CS3244 Tutorial 2

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BL 2

Sean Tay has constructed a dataset of binary-valued input attributes and binary-valued output/target concept with 4 noise-free training examples:
$D = \{\langle\langle 0{,}0\rangle{,}1\rangle, \langle\langle 0{,}1\rangle{,}0\rangle, \langle\langle 1{,}0\rangle{,}1\rangle, \langle\langle 1{,}1\rangle{,}1\rangle\}$.

(a) State the hypothesis h_4 that the FIND-S algorithm would output after learning from all 4 training examples.

(b) Is hypothesis h_4 consistent with D?

Find-S Algorithm

Idea. Start with most specific hypothesis. Whenever it wrongly classifies a +ve training example as –ve, "minimally" generalize it to satisfy its input instance.

1. Initialize h to most specific hypothesis in H
2. For each positive training instance x
   - For each attribute constraint a_i in h

     If x satisfies constraint a_i in h

     Then do nothing

     Else replace a_i in h by the next more general constraint that is satisfied by x
3. Output hypothesis h

BL 2

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$D = \{\langle\langle 0{,}0\rangle{,}1\rangle, \langle\langle 0{,}1\rangle{,}0\rangle, \langle\langle 1{,}0\rangle{,}1\rangle, \langle\langle 1{,}1\rangle{,}1\rangle, \}$.

(a) State the hypothesis h_4 that the FIND-S algorithm would output after learning from all 4 training examples.

(b) Is hypothesis h_4 consistent with D?

Solution

- $h_4 = \langle ?, ? \rangle$
- h_4 is not consistent with D since $h_4(\langle 0{,}1\rangle) = 1 \neq c(\langle 0{,}1\rangle) = 0$.

BL 2

(c) Does your answer to (b) contradict Proposition 2 on page 15 of the "Concept Learning" lecture slides?

Proposition 2: *Suppose that $c \in H$. Then, h_n is consistent with* $D = \{\langle x_k, c(x_k)\rangle\}_{k=1,\ldots,n}$.

Solution

- No, take note of the necessary assumption.
- What is the true target concept? Can it be represented using the H we defined?
  - The true target concept $c(x)$ has a value of 0 if $x = \langle 0,1 \rangle$, and a value of 1 otherwise.
- Note our hypothesis representation is a conjunction of constraints on input attributes.

BL 3

Why, in the CANDIDATE-ELIMINATION algorithm, each minimal generalization h of s is not just consistent with the +ve training example d but also consistent with all +ve and –ve training examples observed thus far?

(a) Prove that h is consistent with all +ve training examples observed thus far, not including d.

BL 3

Why, in the CANDIDATE-ELIMINATION algorithm, each minimal generalization h of s is not just consistent with the +ve training example h but also consistent with all +ve and –ve training examples observed thus far?

(a) Prove that h is consistent with all +ve training examples observed thus far, not including d.

Solution

1. Since $s \in S$, s is consistent with all +ve training examples observed thus far (except d), by definition of S.

2. Then, every +ve training examples (except d) satisfies s.

3. Since $h \geq_g s$, every +ve training examples (except d) also satisfies h.

4. Therefore, h is consistent with all +ve training examples observed thus far.

BL 3

Why, in the CANDIDATE-ELIMINATION algorithm, each minimal generalization h of s is not just consistent with the +ve training example h but also consistent with all +ve and –ve training examples observed thus far?

(b) Prove that h is consistent with all -ve training examples observed thus far, not including d.

Solution

1. Recall in the CANDIDATE-ELIMINATION update step, there should be "some member of G that is more general than h": $\exists g \in G \quad g \geq_g h$.

2. Since $g \in G$, g is consistent with all -ve training examples observed thus far (except d), by definition of G.

3. Then, every -ve training examples (except d) does not satisfy g.

4. Since $g \geq_g h$, every -ve training examples (except d) also does not satisfy h.

5. Therefore, h is consistent with all -ve training examples observed thus far.

TM 2.3

Consider the $EnjoySport$ learning task and the hypothesis space H described on page 7 of the "Concept Learning" lecture slides. Let us define a new hypothesis space H' that consists of all pairwise disjunctions of the hypotheses in H. For example, a typical hypothesis in H' is

$$\langle ?, Cold, High, ?, ?, ? \rangle \vee \langle Sunny, ?, High, ?, ?, Same \rangle.$$

Trace the CANDIDATE-ELIMINATION algorithm for the hypothesis space H'.

