CS3244 Tutorial 4a

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BL 8a (Midterm AY2020/21)

Let $Z = \{0,1,\ldots,10\}$. *Consider the input instance space* $X = \{(x_1, x_2)\}_{x_1,x_2 \in Z}$ *consisting of integer points in the* x_1, x_2 *plane, and the hypothesis space* H *such that each hypothesis* $h \in H$ *is defined as*

$$h(x_1, x_2) = \begin{cases} 1 & \text{if } a \leq x_1 \leq b \text{ and } c \leq x_2 \leq d, \\ 0 & \text{otherwise} \end{cases}$$

where $a, b, c, d \in Z$. *We represent hypothesis h in the form* (a, b, c, d). *For example, a typical hypothesis in* H *is* $(3,5,2,9)$. *Note that for any* $h = (a, b, c, d) \in H$, *if* $a > b$ *or* $c > d$, *then no input instance* $(x_1, x_2) \in X$ *satisfies* h.

Trace the CANDIDATE-ELIMINATION algorithm for the hypothesis space H *given the sequence of positive* $(c(x_1, x_2) = 1)$ *and negative* $(c(x_1, x_2) = 0)$ *training examples from Table 1 below.*

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Solution

We start from

$G_0 = \{(0{,}10{,}0{,}10)\}$

$S_0 = \{(6{,}5{,}3{,}2), (7{,}4{,}4{,}1), (10{,}0{,}10{,}0), \ldots\} = \{(6{,}5{,}3{,}2)\}$

Note that the above hypotheses in S_0 are not semantically distinct

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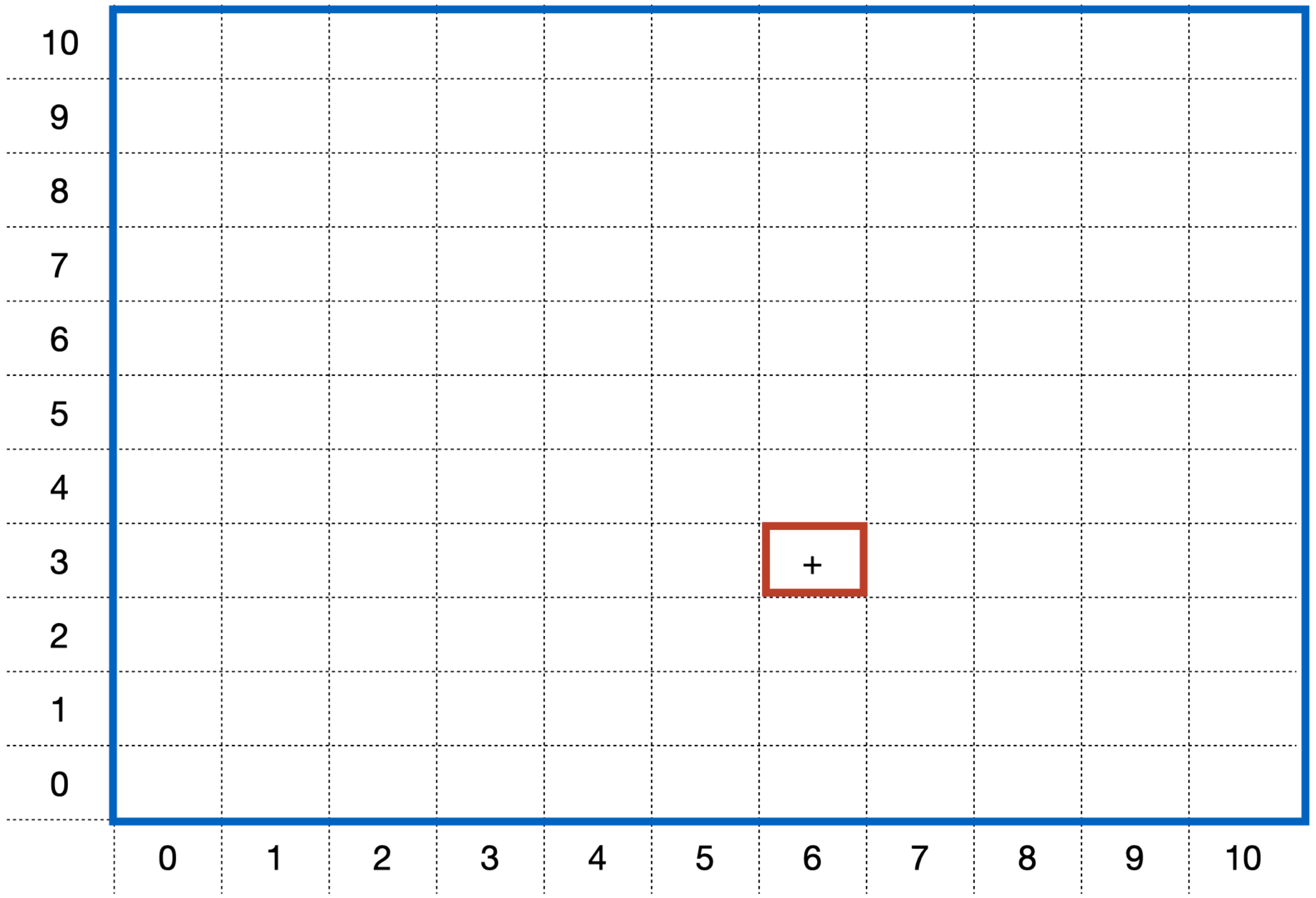
Example	Input Instance		Target Concept
	x_1	x_2	$c(x_1, x_2)$
1	6	3	1
2	8	7	0
3	4	7	1
4	2	1	0
5	3	9	0

$G_0 = \{(0,10,0,10)\}, S_0 = \{(6,5,3,2)\}$

Minimally generalize hypotheses in S_0 to include positive example (6,3)

