Analysis and Design of Algorithms

National University of Singapore

Algorithms

CS3230

Tutorial

Week 06.5
(Video to be uploaded during recess week)

Question 1

Alice and Bob are serving SHN in two separate rooms. They can only communicate through SMS, and they get charged $1 for every bit they transmit to each other.

Alice has with her an integer x and Bob an integer y, both less than 2^n. Bob wants to know whether $x = y$.

Alice and Bob do the obvious thing. Alice sends x to Bob, and Bob compares x to y.

What is the worst-case cost for the communication between Alice and Bob?

(1) $\Theta(n^2)$
(2) $\Theta(n)$
(3) $\Theta(\sqrt{n})$
(4) $\Theta(\lg n)$

Question 2

Alice and Bob are serving SHN in two separate rooms. They can only communicate through SMS, and they get charged $1 for every bit they transmit to each other.

Alice has with her an integer x and Bob an integer y, both less than 2^n. Bob wants to check whether $x = y$.

Alice randomly chooses a prime number p, computes $a = x \ (mod\ p)$, and sends p and a to Bob. Bob computes $b = y \ (mod\ p)$.

What is the cost if they want the false positive probability to be $< 1\%$?

(1) $\Theta(n^2)$

(2) $\Theta(n)$

(3) $\Theta(\sqrt{n})$

(4) $\Theta(\lg n)$

Question 3

This problem is about the 2D pattern matching problem. The text string T is an $n_1 \times n_2$-sized rectangle, and the pattern string P is an $m_1 \times m_2$-sized rectangle. Here $m_1 \leq n_1$ and $m_2 \leq n_2$.

What is the time complexity for the naïve algorithm that checks whether each $m_1 \times m_2$-sized block in T equals P?

Question 4

This problem is about the 2D pattern matching problem. The text string T is an $n_1 \times n_2$-sized rectangle, and the pattern string P is an $m_1 \times m_2$-sized rectangle. Here $m_1 \leq n_1$ and $m_2 \leq n_2$.

Extend the Karp-Rabin algorithm to solve the pattern matching problem in time $O(n_1 n_2)$ with 1% probability of false positives.

Assume that arithmetic on integers of size $O(n_1 + n_2)$ can be done in $O(1)$ time.