Example	*Sky*	*AirTemp*	*Humidity*	*Wind*	*Water*	*Forecast*	*EnjoySport*
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Change	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes

TM 2.3

- Remove from G any hypothesis inconsistent with d
- For each $s \in S$ not consistent with d
  - Remove s from S
  - Add to S all minimal generalizations h of s s.t. h is consistent with d, and some member of G is more general than h

I used the symbol $=_s$ to denote semantic equivalence.

Eg. 1: $\langle Sunny, Warm, Normal, Strong, Warm, Same \rangle$, +ve

- Init $S_0 = \{\langle \varnothing, \varnothing, \varnothing, \varnothing, \varnothing, \varnothing \rangle \vee \langle \varnothing, \varnothing, \varnothing, \varnothing, \varnothing, \varnothing \rangle\} =_s \{\langle \varnothing, \varnothing, \varnothing, \varnothing, \varnothing, \varnothing \rangle\}$
- Init $G_0 = \{\langle ?, ?, ?, ?, ?, ? \rangle \vee \langle ?, ?, ?, ?, ?, ? \rangle\} =_s \{\langle ?, ?, ?, ?, ?, ? \rangle\}$
- $\langle ?, ?, ?, ?, ?, ? \rangle$ is consistent with eg.1, so $G_1 = G_0$
- $\langle \varnothing, \varnothing, \varnothing, \varnothing, \varnothing, \varnothing \rangle$ is not consistent with eg.1, remove it from S and add minimal generalization $h = \langle Sunny, Warm, Normal, Strong, Warm, Same \rangle \vee \langle \varnothing, \varnothing, \varnothing, \varnothing, \varnothing, \varnothing \rangle$ to S. Note $G_1 \geq_g h$.
- $$\begin{aligned} S_1 &= \{\langle Sunny, Warm, Normal, Strong, Warm, Same \rangle \vee \langle \varnothing, \varnothing, \varnothing, \varnothing, \varnothing, \varnothing \rangle\} \\ &=_s \{\langle Sunny, Warm, Normal, Strong, Warm, Same \rangle \vee \langle Sunny, Warm, Normal, Strong, Warm, Same \rangle\} \\ &=_s \{\langle Sunny, Warm, Normal, Strong, Warm, Same \rangle\} \end{aligned}$$

TM 2.3

- Remove from G any hypothesis inconsistent with d
- For each $s \in S$ not consistent with d
  - Remove s from S
  - Add to S all minimal generalizations h of s s.t.
    h is consistent with d, and
    some member of G is more general than h

I used the symbol $=_s$ to denote semantic equivalence.

Eg. 2: $\langle Sunny, Warm, High, Strong, Warm, Same \rangle$, +ve

- $S_1 = \{\langle Sunny, Warm, Normal, Strong, Warm, Same \rangle\}$
- $G_1 = \{\langle ?, ?, ?, ?, ?, ? \rangle\}$
- $\langle ?, ?, ?, ?, ?, ? \rangle$ is consistent with eg.2, so $G_2 = G_1$
- $\langle Sunny, Warm, Normal, Strong, Warm, Same \rangle$ is not consistent with eg.2, remove it from S and add minimal generalization
  $h = \langle Sunny, Warm, Normal, Strong, Warm, Same \rangle \vee \langle Sunny, Warm, High, Strong, Warm, Same \rangle$
  to S. Note $G_2 \geq_g h$.
  - $$\begin{aligned} S_2 &= \{\langle Sunny, Warm, Normal, Strong, Warm, Same \rangle \vee \langle Sunny, Warm, High, Strong, Warm, Same \rangle\} \\ &=_s \{\langle Sunny, Warm, ?, Strong, Warm, Same \rangle\} \end{aligned}$$

TM 2.3

- Remove from S any hypothesis inconsistent with d
- For each $g \in G$ not consistent with d
  - Remove g from G
  - Add to G all minimal specializations h of g s.t.