$G_1 = \{(0,10,0,10)\}, S_1 = \{(6,6,3,3)\}$



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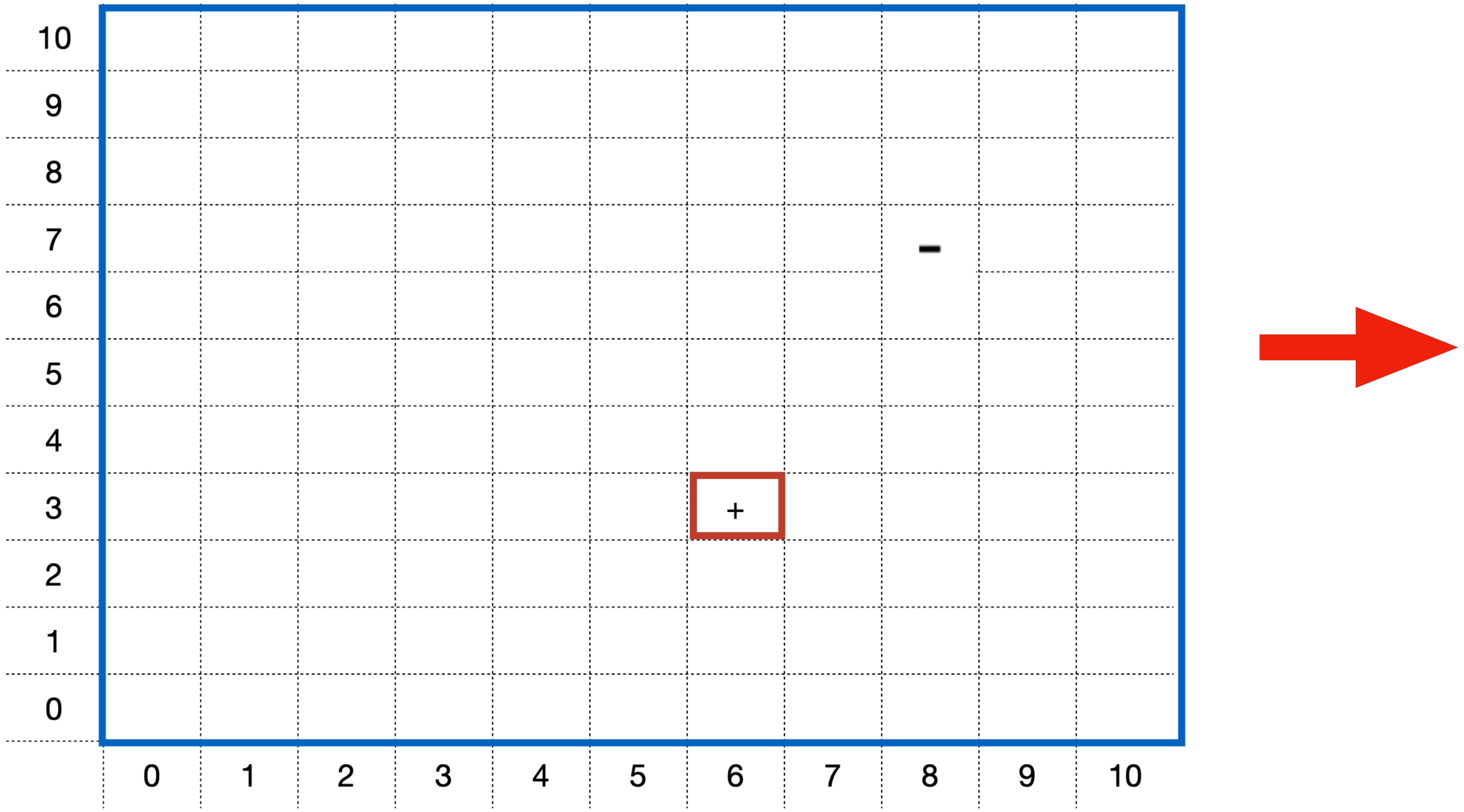
Example	Input Instance		Target Concept
	x_1	x_2	$c(x_1, x_2)$
1	6	3	1
2	8	7	0
3	4	7	1
4	2	1	0
5	3	9	0

