# Informed Search: Incorporating Domain Knowledge

CS3243: Introduction to Artificial Intelligence – Lecture 3

25 January 2023 (Video Lecture)

# Contents



Reference: AIMA 4th Edition, Section 3.5

# Administrative Matters

# Project 1

- Released this week (Week 3)
  - Due Week 6 Sunday,19 February, 2359 hrs (about a month to work on it)
- Consultation session
  - Week 4 Friday, 3 February, Time TBC
  - Online (Zoom) – will be recorded
    - Meeting link = Canvas announcement
  - Reviews FAQ + any new questions raised
- Late submissions
  - Penalties
    - Within deadline +24 hours = 80% of score
    - Within deadline +48 hours = 50% of score
    - Beyond deadline +48 hours = 0% of score

Live Coding Session: TBC
(Probably also in Week 4)

Start on Project 1 early!
Start today!

DO NOT COPY/SHARE CODE!

# Upcoming…

- Tutorials begin this week
  - Only sessions on Monday/Tuesday rescheduled
    - T03 (MON 1400 hrs) → WED 1400 hrs - T03 Zoom Link (Week 3)
    - T04 (MON 1500 hrs) → WED 0800 hrs - T04 Zoom Link (Week 3)
    - T05 (TUE 0900 hrs) → FRI 1400 hrs - T05 Zoom Link (Week 3)
- Deadlines
  - TA1 (released last week)
    - *Due in your Week 3 tutorial session*
    - *Submit the a physical copy (more instructions on the Tutorial Worksheet)*
    - *Those with tutorials on Zoom this week → submit via email to tutors*
  - Remember to prepare for the tutorial
    - Participation marks = 5%

# Reviewing Uninformed Search

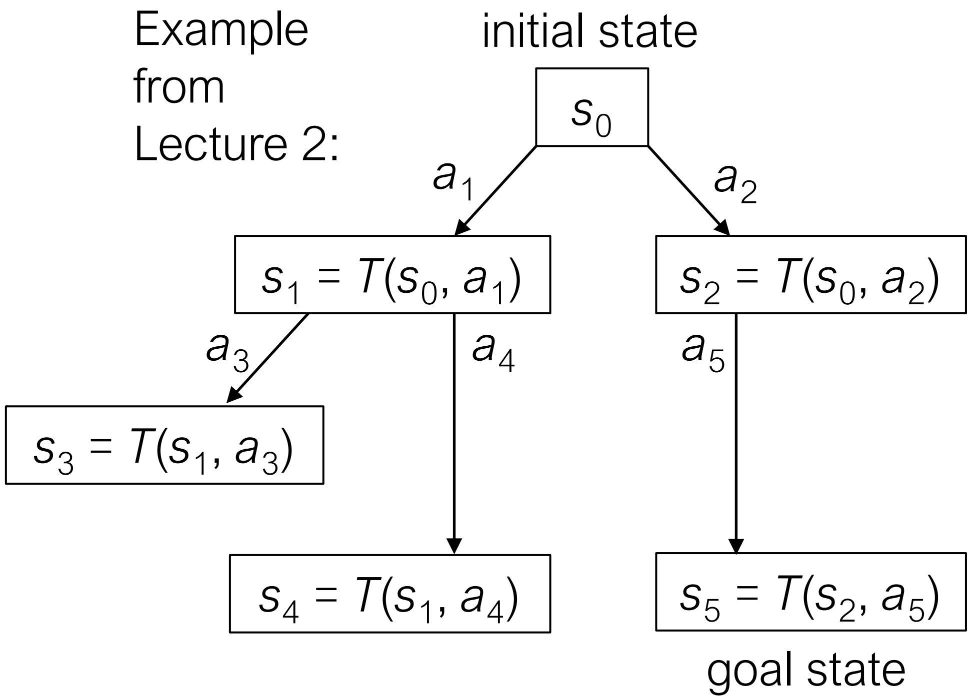
# Problem-solving Agent: General Path Planning

- Assumptions on problem environment
  - Fully observable
  - Deterministic
  - Discrete
  - Episodic

- General definition of a path planning problem
  - State representation, $s_i$
    - Initial state ($s_0$)
  - Goal test, *isGoal* : $s_i \rightarrow \{0, 1\}$
    where 1 denotes a goal state
  - Actions, *actions* : $s_i \rightarrow A = \{a_1, \ldots, a_k\}$
  - Action costs, *cost* : $(s_i, a_j, s_i') \rightarrow v \geq 0$
  - Transition model, $T$ : $(s_i, a_j) \rightarrow s_i'$

General solution for ALL path planning problems – stronger AI (than reflex agent)

Example from Lecture 2:



# Tree Search Algorithm

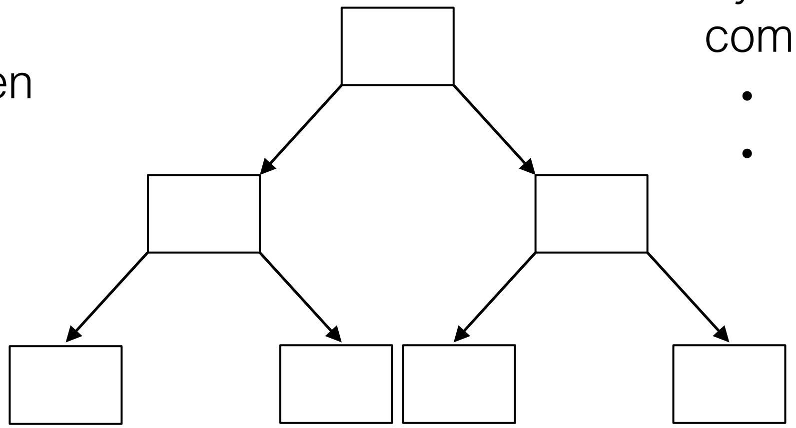
```
frontier = {Node(initial state), NULL}
while frontier not empty:
       current = frontier.pop()
       if isGoal(current.state): return current.getPath()
       for a in actions(current.state):
              successor = Node(T(current.state, a), current)
              frontier.push(successor)
return failure
```

