Local Search: Goal Versus Path Search

CS3243: Introduction to Artificial Intelligence – Lecture 5a

6 February 2023

Contents

Reference: AIMA 4th Edition, Section 4.1 & Section 5.1

Administrative Matters

Midterm Examination

- Schedule
  - Week 7 Lecture Slot
  - Monday (27 FEB), 1030-1130 hrs (Arrive by 1010 hrs)
- Venue
  - MPSH1a (Conducted in-person)
- Format
  - Duration = 1 hour
  - Total = 30 marks
  - Closed-book + Cheat Sheet (1 × Double-sided A4 Sheet)
  - Lectures 1-5 (i.e., everything up to and including this lecture)
- Practice Papers
  - Canvas > CS3244 > Files > Past Papers

Consultations

- Project 1
  - Consultation recording → Canvas
  - Important notes on grid representation → Canvas
  - For more support → Message TA
  - Last resort → Email me (dler@comp.nus.edu.sg)

- Midterm
  - Review past midterm papers
  - Message TAs for clarifications

Upcoming…

- Deadlines
  - TA3 (released last week)
    - *Due in your Week 5 tutorial session*
    - *Submit the a physical copy (more instructions on the Tutorial Worksheet)*
  - Prepare for the tutorial!
    - Participation marks = 5%
  - Project 1
    - *Due next Sunday (19 February), 2359 hrs*

Goal Versus Path Search

Slightly Different Problems

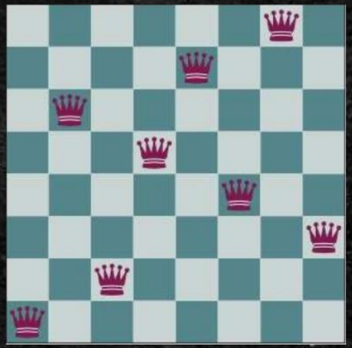
- Thus far: finding a path to a goal
  - Algorithms track paths
  - Systematically search paths
- What if only interested in goal state?
  - Have goal test, but not values to satisfy it
  - Only want goal state values

  - Optimisation problems
    - Vertex cover problems
    - Boolean satisfiability problems (SAT)
    - Travelling salesman problem
    - Timetabling / scheduling problems

- Sudoku

		3					9	
	1			7		2		4
4					1		5	
			9			3		
	8			1			7	
		6			4			
	3		5					7
9		5		8			6	
	7					4		

- n-queens



Path Versus Goal

- Search problems – path planning
  - Path to a goal necessary
  - Path cost is important
- Local search – goal determination
  - Abandon systematic search – ignore path (and path cost)
  - Maintain “best” successor state – greedy approach
- Advantages
  - Only store current and immediate successor states
    - Space complexity: O(b)
      - Note that space complexity may be reduced to O(1) if successors may be processed one at a time
  - Applicable to very large or infinite search spaces

Path planning can satisfy the objective of goal search but does more than it needs to since we don’t need the path

		3					9	
	1			7		2		4
4					1		5	
			9			3		
	8			1			7	
		6			4			
	3		5					7
9		5		8			6	
	7					4		

Local Search is incomplete

Local Search via Hill-Climbing

Hill-Climbing Algorithm

```
current = initial_state
while true:
    neighbour = highest_valued_successor(current)
    if value(neighbour) ≤ value(current): return current
    current = neighbour
```

- How it works (steepest ascent – greedy strategy)
  - Starts with a *random* initial state (typically) – more on this later
  - Only store the current state
  - In each iteration, find a successor that ***improves*** on current state
    - Requires **actions** and **transition** to determine successors
    - Requires **value**; a way to value each state – e.g., f(n) = -h(n)
  - If none exists, return current state as the best option
    - This algorithm *can fail*; may return a non-goal state

Requires heuristic (similar to informed search heuristic)

