NATIONAL UNIVERSITY OF SINGAPORE

SCHOOL OF COMPUTING

FINAL EXAMINATION FOR
Special Term (Part II) AY2021/2022

CS3243: INTRODUCTION TO ARTIFICIAL INTELLIGENCE

July 29, 2022 Time Allowed: 120 Minutes

INSTRUCTIONS TO CANDIDATES

1. This assessment contains SIX (6) questions. All the questions are worth a total of 70 MARKS. It is set for a total duration of 120 MINUTES. You are to complete all 6 questions.
2. This is a CLOSED BOOK assessment. However, you may reference a SINGLE DOUBLE-SIDED A4 CHEAT SHEET.
3. You are allowed to use NUS APPROVED CALCULATORS.
4. If something is unclear, solve the question under a reasonable assumption. State your assumption clearly in the answer. If you must seek a clarification, the invigilators will only answers questions with Yes/No/No Comment answers.
5. You may not communicate with anyone other than invigilators in any way.

STUDENT NUMBER: ___________________________________

EXAMINER'S USE ONLY		
Question	Mark	Score
1	14	
2	12	
3	10	
4	8	
5	8	
6	18	
TOTAL	70	

1a. The Queen of the CS3243 realm buys a new horseless carriage to improve her ability to travel the kingdom. However, the horseless carriage requires a set of controls, specifically: throttle and steering. Assume that the kingdom is a grid-like space (e.g., like a chessboard).

Throttle: $t_i \in \{1, 0, -1\}$, corresponding to {Accelerate, Cruise, Decelerate}. This controls the speed of the car by determining its acceleration. The integer chosen here will be added to his velocity for the next state. For example, if the Queen is currently driving at 5 *u*/*s* (i.e., 5 units per second) and chooses Accelerate, she will be traveling at 6 *u*/*s* in the next state.

Steering: $s_i \in \{1, 0, -1\}$, corresponding to {Turn Left, Constant, Turn Right}. This controls the direction of the car. For example, if the Queen is facing North and chooses Turn Left, she will be facing West in the next state.

Suppose that the CS3243 realm has dimensions m by n, but only $k < mn$ grid cells correspond to legal road where the horseless carriage may travel. Further, assume that the speed limit on all roads where the horseless carriage may travel is 4 *u*/*s*.

For part **(i)** and **(ii)**, assume that the Queen is a law-abiding citizen – i.e., she only drives on legal roads, and never breaks the speed limit.

(i) [2 marks] Without any additional information, what is the tightest upper bound on the size of state space if the Queen wishes to search for a route (not necessarily the shortest) from her current location to anywhere in the realm. Please note that your state space representation must be able to represent all possible states in the given environment.

A. $4mn$	B. $4k$	C. $5mn$	D. $5k$
E. $15mn$	F. $15k$	G. $20mn$	H. $20k$

Solution:

(ii) [2 marks] What is the maximum branching factor?

Solution:

We now remove the constraint that the Queen adheres to the speed limit. The Queen's speed is now limited by the mechanical constraints of the horseless carriage, which is 8 *u*/*s*, double the speed limit. The Queen is now able to travel twice as fast on the route to her destination.

How do the following properties of the search problem change as a result of being able to travel twice as fast?

(iii) [1 mark] Size of the state space.

A. Increases B. Remains unchanged C. Decreases

Solution:

(iv) [1 mark] Maximum branching factor.

A. Increases B. Remains unchanged C. Decreases

Solution:

1b. In this question, we will modify the action costs associated with the search problem being solved. For each search algorithm listed, indicate whether the path returned after the modification to the associated search tree is guaranteed to be identical to the path found using the unmodified action costs. Assume that all action costs are non-negative before modifications.

(i) [1 mark] Adding a constant *c*, where $c > 0$, to each action cost.

