NATIONAL UNIVERSITY OF SINGAPORE

SCHOOL OF COMPUTING

FINAL EXAMINATION FOR
Semester 2 AY2021/2022

CS3243: INTRODUCTION TO ARTIFICIAL INTELLIGENCE

April 25, 2022 Time Allowed: 120 Minutes

INSTRUCTIONS TO CANDIDATES

1. This assessment contains SEVEN (7) questions. All the questions are worth a total of 80 MARKS. It is set for a total duration of 120 MINUTES. You are to complete all 7 questions.
2. This is a CLOSED BOOK assessment. However, you may reference a SINGLE DOUBLE-SIDED A4 CHEAT SHEET.
3. You are allowed to use NUS APPROVED CALCULATORS.
4. If something is unclear, solve the question under a reasonable assumption. State your assumption clearly in the answer. If you must seek a clarification, the invigilators will only answers questions with Yes/No/No Comment answers. To ask for clarification to invigilator, please raise your hands on Zoom.
5. You may not communicate with anyone other than invigilators in any way.

STUDENT NUMBER: _____

| EXAMINER'S USE ONLY | | |
|---|---|---|
| Question | Mark | Score |
| 1 | 19 | |
| 2 | 10 | |
| 3 | 7 | |
| 4 | 11 | |
| 5 | 11 | |
| 6 | 14 | |
| 7 | 8 | |
| TOTAL | 80 | |

**1a. (3 Marks)** An environment is non-deterministic if at every state the agent plays randomly. The random agent plays randomly by simply picking a random action $a$ with uniform probability from the set of possible actions $A$. Here, we assume that every action in $A$ is valid for every state.

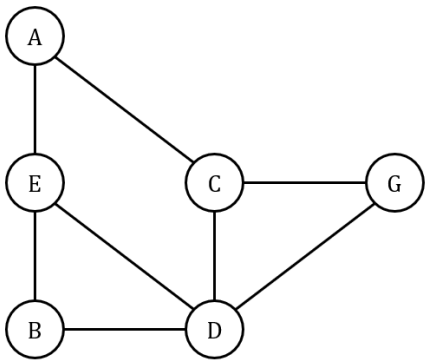
**(i)** State if the above statement is True or False.

**(ii)** Justify your answer to **(i)**.

**(iii)** Contrast the given environment with an environment with a static agent. Is the new environment deterministic? Note that a static agent is an agent that plays the same action $a' \in A$ deterministically for all states.

**Solution:**

**1b. (3 Marks)** Consider the following graph, where the goal node is G.




Assume that the following table lists the heuristic values for each node in the graph. Further assume that the smaller heuristic values are to be evaluated as better.

| Node (n) | Heuristic (h(n)) |
|---|---|
| A | 2 |
| B | 7 |
| C | 2 |
| D | 3 |
| E | 8 |
| G | 0 |

Finally, assuming that 2-beam limited graph search is employed to traverse this graph with the initial nodes A and E. Which of the following statements are true?

Note: explored refers to popping from the frontier; assume that within a given iteration of the beam search, all nodes are added to the frontier simultaneously.

**Option 1:** Node B is explored before Node D.

**Option 2:** Node B is never added to the frontier.

**Option 3:** Node C is explored before node D.

**Option 4:** None of the above.

You may pick more than one option.

**Solution:**

**1c. (3 Marks)** Under tree-based implementations for problems that have finite search trees, the Uniform-Cost Search (UCS) algorithm will always explore a number of nodes that is less than or equals to the number of nodes explored by the Greedy Best-First search (Greedy) algorithm using an admissible heuristic.

**(i)** State if the above statement is True or False.

**(ii)** Justify your answer to **(i)**.

**Solution:**

**1d. (3 Marks)** Every admissible heuristic is always better than any non-admissible heuristics. For example, heuristic $h_2$ is better than heuristic $h_1$ if its effective branching factor $b_2$* is less than $h_1$'s branching factor $b_1$*.