8-Queens Example

- Given an 8×8 chess board
  - Place 8 queens
  - No queen must threaten another
  - Use h: *pairs of queens threatening each other*
- Search problem
  - State: 1 queen per column
  - Action: move 1 queen to different col. position
  - Goal: 0 pairs threatening
- Example h
  - Consider top-most left-most cell (h-value is 18)

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C1 (now in top-most left-most call) attacks C4, C5, C6, C7 [4]

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C6 attacks C7, C8 [2]
C7 attacks C8 [1]

Complete-State Formulations

- States in the 8-Queens search problem have all 8 queens present
- Every state has all components of a solution
  - No *partially completed* states
  - All actions perturb current state by 1 move
- Each state is a potential solution
  - Apt for problems where path is not important
    - Simply "guess" a solution
    - "Check" its value
    - Make a "systemic guess" by moving to states of higher value (e.g., via $f(n) = -h(n)$)
      - Assumes that states with higher f values are closer to the goal (i.e., more likely to reach a goal)
- Most local search problems may be formulated in this manner

Practically, it is fine to use $f(n) = h(n)$ and seek a local minima as well. In such cases, we simply replace the $\leq$ in the algorithm with $\geq$.

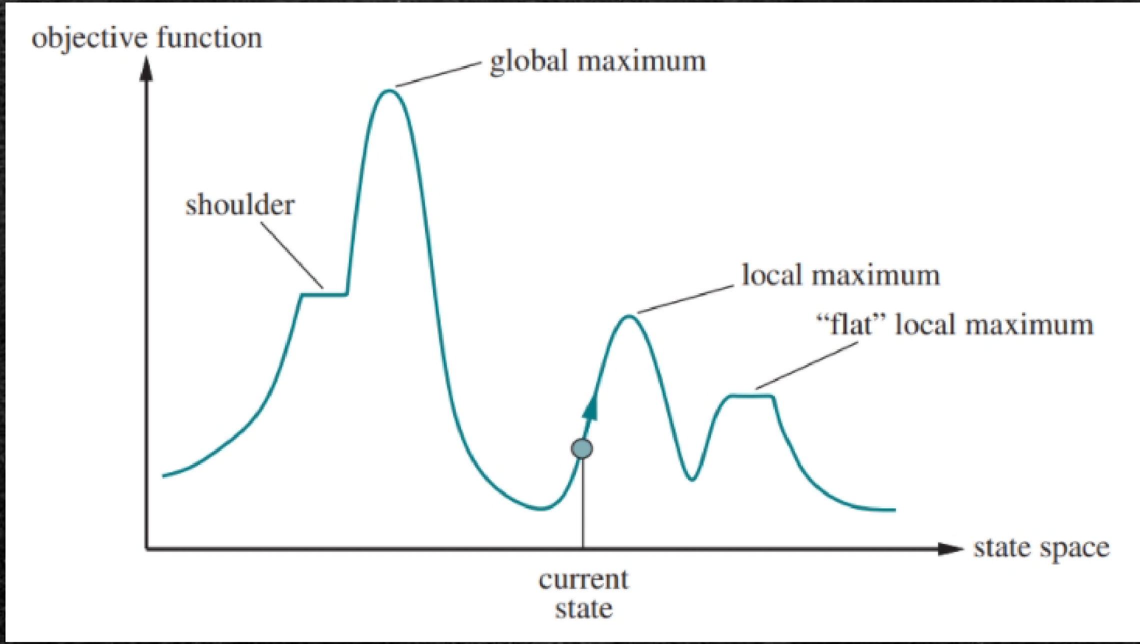
Hill-Climbing Algorithm (Revisited)

```
current = initial_state
while true:
        neighbour = highest_valued_successor(current)
        if value(neighbour) ≤ value(current): return current
        current = neighbour
```

- NOT guaranteed to find a goal!
  - **value** defined by informed search heuristic, h; e.g., f(n) = -h(n)
  - Goal → h(n) = 0
- What happens if the returned state is not a goal state?
- When does this happen?

