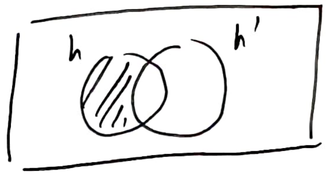
CS3244 19/20 S2 Finals

$h \setminus h' = h(x) = 1 \wedge h'(x) = 0$



Concept Learning 1

$G_0 = \{ (0,10,0,10) \setminus (6,5,3,2), \dots \} = \{ (0,10,0,10) \}$

$S_0 = \{ (6,5,3,2) \setminus (0,10,0,10) \dots \} = \{ (6,5,3,2) \}$

— no instance satisfies 675, 372

$G_1 = G_0$

no instance satisfy

$S_1 = \{ (6,6,3,3) \setminus (10,0,10,0) \} = \{ (6,6,3,3) \}$

$S_2 = S_1$

$G_2 = \{ (0,10,0,10) \setminus (8,8,7,7) \}$ = everything except (8,7)

$G_3 = G_2$

$S_3 = \{ (4,6,3,7) \setminus (5,5,3,7), (4,6,3,7) \setminus (4,6,4,6), (4,6,3,7) \setminus (4,5,3,6), (4,6,3,7) \setminus (5,6,4,7) \}$

$S_4 = S_3$

no S more specific

$G_4 = \{ (0,10,2,10) \setminus (8,8,7,7),$ ~~(0,10,0,6) \ (2,2,1,1)~~

$(0,7,0,10) \setminus (2,2,1,1), (3,10,0,10) \setminus (8,8,7,7) \}$

$S_5 = S_4$

$G_5 = \{ (0,10,2,8) \setminus (8,8,7,7), (4,10,0,10) \setminus (8,8,7,7),$

$(0,7,2,10) \setminus (3,3,9,9), (3,7,0,10) \setminus (3,3,9,9),$

$(0,7,0,8) \setminus (2,2,1,1) \}$

Neural Networks 1

i) Disprove by counterexample

network A models x_1 XOR x_2
network B models x_1 OR x_2

		A	B
x_1	x_2	x_1 XOR x_2	x_1 OR x_2
1	1	-1	1
1	-1	1	1
-1	1	1	1
-1	-1	-1	-1

For network A to be more general than or equal to unit B

$$\forall x \in X \quad (o_B = 1) \rightarrow (o_A = 1)$$

let $x = (1, 1)$

$$(1 \text{ OR } 1) = 1 \quad \wedge \quad (1 \text{ XOR } 1) = -1$$

$$\exists x \in X \text{ st } (o_B = 1) \wedge (o_A = -1) \implies A \not\geq_g B$$

ii) Prove by exhaustion

network C models $(x_1 \text{ OR } x_2)$ XOR $(x_3 \text{ OR } x_4)$

network F symmetric
outputs 1 if 1 of hidden unit outputs 1
For hidden unit to output 1, input to -2, -2 weights must be both '-1'
other inputs must have at least 1 '1'

both implement $(x_1 \text{ OR } x_2)$ XOR $(x_3 \text{ OR } x_4)$
same truth table

x_1	x_2	x_3	x_4	o_C	o_F
1	1	1	1	-1	-1
1	1	1	-1	-1	-1
1	1	-1	1	-1	-1
1	1	-1	-1	1	1
1	-1	1	1	-1	-1
1	-1	1	-1	-1	-1
1	-1	-1	1	-1	-1
1	-1	-1	-1	1	1
-1	1	1	1	-1	-1
-1	1	1	-1	-1	-1
-1	1	-1	1	-1	-1
-1	1	-1	-1	1	1
-1	-1	1	1	1	1
-1	-1	1	-1	1	1
-1	-1	-1	1	1	1
-1	-1	-1	-1	-1	-1

$$\forall x \in X \quad (o_F = 1) \rightarrow (o_C = 1)$$

$$C \geq_g F$$

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Bayesian Inference

i)

A implements OR gate $\quad w_1 \cdots w_4 = 1 \;\&\; 2 < w_0 \le 4$

B implements NAND gate $\quad \sim x_1 \vee \sim x_2 \vee \sim x_3 \vee \sim x_4 \equiv \sim(x_1 \wedge x_2 \wedge x_3 \wedge x_4)$

C & F implement $(x_1 \text{ OR } x_2)$ XOR $(x_3 \text{ OR } x_4)$ see previous question

prior beliefs are equal and sum to 1

$P(w_A) = P(w_B) = P(w_C) = P(w_F) = \frac{1}{4}$

	o_1/t_{d_1}	o_2/t_{d_2}	o_3/t_{d_3}	o_4
A	1/1	1/1	1/−1	−1
B	1/1	1/1	−1/−1	1
C	1/1	1/1	−1/−1	−1
F	1/1	1/1	−1/−1	−1