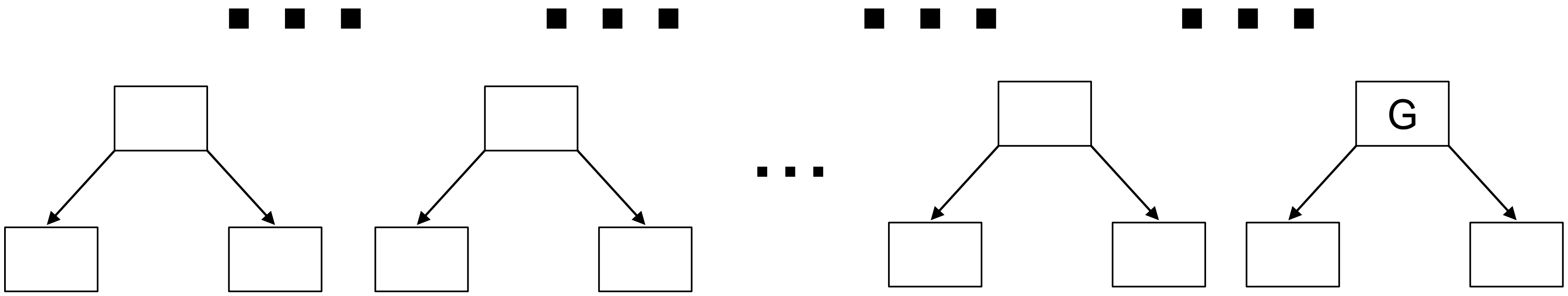
- Frontier
  - Paths to search
  - Considers ALL paths (including redundant ones)
- Concept of a Node for defining paths
  - Referenced *end point of path*, state *s*
  - Parent node, for *Linked List reference*
  - Other attributes
    - Action – facilitate *backtracking* in *DFS* – i.e., O(m) space complexity
    - Path cost – for efficient *UCS* (avoids traversing path Linked List)
    - Depth – for efficient *DLS*/*IDS* (avoids traversing path Linked List)

# Uninformed Search Algorithms

- Systematic traversal of search space
  - BFS: paths in order of number of actions taken
  - DFS: paths to terminal state
  - DLS/IDS: hybrid

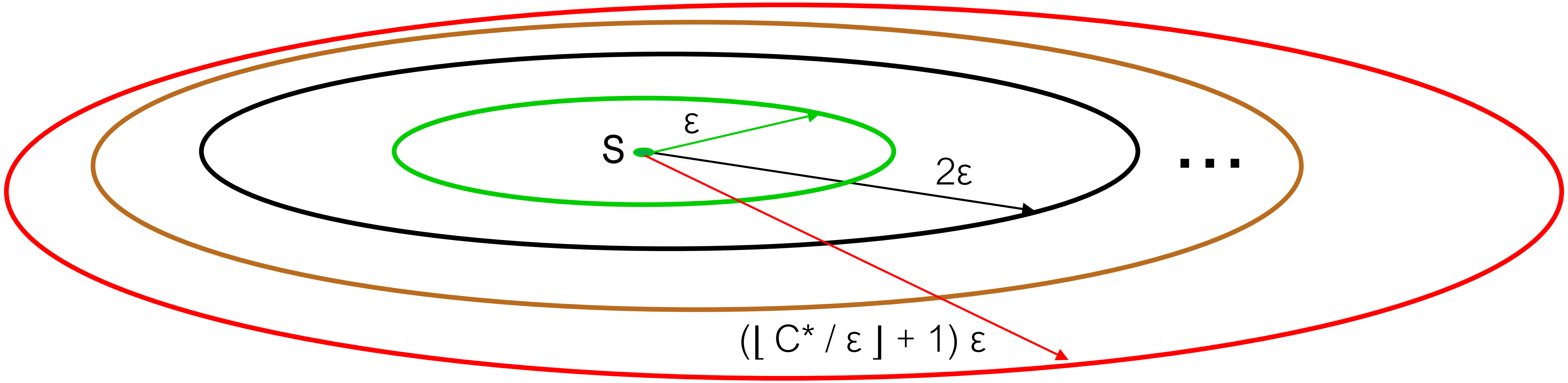
- Systematic search allows completeness
  - Assuming search space finite
  - Assuming solution exists





# UCS and Optimality

- UCS traverses paths in order of path cost




- Ordered traversal of path costs required to ensure optimality

# Summary under tree search

- Performance of search algorithms

| Criterion | BFS | UCS | DFS | DLS | IDS |
|---|---|---|---|---|---|
| Complete? | Yes[1] | Yes[1,2] | No[3] | No | Yes[1] |
| Optimal Cost? | Yes[4] | Yes | No | No | Yes[4] |
| Time | $O(b^d)$ | $O(b^{1+\lfloor C^*/\varepsilon \rfloor})$ | $O(b^m)$ | $O(b^\ell)$ | $O(b^d)$ |
| Space | $O(b^d)$ | $O(b^{1+\lfloor C^*/\varepsilon \rfloor})$ | $O(bm)$ | $O(b\ell)$ | $O(bd)$ |

1. Complete if *b* finite and either has a solution or *m* finite
2. Complete if all actions costs are > ε > 0
3. DFS is incomplete unless the search space is finite – i.e., when *b* finite and *m* finite
4. Cost optimal if action costs are all identical (and several other cases)

- Recall that an Early Goal Test is assumed for BFS
- UCS must perform a Late Goal Test to be optimal (this also accounts for the +1 in the index of its complexity)
- DFS is not complete (even under 1) as even if a solution exists, it may infinitely traverse a path without a solution (note the “or”)
- DFS space complexity may be improved to $O(m)$ with backtracking (similar for DLS and IDS)

# Graph Search Algorithm (Version 1)

```
frontier = {Node(initial state), NULL}
reached = {initial state: Node(initial state)}
while frontier not empty:
      current = frontier.pop()
      if isGoal(current.state): return current.getPath()
      for a in actions(current.state):
            successor = Node(T(current.state, a), current)
            if successor.state not in reached:
                  frontier.push(successor)
                  reached.insert(successor.state: successor)
return failure
```