Issues & the Potential for Failure

- Hill-climbing may not return a solution



- May get stuck at
  - Local Maxima
  - Shoulder or Plateau
  - Ridge (sequence of local maxima)
- Require strategies to counter these problems

Hill-Climbing Variants

- Stochastic hill climbing
  - Changes **`highest_valued_successor`**`(..)`
  - Chooses randomly among states with values better than current
  - May take longer to find a solution but sometimes leads to better solutions

- First-choice hill climbing
  - Changes **`highest_valued_successor`**`(..)`
  - Handles high b by randomly generating successors until one with better value than current is found (instead of generating all possible successors)

Hill-Climbing Variants

- Sideways move
  - Replaces ≤ with <; allows continuation when `value(neighbour) == value(current)`
  - Can traverse shoulders / plateaus

- Random-restart hill climbing
  - Different algorithm
  - Adds an outer loop which randomly picks a new starting state
  - Keeps attempting random restarts until a solution is found

Random Restarts Hill-Climbing Algorithm

```
current = random_initial_state()
while not isGoal(current):
      while true:
            neighbour = highest_valued_successor(current)
            if value(neighbour) < value(current):
                  return current
            current = neighbour
      current = random_initial_state()
```

- Changes from the Hill-Climbing Algorithm
  - Requires function to generate random initial state: ***random_initial_state***()
  - Utilises **isGoal**; if goal not found then loops with a random restart
  - Considers sideways moves since it utilises < instead of ≤

Back to 8-Queens: Analysis

- Hill climbing (via steepest-ascent) with random restarts
  - Solution: p_1 = 14% (expected solution in 4 steps; expected failure in 3 steps)
  - Expected computation = $1 \times$(steps for success) + $((1 - p_1) / p_1) \times$(steps for failure)
    = $1 \times (4)$ + $(0.86/0.14) \times (3)$
    = 22.428571428571427 steps

$(1 - p_1) / p_1$ determines the expected number of failed attempts

- Adding sideways moves
  - Solution: p_2 = 94% (expected solution in 21 steps; expected failure in 64 steps)
  - Expected computation = $1 \times$(steps for success) + $((1 - p_1) / p_1) \times$(steps for failure)
    = $1 \times (21)$ + $(0.06/0.94) \times (64)$
    = 25.085106382978722 steps

- 8-Queens possible states = 8^8 = 16777216

Extremely efficient for such a large space

Expected values taken from AIMA pp. 131

Local Beam Search

Local Beam Search

- Store k states instead of 1
  - Hill climbing just stores the current state
  - Beam (window) stores k
- Algorithm
  - Begins with k random starts
  - Each iteration generates successors for each of the k random start states
  - Repeat with best k among ALL generated successors unless goal found
- Better than k parallel random restarts
  - Since best k among ALL successors taken (not best from each set of successors, k times)
- Stochastic beam search
  - Original variant may still get stuck in a local cluster
  - Adopt stochastic strategy similar to stochastic hill climbing to increase state diversity

Questions about the Lecture?

- Was anything unclear?
- Do you need to clarify anything?

- Ask on Archipelago
  - Specify a question
  - Upvote someone else's question

Invitation Link (Use NUS Email --- starts with E)
https://archipelago.rocks/app/resend-invite/12384352999

Constraint Satisfaction Problems: Generalising Goal Search I

CS3243: Introduction to Artificial Intelligence – Lecture 5b

6 February 2023

Systematic Goal Search

- With local search we apply greedy search strategies
  - Are there more ***systematic*** search strategies applicable?
- Issues with systematic searching
  - Systematic approaches tend to be computationally expensive
    - Incorporating domain knowledge via heuristics helped direct the search such that less was searched
    - Need to reduce the search space to make a systematic search more viable
- A general solution
  - Use a factored representation for each state
    - State: set of variables $X = \{x_1, \ldots, x_n\}$, where each variable x_i has a domain $D_i = \{d_1, \ldots, d_m\}$
  - Divide the goal test into a set of constraints
    - If a state satisfies all constraints, it is a goal state
  - Constraint satisfaction problem (CSP)
    - Any state that does not satisfy a constraint should not be further explored

