CS3230 Assignment 11

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1. a) The problem 3-SAT can be solved in polynomial time

By the Cook-Levin Theorem, any problem in NP poly-time reduces to 3-SAT. Hence, 3-SAT is NP-hard and NP-complete

could be true

3-SAT could be solvable in polynomial time if P=NP or there exists a poly-time algorithm for any of the NP complete problems

b) The problem INDEPENDENT SET cannot be solved in polynomial time

Since there is a polynomial-time reduction from 3-SAT to INDEPENDENT SET, INDEPENDENT SET is also NP-Complete

could be true

The problem INDEPENDENT SET cannot be solved in polynomial time if P≠NP or if any of the NP complete problems are proven to be hard

c) Some NP-complete problems can be solved in polynomial time while others cannot

A poly-time algorithm for any NP complete problem implies poly-time algorithm for all others.

False

NP complete problems are NP hard, which are as hard as every problem in NP. Thus, it is not possible for some NP-complete problems to be less hard than others.

2.

Prove that 2022-LABEL is NP complete

① show 2022-LABEL is in NP

Proof: Let the certificate be the undirected graph G with the vertices labelled

For each vertex, verifier checks if every adjacent vertex has a different label in $O(V+E)$ time

② show 2022-LABEL is NP hard assuming 2021-LABEL is NP-complete

2021-LABEL $\leq_p$ 2022-LABEL

Proof:

Reduction: Given an undirected graph $G(V,E)$ as an instance of 2021-LABEL generate an instance $G'(V',E')$ of 2022-LABEL by adding a new vertex which has an edge to all the vertices in G

reduction can be done in $O(V)$ time (polynomial in input $G(V,E)$)

Suppose G is a YES-instance of 2021-LABEL. Using the same labels in G for the corresponding vertices in G' and labelling the new vertex 2022 which does not conflict with $\{1..2021\}$, G' is also a YES instance for 2022-LABEL

Suppose G' is a YES-instance of 2022-LABEL. Every vertex in G must have a different label than the new vertex with label k since they are all adjacent. Since there are only 2021 possible labels in $\{1..2022\} \setminus \{k\}$, they can be mapped to $\{1..2021\}$ such that G is a YES instance for 2021-LABEL

∴ 2022-LABEL is at least as hard as 2021-LABEL

Since 2022-LABEL is NP and NP-hard, it is NP-Complete

3.

Prove that 3-SATTWICE is NP-complete

① show 3-SATTWICE is in NP

Proof: let the certificate be the 2 assignments of input attributes / literals, verification it both assignments are satisfying can be done in polynomial time

② show 3-SATTWICE is NP hard using 3-SAT which is NP-complete

3-SAT $\leq_P$ 3-SATTWICE

Reduction: Given a 3-CNF function as an instance of 3-SAT ϕ

add the clause $(x \vee x \vee \bar{x})$ to ϕ to form

$\phi' = \phi \wedge (x \vee x \vee \bar{x})$ as instance of 3-SATTWICE

where x does not appear in ϕ

reduction can be done in polynomial time

Suppose ϕ is a YES instance for 3-SAT. Using the same set of assignment of literals in ϕ, we can choose x = False and x = True as 2 satisfying assignments of 3-SATTWICE

ϕ' is a YES instance for 3-SATTWICE

Suppose ϕ' is a YES instance for 3-SATTWICE. There must exist a satisfying assignment such that all the clauses are true. assignment without literal x is still a satisfying assignment for ϕ. ϕ is a YES instance for 3-SAT

since 3-SATTWICE is NP and NP-hard, it is NP-complete