# CS3243 : Introduction to Artificial Intelligence

Tutorial 3

NUS School of Computing

February 6, 2023

# Admin

- Extra material for practice

# Review

- Heuristics, more importantly on how do we come up with them
- Dominance, and how it helps (as we discussed last time)
- Relaxed problems, the recipe for obtaining heuristics

# Review

| | |
|---|---|
| Admissibility | $h$ is admissible iff $\forall n: h(n) \leq h*(n)$<br>An admissible heuristic will never overestimate the cost to reach the goal |
| Consistency | $h$ is consistent, iff for every node $n$ and every successor $n'$ of $n$ generated by action $a$<br>$h(n) \leq c(n, a, n') + h(n')$ |
| Dominance | $h_1(n)$ dominates $h_2(n)$ iff $\forall n: h_1(n) \geq h_2(n)$ |

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- Relaxed Version 1 : A tile can move from square $X$ to square $Y$ ~~if $X$ is adjacent to $Y$ and $Y$ is blank~~
- Relaxed Version 2 : A tile can move from square $X$ to square $Y$ if $X$ is adjacent to $Y$ ~~and $Y$ is blank~~

# Tutorial Question 1

- Design Admissible Heuristics for SameGame Puzzle
- Given information :
  - Initial state : Grid with $n \times m$ tiles with $c$ different colours
  - Neighbors : Directly adjacent tiles (Internal tiles : 4 neighbors, Edge tiles : 3 neighbors, Corner tiles : 2 neighbors)
  - Group : Set of $\geq 2$ neighboring tiles of same color
  - Singleton : Tile not belonging to any group
  - Action/Move : Deleting a group (not singleton). Thereafter, vertical gravity and column shifting (right-to-left) applies (in transition model)
  - Goal state : Empty grid (assume solvable)
  - Transition cost : 1 (or $\infty$ if no groups exist)

# Tutorial Question 1

- An example of the initial state :

- Top-left corner is $(1, 1)$, $1 \leq ROW \leq n$, and $1 \leq COL \leq m$

# Tutorial Question 1

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- Idea - Admissible : $\forall n,\ h(n) \leq h^*(n)$
- We have two approaches
- Approach 1 : By reasoning/inference (ie What information could I use at each state to get a estimate of the number of moves needed to reach the goal? Important : Must be an underestimate!)
- Approach 2 (better approach) : Relaxation of the game rules (ie. I now need less moves than what I originally need to reach the goal because the game has become "easier" = admissible. Important : We should be able to easily obtain the number of moves needed with the relaxed rules)

# Tutorial Question 1

- Proof of admissibility : $h(s) =$ Number of colours remaining.
- Each group contains exactly 1 colour
- Each remaining color has 1 or more groups
- Each move may reduce groups by more than 1 (example when groups of same color combine), but will never reduce remaining groups of same color below 1 (unless that color is removed by the move)
- Hence, each legal move will leave the number of colors remaining unchanged or reduce it by at most 1 if the color is removed by the move, ie. $h(s) \leq h^*(s)$

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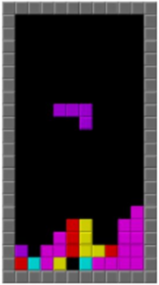
- Hence, each legal move will leave the number of colors remaining unchanged or reduce it by at most 1 if the color is removed by the move, ie. $h(s) \leq h^*(s)$
- Note : You may also prove admissibility by showing that heuristic was derived from a relaxed version of the game (I invite you to try this!)

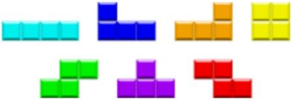
# Tutorial Question 2

- Given information :
  - State : Partially filled tetris fields with a tetrimino that is about to be placed in the field next (but not placed yet).
  - Initial state : Empty field with a starting tetrimino.
  - Action : Sequence of lateral movement(s) and/or rotation(s) of a tetrimino (assume player has enough time to shift the tetrimino to an intended configuration before descent).
  - Action cost : 1
  - Transition model : Takes in a state, applies the sequence of actions on the tetrimino that enters the field, and outputs a state where the tetrimino of that specified configuration descended onto the field.
  - Goal state : Completely filled board where there are no gaps (and every tetrimino fits perfectly). (assume solvable)

# Tutorial Question 2

- An example of an intermediate state :





*7 kinds of Tetriminos*

# Tutorial Question 2

- Assume we start with a fixed number of tetriminos, N (comprising some of each kind), and all are required to be used to fill the board (i.e. there exists a way to place all these tetriminos such that the board is filled).
- Gap : An empty cell on the board.
- Blocked gap : If for the corresponding column where the gap belongs to, there exists an occupied cell somewhere above that gap.
- A row (or column) is complete if there are no gaps in that row (or column respectively).

