A0214561M

CS3230 Midterm Q3

3. a) all functions from N to M $\Rightarrow$ each $n \in N$ has m choices

$\Rightarrow$ m^n function..?

$H = \{1 \ldots m\}^n$

see proof in next page

$\Pr_{h \in H}[h(x) = h(y)] = \frac{1}{m}$ for such a family of function.

For $\Pr[h(x) = h(y)] < \frac{1}{m}$,

just remove $h \in H$ where all n entries are the same.

e.g. $n = 2$ $m = 2$

n	1	2
	1	1
	1	2
	2	1
	~~2~~	~~2~~ X

$\Pr[h(1)=h(2)] = \frac{1}{2} = \frac{1}{m}$

$\rightarrow \Pr[h(1) = h(2)] = \frac{1}{3} < \frac{1}{m}$

$n = 3$ $m = 2$

n	1	2	3
	1	1	1
	1	1	2
	1	2	1
	1	2	2
	2	1	1
	2	1	2
	2	2	1
	~~2~~	~~2~~	~~2~~ X

$\Pr[h(1) = h(2)] = \frac{4}{8} = \frac{1}{2}$

$\Pr[h(2) = h(3)] = \frac{4}{8} = \frac{1}{2}$

$\Pr[h(1) = h(3)] = \frac{4}{8} = \frac{1}{2}$

After removing $\{222\}$

$\Pr[h(x) = h(y)] = \frac{3}{8}$ $< \frac{1}{2}$

proof

$H = \{1..m\}^n$

$\# h \in H$ where $h(x) = h(y) = \sum_{i=1}^{m} h_x = h_y = i$

where x & y are the respective index

$= m \cdot m^{n-2}$

$= m^{n-1}$

$h = \{h_1, h_2, \dots h_n\}$

where h_i represents value from $[1..m]$ corresponds to each value of input n

$P[h(x) = h(y)] = \frac{m^{n-1}}{m^n} = \frac{1}{m}$ //