

# CS3230: Design and Analysis of Algorithms

## Semester 1, 2021-22, School of Computing, NUS

### Practice Problem Set: Reductions

April 13, 2022

## Instructions

- This problem set is **completely optional**. There is no need to submit the solutions.
- Post on the LumiNUS forums if you will face any problem while solving the questions.

**Question 1:** A 2-CNF formula is a CNF formula where each clause consists of at most 2 literals. The MAX-2-SAT problem is: given a 2-CNF formula  $\phi$  and a number  $k$ , is there an assignment that satisfies at least  $k$  clauses of  $\phi$ ? Show that MAX-2-SAT is NP-complete. [**Hint:** Try a reduction from 3-SAT. For each 3-CNF clause involving 3 variables, give a set of ten 2-CNF clauses on 4 variables such that if the 3-CNF clause is satisfiable, then 7 of the 2-CNF clauses can be satisfied whereas otherwise, only at most 6 of the 2-CNF clauses can.]

**Question 2:** Consider the Max-Clique problem: Given an undirected graph  $G = (V, E)$  and an integer  $k$  decide whether there exists a clique of size at least  $k$  in  $G$  (i.e., as a subgraph of  $G$ ). Show that Max-Clique problem is NP-complete. [**Hints:** Try a reduction from the Maximum Independent Set problem.]. Note that this sem (AY21/22 S2), this is a problem in tutorial. Try and derive it again yourself!

**Question 3:** The circuit satisfiability problem is: Given a circuit with boolean gates (AND, OR, and NOT) with several input wires and one output wire, decide if there is an input that makes the output be true.

Give a poly-time reduction from circuit satisfiability to CNF-SAT. [**Hint:** For each gate  $k$  introduce a new variable  $x_k$ . Then create a sub-formula for each gate. e.g. Suppose for  $k$ -th gate the input wires are from (output wire of)  $i$  and  $j$ -th gate. Then the sub-formula will be  $(\overline{x_k} \vee x_i) \wedge (\overline{x_k} \vee x_j) \wedge (x_k \vee \overline{x_i} \vee \overline{x_j})$ . Similarly define sub-formulas for OR and NOT gates.]

**Question 4:** Give a poly-time reduction from CNF-SAT to 3-SAT. [**Hint:** For the clauses with more than three literals try to write them as AND of several clauses by introducing new variables.]