# Tutorial Question 2(a)

- Let's determine the admissibility of different heuristics

# Tutorial Question 2(a)

- $h_1(n)$ : number of unfielded tetriminos
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- $h_1(n)$ : number of unfielded tetriminos
- What do you think?
- Admissible
- Justification : We need to correctly field each of the unfielded tetriminos (with 1 move each) to reach the goal.

# Tutorial Question 2(a)

- $h_2(n)$ : number of gaps
- What do you think?

# Tutorial Question 2(a)

- $h_2(n)$ : number of gaps
- What do you think?
- Inadmissible
- Recall that a gap is *an empty cell*. Can we think of a counter-example where fielding 1 tetrimino (with 1 move) could fill $> 1$ gap-cell?

# Tutorial Question 2(a)

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- $h_3(n)$ : number of incomplete rows
- What do you think?
- Inadmissible
- Recall an incomplete row = Row w/ any empty cell(s) Can we derive a counter-example where fielding 1 tetriminos (with 1 move) could fill $> 1$ incomplete row?

# Tutorial Question 2(a)

- $h_4(n)$ : number of blocked gaps
- What do you think?

# Tutorial Question 2(a)

- $h_4(n)$ : number of blocked gaps
- What do you think?
- Admissible
- Recall that a blocked gap = a gap w/ a cell above it occupied. Problem : Blocked gaps cannot be fielded! (assumption) Tetris fields with $\geq 1$ blocked gap cannot be solved and hence $h^*$ on these nodes is infinite while $h_4$ is finite, hence $h_4 < h^*$ on these nodes. For nodes on the optimal path, $h_4 = 0$ while $h^*$ is $\geq 0$.

# Tutorial Question 2(b)

- $max(h_1, h_2)$ is admissible
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- $max(h_1, h_2)$ is admissible
- What do you think?
- False
- Recall that $h_1$ is admissible and $h_2$ is inadmissible. For nodes where $h_2 > h^*$, $max(h_1, h_2)$ will also be $> h^*$. Hence, $max(h_1, h_2)$ is inadmissible.

# Tutorial Question 2(b)

- $min(h_2, h_3)$ is admissible
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- $min(h_2, h_3)$ is admissible
- What do you think?
- False
- Recall that $h_2$ and $h_3$ are both inadmissible. But this does not mean that their min must be inadmissible as well! If the heuristics "complement" each other, ie. for nodes where $h_2$ overestimates ($h_2 > h^*$), $h_3$ is guaranteed to not overestimate (and vice versa), then we are safe. But is it true in this case?

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- We need to look at the heuristics / previous counter-examples in detail.

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- $max(h_3, h_4)$ is inadmissible
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- $max(h_3, h_4)$ is inadmissible
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- True
- Recall that $h_3$ is inadmissible while $h_4$ is admissible. For nodes where $h_3 > h^*$, $max(h_3, h_4)$ will also be $> h^*$.

# Tutorial Question 2(c)

- $h_1$ dominates $h_2$
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- $h_1$ dominates $h_2$
- What do you think?
- False
- Recall that $h_1$ is admissible while $h_2$ is inadmissible. There are nodes where $h_2 > h^* > h_1$. Hence $h_1$ does not dominate $h_2$.

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- True
- Recall that a blocked gap is an empty cell that has an additional requirement that its top is blocked by an occupied cell above it. Since each blocked gap is also a gap but not all gaps are blocked gaps, blocked gaps has to be a subset of gaps. Hence, $h_2 > h_4$ at all nodes and $h_2$ dominates $h_4$.

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- $h_3$ does not dominate $h_2$
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- $h_3$ does not dominate $h_2$
- What do you think?
- Recall that both $h_3$ and $h_2$ are inadmissible hence we cannot conclude anything from their properties. Let's look at the heuristic in detail.

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- Recall that both $h_3$ and $h_2$ are inadmissible hence we cannot conclude anything from their properties. Let's look at the heuristic in detail.
- True
- Since each incomplete row has to have $\geq 1$ gap, $h_2$ has to be $\geq h_3$ and hence $h_3$ does not dominate $h_2$.
- OR give a counter-example.

# Tutorial Question 2(c)

- $h_4$ does not dominate $h_2/2$
- What do you think?

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- $h_4$ does not dominate $h_2/2$
- What do you think?
- True
- In the initial state, $h_2/2 = \#$ of gaps / 2 = no. of cells in grid / 2. But $h_4 = \#$ of blocked gaps = 0. Since $h_4 < h_2/2$, $h_4$ does not dominate $h_2$.

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- $h_1$ and $h_4$ do not have a dominance relationship
- $h_3$ and $h_4$?
- $h_3$ dominates $h_4$

# Thank you!

If you have any questions, please don't hesitate. Feel free to ask!
We are here to learn together!