CS3243 : Introduction to Artificial Intelligence

Tutorial 8

NUS School of Computing

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Admin

- Regarding T7 Assignment

Review

- Making Inferences
- How do we obtain knowledge from the percepts of an agent?
- Can we infer query α given KB, that we possess?
- Can we show KB $\models \alpha$?

Review

- Several Approaches
- **Truth Table**
  - Check all the 2^n truth value assignments to verify whether KB $\models \alpha$
- **Resolution**
  - Use propositional logic
  - Conjunctive Normal Form (CNF)
  - Resolution Algorithm
  - Soundness and Completeness of Resolution Algorithm

Developing Intuition

- Some relevant definitions:
  - A sentence is valid if it is true in all models
  - A sentence is satisfiable if it is true in some models
  - A sentence is unsatisfiable if it is true in no models
- How is satisfiability related to entailment?
- Satisfiability is connected to Entailment via the following KB $\models \alpha$ if KB $\wedge \neg\alpha$ is unsatisfiable

Tutorial Question 2

- We have a knowledge base with propositional logic statements involving the boolean variables $x_1, x_2, .., x_n$
- Given a logical formula q, let $M(q)$ be the set of all the truth assignments to variables for which q is true (essentially the models)
- Recall that an inference algorithm $\mathcal{A}$ is sound if whenever a statement q is inferred from KB by $\mathcal{A}$, KB entails q
- Also an inference algorithm is complete if whenever KB entails a statement q, $\mathcal{A}$ will eventually infer q

Tutorial Question 2

- Proof by Induction
- Direct Proof

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- Suppose we obtain α by running some resolution operations on the statements in KB
- We prove on the number of resolution operations executed before obtaining α

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- Suppose we obtain α by running some resolution operations on the statements in KB
- We prove on the number of resolution operations executed before obtaining α
- Suppose $\overrightarrow{x} = \langle x_1, x_2, .., x_n \rangle$ is the set of sentences in KB and KB is in CNF form
- First resolution step (case of single resolution step)
  - There exists two clauses $P(\overrightarrow{x}) \vee x$ and $Q(\overrightarrow{x}) \vee \neg x$ which reduces to $P(\overrightarrow{x}) \vee Q(\overrightarrow{x})$
  - Since the two clauses are true (being a part of KB), either of the clauses $P(\overrightarrow{x})$ or $Q(\overrightarrow{x})$ must be true
  - Hence, it is true for any resolvent reached after one resolution step

Tutorial Question 2

- Inductive step
  - Suppose that any resolvent q is achievable after r resolution steps that satisfy $M(\text{KB}) \subseteq M(q)$
  - We are supposed to show that the claim holds for $r + 1$ steps
  - However, this is simply a repitition of the proof for the one-resolution step case, with KB being KB' (the set of resolvents reachable from KB after r resolution steps)

Tutorial Question 2

- Direct Proof?
- Give it a try!

Tutorial Question 3

- Every CNF formula can be converted into 3-CNF formula
- Look at the proof!

Proof

Tutorial Question 4

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- Hints
  - Any clause in a 2-CNF formula can be written in one of these forms : $x \to y, \neg x \to y, x \to \neg y, \neg x \to \neg y$
  - Create a directed (implication) graph whose nodes are variables and their negations
  - If the clause $l_1 \to l_2$ appears in the original formula, we add edges $\langle l_1, l_2 \rangle$ and $\langle \neg l_1, \neg l_2 \rangle$ to the graph!
  - Think about what happens when there is some cycle containing variable x and its negation $\neg x$

Proof

Thank you!

If you have any questions, please don't hesitate. Feel free to ask!
We are here to learn together!