    h is consistent with d, and

    some member of S is more specific than h

<u>I used the symbol $=_s$ to denote semantic equivalence.</u>

Eg. 3: $\langle Rainy, Cold, High, Strong, Warm, Change \rangle$, -ve

- $S_2 = \{\langle Sunny, Warm, ?, Strong, Warm, Same \rangle\}$
- $G_2 = \{\langle ?, ?, ?, ?, ?, ? \rangle\}$
- $\langle Sunny, Warm, ?, Strong, Warm, Same \rangle$ is consistent with eg.3, so $S_3 = S_2$
- $\langle ?, ?, ?, ?, ?, ? \rangle$ is not consistent with eg.3, remove it from G and add some minimal specialization:
  - 1st attribute can take $\{\cancel{?}, Sunny, Cloudy, \cancel{Rainy}\}$.
  - 2nd attribute can take $\{\cancel{?}, Warm, \cancel{Cold}\}$.
  - 3rd attribute can take $\{\cancel{?}, Normal, \cancel{High}\}$.
  - 4th attribute can take $\{\cancel{?}, Weak, \cancel{Strong}\}$.
  - 5th attribute can take $\{\cancel{?}, Cool, \cancel{Warm}\}$.
  - 6th attribute can take $\{\cancel{?}, Same, \cancel{Change}\}$.
- Try $h = \{\langle Sunny, ?, ?, ?, ?, ? \rangle \vee \langle Cloudy, ?, ?, ?, ?, ? \rangle\}$, since $h \geq_g S_2$, we add h to G.
- How about $h = \{\langle Sunny, ?, ?, ?, ?, ? \rangle \vee \langle ?, ?, ?, ?, ?, ? \rangle\}$?

How about $h = \{\langle Sunny, ?, ?, ?, ?, ? \rangle \vee \langle ?, Warm, ?, ?, ?, ? \rangle\}$?

TM 2.3

I used the symbol $=_s$ to denote semantic equivalence.

Eg. 3: $\langle Rainy, Cold, High, Strong, Warm, Change \rangle$, -ve

- $S_3 = \{\langle Sunny, Warm, ?, Strong, Warm, Same \rangle\}$
- $$\begin{aligned} G_3 \quad = \quad \{ & \langle Sunny,?,?,?,?,? \rangle \vee \langle Cloudy,?,?,?,?,? \rangle, \\ & \langle Sunny,?,?,?,?,? \rangle \vee \langle ?,Warm,?,?,?,? \rangle, \\ & \langle Sunny,?,?,?,?,? \rangle \vee \langle ?,?,Normal,?,?,? \rangle, \\ & \langle Sunny,?,?,?,?,? \rangle \vee \langle ?,?,?,Weak,?,? \rangle, \\ & \langle Sunny,?,?,?,?,? \rangle \vee \langle ?,?,?,?,Cool,? \rangle, \\ & \langle Sunny,?,?,?,?,? \rangle \vee \langle ?,?,?,?,?,Same \rangle, \\ & \langle Cloudy,?,?,?,?,? \rangle \vee \langle ?,Warm,?,?,?,? \rangle, \\ & \langle Cloudy,?,?,?,?,? \rangle \vee \langle ?,?,?,?,?,Same \rangle, \\ & \langle ?,Warm,?,?,?,? \rangle \vee \langle ?,?,Normal,?,?,? \rangle, \\ & \langle ?,Warm,?,?,?,? \rangle \vee \langle ?,?,?,Weak,?,? \rangle, \\ & \langle ?,Warm,?,?,?,? \rangle \vee \langle ?,?,?,?,Cool,? \rangle, \\ & \langle ?,Warm,?,?,?,? \rangle \vee \langle ?,?,?,?,?,Same \rangle, \\ & \langle ?,?,Normal,?,?,? \rangle \vee \langle ?,?,?,?,?,Same \rangle, \\ & \langle ?,?,?,Weak,?,? \rangle \vee \langle ?,?,?,?,?,Same \rangle, \\ & \langle ?,?,?,?,Cool,? \rangle \vee \langle ?,?,?,?,?,Same \rangle \} \end{aligned}$$

- Remove from S any hypothesis inconsistent with d
- For each $g \in G$ not consistent with d
  - Remove g from G
  - Add to G all minimal specializations h of g s.t.
    h is consistent with d, and
    some member of S is more specific than h

TM 2.3

- Remove from G any hypothesis inconsistent with d
- For each $s \in S$ not consistent with d
  - Remove s from S
  - Add to S all minimal generalizations h of s s.t.

    h is consistent with d, and

    some member of G is more general than h

I used the symbol $=_s$ to denote semantic equivalence.

