Design and Analysis of Algorithms

National University of Singapore

Algorithms
CS3230

Tutorial

Week 3

Question 1

- Given an unsorted array of n real numbers A[1..n] and a query number x. You need to develop a function search(x, A) which returns an integer i if A[i]=x; and returns -1 otherwise.
- We have two assumptions:
  - Assume comparison model
  - Assume each comparison returns $<$, or $>$, or $=$ between x and an element of A.
- What is the lower bound of the number of comparisons?

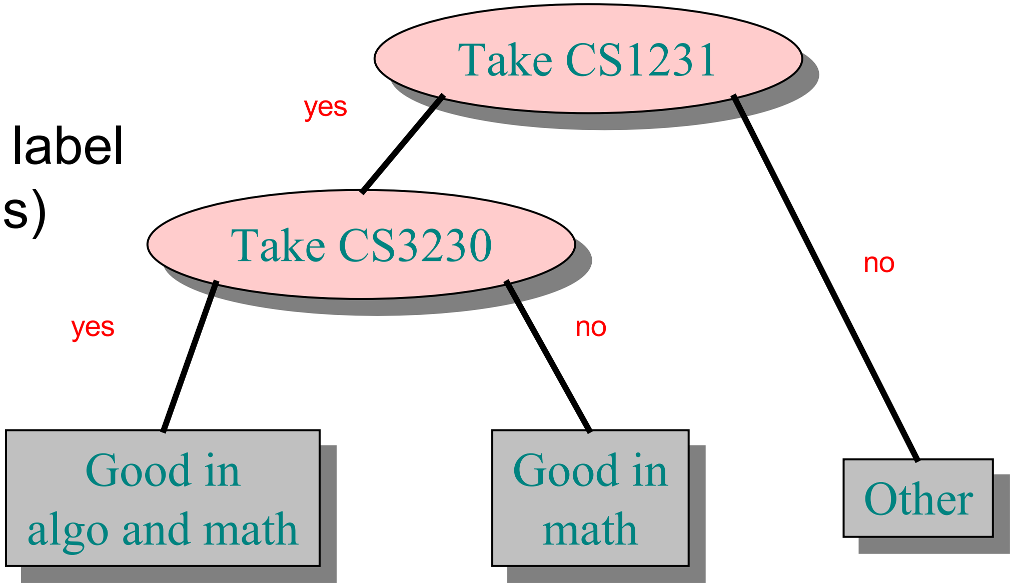
A. n B. $\lfloor \lg n \rfloor + 1$ C. $n \lfloor \lg n \rfloor$

What is a decision tree?

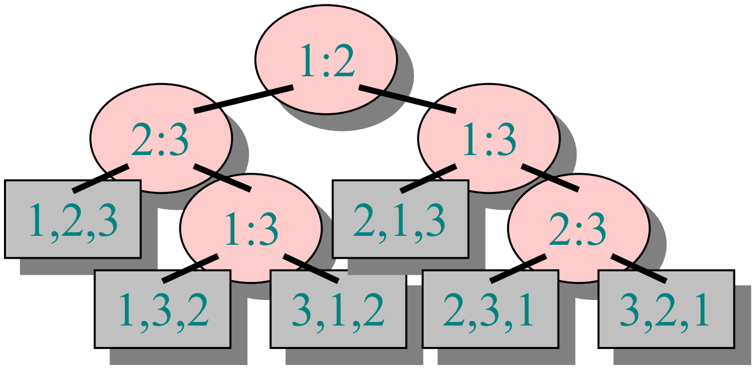
```
if (take CS1231) then
if  (take CS3230) then
              return "Good in Algo & Math"
else
              return "Good in Math"
else
 return "other"
```

- A **decision tree** is a tree-like model.
  - Every node is a comparison.
  - Every branch represents the outcome of the comparison
  - Every leaf represents a class label (decision after all comparisons)

- We can use decision tree to model any comparison-based algorithm.



Decision-tree model



A ***decision tree can model the execution of any comparison sort****:*

- One tree for each input size *n*.
- View the algorithm as splitting whenever it compares two elements.
- The tree contains the comparisons along all possible instruction traces.
- The running time of the algorithm = the length of the path taken.
- Worst-case running time = height of tree.

Lower bound for decision-tree sorting

- **Theorem:** Any decision tree that can sort n elements must have height $\Omega(n \lg n)$.
- *Proof:* The tree must contain $\geq n!$ leaves, since there are $n!$ possible permutations. A height-h binary tree has $\leq 2^h$ leaves. Thus, $n! \leq 2^h$.

$$
\begin{aligned}
\therefore \quad h &\geq \lg(n!) && (\lg \text{ is monotonically increasing}) \\
&\geq \lg((n/e)^n) && (\text{Stirling's formula}) \\
&= n \lg n - n \lg e \\
&= \Omega(n \lg n). \quad \square
\end{aligned}
$$

Question 2

- Given a sorted array of n real numbers A[1..n] and a query number x. You need to develop a function search(x, A) which returns an integer i if A[i]=x; and returns -1 otherwise.
- We have two assumptions:
  - Assume comparison model
  - Assume each comparison returns $<$, or $>$, or $=$.
- What is the lower bound of the number of comparisons?

A. n B. $\lfloor \lg n \rfloor + 1$ C. $n \lfloor \lg n \rfloor$

Question 3

Which of the following is true?

A. $f(n) = o(g(n))$
B. $f(n) = \Theta(g(n))$
C. $f(n) = \omega(g(n))$

when $f(n) = \ln(n)$ and $g(n) = \log_{10}(n)$.

Question 4

Which of the following is true?

A. $f(n) = o(g(n))$
B. $f(n) = \Theta(g(n))$
C. $f(n) = \omega(g(n))$

when $f(n) = n^{2.5}$ and $g(n) = n^2 \log^4 n$.

Question 5

Which of the following is true?

A. $f(n) = o(g(n))$
B. $f(n) = \Theta(g(n))$
C. $f(n) = \omega(g(n))$

when $f(n) = 3^n$ and $g(n) = 2^n$.

Question 6 (If time allows)

- Ali has 81 coconuts, all of which have the same weight, except for one which is heavier. He does not know which is the heavier coconut. Ali's friend has a balance scale, but will charge Ali one dollar for each use of the scale.

- What is the maximum amount of money that Ali has to pay to guarantee that he can find the heaviest coconut, assuming that Ali uses an optimal algorithm?

A. 3 B. 4 C. 5 D. 6