

3. n friends, n documents

cost of having friend i typing document j is $\frac{w_i}{r_i}$ $\frac{\text{words}}{\text{word/min}}$

let D be a subset of $\{1, 2, \dots, n\}$ representing documents completed

let f be friends involved $\{1, \dots, f\}$ $D.\text{size} = f$

let $dp[D, f]$ be minimum cost of documents in D completed by friends $\{1, \dots, f\}$

Base case: $f = 1$

$$dp[\{d\}, 1] = \frac{w_d}{r_1} \quad \text{for all } d \text{ from } 1 \text{ to } n$$

General case:

$$dp[D, f] = \min \left\{ \begin{aligned} &\frac{w_{d_1}}{r_f} + dp[D \setminus \{d_1\}, f-1], \\ &\frac{w_{d_2}}{r_f} + dp[D \setminus \{d_2\}, f-1], \\ &\vdots \\ &\frac{w_{d_f}}{r_f} + dp[D \setminus \{d_f\}, f-1] \end{aligned} \right\} \quad O(f)$$

for all subset D of $\{1, 2, \dots, n\}$ of size f $\binom{n}{f}$

for d from 1 to n

$$dp[\{d\}, 1] = \frac{w_d}{r_1}$$

for f from 2 to n

for all $D \subseteq \{1, \dots, n\}$, $|D| = f$

$$dp[D, f] = \min_{d_i \in D} \left(\frac{w_{d_i}}{r_f} + dp[D \setminus \{d_i\}, f-1] \right)$$

return $dp[D, n]$

$$\# \text{ subproblems} = \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n} = 2^n - 1$$

$$\text{time / subproblem} = O(n)$$

$$\text{total time} = O(n 2^n)$$