

National University of Singapore  
School of Computing  
CS3243 Introduction to AI

**Tutorial 2: Informed Search**

Issued: Week 3

Discussion in: Week 4

**Important Instructions:**

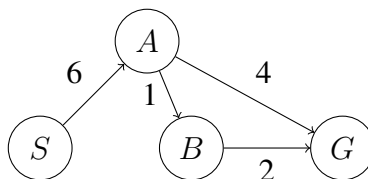
- **Assignment 2** consists of **Question 5** from this tutorial.
- Your solution(s) must be TYPE-WRITTEN, though diagrams may be hand-drawn.
- You are to submit your solution(s) during your **Tutorial Session in Week 4**.

Note: you may discuss the assignment question(s) with your classmates, but you must work out and write up your solution individually. Solutions that are plagiarised will be heavily penalised.

- (a) Provide a counter-example to show that the **tree search** implementation for the **Greedy Best-First Search** algorithm is **incomplete**.
  - (b) Briefly explain why the **graph search** implementation for the **Greedy Best-First Search** algorithm is **complete**.
  - (c) Provide a counter-example to show that neither the **tree search** nor the **graph search** implementations for the **Greedy Best-First Search** algorithm are **optimal**.
- (a) Prove that the **tree search** implementation of the **A\* Search** algorithm is optimal when an **admissible heuristic** is utilised.
  - (b) Prove that the **graph search** implementation of the **A\* Search** algorithm is optimal when a **consistent heuristic** is utilised. Assume graph search **Version 3**.
- (a) Given that a **heuristic**  $h$  is such that  $h(t) = 0$ , where  $t$  is any goal state, prove that if  $h$  is **consistent**, then it must be **admissible**.
  - (b) Give an example of an **admissible heuristic** that is **not consistent**.
4. We have seen various search strategies in class, and analysed their worst-case running time. Prove that *any deterministic search algorithm* will, in the worst case, **search the entire state space**. More formally, prove the following theorem

**Theorem 1.** Let  $\mathcal{A}$  be some complete, deterministic search algorithm. Then for any search problem defined by a finite connected graph  $G = \langle V, E \rangle$  (where  $V$  is the set of possible states and  $E$  are the transition edges between them), there exists a choice of start node  $s_0$  and goal node  $g$  so that  $\mathcal{A}$  searches through the entire graph  $G$ .

5. (a) In the search problem below, we have listed 5 heuristics. Indicate whether each **heuristic** is **admissible** and/or **consistent** in the table below.



|       | $S$ | $A$ | $B$ | $G$ | Admissible | Consistent |
|-------|-----|-----|-----|-----|------------|------------|
| $h_1$ | 0   | 0   | 0   | 0   |            |            |
| $h_2$ | 8   | 1   | 1   | 0   |            |            |
| $h_3$ | 9   | 3   | 2   | 0   |            |            |
| $h_4$ | 6   | 3   | 1   | 0   |            |            |
| $h_5$ | 8   | 4   | 2   | 0   |            |            |

- (b) Write out the order of the nodes that are explored by the **A\* Search** algorithm. Assume a **graph search** implementation that utilises heuristic  $h_4$ . Further, assume graph search **Version 3**.

You should express your answer in the form  $A-B-C$  (i.e., no spaces, all uppercase letters, delimited by the dash (–) character), which, for example, corresponds to the order  $A$ ,  $B$ , and  $C$ .

- (c) Which heuristic would you use? Explain why.

- (d) Prove or disprove the following statement:

The heuristic  $h(n) = \max\{h_3(n), h_5(n)\}$  is admissible.