CSPs systematically search for goal states by pruning invalid subtrees as early as possible

CSP Formulation

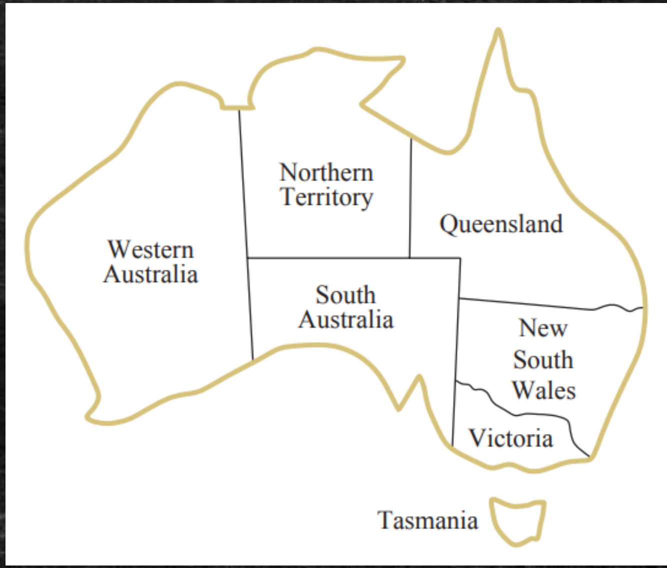
Formulating CSPs

- State representation
  - Variables: $X = \{x_1, ..., x_n\}$
  - Domains: $D = \{d_1, ..., d_k\}$
    - Such that x_i has a domain d_i
  - Initial state: all variables unassigned
  - Intermediate state: partial assignment
- Goal test
  - Constraints: $C = \{c_1, ..., c_m\}$
    - Defined via a constraint language
      - Algebra, Logic, Sets
    - Each c_i corresponds to a requirement on some subset of X
- Actions, costs and transition
  - Assignment of values (within domain) to variables
  - Costs are not utilised

  - Objective is a ***complete*** and ***consistent*** assignment
    - Find a legal assignment $(y_1, ..., y_n)$
      - $y_i \in d_i$ for all $i \in [n]$
    - Complete: all variables assigned values
    - Consistent: all constraints C satisfied

CSP Formulation Example 1: Graph Colouring

- Colour each state of Australia such that no two adjacent states share the same colour
- Variables
  - $X = \{ WA, NT, Q, NSW, V, SA, T \}$
- Domains
  - $d_i = \{ \text{Red, Green, Blue} \}$
- Constraints
  - $\forall (x_i, x_j) \in E$, $\texttt{colour}(x_i) \neq \texttt{colour}(x_j)$



CSP Formulation Example 2: Cryptarithmetic Puzzle

- Given that each letter represents a digit, determine the letter-digit mapping that solves the given sum

$$\begin{array}{cccc} & T & W & O \\ + & T & W & O \\ \hline F & O & U & R \end{array}$$

- Variables
  - $X = \{ T, W, O, F, U, R, B_1, B_2, B_3 \}$
  - Where B_1, B_2, B_3 are carry bits for ($2O$, $2W$, $2T$ respectively)
- Domains
  - $d_i = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 \}$
  - Strictly, B_1, B_2, B_3 should have domain $\{0, 1\}$
- Constraints
  - **`alldiff`**(T, W, O, F, U, R)
  - $O + O = R + 10.B_1$
  - $B_1 + W + W = U + 10.B_2$
  - $B_2 + T + T = O + 10.B_3$
  - $B_3 = F$
  - $T, F \neq 0$

