National University of Singapore
School of Computing
CS3243 Introduction to AI

Tutorial 7: Logical Agents I

Issued: Week 9 Discussion in: Week 10

Important Instructions:

- **Assignment 7** consists of **Question 4** from this tutorial.
- Your solution(s) must be TYPE-WRITTEN, though diagrams may be hand-drawn.
- You are to submit your solution(s) during your **Tutorial Session in Week 10**.

Note: you may discuss the assignment question(s) with your classmates, but you must work out and write up your solution individually. Solutions that are plagiarised will be heavily penaltised.

Please refer to **Appendix A** for notes on Knowledge Bases, and **Appendix B** for Propositional Logic Laws.

1. Verify the following logical equivalences. Cite the equivalence law used with each step of your working (refer to Appendix B for a list of these laws).

   (a) $\neg(p \lor \neg q) \lor (\neg p \land \neg q) \equiv \neg p$

   (b) $(p \land \neg(\neg p \lor q)) \lor (p \land q) \equiv p$

2. Victor would like to invite three friends, Alice, Ben, and Cindy to a party, but must satisfy the following constraints:

   (a) Cindy comes only if Alice does not come.

   (b) Alice comes if either Ben or Cindy (or both) comes.

   (c) Cindy comes if Ben does not come.

   Victor would like to know who will come to the party, and who will not. Help Victor by expressing each of the above three constraints in propositional logic, and then, using these constraints, determine who will attend his party.

3. Consider the following knowledge base.

   "All firetrucks are red"

   "All firetrucks are cars"

   "All cars have four wheels"

   (a) Assume that an inference algorithm, A_1, that takes the query sentence "a ferrari is a red car" and infers "a ferrari is a firetruck". Determine which of the following properties **does not** apply to A_1.

   Option 1: complete
   Option 2: sound
   Option 3: both of the above

   (b) Assume that an inference algorithm, A_2 is given the query sentence "a ferrari is a red car". Determine which of the following properties **would guarantee** that A_2 would infer the sentence "a ferrari has four wheels".

   Option 1: completeness
   Option 2: soundness
   Option 3: both of the above need to be combined

   (c) Determine if the following statement is True or False.

   "Two agents with the same knowledge base and different inference engines, both of which are complete and sound, always behave in the same way".

   Justify your answer.

4. Given the following logical statements, use truth-table enumeration to show that $KB \models \alpha$. In other words, write down all possible true/false assignments to the variables, the ones for which KB is true and the one for which α is true, and see whether one is a subset of the other.

   (a)
   $$\begin{aligned} KB =& (x_1 \vee x_2) \wedge (x_1 \Rightarrow x_3) \wedge \neg x_2 \\ \alpha =& x_3 \vee x_2 \end{aligned}$$

   (b)
   $$\begin{aligned} KB =& (x_1 \vee x_3) \wedge (x_1 \Rightarrow \neg x_2) \\ \alpha =& \neg x_2 \end{aligned}$$

Appendix A: Notes on Knowledge Bases

A knowledge base KB is a set of logical rules that model what the agent knows. These rules are written using a certain language (or *syntax*) and use a certain truth model (or *semantics* which say when a certain statement is true or false). In propositional logic sentences are defined as follows

1. Atomic Boolean variables are sentences.
2. If S is a sentence, then so is $\neg S$.
3. If S_1 and S_2 are sentences, then so is:
   (a) $S_1 \wedge S_2$ "S_1 and S_2"
   (b) $S_1 \vee S_2$ "S_1 or S_2"
   (c) $S_1 \Rightarrow S_2$ "S_1 implies S_2"
   (d) $S_1 \Leftrightarrow S_2$ "S_1 holds if and only if S_2 holds"

We say that a logical statement a models b ($a \models b$) if b holds whenever a holds. In other words, if $M(q)$ is the set of all value assignments to variables in a for which a holds true, then $M(a) \subseteq M(b)$.

An inference algorithm $\mathcal{A}$ is one that takes as input a knowledge base KB and a query α and decides whether α is derived from KB, written as $KB \vdash_{\mathcal{A}} \alpha$. $\mathcal{A}$ is sound if $KB \vdash_{\mathcal{A}} \alpha$ implies that $KB \models \alpha$; $\mathcal{A}$ is complete if $KB \models \alpha$ implies that $KB \vdash_{\mathcal{A}} \alpha$.

Appendix B: Propositional Logic Laws

De Morgan's Laws	$\neg(p \vee q) \equiv \neg p \wedge \neg q$	$\neg(p \wedge q) \equiv \neg p \vee \neg q$
Idempotent laws	$p \vee p \equiv p$	$p \wedge p \equiv p$
Associative laws	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Commutative laws	$p \vee q \equiv q \vee p$	$p \wedge q \equiv q \wedge p$
Distributive laws	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
Identity laws	$p \vee False \equiv p$	$p \wedge True \equiv p$
Domination laws	$p \wedge False \equiv False$	$p \vee True \equiv True$
Double negation law	$\neg\neg p \equiv p$	
Complement laws	$p \wedge \neg p \equiv False \wedge \neg True \equiv False$	$p \vee \neg p \equiv True \vee \neg False \equiv True$
Absorption laws	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Conditional identities	$p \Rightarrow q \equiv \neg p \vee q$	$p \Leftrightarrow q \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$