DQ10 (L10)

Due 2 Apr at 23:59 **Points** 20 **Questions** 6
Available after 27 Mar at 12:00 **Time limit** None
Allowed attempts Unlimited

Instructions

- This quiz is NOT GRADED. However, it is HIGHLY RECOMMENDED that you use these questions to complement your review of the lecture content.
- The questions are based on content from the Lecture 10 and from part of Chapter 12 & 13 of the AIMA (4th Ed.) textbook (i.e., 12; 13.1-13.2).

Take the quiz again

Attempt history

	Attempt	Time	Score
KEPT	**Attempt 2**	7 minutes	20 out of 20
LATEST	**Attempt 2**	7 minutes	20 out of 20
	Attempt 1	33 minutes	11 out of 20

Submitted 27 Mar at 13:12

Question 1

2 / 2 pts

What is the size of a joint probability table given n random variables, each with a domain of size d? The variables are dependent on one another.

Answer in terms of n and d.

Size of joint probability table = nd

Correct!

Size of joint probability table = d^n

Size of joint probability table = n^d

Size of joint probability table = 2^{nd}

Question 2

3 / 3 pts

Consider the joint distribution table of <**toothache**, **cavity**, **catch**> as described in the lecture.

	toothache		¬toothache	
	catch	¬catch	catch	¬catch
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

Part 1: ***What is the probability of getting a cavity, given a toothache?***

Answer: 0.6

Part 2: ***What is the probability of having no cavity, given a toothache?***

Answer: 0.4

Answer 1:

Correct!

0.6

Answer 2:

Correct!

0.4

$$P(cavity \mid toothache) = \frac{P(cavity \wedge toothache)}{P(toothache)} = \frac{0.108 + 0.012}{0.108 + 0.012 + 0.016 + 0.064} = 0.6$$

$$P(\neg cavity \mid toothache) = \frac{P(\neg cavity \wedge toothache)}{P(toothache)} = \frac{0.016 + 0.064}{0.108 + 0.012 + 0.016 + 0.064} = 0.4$$

Question 3

5 / 5 pts

Determine is the follow is true or false.

If P(a | b, c) = P(b | a, c), then P(a | c) = P(b | c)

Note:

- P(a,b) ≡ P(a ∧ b)
- P(b,c), P(a,c) > 0

- True

Correct!

- False

The statement is **False**, notably when there exists a probability = 0.

Note that while P(b,c) and P(a,c) should be non-zero for the conditional probabilities in the question P(a | b, c) and P(b | a, c) to be valid, P(a,b,c) could be zero.

Let us consider an example of rolling 2 distinct dices d1 and d2, with values 1-6 each.

- P(a): d1 = 1
- P(b): d1+d2 = 11
- P(c): d1+d2 > 6

We have -

- P(a | b, c) = P(b | a, c) = 0 // since P(a,b,c) = 0

However -

- P(a|c) = P(a,c) / P(c) = (1/36) / (21/36) = 1/21
- P(b|c) = P(b,c) / P(c) = (2/36) / (21/36) = 2/21

Hence, P(a|c) != P(b|c).

The statement is true ***when assuming non-zero probabilities***.

- P(a | b, c) = P(b | a, c) means, by definition, that P(a, b, c)/P(b, c) = P(b, a, c)/P(a, c).
- This evaluates to P(b, c) = P(a, c). (After simplification)
- We then divide both sides by P(c) and you get P(a | c) = P(b | c).

Question 4

4 / 4 pts

Suppose you are given a bag containing 9 unbiased coins. You are told that 8 of these coins are normal, with heads on one side and tails on the other, whereas one coin is a fake, with heads on both sides.

Suppose you reach into the bag, pick out a coin at random, flip it, and get a head. What is the (conditional) probability that the coin you chose is the fake coin?

Answer: 0.2

Answer 1:

Correct!

0.2

P(fake | heads)

= P(heads | fake) · P(fake) / P(heads)

= P(heads | fake) · P(fake) / [P(heads | fake) · P(fake) + P(heads | −fake) · P(−fake)]

= 1 * (1/n) / [1 * (1/n) + 0.5 * ((n−1) / n)]

= 2 / (n+1)

Sub n = 9.

P(fake | heads) = 2/(9 + 1)

= 2/10

= 0.2

Question 5

2 / 2 pts

Select the following statement(s) that is/are true.

- [ ]
- [ ]
- [ ] A Bayesian network does not have to be directed acyclic graph.

Correct!

- [x] An edge from a variable *x* to a variable *y* in a Bayesian network denotes that variable *x* directly influences variable *y*, and *x* is a parent of *y*.

Correct!

- [x] Each node in a bayesian network corresponds to a random variable.

Correct!

- [x] Each node in a bayesian network has a conditional distribution for the node, given its parents.

Question 6

4 / 4 pts

What is the maximum number of edges in a Bayesian network with **n** nodes?

- n^2

Correct!

- (•) $n(n-1)/2$

- $2n - 1$

- $2^n/n$

$n(n-1)/2$.

Proof by construction: Consider a Bayesian Network (BN) over $X_1, X_2, \ldots X_n$ such that $\forall j > i$ there is an edge between X_i and X_j. The total number of edges in this graph is $n-1+(n-2)+\ldots+0 = n(n-1)/2$.

To show that you cannot have a directed cycle in this graph, assume the contrary and suppose there is a cycle of the form $X_{i\_1}, X_{i\_2}, \ldots X_{i\_m}, X_{i\_1}$; by construction of graph, we have i_1 < i_2 . . . i_m < i_1 leading to i_1 < i_1 which is a contradiction.

Therefore, no cycle exists. Additionally, you cannot construct a BN with more than $n(n-1)/2$ edges, since any directed graph with more than $n(n-1)/2$ edges should have at least one pair of vertices for which there is more than one edge implying that you have at least one edge in both directions resulting in a cycle.