- Reached (sometimes also labelled: Visited)
  - Hash table that tracks all nodes already reached or visited via prior searching
- This version ensures that nodes are never revisited
  - Omits all redundant paths
  - May omit optimal path
  - No impact on completeness

# Graph Search Algorithm (Version 2)

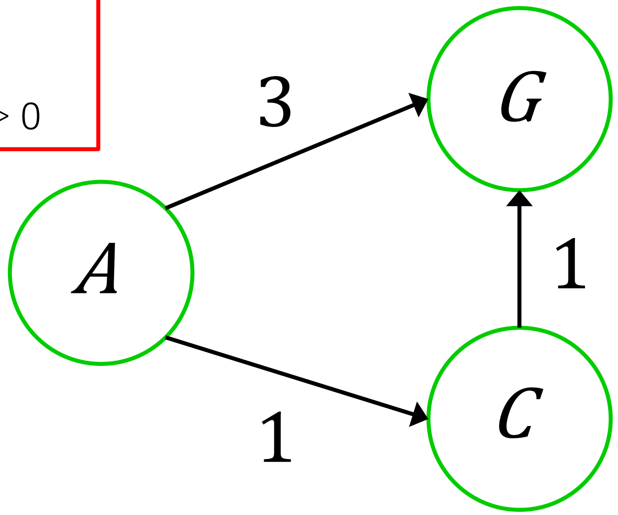
```
frontier = {Node(initial state), NULL}
reached = {initial state: Node(initial state)}
while frontier not empty:
      current = frontier.pop()
      if isGoal(current.state): return current.getPath()
      for a in actions(current.state):
            successor = Node(T(current.state, a), current)
            if successor.state not in reached or
               successor.getCost() < reached[successor.state].getCost():
                  frontier.push(successor)
                  reached.insert(successor.state: successor)
return failure
```

- More relaxed constraint on paths that are considered
  - Also considers paths with lower path cost

# Tree Search Versus Graph Search

- Tree search will try ALL paths
  - No paths excluded
  - No issues with optimality

Completeness assumptions:
- *b* finite, and *m* finite
  OR has a solution
- All action costs are > ε > 0




- What about graph search?
  - Ensure optimal path not among excluded paths
  - Consider this example
    - F = {A(0)}; R = {A}
      - pop A(0), push C(1) and G(3)
    - F = {C(1), G(3)}; R = {A, C, G}
      - pop C(1), push G(2) since lower cost
    - F = {G(2), G(3)}; R = {A, C, G}
      - pop G(2), path is A → C → G

Notice that without the update to G while it was on the frontier, we would not have returned the optimal path (similar to applying Early Goal Test)

# Graph Search Properties

- Performance under graph search implementations

<table>
<tr><th>Criterion</th><th>BFS</th><th>UCS</th><th>DFS</th><th>DLS</th><th>IDS</th></tr>
<tr><td>Complete?</td><td>Yes[1]</td><td>Yes[1,2]</td><td>No[3]</td><td>No</td><td>Yes[1]</td></tr>
<tr><td>Optimal Cost?</td><td>Yes[4]</td><td>Yes</td><td>No</td><td>No</td><td>Yes[3]</td></tr>
<tr><td>Time</td><td colspan="5" rowspan="2">$O(|V| + |E|)$</td></tr>
<tr><td>Space</td></tr>
</table>

1. Complete if $b$ finite and either has a solution or $m$ finite
2. Complete if all actions costs are $> \varepsilon > 0$
3. DFS is incomplete unless the search space is finite – i.e., when $b$ finite and $m$ finite
4. Cost optimal if action costs are all identical (and several other cases)

For CS3243, assume graph search Version 1 is used unless otherwise mentioned

DONE? SOLVED?

- Time and space complexities are now bounded by the size of the search space - i.e., the number of vertices (nodes) and edges, $|V| + |E|$
- Note that we ***do not*** need to check for cheaper paths under graph search for BFS and DFS since costs play no part in those algorithms and they cannot guarantee an optimal solution anyway

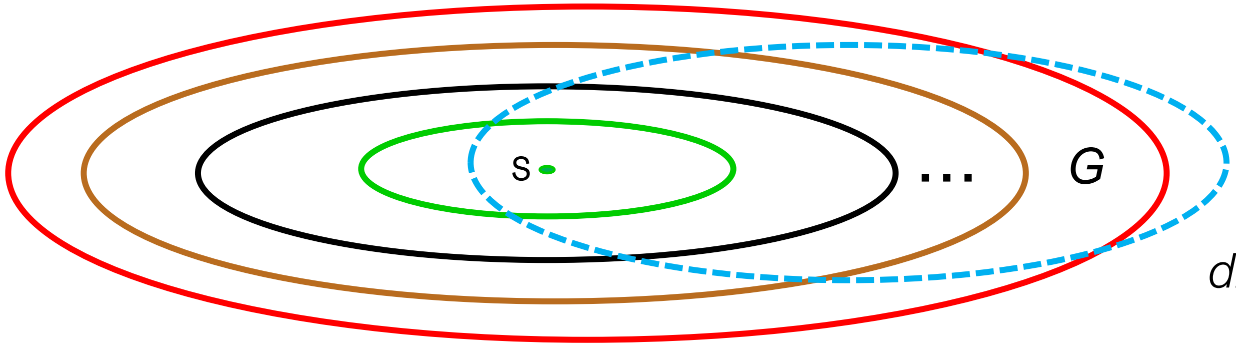
# Informed Search

# Searching Less?

- Uninformed search algorithms are systematic
  - Search outward from the initial state
  - All directions
- Search spaces are LARGE
- Can we search more efficiently?