Eg. 4: $\langle Sunny, Warm, High, Strong, Cool, Change\rangle$, +ve

- $S_3 = \{\langle Sunny, Warm, ?, Strong, Warm, Same\rangle\}$
- $$\begin{aligned} G_3 \quad = \quad & \{\langle Sunny,?,?,?,?,?\rangle \vee \langle Cloudy,?,?,?,?,?\rangle, \\ & \langle Sunny,?,?,?,?,?\rangle \vee \langle ?,Warm,?,?,?,?\rangle, \\ & \langle Sunny,?,?,?,?,?\rangle \vee \langle ?,?,Normal,?,?,?\rangle, \\ & \langle Sunny,?,?,?,?,?\rangle \vee \langle ?,?,?,Weak,?,?\rangle, \\ & \langle Sunny,?,?,?,?,?\rangle \vee \langle ?,?,?,?,Cool,?\rangle, \\ & \langle Sunny,?,?,?,?,?\rangle \vee \langle ?,?,?,?,?,Same\rangle, \\ & \langle Cloudy,?,?,?,?,?\rangle \vee \langle ?,Warm,?,?,?,?\rangle, \\ & \langle Cloudy,?,?,?,?,?\rangle \vee \langle ?,?,?,?,?,Same\rangle, \times \\ & \langle ?,Warm,?,?,?,?\rangle \vee \langle ?,?,Normal,?,?,?\rangle, \\ & \langle ?,Warm,?,?,?,?\rangle \vee \langle ?,?,?,Weak,?,?\rangle, \\ & \langle ?,Warm,?,?,?,?\rangle \vee \langle ?,?,?,?,Cool,?\rangle, \\ & \langle ?,Warm,?,?,?,?\rangle \vee \langle ?,?,?,?,?,Same\rangle, \\ & \langle ?,?,Normal,?,?,?\rangle \vee \langle ?,?,?,?,?,Same\rangle, \times \\ & \langle ?,?,?,Weak,?,?\rangle \vee \langle ?,?,?,?,?,Same\rangle, \times \\ & \langle ?,?,?,?,Cool,?\rangle \vee \langle ?,?,?,?,?,Same\rangle\} \end{aligned}$$

TM 2.3

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  - Remove s from S
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    h is consistent with d, and
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Eg. 4: $\langle Sunny, Warm, High, Strong, Cool, Change \rangle$, +ve

- $S_3 = \{\langle Sunny, Warm, ?, Strong, Warm, Same \rangle\}$
- $\langle Sunny, Warm, ?, Strong, Warm, Same \rangle$ is not consistent with eg.4, remove it from S and add minimal generalization $h = \langle Sunny, Warm, ?, Strong, Warm, Same \rangle \vee \langle Sunny, Warm, High, Strong, Cool, Change \rangle$ to S. Note $\langle Sunny, ?, ?, ?, ?, ? \rangle \vee \langle Cloudy, ?, ?, ?, ?, ? \rangle \in G_4 \geq_g h$.

TM 2.3

- Final answer (yes.. finally..)

$$
\begin{aligned}
G_4 \;=\; \{ & \langle Sunny,?,?,?,?,?\rangle \vee \langle Cloudy,?,?,?,?,?\rangle, \\
& \langle Sunny,?,?,?,?,?\rangle \vee \langle ?,Warm,?,?,?,?\rangle, \\
& \langle Sunny,?,?,?,?,?\rangle \vee \langle ?,?,Normal,?,?,?\rangle, \\
& \langle Sunny,?,?,?,?,?\rangle \vee \langle ?,?,?,Weak,?,?\rangle, \\
& \langle Sunny,?,?,?,?,?\rangle \vee \langle ?,?,?,?,Cool,?\rangle, \\
& \langle Sunny,?,?,?,?,?\rangle \vee \langle ?,?,?,?,?,Same\rangle, \\
& \langle Cloudy,?,?,?,?,?\rangle \vee \langle ?,Warm,?,?,?,?\rangle, \\
& \langle ?,Warm,?,?,?,?\rangle \vee \langle ?,?,Normal,?,?,?\rangle, \\
& \langle ?,Warm,?,?,?,?\rangle \vee \langle ?,?,?,Weak,?,?\rangle, \\
& \langle ?,Warm,?,?,?,?\rangle \vee \langle ?,?,?,?,Cool,?\rangle, \\
& \langle ?,Warm,?,?,?,?\rangle \vee \langle ?,?,?,?,?,Same\rangle, \\
& \langle ?,?,?,?,Cool,?\rangle \vee \langle ?,?,?,?,?,Same\rangle \} \\
S_4 \;=\; \{ & \langle Sunny,Warm,?,Strong,Warm,Same\rangle \vee \langle Sunny,Warm,High,Strong,Cool,Change\rangle \}
\end{aligned}
$$

TM 2.3

Some takeaways

- The hypothesis space H you defined matters a lot
- Remember dual treatment for S and G
- Take note of how you should choose combinations to enumerate all possibilities

Thank you!

- Any questions?