Analysis and Design of Algorithms

National University of Singapore

Algorithms

CS3230

Tutorial

Week 6

Question 1

A **pairwise independent** family $\mathcal{H}$ of hash functions mapping $\mathcal{U}$ to $\{1, \dots, M\}$ has the property that for any two distinct universe elements x, y and for any two hash values i_1, i_2:

$$\Pr_{h \sim \mathcal{H}}[h(x) = i_1, h(y) = i_2] \le \frac{1}{M^2}.$$

Is a pairwise independent family always universal?

Question 2

A **pairwise independent** family $\mathcal{H}$ of hash functions mapping $\mathcal{U}$ to $\{1, \dots, M\}$ has the property that for any two distinct universe elements x, y and for any two hash values i_1, i_2:

$$\Pr_{h \sim \mathcal{H}}[h(x) = i_1, h(y) = i_2] \leq \frac{1}{M^2}.$$

Is a universal family always pairwise independent?

Question 3

A **pairwise independent** family $\mathcal{H}$ of hash functions mapping $\mathcal{U}$ to $\{1, \ldots, M\}$ has the property that for any two distinct universe elements x, y and for any two hash values i_1, i_2:

$$\Pr_{h \sim \mathcal{H}}[h(x) = i_1, h(y) = i_2] \leq \frac{1}{M^2}.$$

Suppose you hash N distinct elements using h randomly drawn from a pairwise independent family. The expected number of elements which hash to slot j is at most?

(A) N/M^2
(B) N/M
(C) $N/2M$
(D) $N/4M$

Question 4

The same upper bound as the previous question holds for a hash function drawn from a universal (instead of pairwise independent) family.

True or False?

Question 5

Let G be an undirected graph with n nodes and m edges. Partition the graph into two parts A and B randomly as follows. For each node v, toss an independent fair coin. If heads, put v in part A. Else if tails, put v in part B.

What is the expected number of edges which cross the cut (meaning, one endpoint in A & other in B)?