CS3244 18/19 sem 2 Finals

Concept Learning 1

-ve example

i) Prove that each minimal specialization h of $g \in G$ (g not consistent with d)
is consistent with all +ve training examples observed thus far

1. some member of S is more specific than or equal to h (candidate elimination algorithm)
   $\exists s \in S \quad h \geq_g s$
2. since $s \in S$, s is consistent with all training examples so far (given in assumption)
3. Therefore, every +ve training example satisfies s (proposition 1)
4. since $h \geq_g s$, $\forall x \in X$, $s(x) = 1 \rightarrow h(x) = 1$ (def of $\geq_g$)
5. Every positive training observed thus far satisfies h
6. Thus, h is consistent with all +ve training examples observed thus far

ii) Prove that each minimal specialization h of $g \in G$ is consistent with all -ve training examples observed thus far, including d

1. $\forall g \in G$, g is consistent with all training examples observed so far (not including d)
2. Therefore, every -ve training example not including d does not satisfy g (prop 1)
3. since h is a minimal specialization of g, $g \geq_g h$
   $\forall x \in X$, $g(x) = 0 \rightarrow h(x) = 0$ (contrapositive of $\geq_g$)
4. every negative training example thus far does not satisfy h (not including d)
5. since h is consistent with d, h is consistent with all negative training examples observed thus far.

concept learning 2

i) proof by contradiction

suppose $\sim [(\forall g \in G \; g \not\geq_g h') \rightarrow (\forall h \in VS_{H,D} \; h \not\geq_g h')]$

$\equiv (\forall g \in G \; g \not\geq_g h') \wedge \sim (\forall h \in VS_{H,D} \; h \not\geq_g h')$ $\quad \sim(p \rightarrow q) \equiv \sim(\sim p \vee q) \equiv p \wedge \sim q$

$\equiv (\forall g \in G \; g \not\geq_g h') \wedge (\exists h \in VS_{H,D} \; h \geq_g h')$

1. $\exists h \in VS_{H,D}$ such that $h \geq_g h'$
2. $\exists g \in G$ such that $g \geq_g h$ (VSRT)
3. By transitive property of $\geq_g$ relation, $\exists g \in G \quad g \geq_g h \geq_g h'$
   $g \geq_g h'$
4. contradiction with $\forall g \in G \; g \not\geq_g h'$, assumption must be false
5. Thus $(\forall g \in G \; g \not\geq_g h') \rightarrow (\forall h \in VS_{H,D} \; h \not\geq_g h')$ must be true

ii) Proof by contradiction

Suppose $\sim [(\forall s \in S \; h' \not\geq_g s) \rightarrow (\forall h \in VS_{H,D} \; h' \not\geq_g h)]$

$\equiv \forall s \in S \; h' \not\geq_g s \wedge \exists h \in VS_{H,D} \; h' \geq_g h$

1. $\exists h \in VS_{H,D}$ such that $h' \geq_g h$
2. $\exists s \in S$ such that $h \geq_g s$ (VSRT)
3. By transitive property of $\geq_g$ relation, $\exists s \in S \; h' \geq_g h \geq_g s$
   $h' \geq_g s$
4. contradiction with $\forall s \in S \; h' \not\geq_g s$, initial assumption is false
5. Thus $(\forall s \in S \; h' \not\geq_g s) \rightarrow \forall h \in VS_{H,D} \; h' \not\geq_g h$ is true

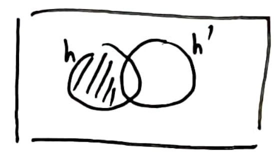
Concept Learning 3

$G_0 = \{\langle ????\rangle \,\Delta\, \langle \emptyset,\emptyset,\emptyset,\emptyset\rangle\} = \{\langle ????\rangle\}$

$S_0 = \{\langle \emptyset,\emptyset,\emptyset,\emptyset\rangle \,\Delta\, \langle \emptyset,\emptyset,\emptyset,\emptyset\rangle, \langle ????\rangle \,\Delta\, \langle ????\rangle, \ldots\} = \{\langle \emptyset,\emptyset,\emptyset,\emptyset\rangle\}$

$h \setminus h' = h(x)=1 \wedge h'(x)=0$

X

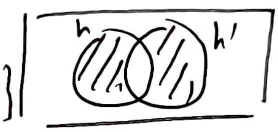


$G_1 = G_0$

$S_1 = \{\langle \text{Sometimes, Good, Average, Yes}\rangle \,\Delta\, \langle \emptyset\emptyset\emptyset\emptyset\rangle\}$

$h \,\Delta\, h' = h\setminus h' \vee h'\setminus h$

X



$G_2 = G_1$

$S_2 = \{\langle \text{Sometimes, Good, Average, Yes}\rangle \,\Delta\, \langle \text{Always, Good, Average, Yes}\rangle\}$

$G_3 = G_2$

$S_3 = \{\langle ?, \text{Good, Average, Yes}\rangle \,\Delta\, \langle \text{Rarely, Good, Good, Yes}\rangle\}$

$G_4 = G_3$

$S_4 = \{\langle ?, \text{Good, Average, Yes}\rangle \,\Delta\, \langle ?, \text{Good, Good, Yes}\rangle\}$

$S_5 = S_4$

$G_5 = \{\langle ????\rangle \,\Delta\, \langle \text{Rarely, Poor, Good, Yes}\rangle\}$

Part 4 EM

$\langle x_1, x_2, x_3 \rangle = \langle 4, 5, 6 \rangle$, $M = 2$, $\sigma^2 = 0.5$

