Analysis and Design of Algorithms

NUS
National University of Singapore

Algorithms
CS3230

Tutorial

Week 11

Paradigm for greedy algorithms

1. Cast the problem where we have to make a choice and are left with one subproblem to solve.

2. Prove that there is always an optimal solution to the original problem that makes the greedy choice, so the greedy choice is safe.

3. Use optimal substructure to show that we can combine an optimal solution to the subproblem with the greedy choice to get an optimal solution to the original problem.

Question 1

Suppose Bob has a collection of music files that he wants to burn into CDs. Each CD has a storage capacity of 100MB. In addition, Bob does not want to store more than two music files per CD. Note that each music file cannot be split and hence cannot be burned on more than 1 CDs. Given a set *A* of file sizes, each smaller than 100MB, let *MinCD*(*A*) denote the minimum number of CDs required to fit the files described in *A*.

Which of the following correctly describes an optimal substructure property of the problem, assuming that **at least one pair of files fit onto a CD**?

1. For any pair of files f_1, f_2 in A
   $MinCD(A) = 1 + MinCD(A \setminus \{f_1, f_2\})$
2. For any pair of files f_1, f_2 in A that belong on a single CD in an optimal solution,
   $MinCD(A) = 1 + MinCD(A \setminus \{f_1, f_2\})$
3. If f_1 and f_2 are the largest and smallest files in A,
   $MinCD(A) = 1 + MinCD(A \setminus \{f_1, f_2\})$

Question 2

Suppose Bob has a collection of music files that he wants to burn into CDs. Each CD has a storage capacity of 100MB. In addition, Bob does not want to store more than two music files per CD. Note that each music file cannot be split and hence cannot be burned on more than 1 CDs. Given a set *A* of file sizes, each smaller than 100MB, let *MinCD*(*A*) denote the minimum number of CDs required to fit the files described in *A*.

Assume that any optimal solution contains a pair that is burned onto a CD. Select all true statements about the problem.

1. The smallest file must be included in a pair in some optimal solution.
2. The pair $\{f_1, f_2\}$ where f_1 is the smallest file and f_2 is the largest file such that f_1 and f_2 fit onto one CD must be included in a pair in some optimal solution.
3. The pair $\{f_1, f_2\}$ where f_1 is the smallest file and f_2 is the largest file must be included in a pair in some optimal solution.

Question 3

Suppose Bob has a collection of music files that he wants to burn into CDs. Each CD has a storage capacity of 100MB. In addition, Bob does not want to store more than two music files per CD. Note that each music file cannot be split and hence cannot be burned on more than 1 CDs. Derive the algorithm for obtaining the the minimum number of CDs that Bob needs to burn his music files on the discs and apply it to the following array of file sizes. $filesizes = [89, 59, 32, 74, 81, 12, 7, 49, 43, 51, 62, 91, 27]$ What is the minimum number of CDs required?

Activity Selection Problem

Given a set of activities $S = \{a_1, a_2, \ldots, a_n\}$:

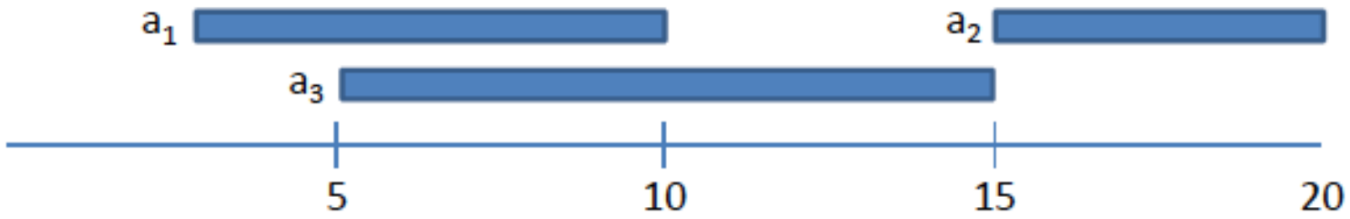
- Each activity takes place during $[s_i, f_i)$
- Two activities a_i and a_j are **compatible** if their time intervals don't overlap: $s_i \geq f_j$ or $s_j \geq f_i$.

Problem: Find a largest subset of mutually compatible activities.

Activity Selection Problem

Example: a_1=[3, 10), a_2=[15, 20), a_3=[5, 15)

- {a_1 and a_2} and {a_2 and a_3} are compatible
- {a_1 and a_3} are not compatible



Question 4

Which of these greedy strategies work for the activity selection problem?

1. Choose the activity a that **starts last**, discard those that conflict with a, and recurse.

2. Choose the activity a that **ends last**, discard those that conflict with a, and recurse.

3. Choose the **shortest activity** a, discard those that conflict with a, and recurse.