CS3230: Assignment for Week 9

Due: Sunday, 3rd Apr 2022, 11:59 pm SGT.

Please upload PDFs containing your solutions (hand-written & scanned, or typed) by 3rd Apr, 11:59 pm to `Assignments/Assignment9/Submissions`. Name the file **Assignment9_SID.pdf**, where SID should be replaced by your student ID.

You may discuss the problems with your classmates or read material online, but you should write up your solutions on your own. Please note the names of your collaborators or online sources in your submission; failure to do so would be considered plagiarism.

1. (7 points, graded for correctness) Let $A = \{a_1, \ldots, a_n\}$ and B be sets of n distinct positive integers each, where $a_1 < a_2 < \cdots < a_n$. You are allowed to reorder the numbers in B as you like. If after your reordering the set B is $\{b_1, \ldots, b_n\}$, then your *score* is the product $a_1^{b_1} a_2^{b_2} \ldots a_n^{b_n}$. Your goal is to find a reordering of B so as to maximize your score.
   (a) (5 points) Design and analyze an algorithm running in time $O(n \lg n)$ for this problem.
   (b) (2 points) Is there a comparison-based algorithm running in time $o(n \lg n)$? Justify your answer.

2. (1 point) In an undirected graph G, an *independent set* is a subset S of vertices such that there is no edge between any pair of vertices in S. Consider the *maximum independent set* problem: given a graph G, find an independent set of the largest size possible.
   (a) Suppose v is a leaf node in G, meaning it has only one neighbor. Show that there is an optimal solution to the maximum independent set problem containing v.
   (b) Use the observation in part (a) to design an efficient greedy algorithm for the maximum independent set problem on trees. Explicitly describe the optimal substructure property.

3. (1 point) Recall that the coin-changing problem is to make change for n cents using the smallest number of coins, where each coin has denomination in the set $\{d_1, \ldots, d_k\}$ and all d_i's are integers.

(a) Suppose that the denominations are $d_1 = 1, d_2 = c, d_3 = c^2, \ldots, d_k = c^{k-1}$ for some integers $c > 1$ and $k \geq 1$. Design a greedy algorithm to solve the problem. You should analyze the correctness of the algorithm, but do not need to state its running time.

(b) Give a set of coin denominations for which the greedy algorithm from part (a) does not always return an optimal solution. Your set should include a 1-cent coin.