CSP Formulation Example 3: Sudoku

- Variables
  - $X = \{ A_1, \ldots, A_9, \ldots, I_1, \ldots, I_9 \}$
  - 81 variables
- Domains
  - $d_i = \{ 1, 2, 3, 4, 5, 6, 7, 8, 9 \}$
- Constraints
  - **`alldiff`**(...)
    - 27 cases
      - 9 columns
      - 9 rows
      - 9 boxes

	1	2	3	4	5	6	7	8	9
A			3		2		6		
B	9			3		5			1
C			1	8		6	4		
D			8	1		2	9		
E	7								8
F			6	7		8	2		
G			2	6		9	5		
H	8			2		3			9
I			5		1		3		

	1	2	3	4	5	6	7	8	9
A	4	8	3	9	2	1	6	5	7
B	9	6	7	3	4	5	8	2	1
C	2	5	1	8	7	6	4	9	3
D	5	4	8	1	3	2	9	7	6
E	7	2	9	5	6	4	1	3	8
F	1	3	6	7	9	8	2	4	5
G	3	7	2	6	8	9	5	1	4
H	8	1	4	2	5	3	7	6	9
I	6	9	5	4	1	7	3	8	2

Variable Domain Types & Constraint Types

- Variable domain types
  - Continuous
  - Discrete
  - Finite
  - Infinite

  - Continuous and Infinite
    - Real values
  - Discrete and Infinite
    - All integers
  - Discrete and finite
    - Sudoku

CS3243 focuses on discrete, finite domains

- Constraint types
  - Linear
  - Nonlinear

Continuous domain and linear constraints → linear programming

Not covered in CS3243

More on Constraints

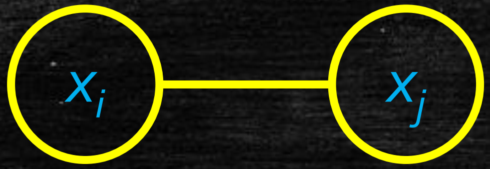
- A language is necessary to express the constraints
  - Arithmetic
  - Sets (of legal values)
  - Logic
  - For example, x_1 greater than x_2 given $d = \{1, 2, 3\}$ may be written
    - $\langle (x_1, x_2), x_1 > x_2 \rangle$
    - $\langle (x_1, x_2), \{ (2, 1), (3, 1), (3, 2) \} \rangle$
- Each constraint, c_i,
  - Describes the necessary relationship, ***rel***, between a set of variables, ***scope***
    - For the example above, ***scope*** = (x_1, x_2). ***rel*** = $x_1 > x_2$
- Types of constraints
  - Unary: | ***scope*** | = 1
  - Binary: | ***scope*** | = 2
  - Global: | ***scope*** | > 2 (i.e., higher-order constraints)

