CS3243 : Introduction to Artificial Intelligence

Tutorial 9

NUS School of Computing

April 3, 2023

Admin

- Project 3

Review

- Basics of Probability
- Independence
- Conditional Independence
- Bayes' Rule

Review

- Bayesian Networks
- Determining independence between two nodes given a Bayesian Network
- Why? Simplify/reduce the size of our probability calculations!

Review

- Bayesian Networks
- Determining independence between two nodes given a Bayesian Network
- Why? Simplify/reduce the size of our probability calculations!
- How?
- Constructing a Bayesian network from cause and effect relationships “Domain knowledge” or simply specified by the question
- Factorising the joint probability distribution given the Bayesian network
- Computing joint probabilities (or other conditional probabilities) using the (smaller) CPTs and probability laws (example chain rule)

Review

- A Bayesian Network is a probabilistic graphical model that represents conditional dependencies between random variables through a Directed Acyclic Graph (DAG)
- The graph consists of nodes (variables) and edges ($X \rightarrow Y$ means that X directly influences Y)
- We also include conditional distribution for each node given its parents: $Pr\{X|parents(X)\}$
- Conditional distribution can be represented as a CPT
- The distribution over X for each combination of parent values:
- Given $X_1, X_2, ..., X_n$:
  $Pr\{X_1 \wedge X_2 \wedge ... \wedge X_n\} = \prod_i Pr\{X_i|parents(X_i)\} \rightarrow$ Chain rule!

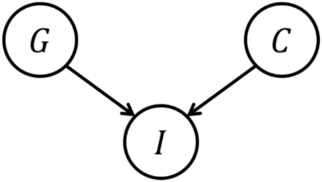
Tutorial Question 1

- Given
- G : Good Grades
- C : Good Communication Skills
- $Pr\{G = 1\} = 0.7$
- $Pr\{C = 1\} = 0.2$

G	C	$\Pr\{I = 1 \mid G, C\}$
1	1	0.9
1	0	0.5
0	1	0.5
0	0	0.1

Tutorial Question 1

- The network



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- Quite straightforward right?
- 0.1 (from the table itself)
- Extra Practice
- What do you think is the value of $Pr\{I = 1, G = 0, C = 0\}$?

Tutorial Question 1(b)

- What is the probability that Bob is a student with great communication skills, assuming we do not know anything about him?

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- $Pr\{C = 1\}$
- 0.2 (given)

Tutorial Question 1(c)

- What is the probability that a student has good communication skills, given that he or she has performed well in an interview? Are good communication skills independent of good performance in an interview?

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- $Pr\{C = 1|I = 1\}$
- $\frac{Pr\{C=1,I=1\}}{Pr\{I=1\}}$
- Apply Bayes' Rule

Tutorial Question 2

- Given
- $Pr[\text{the kit shows positive} \mid \text{the patient is a carrier}] = 0.998$
- $Pr[\text{the kit shows negative} \mid \text{the patient is not a carrier}] = 0.996$

Tutorial Question 2

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- $Pr[\text{the kit shows positive} \mid \text{the patient is a carrier}] = 0.998$
- $Pr[\text{the kit shows negative} \mid \text{the patient is not a carrier}] = 0.996$
- Let's define some variables
- $X/\neg X$: Positive/negative test result
- $Y/\neg Y$: Patient is/isn't a carrier

Tutorial Question 2

- Based on the new variables, let's rewrite the probabilities
- $Pr\{Y\} = 0.02$
- $Pr\{X|Y\} = 0.998$
- $Pr\{\neg X|\neg Y\} = 0.996$

Tutorial Question 2(a)

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- In this question, we need to find posterior probability that patient is not a carrier given positive test results, ie $Pr\{\neg Y|X\}$

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- Apply Bayes' Rule
- $Pr\{\neg Y|X\}$
- $\frac{Pr\{X|\neg Y\}\ Pr\{\neg Y\}}{Pr\{X\}}$

Tutorial Question 2(b)

- Suppose that the patient doesn't entirely trust the result offered by the first kit (perhaps because it has expired) and decides to use another test kit. If the patient is again tested to be positive using this second kit, what is the (updated) likelihood that he is not a carrier? You can assume conditional independence between results of different test kits given the patient's state of virus contraction. Give your answer to 4 decimal places.

Tutorial Question 2(b)

- Suppose that the patient doesn't entirely trust the result offered by the first kit (perhaps because it has expired) and decides to use another test kit. If the patient is again tested to be positive using this second kit, what is the (updated) likelihood that he is not a carrier? You can assume conditional independence between results of different test kits given the patient's state of virus contraction. Give your answer to 4 decimal places.
- We need to find the probability that patient is not a carrier given two positive test results, ie $Pr\{\neg Y | X_1 \wedge X_2\}$
- Apply Bayes' Rule
- $\frac{Pr\{\neg Y \wedge X_1 \wedge X_2\}}{Pr\{X_1 \wedge X_2\}}$

Tutorial Question 3

- Constructing a Bayesian Network from given CPT

P(Wet_Grass | Sprinkler $\wedge$ Rain)	0.95
P(Wet_Grass | Sprinkler $\wedge \neg$ Rain)	0.9
P(Wet_Grass | $\neg$ Sprinkler $\wedge$ Rain)	0.8
P(Wet_Grass | $\neg$ Sprinkler $\wedge \neg$ Rain)	0.1
P(Sprinkler | Rainy_Season)	0.0
P(Sprinkler | $\neg$ Rainy_Season)	1.0
P(Rain | Rainy_Season)	0.9
P(Rain | $\neg$ Rainy_Season)	0.1
P(Rainy_Season)	0.7

Tutorial Question 3

- One key observation to reduce the number of nodes in the tree
- Observe the variables, $Sprinkler$ and $Rainy\ Season$
- The conditional probabilities between them tell something about their relationship

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- Observe the variables, $Sprinkler$ and $Rainy\ Season$
- The conditional probabilities between them tell something about their relationship
- Given two random variables A and B, if $Pr\{A|B\} = 0$ and $Pr\{A|\neg B\} = 1$, A and B are actually complementing variables
- Let's look at the tree!

Thank you!

If you have any questions, please don't hesitate. Feel free to ask!
We are here to learn together!