CS3230: Assignment for Week 3

Due: Sunday, 6th Feb 2022, 11:59 pm SGT.

Please upload PDFs containing your solutions (hand-written & scanned, or typed) by 6th Feb, 11:59 pm to `Assignments/Assignment3/Submissions`. Name the file **Assignment3_SID.pdf**, where SID should be replaced by your student ID.

You may discuss the problems with your classmates or read material online, but you should write up your solutions on your own. Please note the names of your collaborators or online sources in your submission; failure to do so would be considered plagiarism.

1. (7 points, graded for correctness) Solve the following recurrences by providing tight asymptotic bounds and justify your answers. You can ignore the fact that numbers like $n/2$, $n/4$, etc. may not be integers.

   (a) (2 points)
   $$T(n) = 3T(n/2) + n^2$$

   (b) (2 points)
   $$T(n) = T(n/4) + T(3n/4) + 2n$$

   (c) (3 points)
   $$T(n) = T(n/2) + T(\sqrt{n}) + n$$

   (Hint for (c): To get some intuition for what the answer could be, note that $T(n) \geq n$ and $T(n) \leq 2T(n/2) + n$.)

2. (1 point) Let $n = 2^k$ for some positive integer k. Determine, with proof, the maximum and the minimum number of comparisons that the Merge Sort algorithm makes when sorting an array of n distinct numbers.

3. (1 point) You are given an array of n distinct numbers. Design and analyze an algorithm that finds a number that is less than all numbers adjacent to it using $O(\lg n)$ comparisons.

   (If there is more than one such number, the algorithm may return any of them. You should analyze both the correctness and the number of comparisons made by the algorithm.)