CS3244 Tutorial 4

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T13

BL6

$w_1, w_2 = \frac{1}{2}$

inputs x_1, x_2 and output $o(x_1, x_2)$ are boolean $\in \{-1, 1\}$

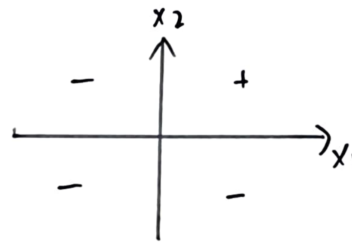
x_1	x_2	$AND(x_1, x_2)$
1	1	1
1	-1	-1
-1	1	-1
-1	-1	-1

$o(1,1) = 1 \Rightarrow w_0 + \frac{1}{2}(1) + \frac{1}{2}(1) > 0$

$o(1,-1) = -1 \Rightarrow w_0 + \frac{1}{2}(1) + \frac{1}{2}(-1) \leq 0$

$o(-1,1) = -1 \Rightarrow w_0 + \frac{1}{2}(-1) + \frac{1}{2}(1) \leq 0$

$o(-1,-1) = -1 \Rightarrow w_0 + \frac{1}{2}(-1) + \frac{1}{2}(-1) \leq 0$



$w_0 + 1 > 0 \Rightarrow w_0 > -1$

$\left.\begin{array}{l} w_0 \leq 0 \\ w_0 - 1 \leq 0 \end{array}\right\} w_0 \leq 0$

$\boxed{-1 < w_0 \leq 0}$

BL7

$w_1, w_2 = -1$

inputs x_1, x_2 and output $o(x_1, x_2)$ are boolean $\in \{-1, 1\}$

x_1	x_2	$NAND(x_1, x_2)$
1	1	-1
1	-1	1
-1	1	1
-1	-1	1

$o(1,1) = -1 \Rightarrow w_0 - 1(1) - 1(1) \leq 0$

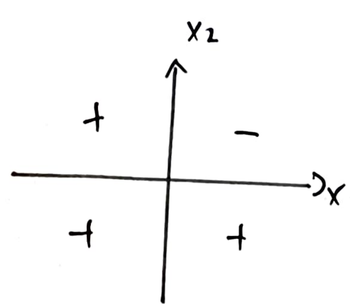
$o(1,-1) = 1 \Rightarrow w_0 - 1(1) - 1(-1) > 0$

$o(-1,1) = 1 \Rightarrow w_0 - 1(-1) - 1(1) > 0$

$o(-1,-1) = 1 \Rightarrow w_0 - 1(-1) - 1(-1) > 0$

$w_0 - 2 \leq 0 \Rightarrow w_0 \leq 2$

$\left.\begin{array}{l} w_0 > 0 \\ w_0 + 2 > 0 \end{array}\right\} w_0 > 0$



$\boxed{0 < w_0 \leq 2}$

B2 8

a) x_1 OR x_2

x_1	x_2	x_1 OR x_2
1	1	1
1	-1	1
-1	1	1
-1	-1	-1

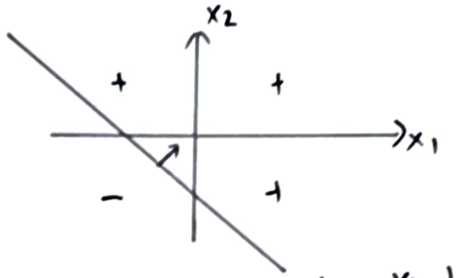
$o(1,1) = 1 \Rightarrow w_0 + w_1 + w_2 > 0$

$o(1,-1) = 1 \Rightarrow w_0 + w_1 - w_2 > 0$

$o(-1,1) = 1 \Rightarrow w_0 - w_1 + w_2 > 0$

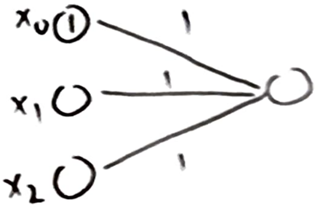
$o(-1,-1) = -1 \Rightarrow w_0 - w_1 - w_2 \leq 0$

$w_0 = 1,\ w_1 = 1,\ w_2 = 1$ works



$x_2 = -x_1 - 1$

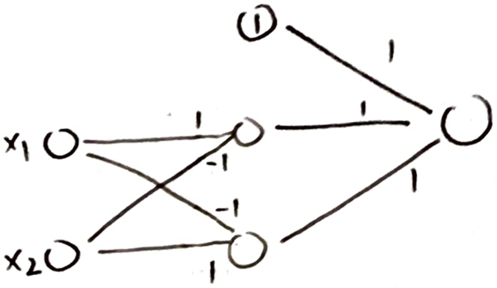
$1 + x_1 + x_2 = 0$



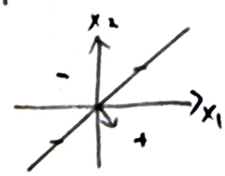
b) x_1 XOR x_2 with 1 hidden layer

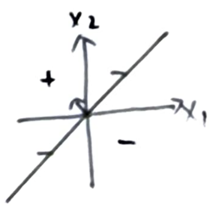
$= (x_1 \text{ AND } \sim x_2) \text{ OR } (\sim x_1 \text{ AND } x_2)$

x_1	x_2	x_1 XOR x_2
1	1	-1
1	-1	1
-1	1	1
-1	-1	-1



x_1	x_2	x_1 AND $\sim x_2$	$\sim x_1$ AND x_2
1	1	-1	-1
1	-1	1	-1
-1	1	-1	1
-1	-1	-1	-1