Branching Factor. If the total number of nodes generated by A* for a particular problem is $N$ and the solution depth is $d$, then $b$* is the branching factor that a uniform tree of depth $d$ would have to have in order to contain all $N$+1 nodes, i.e., $N+1 = 1 + b^* + (b^*)^2 + \cdots + (b^*)^d$.

**(i)** State if the above statement is True or False.

**(ii)** Justify your answer to **(i)**.

**Solution:**

**1e. (2 Marks)** Consider the following knowledge base (KB):

"All dogs are cute"

"All dogs are mammals"

"All mammals have four limbs"

An inference algorithm that receives the query "A Shih Tzu is a cute mammal". From this query, the algorithm then infers the statement "A Shih Tzu is a dog". Based on this observation, the inference algorithm cannot be _____?

**Option 1:** Complete

**Option 2:** Sound

**Option 3:** Complete and Sound

Note: No marks will be given without a valid rationale or the correct answer.

**Solution:**

**1f. (2 Marks)** Consider the following knowledge base (KB):

"All dogs are cute"

"All dogs are mammals"

"All mammals have four limbs"

Given the knowledge base and the query "Shih Tzus are cute mammals", which of the properties would guarantee that an inference algorithm can infer the sentence "Shih Tzus have four limbs"?

**Option 1:** Complete

**Option 2:** Sound

**Option 3:** Both of the above

Note: No marks will be given without a valid rationale or the correct answer.

**Solution:**

**1g. (3 Marks)** Suppose we are constructing a Bayesian Network for a set of random variables $\{ X_1, \ldots, X_n \}$, where $n > 1$. We do not have any additional information on the independence relationships between the variables. How many (directed) edges would the Bayesian Network have?

**Option 1:** $(n - 1)^2$

**Option 2:** $(n - 1)^2 - 1$

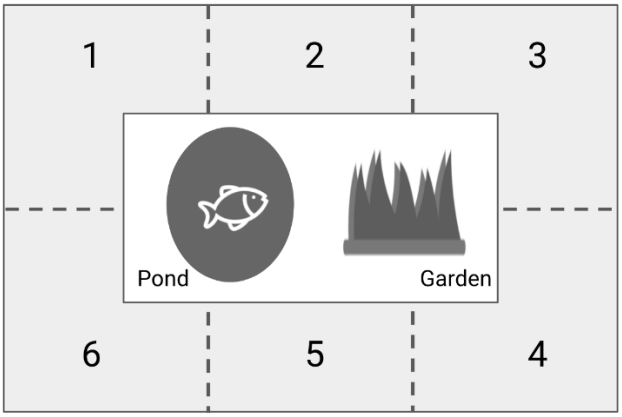
**Option 3:** $0.5n(n - 1)$

**Option 4:** $n(n - 1)$

**Solution:**

**2. (10 Marks)** The SoC COM3 building is opening soon and the school's administrator is tasked to allocate six research teams (teams A, B, C, D, E and F) to each of the six offices (offices 1-6) based on the research teams' preferences.

A floor plan of the building is given below with the locations of offices 1-6 labelled:




The following information is also given:

- Two offices are considered next to each other if they share a wall represented by dotted lines in the floor plan (for example, offices 1 and 6 are next to each other).
- Two offices are considered opposite to each other if they are directly across the building. Specifically, offices 1 and 4, offices 2 and 5, and offices 3 and 6 are opposite to each other.

- Offices 1 and 6 have a view of the pond only. Offices 3 and 4 have a view of the garden only. Offices 2 and 5 have a view of both the pond and garden.

The administrator gathered the following preferences from the research teams:

- Team A wants a view of the garden and not the pond, while Team C wants a view of both the pond and garden.
- Team B does not want to be opposite to team D's office.
- Team F wants to be next to teams A and C.
- No two teams may occupy the same office.

**(i)** The administrator formulates the above as a Constraint Satisfaction Problem (CSP). The variables are $O_A$, $O_B$, $O_C$, $O_D$, $O_E$ and $O_F$ (representing the office allocated to each research team), and the initial domain of each variable is {1, 2, 3, 4, 5, 6} (representing offices 1-6 respectively).