General idea

Use domain knowledge about the problem environment to determine the cost required to go from a particular state to its nearest goal



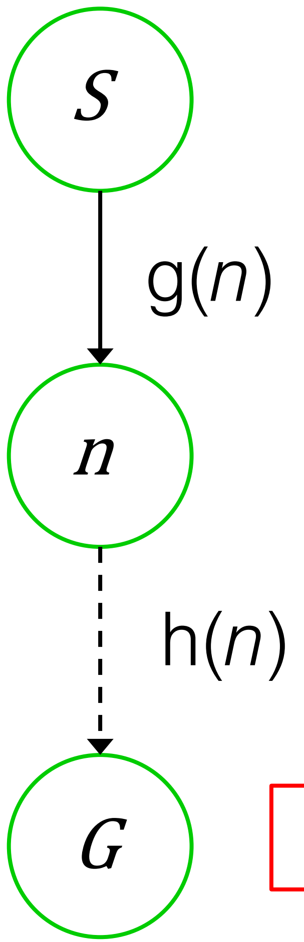

*try to focus on searching in the direction of the goal*

# Path Costs & Heuristics

- UCS
  - Frontier = Priority Queue
  - Priority for node *n* = g(*n*)
    - g(*n*) quantifies the path cost from the initial state *S* to *n.state* as defined by *n.path*

- General idea: use domain knowledge to estimate cost from *n.state* to *G*

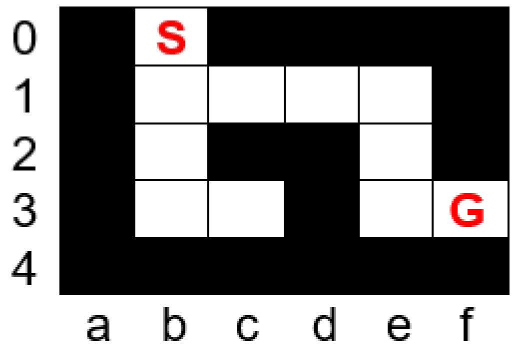
- Define a *heuristic* function h
  - h approximates the path cost from *n.state* to its nearest goal *G*




h(*n*) ≥ 0 requirement

# Ideas on Deriving Heuristic Functions

- Consider a Maze Puzzle problem
  - Layout known
  - Moves ←,↑,→,↓
  - Find path from ***S*** to ***G***




- Example: Euclidean distance
  - h(*n*) = Euclidean distance from *n* to ***G***

h(*G*) = 0 requirement

- General requirements
  - Efficient – e.g., Euclidean distance is O(*m*), where *m* = no. dimensions
  - More properties discussed later

General idea

Use domain knowledge about the costs (e.g., distances) between a given node and its closest goal – i.e., think about how to define the function h

More on how to define h in the next lecture

# Implementation with Evaluation Functions

- Keep using a priority queue for frontier
  - Use different priorities
- Define an evaluation function f
  - Priority for priority queue
  - Priority for node $n$ = f($n$)
  - UCS: priority = f($n$) = g($n$)
- Now consider different evaluation functions
  - *Greedy Best-First Search*: priority = f($n$) = h($n$)
  - *A* Search*: priority = f($n$) = g($n$) + h($n$)

# Best-First Search Algorithm

- General graph-search implementation

```
function BEST-FIRST-SEARCH(problem, f) returns a solution node or failure
  node ← NODE(STATE=problem.INITIAL)
  frontier ← a priority queue ordered by f, with node as an element
  reached ← a lookup table, with one entry with key problem.INITIAL and value node
  while not IS-EMPTY(frontier) do
    node ← POP(frontier)
    if problem.IS-GOAL(node.STATE) then return node
    for each child in EXPAND(problem, node) do
      s ← child.STATE
      if s is not in reached or child.PATH-COST < reached[s].PATH-COST then
        reached[s] ← child
        add child to frontier
  return failure

function EXPAND(problem, node) yields nodes
  s ← node.STATE
  for each action in problem.ACTIONS(s) do
    s′ ← problem.RESULT(s, action)
    cost ← node.PATH-COST + problem.ACTION-COST(s, action, s′)
    yield NODE(STATE=s′, PARENT=node, ACTION=action, PATH-COST=cost)
```

Late Goal Test

Graph-search (v2)

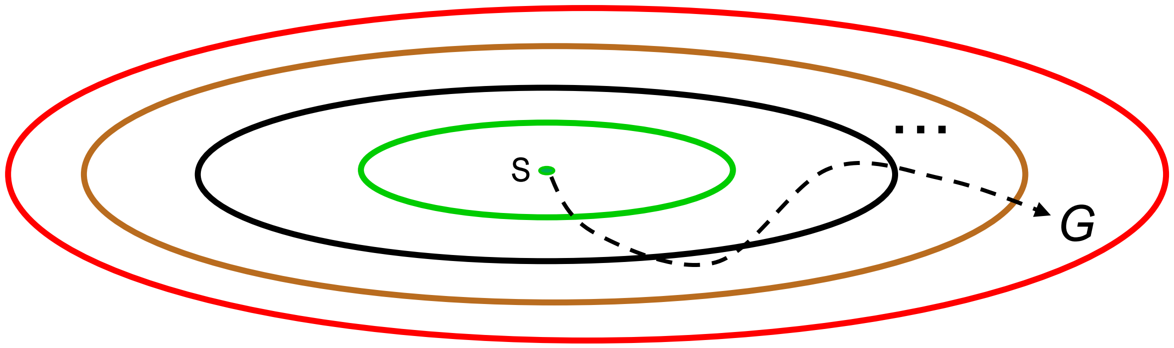
Utilises search problem definitions

# Greedy Best-First Search

# The Greedy Best-First Search Algorithm

- Implemented like UCS except
  - f($n$) = h($n$)

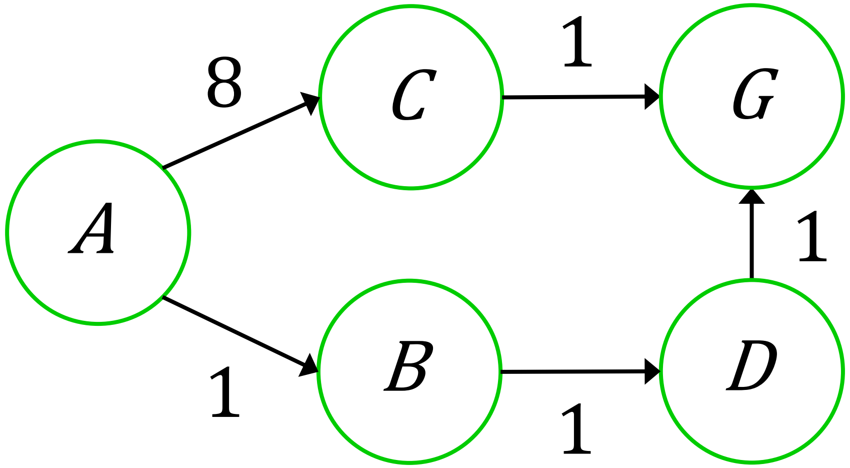
- General idea
  - Given all nodes along the frontier
  - Explore next reachable state that you estimate is closest to a goal




keep picking states estimated to be closest to the goal (based on candidate paths on the frontier)