$x_1 - x_2 = 0$

$w_1 = 1$

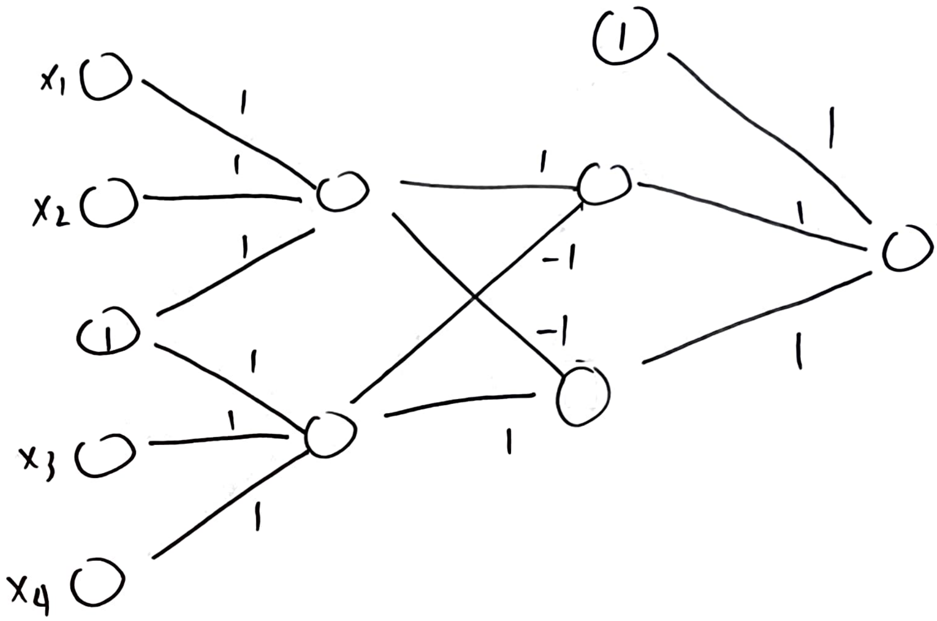
$w_2 = -1$

$x_2 - x_1 = 0$

$w_1 = -1$

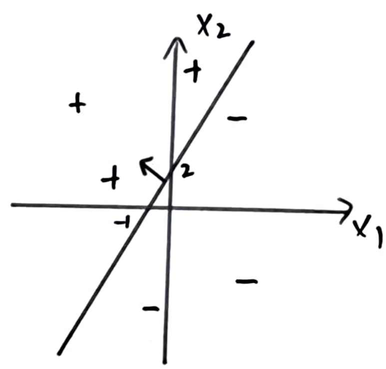
$w_2 = 1$

c) $(x_1 \text{ OR } x_2)$ XOR $(x_3 \text{ OR } x_4)$ 2 hidden layers, 2 nodes each



TM 4.1

linear separator : $w_0 + w_1 x_1 + w_2 x_2 = 0$



$x_2 = 0 \Rightarrow w_0 + w_1(-1) = 0$

$w_0 = w_1$

$x_1 = 0 \Rightarrow w_0 + w_2(2) = 0$

$w_0 = -2w_2$

$w_0 = w_1 = -2w_2$

normal vector points in positive direction $\Rightarrow$ $w_1 < 0,\ w_2 > 0$

$w_0 < 0$

e.g $w_2 = 1$, $w_0 = w_1 = -2$

TM 4.3

$w_0 + w_1 x_1 + w_2 x_2 > 0$

Perception A

$\left.\begin{array}{l} w_0 = 1 \\ w_1 = 2 \\ w_2 = 1 \end{array}\right\} \Rightarrow 1 + 2x_1 + x_2 > 0$

Perception B

$\left.\begin{array}{l} w_0 = 0 \\ w_1 = 2 \\ w_2 = 1 \end{array}\right\} \Rightarrow 2x_1 + x_2 > 0$

A is more general than B

iff $\forall x \in X \quad O_B(x) = 1 \rightarrow O_A(x) = 1$ $\quad (A \geq_g B)$

and $\exists x \in X \quad O_A(x) = 1 \wedge O_B(x) = 0$ $\quad (B \ngeq_g A)$

$\forall (x_1, x_2) \in X, \quad 2x_1 + x_2 > 0 \Rightarrow 1 + 2x_1 + x_2 > 0$ $\quad (A \geq_g B)$

$\exists (0, -\frac{1}{2}) \in X$ st $1 - \frac{1}{2} > 0$ but $-\frac{1}{2} \leq 0$ $\quad (B \ngeq_g A)$

$\therefore$ A is more general than B