$\langle \mu_1, \mu_2 \rangle = \langle 4, 6 \rangle$

Run EM for 3 iterations, estimate μ_1, μ_2 up to 6 dp — intermediate values 8dp

Iteration 1

E step

$$E[z_{11}] = \frac{e^{-\frac{1}{1}(4-4)^2}}{\sum_{\ell=1}^{2} e^{-\frac{1}{1}(4-\mu_\ell)^2}} = \frac{1}{1.01831564} = 0.98201379$$

$$\sum_{\ell=1}^{2} e^{-(4-\mu_\ell)^2} = e^{-0} + e^{-4} = 1 + 0.01831564 = 1.01831564 = \sum_{\ell=1}^{2} e^{-(6-\mu_\ell)^2}$$

$$E[z_{12}] = \frac{e^{-(4-6)^2}}{1.01831564} = 0.01798621$$

$$E[z_{21}] = \frac{e^{-(5-4)^2}}{e^{-(5-4)^2} + e^{-(5-6)^2}} = \frac{1}{2}$$

$$E[z_{22}] = \frac{1}{2}$$

$$E[z_{31}] = \frac{e^{-(6-4)^2}}{1.01831564} = 0.01798621$$

$$E[z_{32}] = 0.98201379$$

M step

$$\mu_1' = \frac{E[z_{11}]\cdot x_1 + E[z_{21}]\cdot x_2 + E[z_{31}]\cdot x_3}{E[z_{11}] + E[z_{21}] + E[z_{31}]} = 4.35731495$$

$$\mu_2' = 5.64268505$$

Iteration 2

$$\sum_{\ell=1}^{2} e^{-(4-\mu_\ell)^2} = 0.88014028 + 0.06731217 = 0.94745245 = \sum_{\ell=1}^{2} e^{-(6-\mu_\ell)^2}$$

E step

$E[z_{11}] = 0.92895457$

$E[z_{12}] = 0.07104543$

$E[z_{21}] = \frac{1}{2}$

$E[z_{22}] = \frac{1}{2}$

$E[z_{31}] = 0.07104543$

$E[z_{32}] = 0.92895457$

M step

$\mu_1' = 4.42806057$

$\mu_2' = 5.57193943$

Iteration 3

$$\sum_{\ell=1}^{2} e^{-(4-\mu_\ell)^2} = 0.83257177 + 0.08450086 = 0.91707263$$

E step

$E[z_{11}] = 0.90785805$

$E[z_{21}] = 0.09214195$

$E[z_{21}] = E[z_{22}] = \frac{1}{2}$

$E[z_{31}] = 0.09214195$

$E[z_{32}] = 0.90785805$

M step

$$\boxed{\begin{aligned} \mu_1' &= 4.45618927 \\ \mu_2' &= 5.54381073 \end{aligned}}$$

Part 5 CLT not-tested ?

Part 6 Neural Networks

i) $w_0 = -1$, $w_2 = 1$ AND(x_1, x_2)

Case 1: perceptron outputs true if both inputs are true

$$w_0 + w_1 x_1 + w_2 x_2 = -1 + w_1 + 1$$
$$= w_1 > 0$$

Case 2: perceptron outputs false if at least 1 input is false

$(1,-1) \Rightarrow -1 + w_1 - 1 = w_1 - 2 \leq 0 \Rightarrow w_1 \leq 2$

$(-1,1) \Rightarrow -1 - w_1 + 1 = -w_1 \leq 0 \Rightarrow w_1 \geq 0$

$(-1,-1) \Rightarrow -1 - w_1 - 1 = -w_1 - 2 \leq 0 \Rightarrow w_1 \geq -2$

restricted by $w_1 > 0$ in case 2

$\therefore \quad 0 < w_1 \leq 2$

ii) $w_1, w_2 \ldots w_n = -1$ NOR function

Case 1: perceptron outputs true if all n Boolean inputs are false

$$\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} -1 = -n$$

$$w_0 + (-1)\sum_{i=1}^{n} x_i = w_0 + n > 0 \Rightarrow w_0 > -n$$

Case 2: perceptron outputs false if at least 1 input is true

$$\sum_{i=1}^{n} x_i \geq 1 + n-1(-1) = 2-n$$

$$-\sum_{i=1}^{n} x_i \leq n-2$$

$$w_0 + (-1)\sum_{i=1}^{n} x_i \leq w_0 + n - 2 \leq 0 \Rightarrow w_0 \leq 2-n$$

$$-n < w_0 \leq 2-n$$

iii)

x_1	x_2	t_k
1	1	1
1	-1	-1
-1	1	-1
-1	-1	1

XNOR

x	weights	t_k	
(1,1)	$(w_1+w_2)>0?\ w_3 : -w_3$ $+w_4+w_5$	>0	$2>0$
(1,-1)	$(w_1-w_2)>0?\ w_3:-w_3$ $+w_4-w_5$	≤ 0	$0\le 0$
(-1,1)	$(-w_1+w_2)>0\ \ w_3:-w_3$ $-w_4+w_5$	≤ 0	$0\le 0$
(-1,-1)	$(-w_1-w_2)>0\ \ w_3:-w_3$ $-w_4-w_5$	>0	$2>0$

$w_4 \neq w_5$ else either (1,1) (2,2) or (-1,-1) (-2,-2)

have a -2 or -4. $\Rightarrow \neq 0$ with +2

try (-1,1) $\left.\begin{array}{l} w_4+w_5 = 0 \\ -w_4-w_5 = 0 \end{array}\right\}$ same as w_1+w_2, $-w_1-w_2$

$\Rightarrow w_3$ -1 or -2

$-w_4+w_5 = 2$

$\boxed{w_3 = -2}$

$(-1, 1, -2, -1, 1)$