$G_1 = \{(0,10,0,10)\}$, $S_1 = \{(6,6,3,3)\}$

Minimally specialize hypotheses in G_0 to exclude negative example $(8,7)$



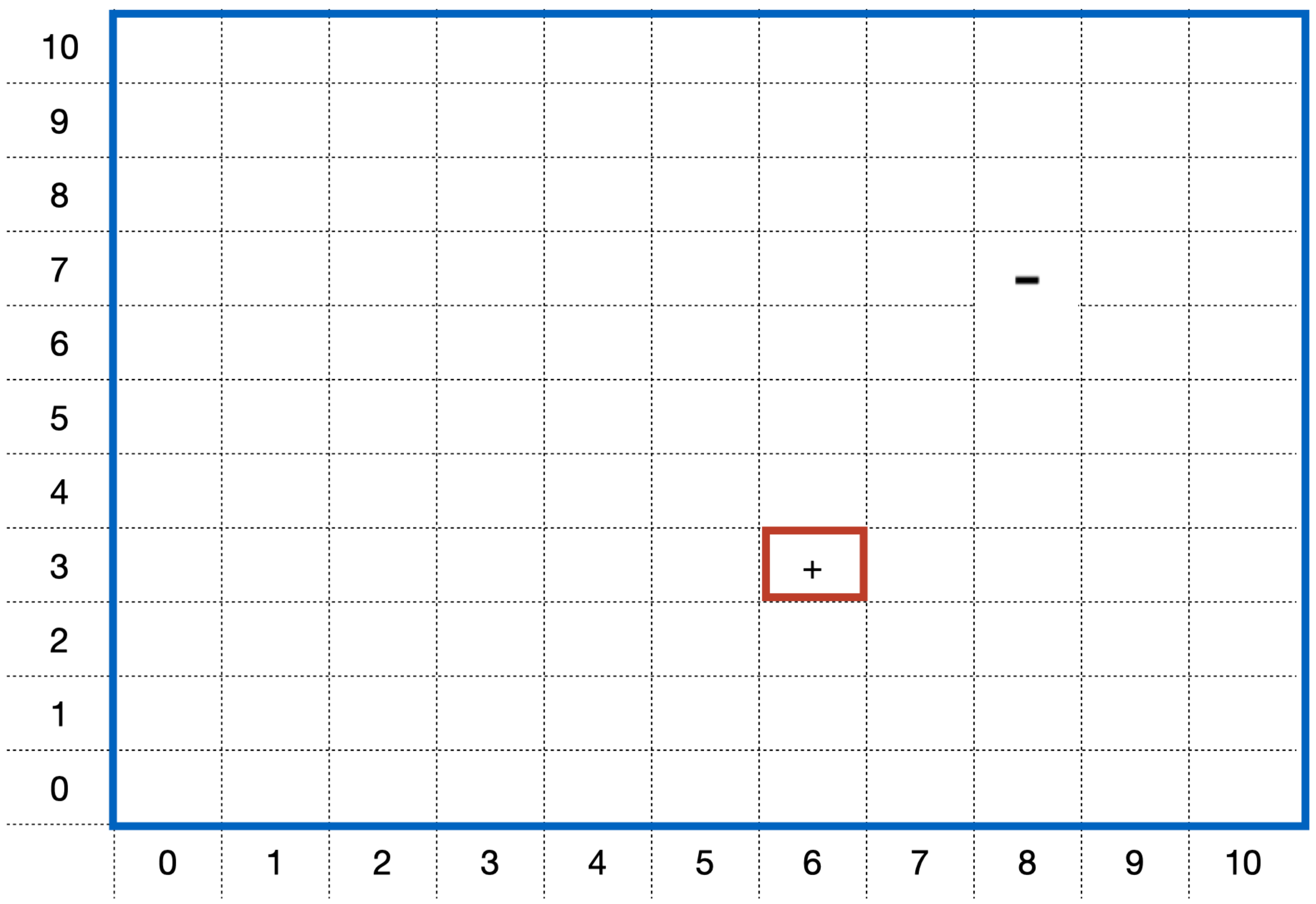
BL 8a

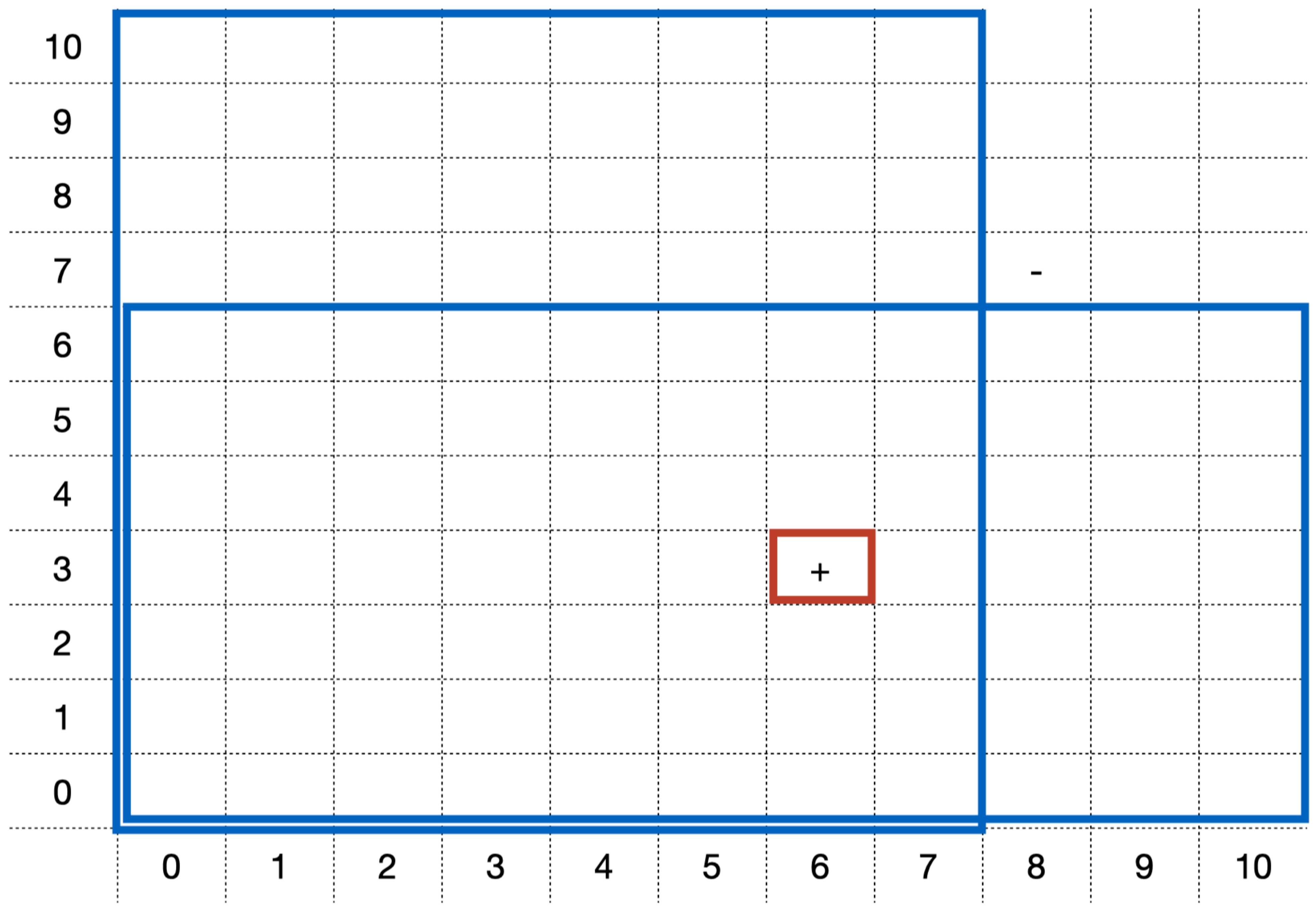
Example	Input Instance		Target Concept
	x_1	x_2	$c(x_1, x_2)$
1	6	3	1
2	8	7	0
3	4	7	1
4	2	1	0
5	3	9	0