Specify the set of constraints that model the teams' preferences listed above. You are to formulate them as unary or binary constraints only (i.e., global constraints are not allowed) and use the smallest set of constraints possible. You are also to use the given variables and arithmetic/set/logic expressions only in the formulated constraints.

**(ii)** The administrator performs the backtracking search algorithm to solve the CSP.

Before beginning the search, we first apply the unary constraint(s) to filter the initial domain of the variables. Specify the domain of each variable after this step.

**(iii)** In the first iteration of the backtracking search, variable $O_A$ is assigned the first value in its updated domain after **(ii)** (domain values are sorted in ascending order). We then perform Forward Checking based on this partial assignment. Specify the domain of each of the other variables after this step.

**(iv)** Suppose we wish to use AC-3 instead of Forward Checking in **(iii)**. How many arcs will there be for this CSP?

**(v)** The administrator's colleague suggests using local search to solve this CSP instead. Can this be done? Provide an answer starting with a yes or no. If yes, suggest a suitable state representation and valuation function to be used for local search. If no, explain why this problem cannot be solved using local search.

**Solution:**

**3. (7 Marks)** Consider 4 variables (A, B, C, D) and the following constraints:

- $2A = 2B + 4$
- $3A < 2C$
- $D < 2A$
- $3A – 6 = 3B$
- $3D < 4C$

We are given the knowledge of the respective domains of variables A, B, C, D, which are, respectively, $D_A$, $D_B$, $D_C$, $D_D$, where $D_A = D_B = D_C = \{0, 1, 2, 3, 4, 5\}$ and $D_D = \{4,5\}$.

**(i)** Trace the AC-3 algorithm and determine the remaining values left in the domain of each variable after the completion of AC-3 algorithm. Input the values of the domain of each variable in the boxes below. You do not have to add any arcs that are in the queue already.

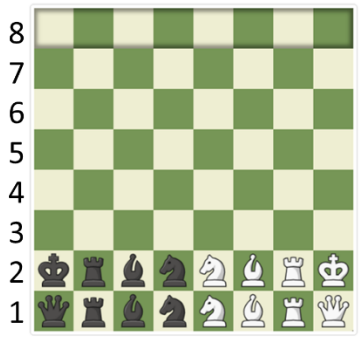
**(ii)** With reference to **(i)**, provide a valid assignment of values to A, B, C and D such that all constraints are satisfied, and $A + B + D = C + 3$.

**Solution:**

**4a. (6 Marks)** Racing Kings is a chess variant where the ultimate goal of each player is to take their kings to the eighth rank (row). Both players see the board from the same perspective and start the game with a mirrored setup of pieces. The pieces consist of: King, Queen, Knight, Rook, Bishop.

In this game, checks are entirely forbidden: not only is it forbidden to move one's king into check, but it is also forbidden to check the opponent's king. When a King is under immediate attack (threatened by an opponent piece), it is said to be in check.

The purpose of the game is to be the first player to move their king to the eighth row. When White moves their king to the eighth row, and Black, immediately on the next move, moves their king to the eighth rank, the game is declared a draw (this rule is to compensate for White's first-move advantage).




The figure above depicts the Initial State of Racing Kings game.

Apart from the rules described above, pieces move and capture exactly in the same manner as regular chess. Additionally, there are no special moves in racing chess (i.e., no castling, promotion, etc.).

You are to model a variant of this game as a zero-sum game. Instead of purely winning or losing the game, scores are given to the players when the game ends (Win/Lose/Draw).

The rules of the game remain unchanged but the players receive scores at the end of the game based on several conditions. Both players want to maximise their own scores.

The conditions and their respective descriptions are as follows.

| **Condition** | **Description** |
|---|---|
| Win/lose | White/Black King reaching the 8th row |
| Draw | This condition can only be initiated by the Black player. When White moves their king to the eighth row, and Black, immediately on the next move, moves their king to the eighth rank, the game is declared a draw. |
| Type of pieces left on the board | Material values are tagged to each piece. The more valuable a piece is, the higher the material value. In general, King > Queen > Rook > Bishop > Knight. |
| Number of **own** moves already made | Number of **own** moves made by **the player** before the game ends. Do not consider opponent's moves. |
| King Rank | King distance to last row |
| Mobility | Number of legal moves available at any state |

For example, a win with a shorter number of moves made should have a higher scoring than a win with a greater number of moves made.