Breadth-first Search	Yes / No
Depth-first Search	Yes / No
Uniform-cost Search	Yes / No

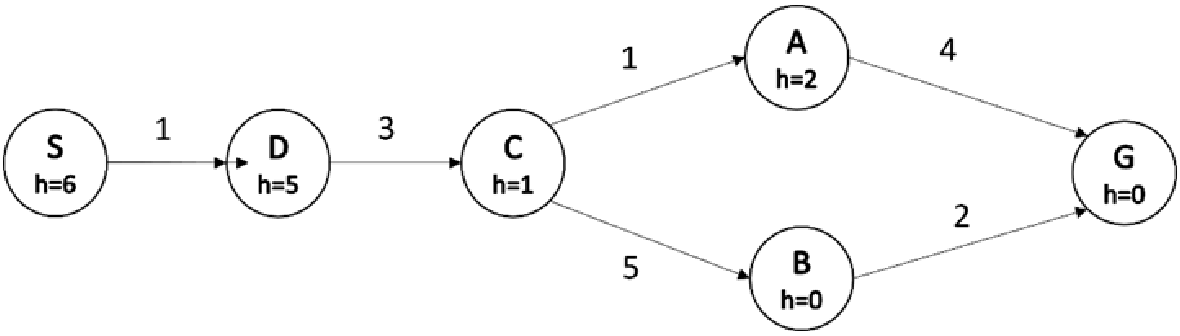
Solution:

(ii) [1 mark] Multiplying each action cost with a constant *w*, where $w > 0$.

Breadth-first Search	Yes / No
Depth-first Search	Yes / No
Uniform-cost Search	Yes / No

Solution:

1c. [2 marks] Consider the following graph where all heuristic values are assumed to be admissible and consistent. There exists a node whose heuristic value is incorrect.



Determine the node whose heuristic value is incorrect and find the maximum range of heuristic values for this node such that the heuristic is both admissible and consistent at every node.

Solution:

1d. [2 marks] Give an advantage that an inadmissible heuristic might have over an admissible one. Your explanation should be generalised for all heuristics and not just trivial heuristics (e.g., using h(n) = 0 for an admissible heuristic).

Solution:

1e. Determine if the following properties of the Hill-climbing algorithm are True or False.

(i) [1 mark] Hill-climbing is complete.

Solution:

(ii) [1 mark] Hill-climbing is optimal.

Solution:

2. In the realm of CS3243, there are two linked ports: a ferry port and a cruise port. There are also five passenger ships, denoted *A*, *B*, *C*, *D*, and *E*. You are tasked to schedule a time slot and a port for each ship to either dock or sail. We have four time slots, {1, 2, 3, 4}, for each port, during which we can schedule a ship to either dock or sail.

Additionally, we have the following constraints:

- Ship *B* has been damaged and must dock in time slot 1.
- Ship *D* can only dock at the port during or after time slot 3.
- Ship *A* has a medical emergency and must dock at latest in time slot 2.
- Ship *D* must dock before Ship *C* sails as some passengers must transfer from Ship *D* to Ship *C*.
- Ships *A*, *B*, and *C* can only use the cruise port.
- Ships *D* and *E* can only use the ferry port.
- No two ships can reserve the same time slot at the same port.

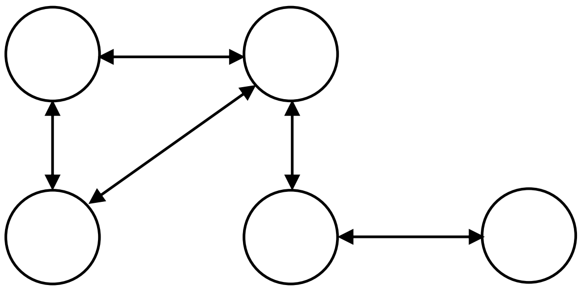
(i) [8.5 marks] Complete the formulation of this problem as a Constraint Satisfaction Problem by specifying the domains for the variables *A*, *B*, *C*, *D*, and *E* (corresponding to the five passenger ships), and the associated constraints (both unary and binary).