$G_1 = \{(0,10,0,10)\}$, $S_1 = \{(6,6,3,3)\}$

Minimally specialize hypotheses in G_0 to exclude negative example $(8,7)$

$G_2 = \{(0,7,0,10), (0,10,0,6)\}$, $S_2 = S_1 = \{(6,6,3,3)\}$



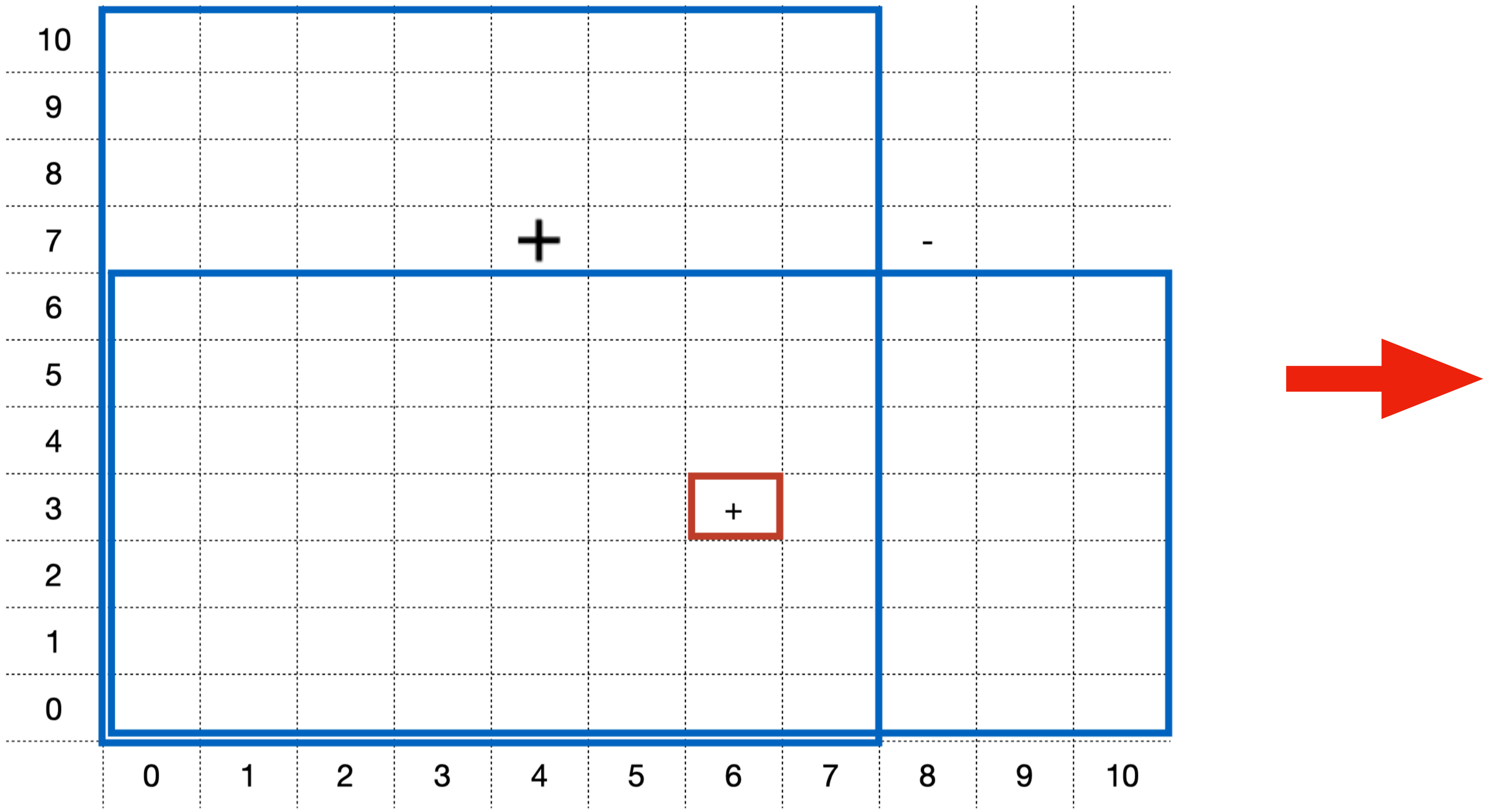


BL 8a

Example	Input Instance		Target Concept
	x_1	x_2	$c(x_1, x_2)$
1	6	3	1
2	8	7	0
3	4	7	1
4	2	1	0
5	3	9	0

$G_2 = \{(0,7,0,10), (0,10,0,6)\}$, $S_2 = \{(6,6,3,3)\}$

Minimally generalize hypotheses in S_2 to include positive example $(4,7)$



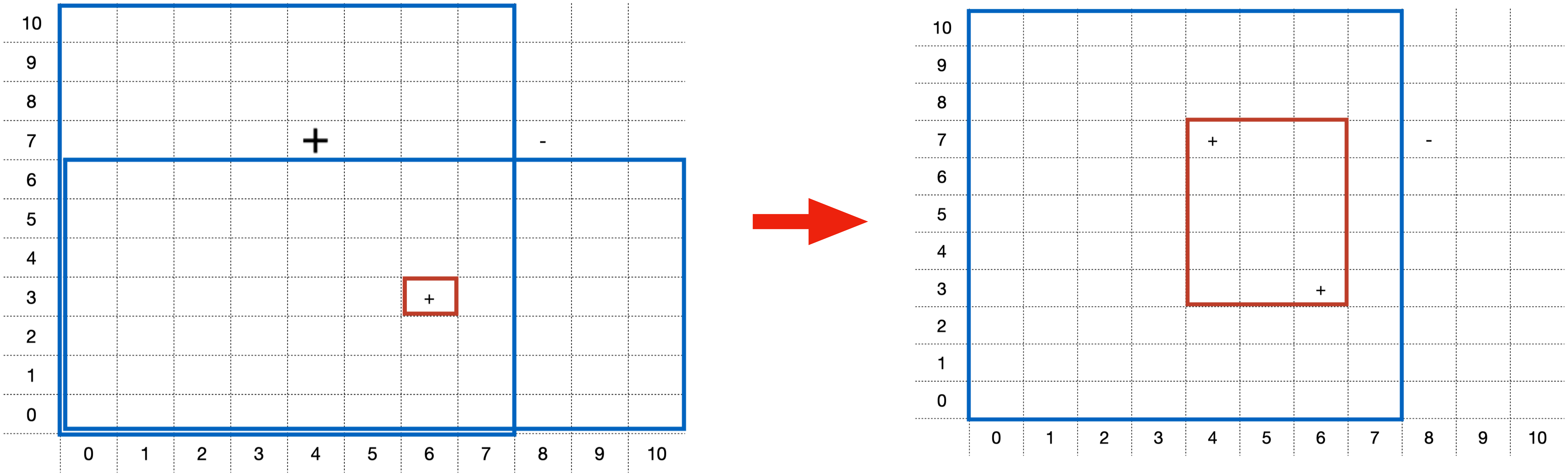
BL 8a

Example	Input Instance		Target Concept
	x_1	x_2	$c(x_1, x_2)$
1	6	3	1
2	8	7	0
3	4	7	1
4	2	1	0
5	3	9	0