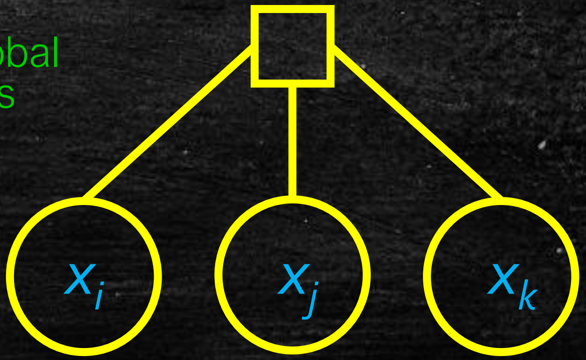
Constraint Graphs

Drawing Constraint Graphs and Hypergraphs

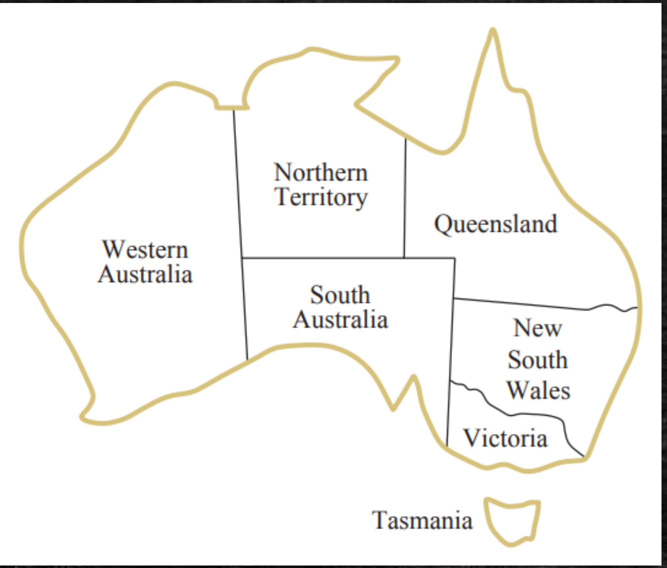
- Constraint graphs represent the constraints in a CSP
  - Simple Vertex: variable 

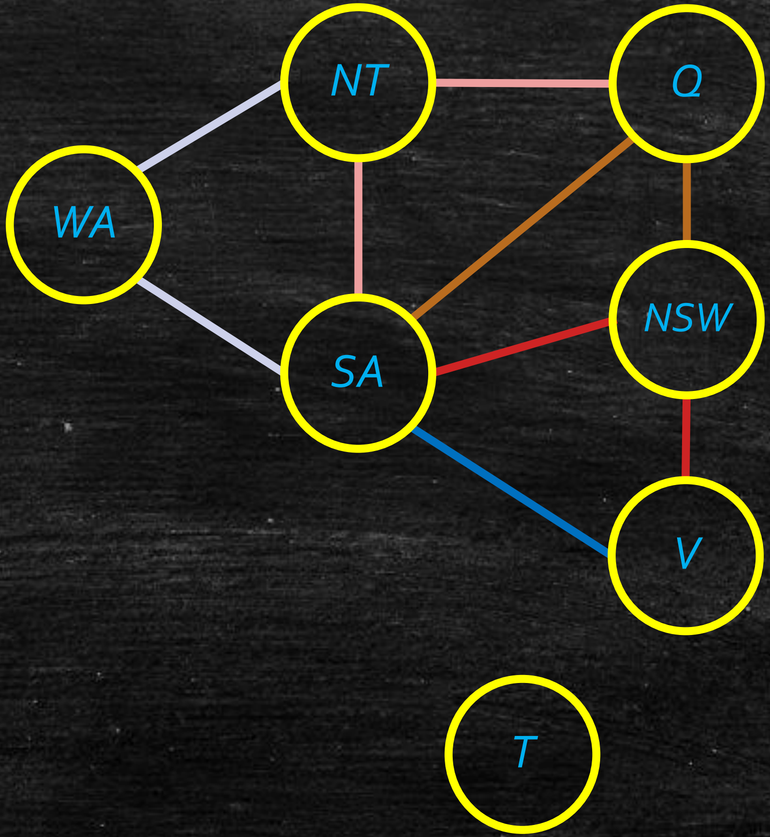
  - Linking Vertex: for global constraints 
  - Edge: links all variables in the scope of a constraint (*rel*)
    - Unary constraints 