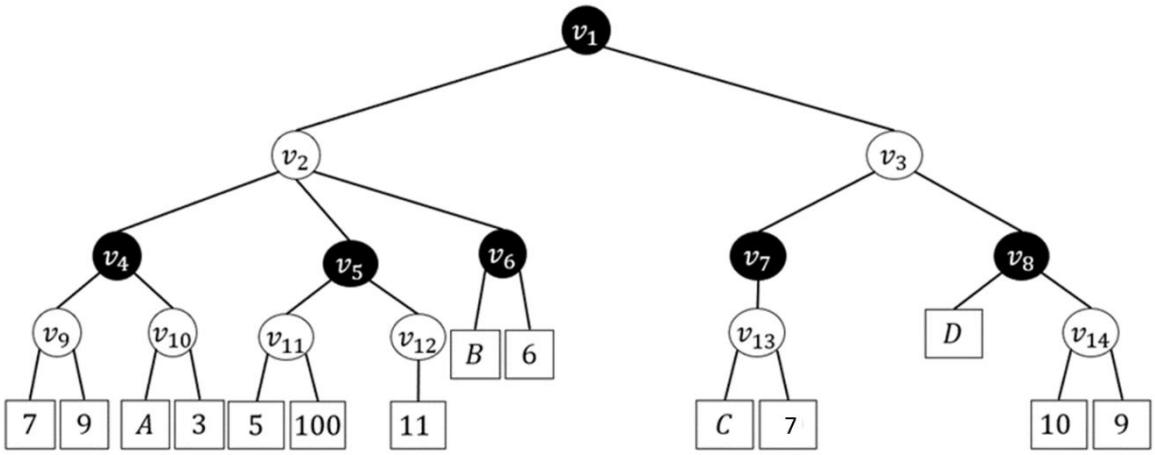
**(i)** Formulate the game of racing kings as an adversarial search problem. Specifically, describe the following:

- Initial state of the game and the state space
- Player(s) function
- Actions function
- Transition model
- Terminal test

**(ii)** Additionally, formulate the utility function given the different conditions stated above. A high-level description with some details should suffice. The utility function calculates the score for each player at the end of each game.

**Solution:**

**4b. (5 Marks)** The following is a game tree, where black nodes correspond to the MAX player while white nodes correspond to the MIN player. For all leaf nodes X (a, b, c, d), their utility values range from $-100 \leq X < 101$.




Assuming that the alpha-beta pruning algorithm is run on the above tree from left to right, find the largest range values of A, B, C and D such that no arcs are pruned.

**Solution:**

**5. (11 Marks)** The police are investigating five suspects to identify the thief that stole a priceless painting. It is well-known that these five suspects all work independently but have knowledge about the theft in question. During the interrogations conducted the police leaned the following:

1. Suspect A stated:
    a. Suspect B stole the painting
    b. Suspect E did not steal the painting
2. Suspect B stated:
    a. Suspect C did not steal the painting
    b. Suspect E did not steal the painting
3. Suspect C stated:
    a. Suspect A did not steal the painting
    b. Suspect E stole the painting
4. Suspect D stated:
    a. Suspect B stole the painting
    b. Suspect C stole the painting
5. Suspect E stated:
    a. Suspect A did not steal the painting
    b. Suspect D stole the painting

Further, via the use of state-of-the-art lie-detecting tools, the police also determined that:

- Each suspect told exactly one lie

Specify the knowledge base (KB) corresponding to all facts known by the police after the interrogations conducted over the five suspects. Each sentence in the KB should be expressed using propositional logic. With your specification of the KB, you must comply with the following.

**(i)** Express the KB based on the facts before any conversion to conjunctive normal form (CNF). You must cite the source of the information pertaining to each such sentence (e.g., Interrogation 1, Interrogation 1a, etc.).

**(ii)** Convert the KB to CNF. You must link the new sentences (now in CNF) to the original sentences specified in **(i)**.