Note that you may use English in place of certain symbols, e.g., “element of” instead of $\in$, “not”, “and”, and “or” instead of $\neg$, $\wedge$, and $\vee$ respectively, “implies” instead of $\Rightarrow$, etc.

Solution:

(ii) **[1 mark]** The diagram below is an incomplete constraint graph representing the constraints for this problem. Fill in the circles with variables *A*, *B*, *C*, *D*, and *E* such that the constraint graph successfully shows the constraints between each variable.

Solution:



(iii) [2.5 marks] Since unary constraints determine which port each ship must use, in this part of the question we will only consider candidate timeslots. What are the domains of the variables ***after*** enforcing unary constraints and arc-consistency?

Format your answer such that the domain values are specified as follows: (*x*,*y*,*z*). Note that your answer should NOT CONTAIN ANY WHITESPACES. For example, if the resultant domain for variable *A* is 10 and 11, then your answer should be: (10,11). If the domain is empty, simply specify: ().

Solution:

Domain of *A*:

Domain of *B*:

Domain of *C*:

Domain of *D*:

Domain of *E*:

3. The following corresponds to a Constraint Satisfaction Problem.

There are 4 variables, A, B, C, and D, as well as the following constraints.

- $2C \leq 2D$
- $B - 1 = C$
- $2A \geq 4B$
- $C \leq D - 1$
- $3C + 3 = 3B$

Further, let the domains for the variable correspond to the following.

- Domain of A: {1, 2, 3, 4, 5, 6}
- Domain of B: {1, 2, 3, 4, 5}
- Domain of C: {1, 2, 3, 4, 5}
- Domain of D: {1, 2, 3}

(i) [8 marks] Suppose that the AC-3 algorithm is run on the problem above. Complete the execution of the AC-3 algorithm assuming that the queue is initialised with the following directed arcs: (A, B), (B, A), (B, C), (C, B), (C, D), (D, C).

You may leave the working for your trace in the rationale box provided. You must specify the final domains for the four variables at the end of the AC-3 execution.

Format your answer such that the domain values are specified as follows: (x,y,z). Note that your answer should NOT CONTAIN ANY WHITESPACES. For example, if the resultant domain for variable A is 10 and 11, then your answer should be: (10,11). If the domain is empty, simply specify: ().

Solution:

Domain of A: Domain of B:

Domain of C: Domain of D:

(ii) [2 marks] Based on the problem formulation above, provide a complete and consistent assignment to the values *A*, *B*, *C*, and *D*, that also includes the additional constraint:

- $A + B = 2C + 2D$

Solution:

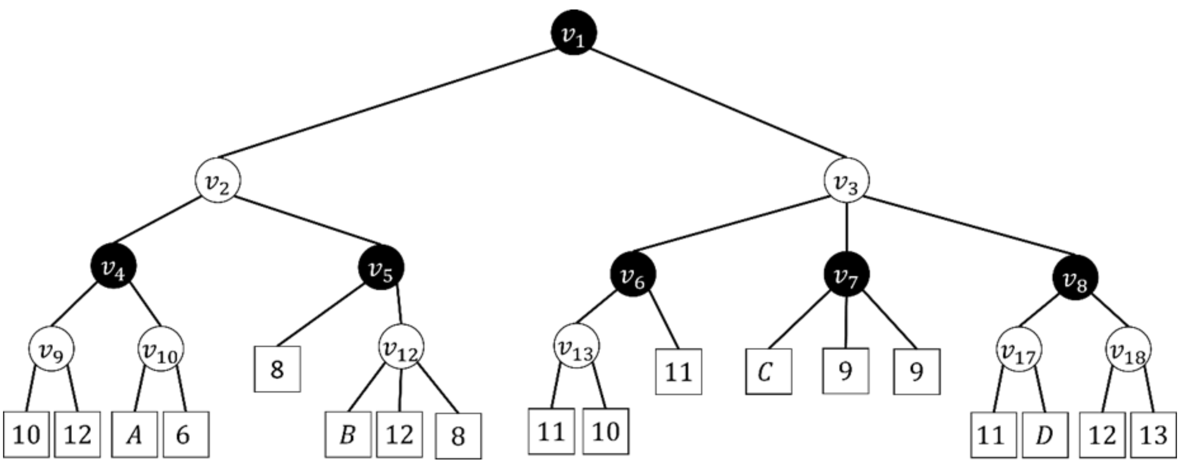
4. [8 marks] The AC-3 algorithm is defined as follows.

```
function AC-3(csp) returns false if an inconsistency is found and true otherwise
  queue ← a queue of arcs, initially all the arcs in csp

  while queue is not empty do
    (Xi, Xj) ← POP(queue)
    if REVISE(csp, Xi, Xj) then
      if size of Di = 0 then return false
      for each Xk in Xi.NEIGHBORS - {Xj} do
        add (Xk, Xi) to queue
  return true

function REVISE(csp, Xi, Xj) returns true iff we revise the domain of Xi
  revised ← false
  for each x in Di do
    if no value y in Dj allows (x,y) to satisfy the constraint between Xi and Xj then
      delete x from Di
      revised ← true
  return revised
```