    - Binary constraints 

    - Binary/Global constraints 

Constraint Graph for Example 1: Graph Colouring





Constraint Graph for Example 2: Cryptarithmetic Puzzle

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- Constraints
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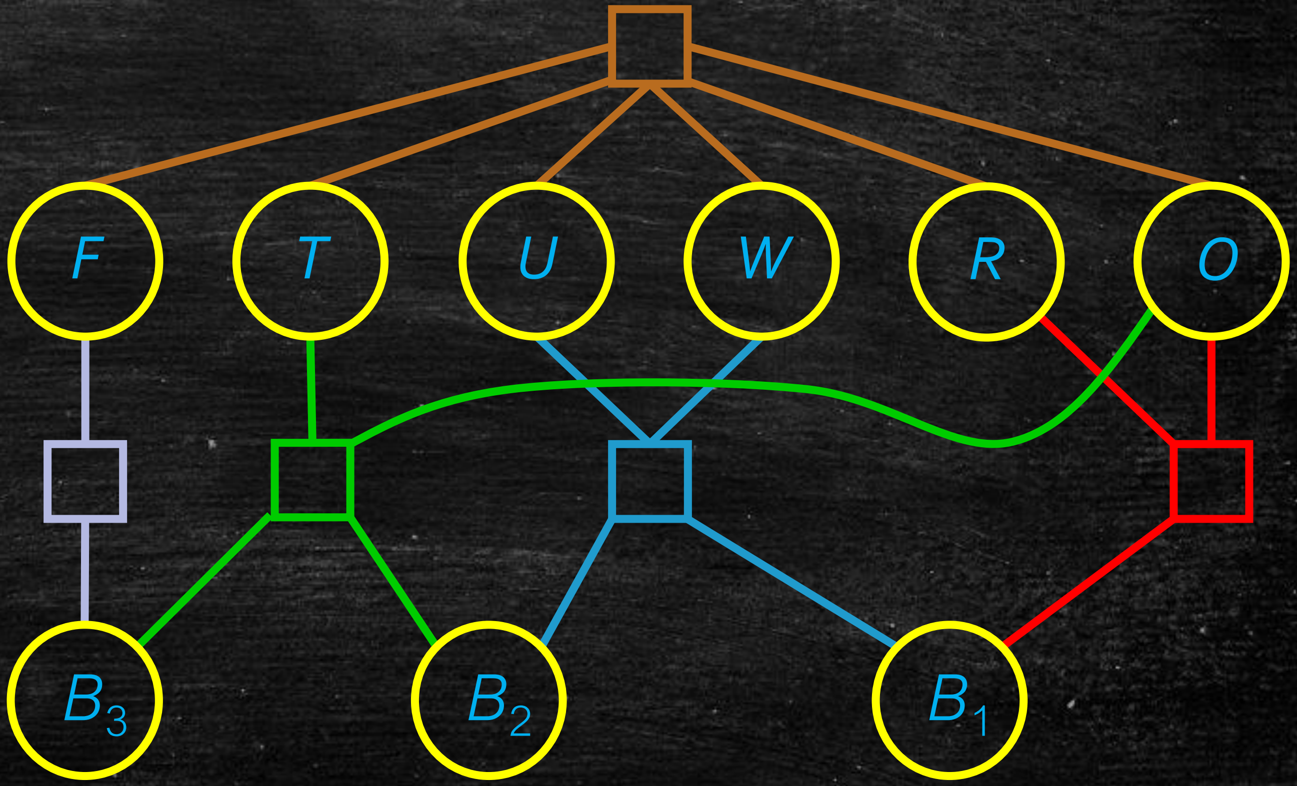


A First Look at an Algorithm for CSPs

General Idea for the Algorithm

```
assignments = initial state (no assignments made)
while assignments incomplete:
      if no possible assignments left return failure
      current = assign a value to non-assigned variable
      if current consistent then assignments.store(current)
return assignments
```

- Applicable to all CSPs
- Search path irrelevant
  - May use complete-state formulation
- All solutions require |X| = n assignments

Which algorithm should be used?

DFS

Search Tree Size

- Example CSP
  - X = {A, B, C, D}
  - All domains: d = {1, 2, 3}
  - No constraints