$G_2 = \{(0{,}7{,}0{,}10), (0{,}10{,}0{,}6)\}$, $S_2 = \{(6{,}6{,}3{,}3)\}$

Minimally generalize hypotheses in S_2 to include positive example $(4{,}7)$

$S_3 = \{(4{,}6{,}3{,}7)\}$, $G_3 = \{(0{,}7{,}0{,}10)\}$

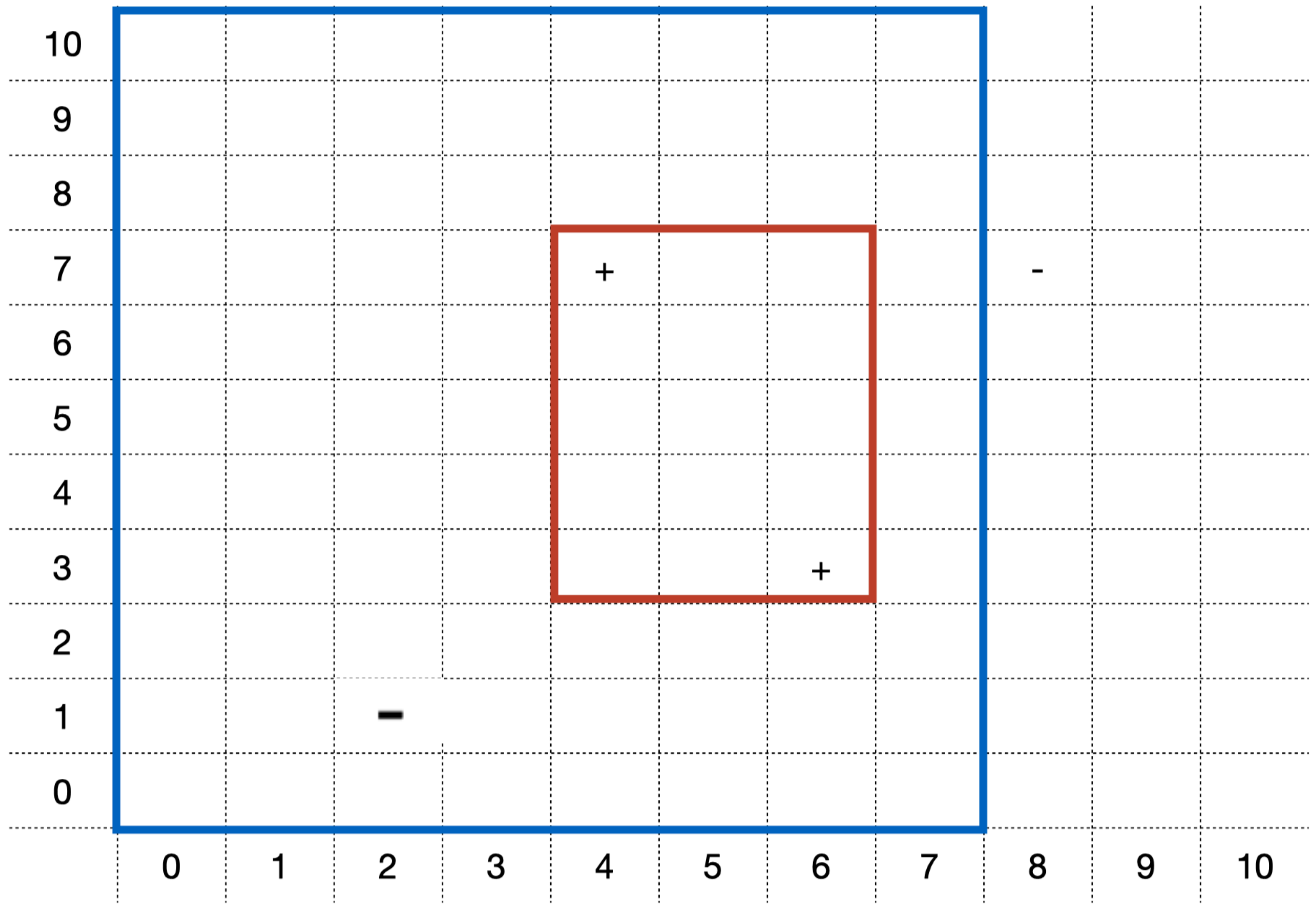


BL 8a

Example	Input Instance		Target Concept
	x_1	x_2	$c(x_1, x_2)$
1	6	3	1
2	8	7	0
3	4	7	1
4	2	1	0
5	3	9	0