**(iii)** Prove by resolution, that Suspect C is the thief.

**Solution:**

**6a. (10 Marks)** It's 2023 and there are 2 strains of the virus COVID-23.

1. Fatal strain, with 100% mortality rate (chance of death when contracted).
2. Non-fatal strain, with 0% mortality rate.

From the MOH website, you know the following information.

- The fatal strain is only found in 540 patients out of 5400 COVID-23 patients.
- 2 in 10 in the population have contracted COVID-23.
- Patients of COVID-23 are evenly split between the fatal and non-fatal strain.
- MOH's testing protocol requires repeated re-testing with another X-test kit until the posterior belief that the patient is a carrier is > 0.99999 given the results of the X-test taken.

The X-test is used to determine if a patient has COVID-23, with information on the various kits from the varying brands as follows.

| Brand | Cost | Positive COVID-23 cases correctly identified | Negative COVID-23 cases correctly identified |
|---|---|---|---|
| Rache | $3 | 49.4% | 99.99% |
| Abbat | $2 | 44.6% | 99.99% |
| MADsan | $1 | 45.8% | 97% |
| Siemans | $5 | 54.9% | 99.99% |

Your friend Eric wants to purchase the X-test kits from a single brand, and he asks you to help.

**(i)** Determine the best brand(s) to purchase. Here, the best would be the brand that gives the lowest false positive rate (prioritised) while also giving the lowest false negative rate (next priority).

**(ii)** MADsan's documentation claims that the results from every test kit is always conditional independent, given the state of virus contraction.

- Express the posterior belief that he is a carrier given $M$ 'positive' results, in terms of $M$.
- You need to test 'positive' $M$ times using MASsan's kits to ensure that posterior belief that the patient is a carrier is > 0.99999 (according to MOH testing protocol), solve for the value of $M$.
- For your value of $M$, calculate (exactly) the posterior belief that he is a carrier given $M$ 'positive' results?

**(iii)** Assume all test kits (regardless of brand) are always conditional independent, given the state of virus contraction. Considering only 'positive' results and that we also do not mix brands.

Determine the cost for each brand's test kits, where the tests are repeatedly applied until the posterior belief that the patient is a carrier is > 0.99999 given the results of the X-test taken.

Now, determine the most cost-effective brand(s) that can satisfy the MOH testing protocol.

**Solution:**

**6b. (4 Marks)** Eric contracted COVID-23 and wonders if the variant he has is the fatal strain. The Ministry of Health (MOH) informs him that there is a test, called Y-test, that indicates 'fatal' with probability 0.9 if he indeed has the fatal strain. MOH also says that the Y-test indicates 'non-fatal' with probability 0.6 if in fact he has the non-fatal strain.

**(i)** Draw the Bayesian network (just the graph, without probability tables) using the following variables; if there are equivalent variables, they must be grouped together.

$C$: Contracted COVID-23
$F$: Contracted the fatal strain
$Y$: Positive Y-test result
$X_1$: Positive X-test result for the 1st time
$X_2$: Positive X-test result for the 2nd time
$D$: Dies

**(ii)** What is the probability that his Y-test will give a false positive?

**(iii)** Unfortunately, his Y-test comes back 'fatal'. What is the probability he will survive?

This question follows from the context given in **Question 6a**.

**Solution:**

**7. (8 Marks)** In tutorials, we see that the speed-up achieved by the $\alpha$-$\beta$ algorithm is dependent on the sequence of leaf nodes. Here, the optimal sequence is the arrangement of leaf nodes such that $\alpha$-$\beta$ algorithm prunes nodes maximally. In other words, an optimal sequence of leaf nodes that yields the maximum speed-up.

We assume that nodes are evaluated from left to right.

**(i)** Describe in one sentence, the main condition for $\alpha$-$\beta$ pruning.

**(ii)** Explain and derive the best case (given an optimal sequence) time complexity for the following the Minimax algorithm and the $\alpha$-$\beta$ algorithm.

Express your answer only in terms of branching factor $b$ and depth $d$.

**Solution:**

END OF PAPER