Consider the code in the box. Notice that the AC-3 algorithm does not check arc (X_j, X_i) after a domain revision to X_i (i.e., it only re-checks arcs (X_k, X_i) for each X_k in X_i.NEIGHBOURS - $\{X_j\}$). Prove that the arc (X_j, X_i) never needs to be rechecked after a domain update to X_i.

Solution:

5. [8 marks] The following is an adversarial search game tree where black nodes correspond to the MAX player and white nodes correspond to the MIN player. The utility values range from -100 to 100.



Given that the Alpha-beta pruning algorithm is applied to the above tree from ***left to right***, find the ***largest possible range of values*** for A, B, C, and D such that ***no arcs are pruned***.

Solution:					
____________	<	A	<	____________
____________	<	B	<	____________
____________	<	C	<	____________
____________	<	D	<	____________

6a. [4 marks] Given a knowledge base (KB) that is represented by the following logical statements, use truth-table enumeration to determine if the query α may be inferred – i.e., prove that KB ⊨ α.

Let the KB = $(x_1 \vee x_2) \wedge \neg x_1 \wedge (x_2 \Rightarrow x_3)$ and $\alpha = \neg x_2 \vee \neg x_3$.

Define the truth table and then prove/disprove that KB ⊨ α.

Solution:

6b. In the realm of CS3243, the Queen decides to test your powers of deduction. She describes a game was played at one of her parties.

"Four hats were distributed among four volunteers. Two of these hats were red, and the other two were blue. Let this be the first piece of information (i.e., Rule 0, denoted R_0) about the Queen's game.

Only use the variables B_1, B_2, B_3, and B_4, such that B_i has a truth value of True if Volunteer i was wearing a blue hat."

The Queen's game was set up such that the only other pieces of information were as follows.

- Volunteer 1 reported: "Volunteers 2 and 3 have different-coloured hats. But I can't tell which has which colour." Let this be Rule 1 (i.e., R_1).
- Volunteer 2 reported: "Volunteers 3 and 4 have different-coloured hats. But I can't tell which has which colour." Let this be Rule 2 (i.e., R_2).
- Volunteer 3 reported: "Volunteer 4 has a blue hat." Let this be Rule 3 (i.e., R_3).

Note that there is no error-carried-forward (ECF) considered in this question. Form and manipulate your expressions carefully.

(i) [4 marks] Define R_0, R_1, R_2, and R_3 using propositional logic.

Solution:

(ii) [4 marks] Assuming that R_0, R_1, R_2, and R_3 correspond to the knowledge base (KB) for this problem, represent the KB in Conjunctive Normal Form (CNF).

Solution:

(iii) [6 marks] Use resolution to determine the kind of hat that Volunteer 1 is wearing.

Solution:

END OF PAPER