

## NATIONAL UNIVERSITY OF SINGAPORE

SCHOOL OF COMPUTING

MIDTERM ASSESSMENT FOR  
Semester 2 AY2017/18

CS3243: INTRODUCTION TO ARTIFICIAL INTELLIGENCE

March 5, 2018

Time Allowed: 1 Hour 30 Minutes

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**INSTRUCTIONS TO CANDIDATES**

1. This examination paper contains **THREE (3)** parts and comprises **7** printed pages, including this page.
2. Answer **ALL** questions as indicated.
3. This is a **RESTRICTED OPEN BOOK** examination.
4. Please fill in your **Matriculation Number** below; **DO NOT WRITE YOUR NAME**.

MATRICULATION NUMBER: \_\_\_\_\_

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EXAMINER'S USE ONLY		
Part	Mark	Score
I	14	
II	20	
III	16	
TOTAL	50	

In Parts I, II, and III, you will find a series of short essay questions. For each short essay question, give your answer in the reserved space in the script.

## Part I

### Constraint Satisfaction Problems

(14 points) Short essay questions. Answer in the space provided on the script.

Suppose that we are given a binary CSP problem with variables  $X_1, \dots, X_n$ . The domains of  $X_1, \dots, X_n$  are all binary (so  $D_i = \{0, 1\}$  for all  $i = 1, \dots, n$ ); we have  $n - 1$  constraints such that the constraint  $C_i$  ( $i \in \{1, \dots, n - 1\}$ ) depends only on  $X_i$  and  $X_{i+1}$ .

1. (2 points) Draw the constraint graph for the above CSP, assuming that  $n = 5$ .

**Solution:**

2. (2 points) What is the most constraining variable in the above CSP?

**Solution:**

3. (10 points) Show that there exists a poly-time (in  $n$  the number of variables) algorithm that computes a complete and consistent assignment (or outputs that no such assignment exists) for the CSP described above.

**Solution:**

## Part II

### Adversarial Search

(20 points) Short essay questions. Answer in the space provided on the script.

- (5 points) Consider the minimax search tree shown in the solution box below. In the figure, black nodes are controlled by the MAX player, and white nodes are controlled by the MIN player. Payments (terminal nodes) are squares; the number within denotes the amount that the MIN player pays to the MAX player (an amount of 0 means that MIN pays nothing to MAX). Naturally, MAX wants to maximize the amount they receive, and MIN wants to minimize the amount they pay.

Suppose that we use the  $\alpha$ - $\beta$  pruning algorithm, given in Figure 5.7 of AIMA 3rd edition (reproduced in Figure 1), and go over nodes from **right to left** in the search tree. **Mark (with an 'X')** all ARCS that are pruned by  $\alpha$ - $\beta$  pruning, if any.

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```

function ALPHA-BETA-SEARCH(state) returns an action
   $v \leftarrow \text{MAX-VALUE}(\text{state}, -\infty, +\infty)$ 
  return the action in ACTIONS(state) with value v

```

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```

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
   $v \leftarrow -\infty$ 
  for each a in ACTIONS(state) do
     $v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(\text{RESULT}(s, a), \alpha, \beta))$ 
    if  $v \geq \beta$  then return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v

```

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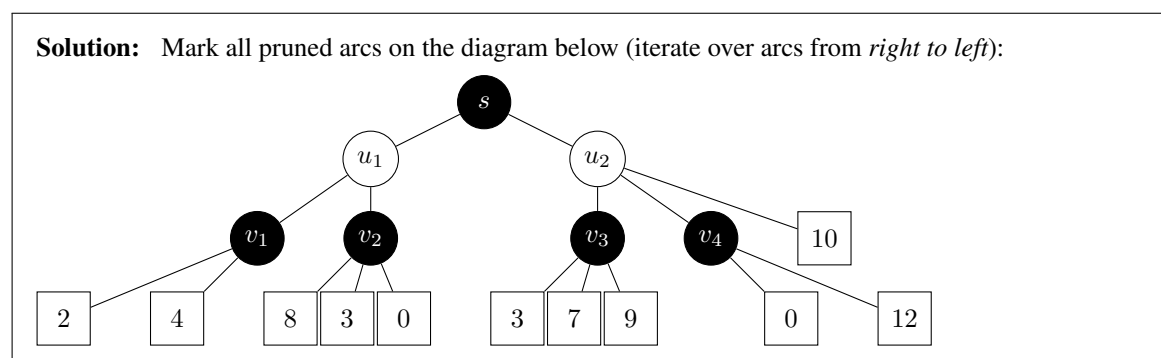
```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
   $v \leftarrow +\infty$ 
  for each a in ACTIONS(state) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(\text{RESULT}(s, a), \alpha, \beta))$ 
    if  $v \leq \alpha$  then return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v

```

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Figure 1: Alpha-beta pruning algorithm (note that  $s = \text{state}$ ).

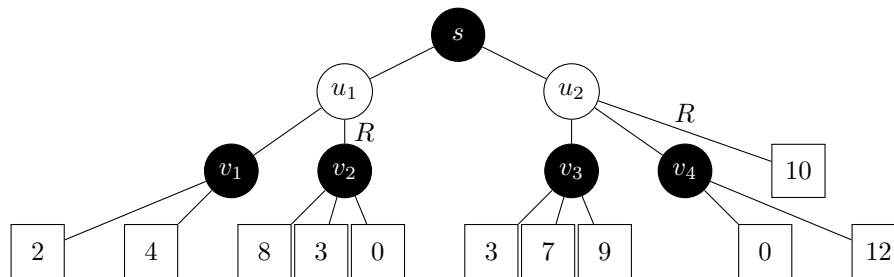


2. (3 points) State the **EXACT** minimax value at the root node.