$S_3 = \{(4,6,3,7)\}, G_3 = \{(0,7,0,10)\}$

Minimally specialize hypotheses in G_3 to exclude negative example $(2,1)$



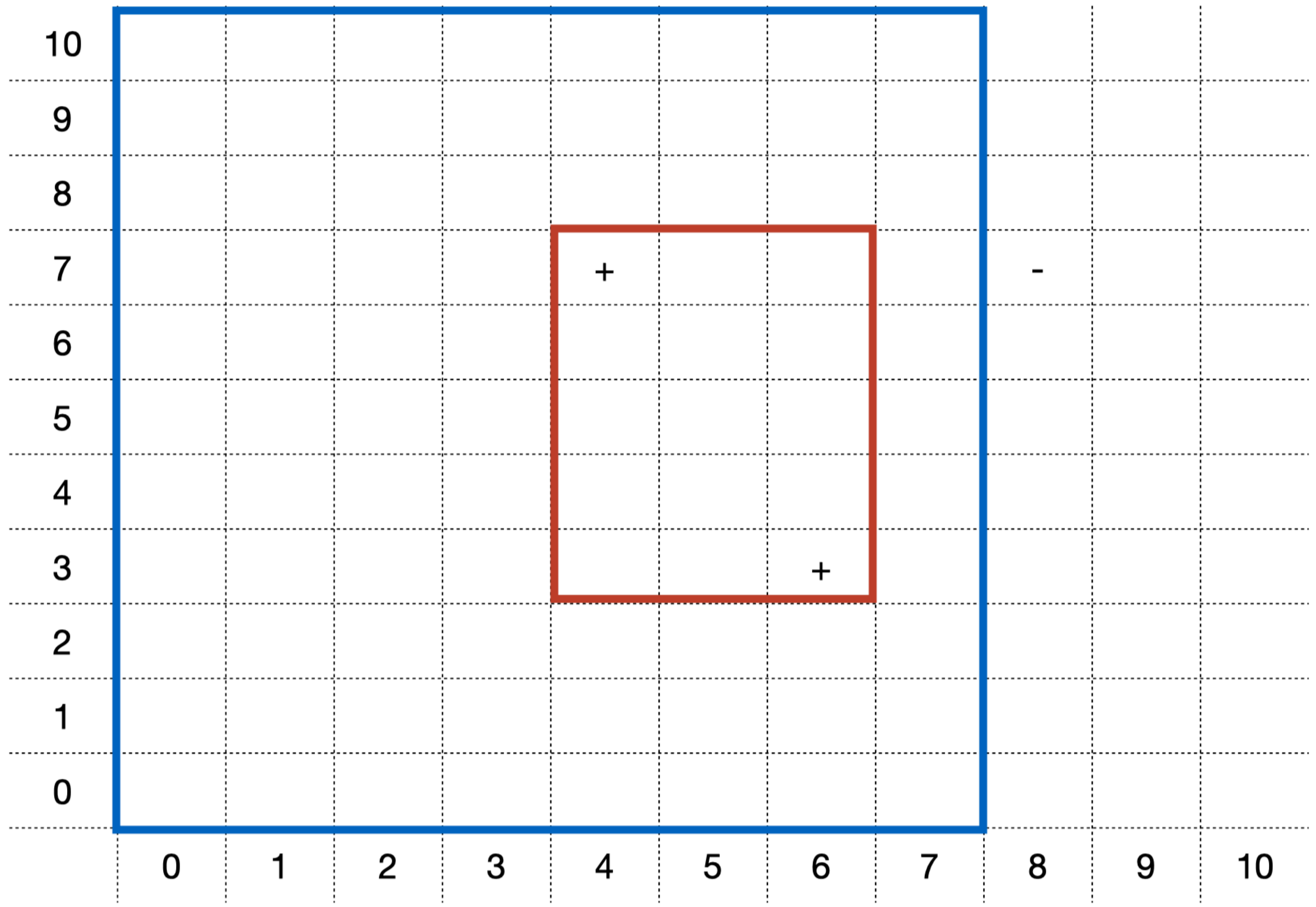
BL 8a

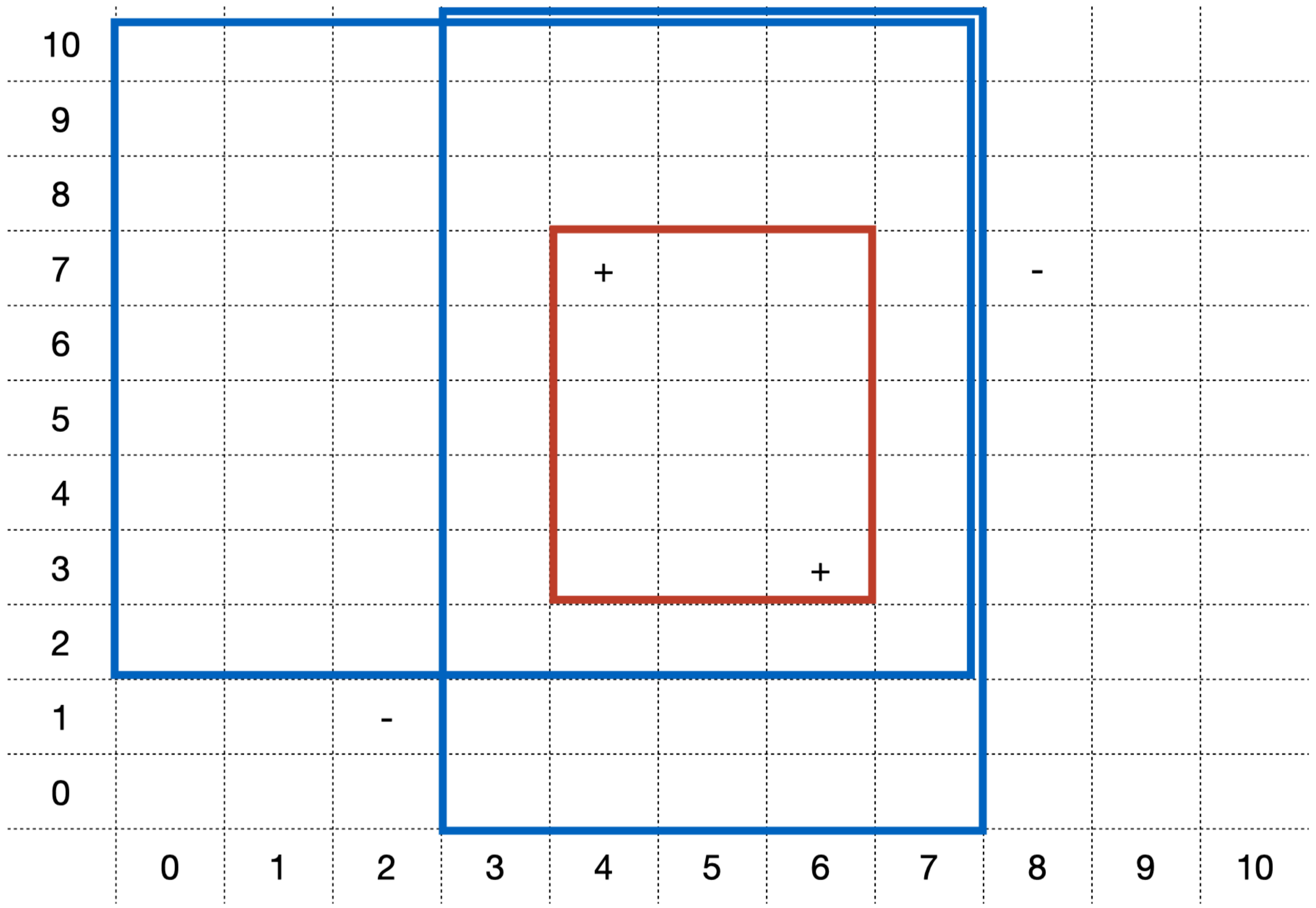
Example	Input Instance x_1	x_2	Target Concept $c(x_1, x_2)$
1	6	3	1
2	8	7	0
3	4	7	1
4	2	1	0
5	3	9	0

$S_3 = \{(4,6,3,7)\}$, $G_3 = \{(0,7,0,10)\}$

Minimally specialize hypotheses in G_3 to exclude negative example $(2,1)$

$G_4 = \{(0,7,2,10), (3,7,0,10)\}$, $S_4 = S_3 = \{(4,6,3,7)\}$



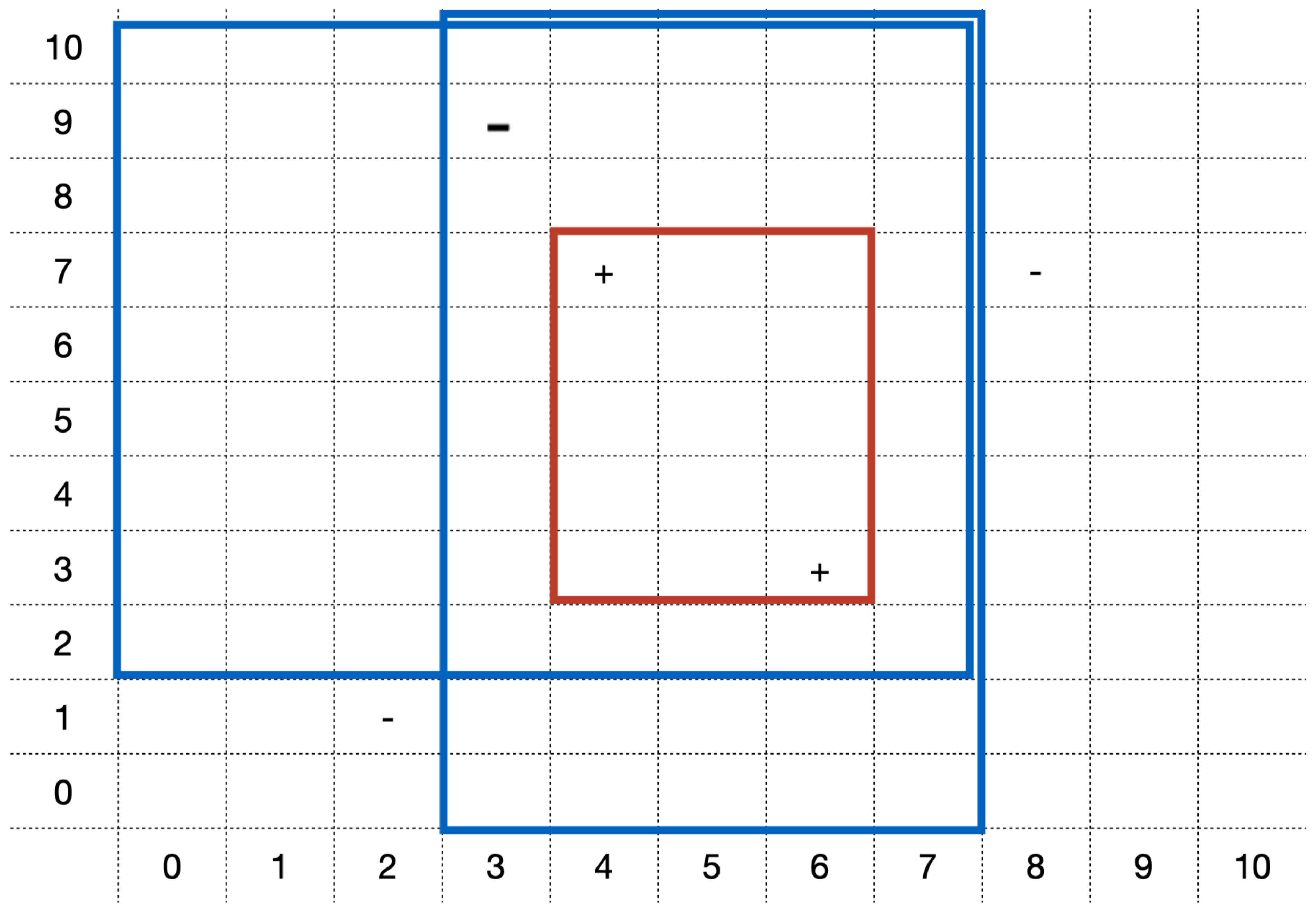


BL 8a

Example	Input Instance		Target Concept
	x_1	x_2	$c(x_1, x_2)$
1	6	3	1
2	8	7	0
3	4	7	1
4	2	1	0
5	3	9	0