# The Greedy Best-First Search Algorithm

- Example (tree-search)




Assume this h:

| $n$ | h($n$) | h*($n$) |
|---|---|---|
| $A$ | 3 | 3 |
| $B$ | 2 | 2 |
| $C$ | 1 | 1 |
| $D$ | 1 | 1 |
| $G$ | 0 | 0 |

Trace:
ITR1 = [A((-),3)]
ITR2 = [C((A),1), B((A),2)]
ITR3 = [G((A,C),0), B((A),2)]
ITR4 = DONE (A,C,G)

Notice that even with the perfect heuristic, we may not get the optimal solution. Why?

h*($n$) = true path cost from $n$ to nearest goal

Algorithm never exploits information on path already travelled.

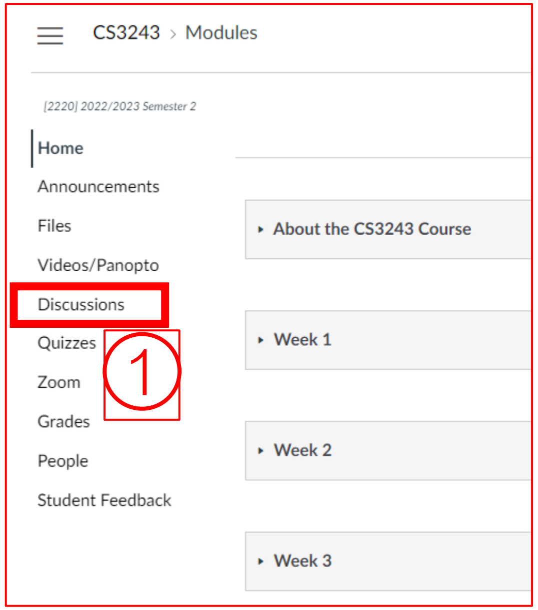
# Completeness and Optimality

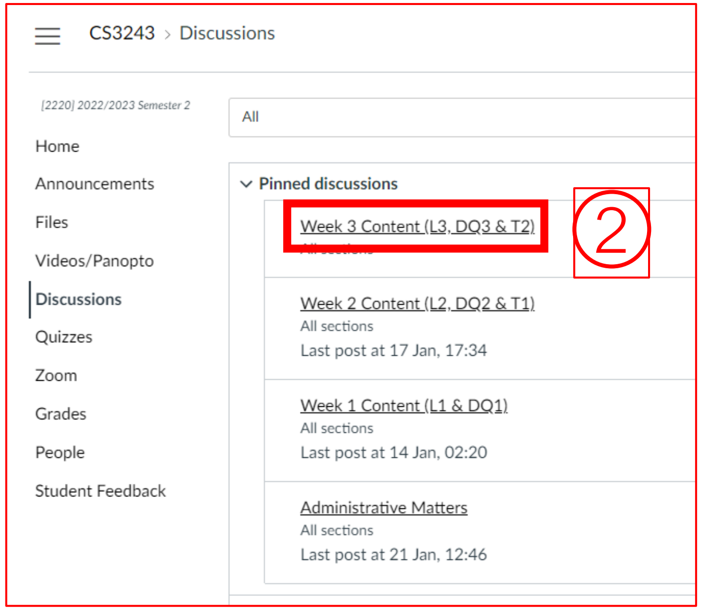
- Tree-search implementation is incomplete!
  - General idea
    - Can get stuck in a loop between nodes where h values are lowest
  - Prove with completeness counter example - T02 Q1a
- Graph-search implementation is complete if search space is finite
  - General idea
    - With no revisits, in finite state space, will visit entire space
  - Prove – T02 Q1b
- Not optimal under either tree search or graph search
  - As shown in example on previous slide
  - Find another example – T02 Q1c

Ordered traversal of path in order of path costs required to ensure optimality

# Questions about the Lecture?

- Do you need to clarify anything?
- Ask on Canvas > CS3243 > Discussion > Week 3 Content