examples are conditionally independent given weights

fixed input instances

$$P(D|w_A) = \prod_{i=1}^{3} P(t_{d_i} | w_A, x_{d_i})$$

$$= 1 \cdot 1 \cdot 0 = 0$$

$$P(D|w_B) = 1 \cdot 1 \cdot 1 = 1$$

$$P(D|w_C) = 1 \cdot 1 \cdot 1 = 1$$

$$P(D|w_F) = 1 \cdot 1 \cdot 1 = 1$$

$$P(w_A|D) = \frac{P(D|w_A) \cdot P(w_A)}{P(D)} = \frac{0 \cdot \frac{1}{4}}{\frac{3}{4}} = 0 \qquad P(D) = \sum_{h \in w_A, w_B, w_C, w_F} P(D|h) \cdot P(h) = \frac{3}{4}$$

$$P(w_B|D) = \frac{1 \cdot \frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

$$P(w_C|D) = \frac{1}{3}$$

$$P(w_F|D) = \frac{1}{3}$$

$$P(t_{d_4} = -1 | D, x_{d_4}) = \sum_{h \in w_A, w_B, w_C, w_F} P(t_{d_4} = -1 | h, x_{d_4}) \cdot P(h|D)$$

$$= 1 \cdot 0 + 0 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} = \frac{2}{3} \quad (\text{argmax})$$

$$P(t_{d_4} = 1 | D, x_{d_4}) = \frac{1}{3}$$

Bayes optimal classifier for new input instance x_{d_4} is -1

ii)

	o_1/td_1	o_2/td_2	o_3/td_3	o_4
A	1/1	1/1	-1/-1	-1
B	1/1	1/1	1/-1	1
C	1/1	-1/1	-1/-1	-1
F	1/1	-1/1	-1/-1	-1

$$P(D'|w_A) = \prod_{i=1}^{3} P(td_i | w_A, xd_i)$$

$$= 1\cdot1\cdot1 = 1$$

$$P(D'|w_B) = 1\cdot1\cdot0 = 0$$

$$P(D'|w_C) = 1\cdot0\cdot1 = 0$$

$$P(D'|w_F) = 1\cdot0\cdot1 = 0$$

$$P(D) = \sum P(D|h)\cdot P(h) - \tfrac{1}{4}$$

$$= \tfrac{1}{4}$$

$$P(w_A|D') = \frac{P(D'|w_A)\cdot P(w_A)}{P(D)}$$

$$= \frac{1\cdot\frac{1}{4}}{\frac{1}{4}} = 1$$

$$P(w_B|D') = \frac{0\cdot\frac{1}{4}}{\frac{1}{4}} = 0$$

$$P(w_C|D') = 0$$

$$P(w_F|D') = 0$$

$$P(td_4=-1 | D', xd_4) = \sum_{h\in H} P(td_4=-1 | h, xd_4)\cdot P(h|D)$$

$$= 1\cdot1 + 0\cdot0 + 1\cdot0 + 1\cdot0 = 1 \quad \text{(argmax)}$$

$$P(td_4=1 | D', xd_4) = 0$$

Bayes optimal classification for new input instance xd_4 is -1

Neural Networks 2

Proof by contradiction

Since there is only 1 unit in the hidden layer, any combination of input attributes $2^4 = 16$ can only produce 2 values in the hidden layer $(0, 1)$

Suppose weights $w_5, w_6, w_7, w_8, w_9, w_{10}, w_{11}, w_{12}$ have been chosen such that the network of perceptron units is consistent with 1)

WLOG, let output of hidden unit with input $d_1 = (1,0,0,0)$ be 0

k_1: $w_9 + 0 \cdot w_5 = w_9 > 0 \Rightarrow t_{k_1} = 1$

k_2: $w_{10} \leq 0 \Rightarrow t_{k_2} = 0$

k_3: $w_{11} \leq 0 \Rightarrow t_{k_3} = 0$

k_4: $w_{12} \leq 0 \Rightarrow t_{k_4} = 0$

$(1,0,0,0)$ ✓

and let output of hidden unit with input $d_2 = (0,1,0,0)$ be 1

k_1: $w_9 + w_5 \leq 0$, since $w_9 > 0$ in previous input, $w_9 = 1,\ w_5 = -1 \Rightarrow t_{k_1} = 0$

k_2: $w_{10} + w_6 > 0$, since $w_{10} \leq 0$ in previous input, $w_{10} = 0,\ w_6 = 1 \Rightarrow t_{k_2} = 1$

k_3: $w_{11} + w_7 \leq 0$,

k_4: $w_{12} + w_8 \leq 0$

$(0,1,0,0)$

Case 1) If output of hidden unit with input $d_3 := (0,0,1,0)$ is 0

k_1: $w_9 = 1 > 0 \Rightarrow t_{k_1} = 1$ ✗ contradiction

Case 2) If output of hidden unit with input $d_4 := (0,0,1,0)$ is 1

k_2: $w_{10} + w_6 = 0 + 1 > 0 \Rightarrow t_{k_2} = 1$ ✗ contradiction

For all possible of outputs of hidden unit, network is not consistent with 1)

Contradiction.

Thus there are no weights which allow the network to be consistent with 1)