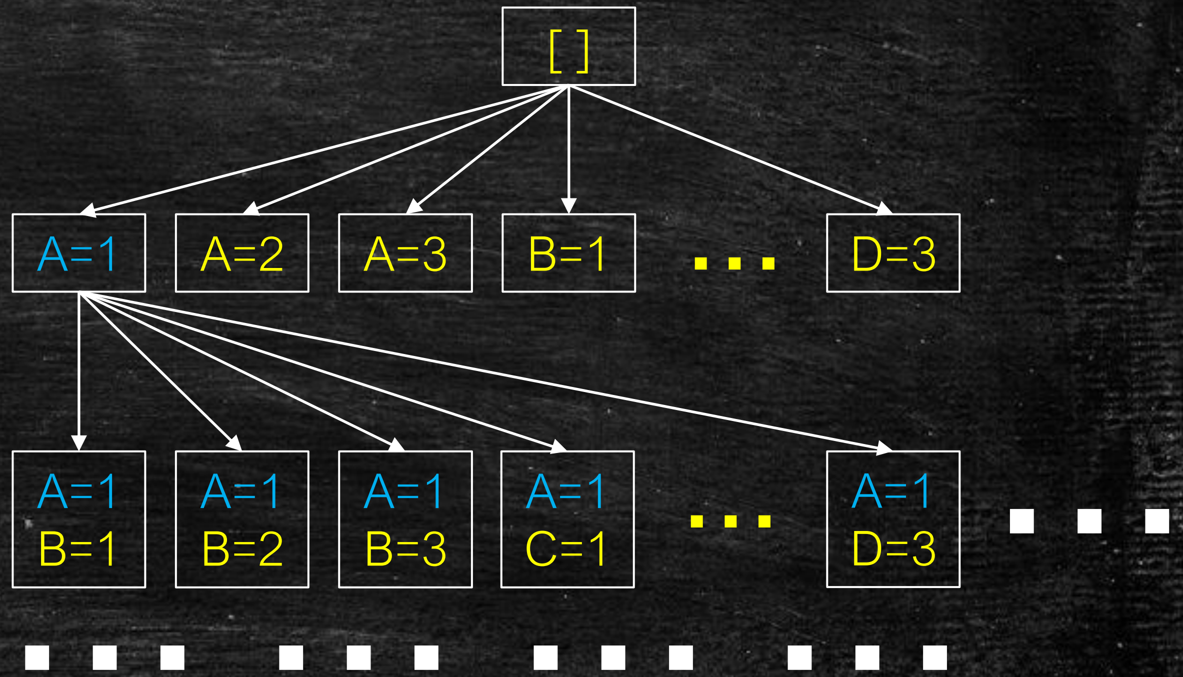
- Analysis

  b at depth 1: 4 variables × 3 values = 12 states
  b at depth 2: 3 variables × 3 values = 9 states
  b at depth 3: 2 variables × 3 values = 6 states
  b at depth 4: 1 variables × 3 values = 3 states

  At depth ℓ : $(|X| - \ell).|d|$ states

  Total number of leaf states:
  $nm \times (n-1)m \times (n-2)m \times \ldots \times 2m \times m = n!m^n$

  where $n = |X|$ and $m = |d|$



Order of variable assignments not important
Just consider assignments to ONE variable per level (m^n leaves)

Basic uninformed search for CSPs: Backtracking
Backtrack when no legal assignments

Backtracking Algorithm for CSPs

```
function BACKTRACKING-SEARCH(csp) returns a solution or failure
  return BACKTRACK(csp, { })

function BACKTRACK(csp, assignment) returns a solution or failure
  if assignment is complete then return assignment
  var ← SELECT-UNASSIGNED-VARIABLE(csp, assignment)
  for each value in ORDER-DOMAIN-VALUES(csp, var, assignment) do
     if value is consistent with assignment then
        add {var = value} to assignment
        inferences ← INFERENCE(csp, var, assignment)
        if inferences ≠ failure then
           add inferences to csp
           result ← BACKTRACK(csp, assignment)
           if result ≠ failure then return result
           remove inferences from csp
        remove {var = value} from assignment
  return failure
```

Determine the variable to assign to

Determine the value to assign

Trying to determine if the chosen assignment will lead to a terminal state

Continues recursively as long as the ***assignment*** is ***viable***

We will look into making these choices in the next lecture

Questions about the Lecture?

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Invitation Link (Use NUS Email --- starts with E)
https://archipelago.rocks/app/resend-invite/12384352999