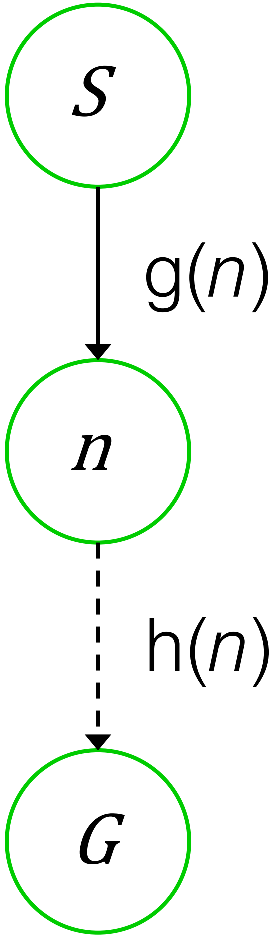


# A* Search

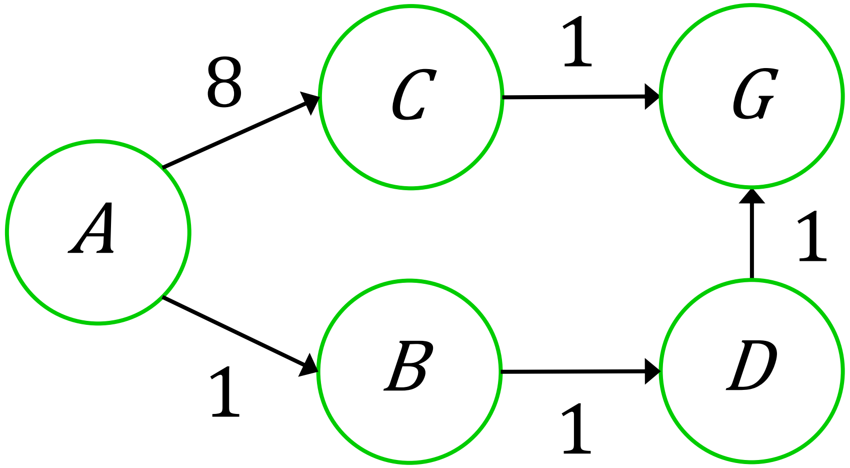
# The A* Search Algorithm

- Greedy Best-First Search
  - With greedy, f(*n*) = h(*n*)
  - Does not consider cost of path already taken
- Accounting for costs already incurred: A*
  - With A*, f(*n*) = g(*n*) + h(*n*)
    - g(*n*): actual path cost from *S* to *n*
    - h(*n*): estimated cheapest path cost from *n* to *G*
- A* priorities
  - Total path cost estimates from *S* to *G*
  - Gets more accurate as paths get explored



# The A* Search Algorithm

- Example (tree search)
  same example as used on greedy (Slide 24)




A* outputs the optimal solution, unlike the Greedy Best-First Search

Will it always be optimal?
What about graph search?

Assume this h:
again, same as before (Slide 24)

| *n* | h(*n*) | h*(*n*) |
|---|---|---|
| *A* | 3 | 3 |
| *B* | 2 | 2 |
| *C* | 1 | 1 |
| *D* | 1 | 1 |
| *G* | 0 | 0 |

Trace:
ITR1 = [A((-),0+3)]
ITR2 = [B((A),1+2), C((A),8+1)]
ITR3 = [D((A,B),2+1), C((A),8+1)]
ITR4 = [G((A,B,D),3+0), C((A),8+1)]
ITR5 = DONE (A,B,D,G)

h*(*n*) = true path cost from *n* to nearest goal

# Completeness and Optimality

- Completeness
  - Same criteria as UCS
    - *b* finite, and *m* finite OR has a solution
    - All action costs > ε > 0
- Optimality
  - Depends on the properties of h

Ordered traversal of path in order of path costs required to ensure optimality

# Admissible Heuristics

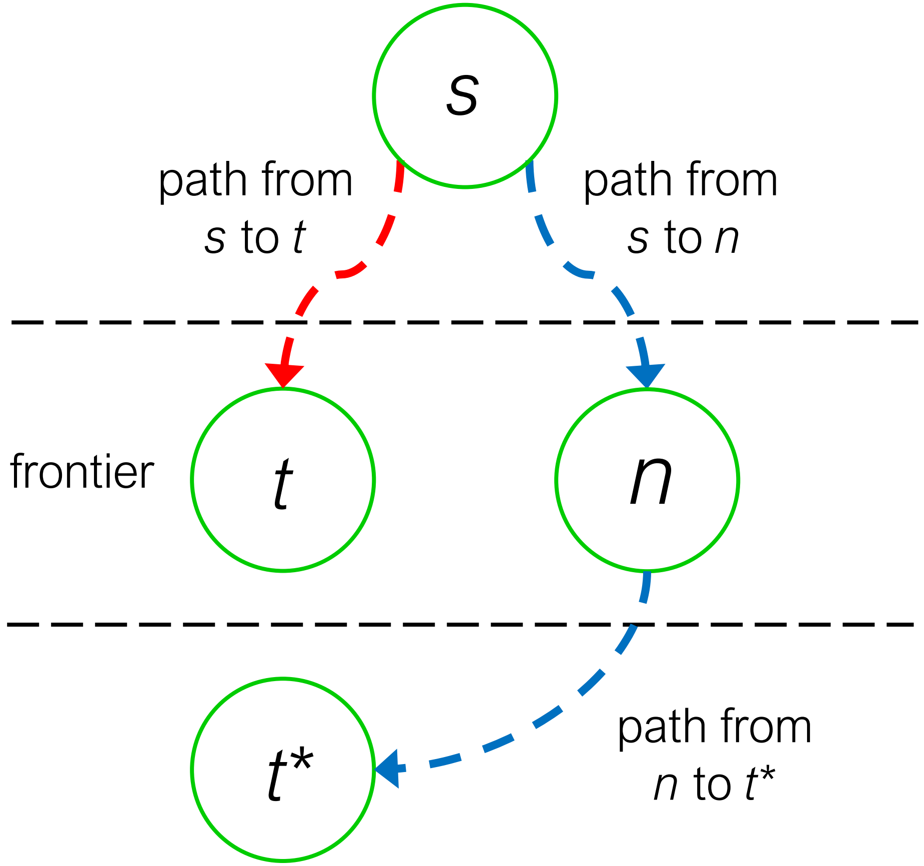
- h($n$) is ***admissible*** if $\forall n$, h($n$) $\leq$ h*($n$)
  - h($n$) never overestimates the cost
    - Implications
      - Paths not ending at a goal are never over-estimated
        - Evaluation function for non-goal is never over-estimated
        - At non-goal n, f($n$) = g($n$) + h($n$) $\leq$ g($n$) + h*($n$)
      - Paths ending at a goal are exact
        - Evaluation function of value of a goal is exact
        - At goal $m$, f($m$) = g($m$) + h($m$), where h($m$) = 0

Main idea, by the time we visit a path to a goal, $P$, all paths with actual costs less than $P$ must be searched

- Examples:
  - Euclidean distance in the maze environment (always underestimates)
- Theorem: If h($n$) is admissible, then A* using tree search is optimal

