P_1 beat P_2 $\Rightarrow$ set includes all players
A' be outcome where P_2 beat P_1 while all other matches remain the same, hence P_2 would now have 8 wins and P_1 have 6 wins $\Rightarrow$ set only has P_2

since M cannot differentiate A and A' but both give different output, it must err on one of them.

$\therefore$ smallest # queries is $\underline{105}$ //