**Solution:**

3. (4 points) Suppose that the MIN player has decided before playing the game to perform action  $R$  (i.e. choose the rightmost action) in its turn, as shown below. However, the MAX player does not know about this before playing the game and assumes that the MIN player is acting optimally. State MAX player's **EXACT** payoff value when starting from the root of the tree.

**Solution:**



**Payoff to MAX:** \_\_\_\_\_

4. (5 points) Consider the minimax search tree shown below; the utility function values are specified with respect to the MAX player and indicated at all the leaf (terminal) nodes. Suppose that we use  $\alpha$ - $\beta$  pruning algorithm, given in Figure 5.7 of AIMA 3rd edition (reproduced in Figure 1), in the direction from **right to left** to prune the search tree. State the **largest possible integer values** for  $A$  and  $B$ , and the **smallest possible integer value** for  $C$  such that **NO** arcs/nodes are pruned by  $\alpha$ - $\beta$  pruning.

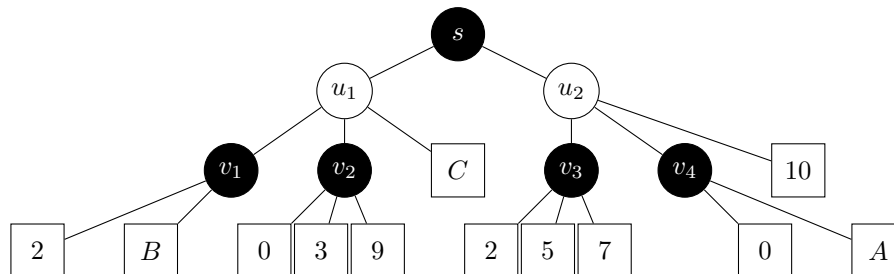


Figure 2: Minimax search tree.

**Solution:**  $A \leq$  \_\_\_\_\_  $B \leq$  \_\_\_\_\_  $C \geq$  \_\_\_\_\_

5. (3 points) Using the **largest possible integer values** for  $A$  and  $B$ , and the **smallest possible integer value** for  $C$  in the minimax search tree shown in Figure 2, state the **EXACT** minimax value at the root node.

**Solution:**

## Part III

### Uninformed and Informed Search

(16 points) Short essay questions. Answer in the space provided on the script.

We are given a graph  $G = \langle V, E \rangle$  with weighted edges;  $G$  is a road map, so given an edge  $(n, n') \in E$ , the cost of moving from  $n$  to  $n'$  is the length of the road connecting  $n$  and  $n'$  (assumed to be  $\infty$  if there is no edge between  $n$  and  $n'$ ). In addition, each node  $n$  has a coordinate  $(x_n, y_n)$  denoting its physical location on the map. The graph  $G$  has a unique goal node  $G^* \in V$  whose coordinates are  $(x^*, y^*)$ ; we have seen the heuristic

$$h_{SLD}(n) = \sqrt{(x_n - x^*)^2 + (y_n - y^*)^2}.$$

Consider the following two heuristic functions

1.  $h_1(n) = \max\{|x_n - x^*|, |y_n - y^*|\}.$
  2.  $h_2(n) = |x_n - x^*| + |y_n - y^*|$
1. (4 points) What is the relationship between  $h_1$  and  $h_{SLD}$ ? In other words, is it the case that: (a) for all  $n \in V$ ,  $h_1(n) \leq h_{SLD}(n)$  (b) for all  $n \in V$ ,  $h_1(n) \geq h_{SLD}(n)$  (c) neither always holds for all  $n$ ?

**Solution:**

2. (4 points) Is  $h_1(n)$  is an admissible heuristic? If yes, prove it; otherwise, provide a counterexample.

**Solution:**

3. (3 points) Is  $h_2$  an admissible heuristic? If so, prove it; otherwise, provide a counterexample.

**Solution:**

4. (5 points) Suppose next that all roads on the map are on a square grid; in other words, a road between any two nodes comprises of horizontal (going east-west) or vertical (going north-south) sections (see Figure 3). You can assume that every such road has an integer length (measured in kilometers). Does this change your answer regarding the admissibility of  $h_2$ ? Explain your answer.

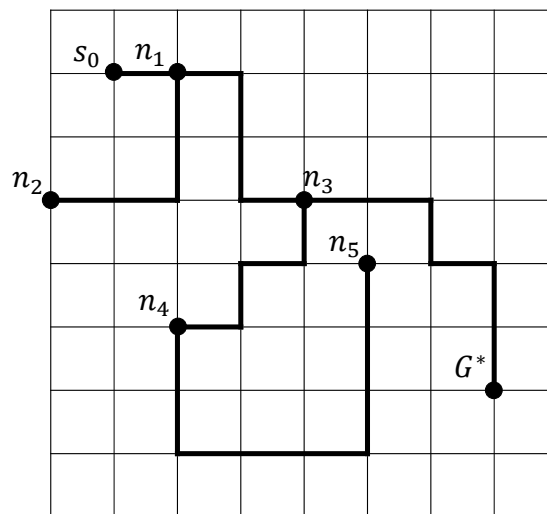


Figure 3: An example of a graph where all roads (thick lines) follow a grid (assume that every square in this example is  $1 \times 1$  km). For example, the road distance between  $n_3$  and  $G^*$  is 6km.

**Solution:**