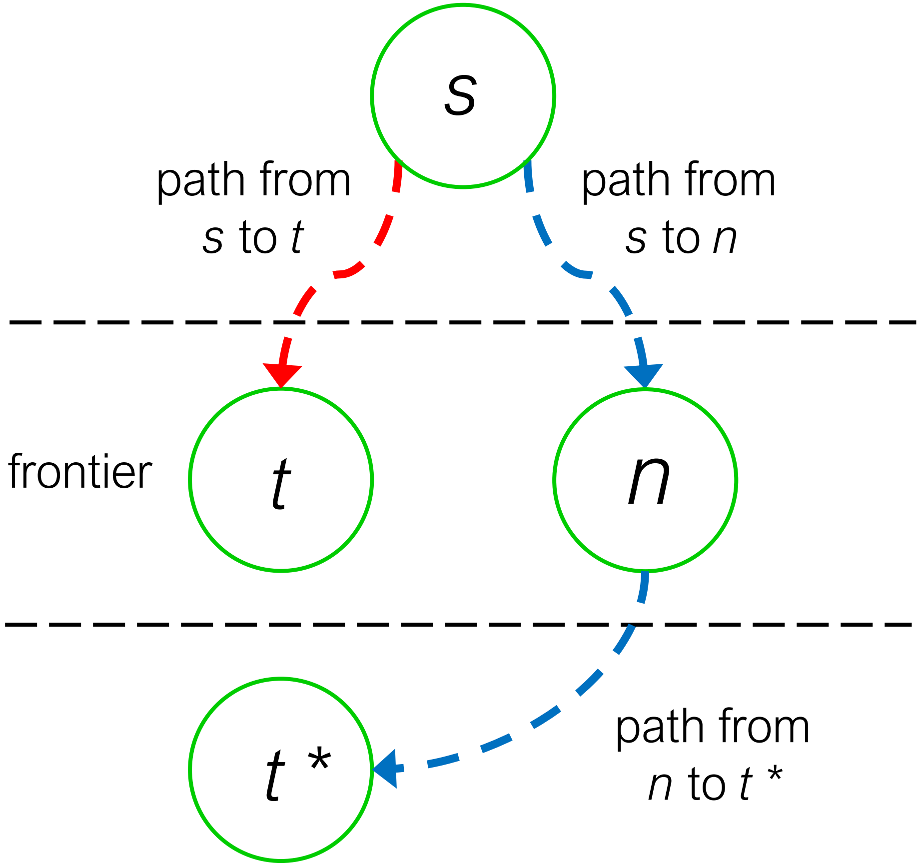
# Optimality of A* Using Tree Search

- Proving the Theorem
  - T02 Q2a
- Consider the following
  - A* is optimal → returns optimal path
    - Let *s* to *n* to *t** be the optimal path
  - If not optimal:
    - Must explore path to *t* first
    - Where *t* is a goal
  - Not optimal → explore *t* before *n*



# Optimality of Admissible A* Under Tree Search

- Assume non-optimal $t$ expanded before $n$ (on optimal path)
  - $f(t) < f(n)$
- Assuming tree-search
  - All paths searched
  - All sub-paths* along a single path to a goal must be searched before path including that goal
    - For non-goal $m$, $f(m) \leq g(m) + h^*(m)$ (since admissible)
    - If goal $m^*$ on path from $m$, $f(m) \leq f(m^*)$ (given ε)
  - Since $t^*$ is goal on optimal path
    - $f(n) < f(t^*) < f(t)$
    - *CONTRADICTION*




* Consider a path, $P$, from an initial state $s$ to a goal state $t$, to be $s > n_1 > n_2 > \ldots > n_k > t$
Let a sub-path to $P$, $P'$ be any path $s > n_1 > n_2 > \ldots > n_i$, where $1 \leq i \leq k$

# A* Using Graph Search

- Difference between tree search and graph search
  - Under admissibility and tree search
    - All nodes leading to a goal are expanded before the node representing the goal
    - Optimal path will be found
  - Under graph search we may skip some paths (due to no revisiting)
- Under graph search (Version 2), non-redundant paths never skipped
  - Graph search checks and allows revisits if path is non-redundant
    - As long as a path is cheaper, allow it onto the frontier even if must revisit
  - Still optimal since equivalent to tree search
- Under graph search (Version 1) may skip optimal path

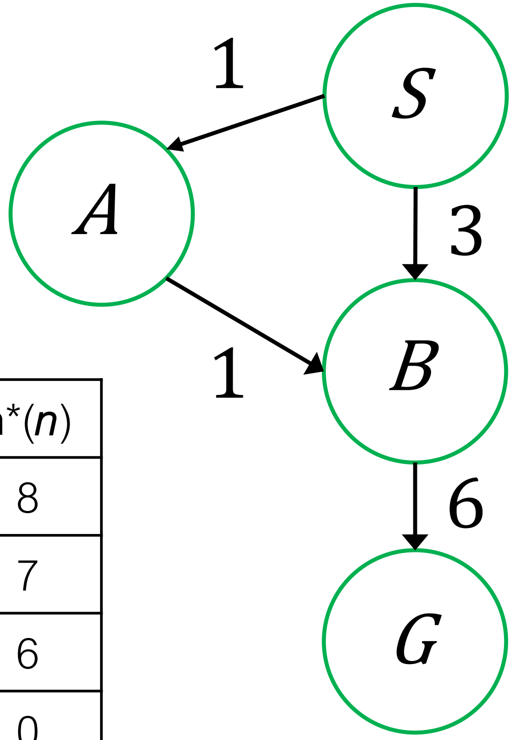
# Ensuring A* Optimal Under Graph Search (Version 1)?

- Will A* be optimal under graph search Version 1?
- Example

T02 Q3b - construct an alternative example

Assume this admissible h:

| *n* | h(*n*) | h*(*n*) |
|---|---|---|
| *S* | 8 | 8 |
| *A* | 7 | 7 |
| *B* | 0 | 6 |
| *G* | 0 | 0 |



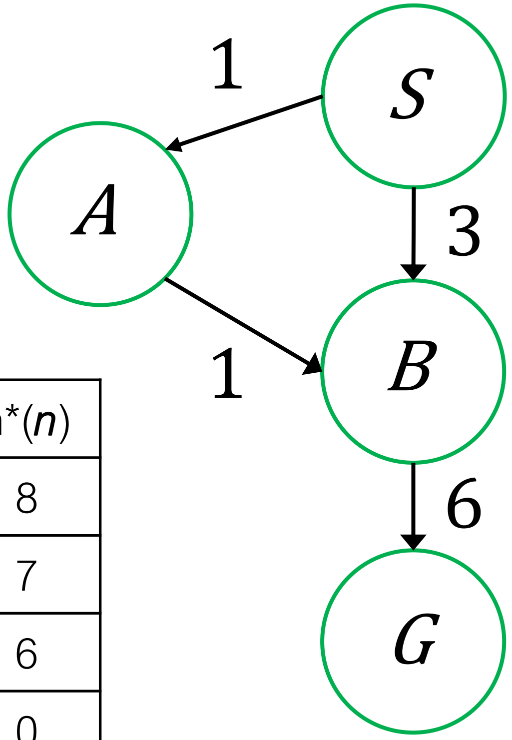

Trace:
ITR1 = [S((-),0+8)]
ITR2 = [B((S),3+0), A((S),1+7)]
ITR3 = [A((S),1+7), G((S,B),9+0)]
ITR4 = [G((S,B),9+0)] as B popped before, do not revisit
ITR5 = DONE (S,B,G) <u>not</u> the optimal path!

We need a tighter constraint on h

Similar to what UCS offers → contours of search progression - i.e., stricter ordering of paths

# Why Not Optimal?

- Consider the previous example




Assume this admissible h:

| *n* | h(*n*) | h*(*n*) |
|---|---|---|
| *S* | 8 | 8 |
| *A* | 7 | 7 |
| *B* | 0 | 6 |
| *G* | 0 | 0 |

Observe the sequence of f(*n*) = g(*n*)+ h(*n*) values along each path:

| path to ***n*** (from ***S***) | g(***n***)+ h(***n***) | g(***n***) + h*(***n***) | |
|---|---|---|---|
| *S* | 0+8 | 0+8 | |
| *S* > *A* | 1+7 | 1+7 | |
| *S* > *A* > *B* | 2+0 | 2+6 | Dip! |
| *S* > *A* > *B* > *G* | 8+0 | 8+0 | |

| path to ***n*** (from ***S***) | g(***n***)+ h(***n***) | g(***n***) + h*(***n***) | |
|---|---|---|---|
| *S* | 0+8 | 0+9 | |
| *S* > *B* | 3+0 | 3+6 | Dip! |
| *S* > *B* > *G* | 9+0 | 6 | |

Want h that ensures paths traversed in order of true path cost!

# Consistent Heuristics

- Forming contours
    - Under tree search g costs are monotonically increasing
        - f costs are not since we can underestimate h by differing amounts
        - But ALL paths with path costs less than the optimal path cost will be explored first
    - For f costs to be monotonically increasing along a path
        - Assume $n$ is a predecessor of $n'$ along a path
        - We need: $g(n) + h(n) \leq g(n) + cost(n, a, n') + h(n')$
        - And thus $h(n) \leq cost(n, a, n') + h(n')$
    - We will use the above requirement

Since true path to $n$ must be less than true path cost to $n'$, we want to ensure h maintains this ordering

Note that:
consistency ⇒ admissibility
- Proof – T02 Q3a

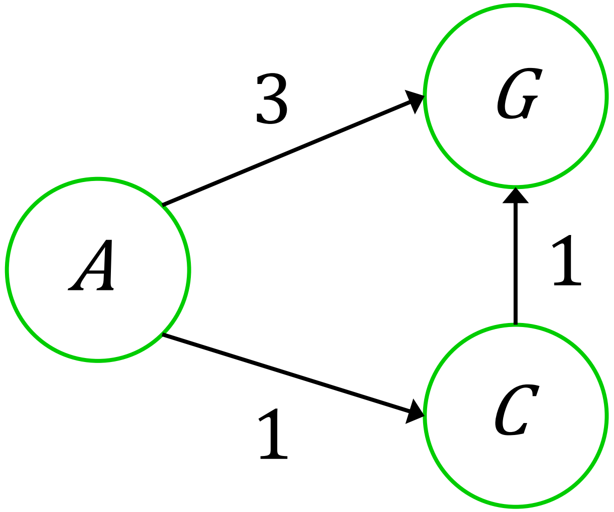
- h($n$) is ***consistent*** if $\forall n$, and successor of $n$, $n'$,

$$h(n) \leq cost(n, a, n') + h(n')$$

- Theorem: If h($n$) is consistent, then A* using graph search is optimal

# Still an Issue!

- Refer to the UCS example on Slide 14
- Assume consistent heuristic h*
  - With this example, we have
    - F = {A(g(A)=0 + h(A)=2)(-)}; R = {A}
      - pop A(0+2)(-), push C(1+1)(A) and G(3+0)(A)
    - F = {C(2)(A), G(3)(A)}; R = {A, C, G}
      - pop C(2)(A), cannot push G(2+0)(A,C) since already in R!
    - F = {G(3)(A)}; R = {A, C, G}
      - pop G(3)(A), path is A → G




Considering G as reached too early under graph search for A* is similar to applying Early Goal Test to UCS!

# Graph Search Algorithm (Version 3)

```
frontier = {Node(initial state), NULL}
reached = {}
while frontier not empty:
      current = frontier.pop()
      reached.insert(current.state: current)
      if isGoal(current.state): return current.getPath()
      for a in actions(current.state):
            successor = Node(T(current.state, a), current)
            if successor.state not in reached:
                  frontier.push(successor)
return failure
```

- Only adds a node to reached when it is popped
- Proof requires this version (given counterexample for Version 1 on Slide 38)
- Prove this in a similar manner to the UCS proof (contours) – T02 Q2b

# Dominant Heuristics

# Efficiency & Dominance

- Efficiency of A* depends on the accuracy of its heuristics
  - Higher heuristic accuracy means we need to try fewer paths
  - Specifics not covered in CS3243

- Which heuristics are better?

- If $h_1(n) \geq h_2(n)$ for all $n$, then $h_1$ ***dominates*** $h_2$
  - If $h_1$ is also *admissible*
    - $h_1$ must be closer to $h^*$ than $h_2$
    - $h_1$ must be more efficient than $h_2$

Note: with typical interpretations, dominance requires admissibility. We apply this in CS3243.

# Questions about the Lecture?

- Do you need to clarify anything?
- Ask on Canvas > CS3243 > Discussion > Week 3 Content