$G_4 = \{(0,7,2,10), (3,7,0,10)\}$, $S_4 = \{(4,6,3,7)\}$

Minimally specialize hypotheses in G_4 to exclude negative example $(3,9)$



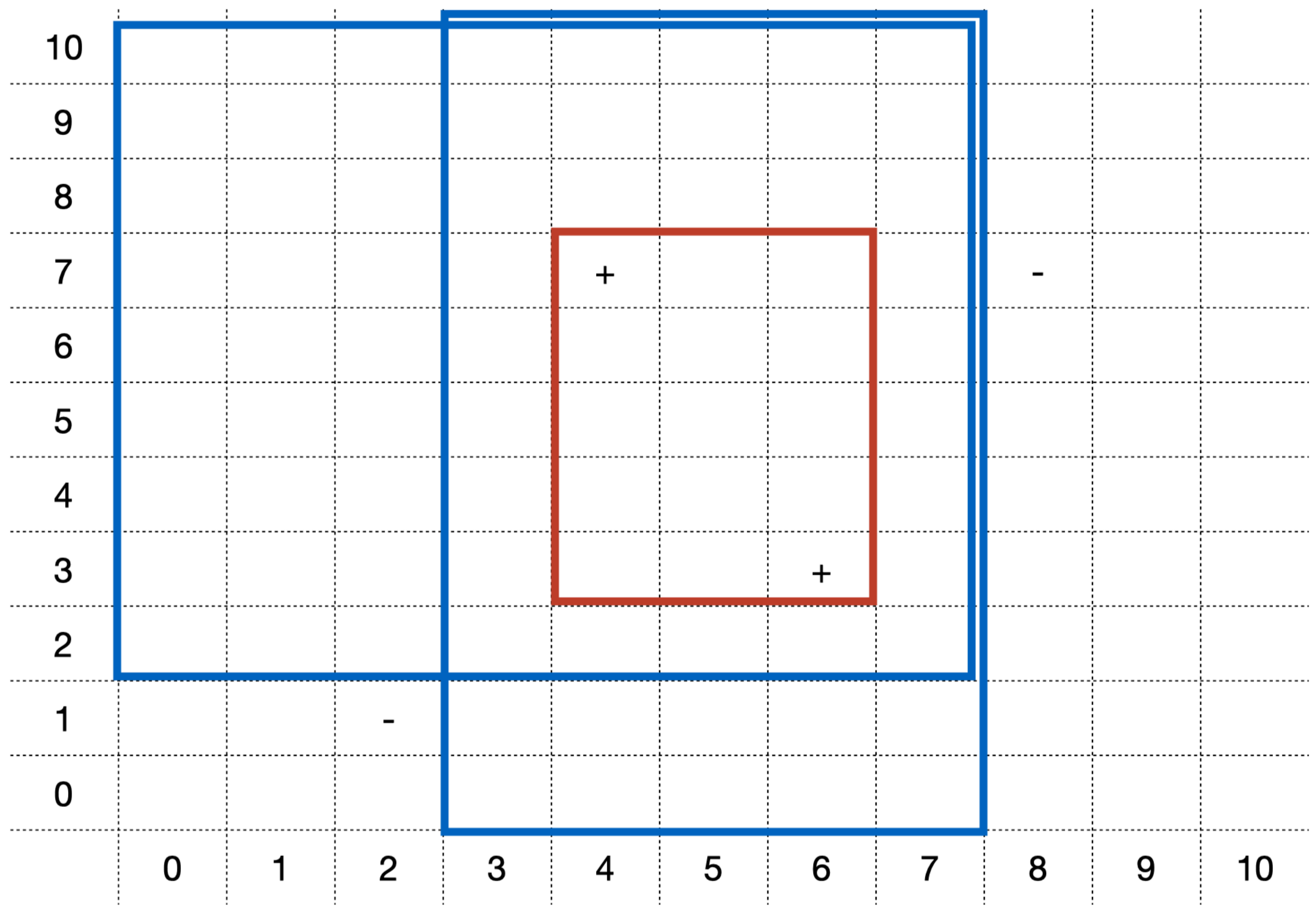
BL 8a

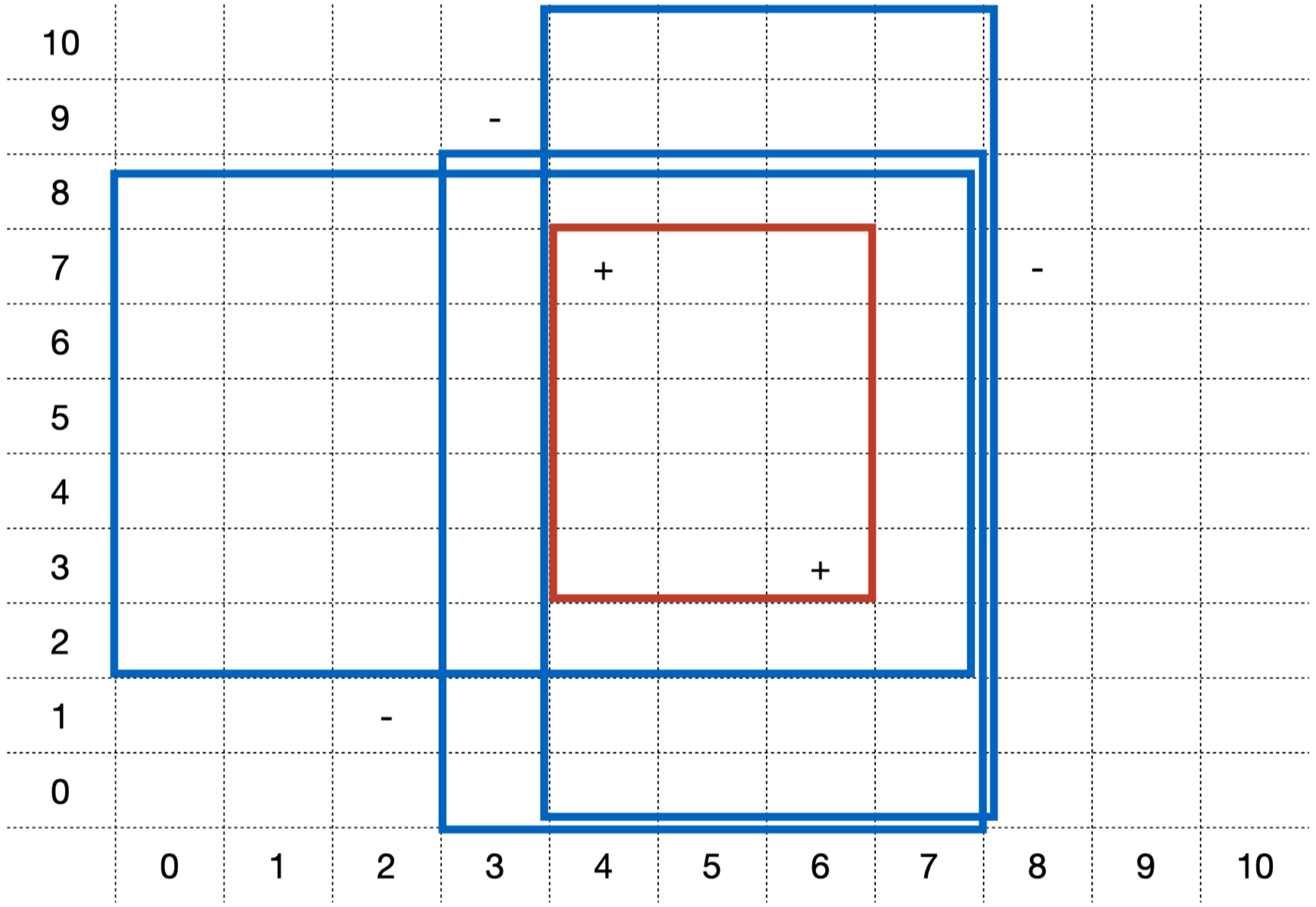
Example	Input Instance		Target Concept
	x_1	x_2	$c(x_1, x_2)$
1	6	3	1
2	8	7	0
3	4	7	1
4	2	1	0
5	3	9	0

$G_4 = \{(0,7,2,10), (3,7,0,10)\}$, $S_4 = \{(4,6,3,7)\}$

Minimally specialize hypotheses in G_4 to exclude negative example $(3,9)$

$G_5 = \{(0,7,2,8), (3,7,0,8), (4,7,0,10)\}$





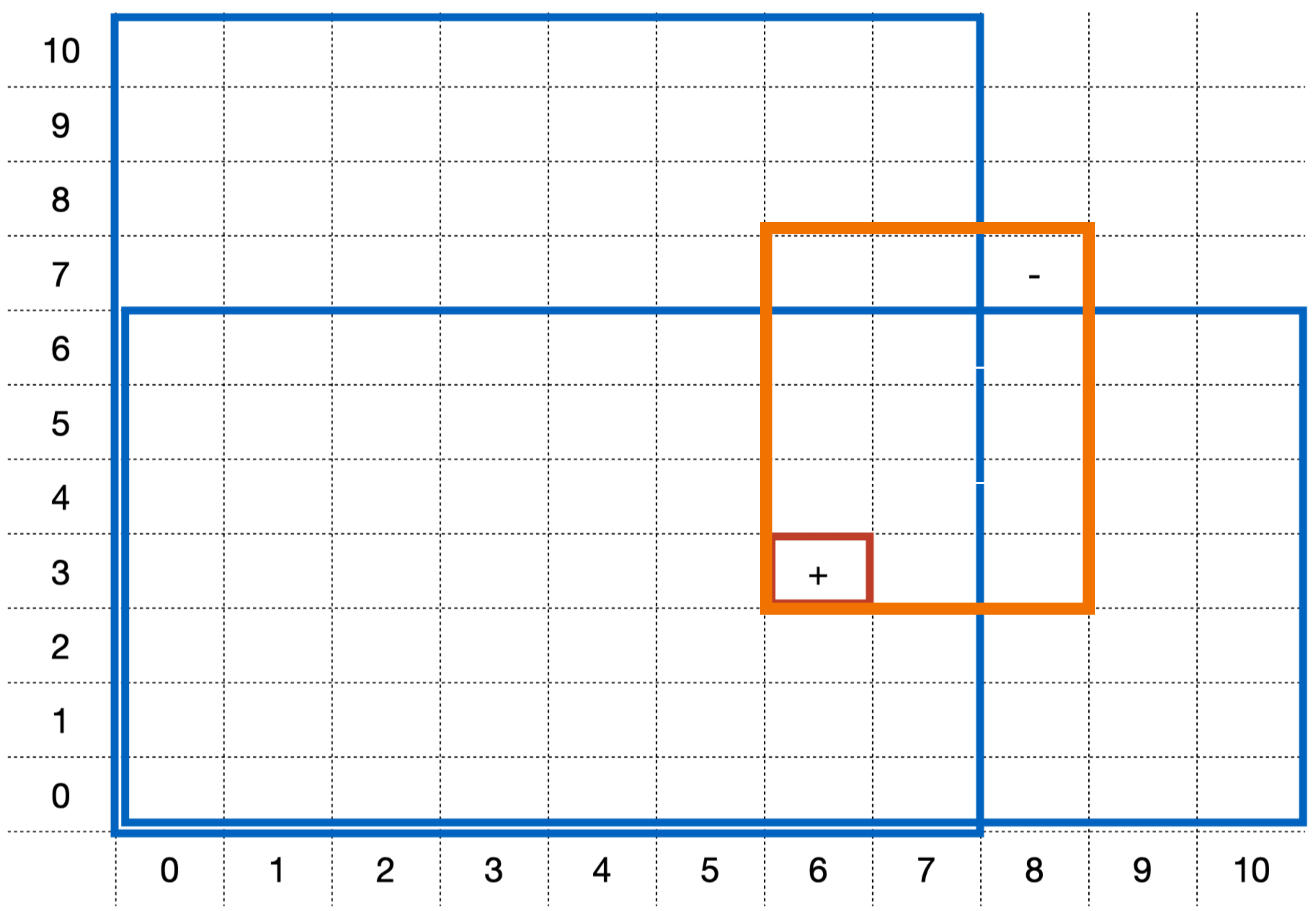
BL 8b

Let G and S be the general and specific boundaries of the version space $VS_{H,D}$, respectively. Prove formally or disprove that for any $h \in H$, if none of $g \in G$ is more general than or equal to h, then h is not more general than or equal to any $s \in S$, that is,

$$\forall h \in H \quad (\forall g \in G \quad g \ngeq_g h) \rightarrow (\forall s \in S \quad h \ngeq_g s).$$

- Hint: You may wish to consider using the content of question BL 8a to help you establish an informal intuition for solving this question

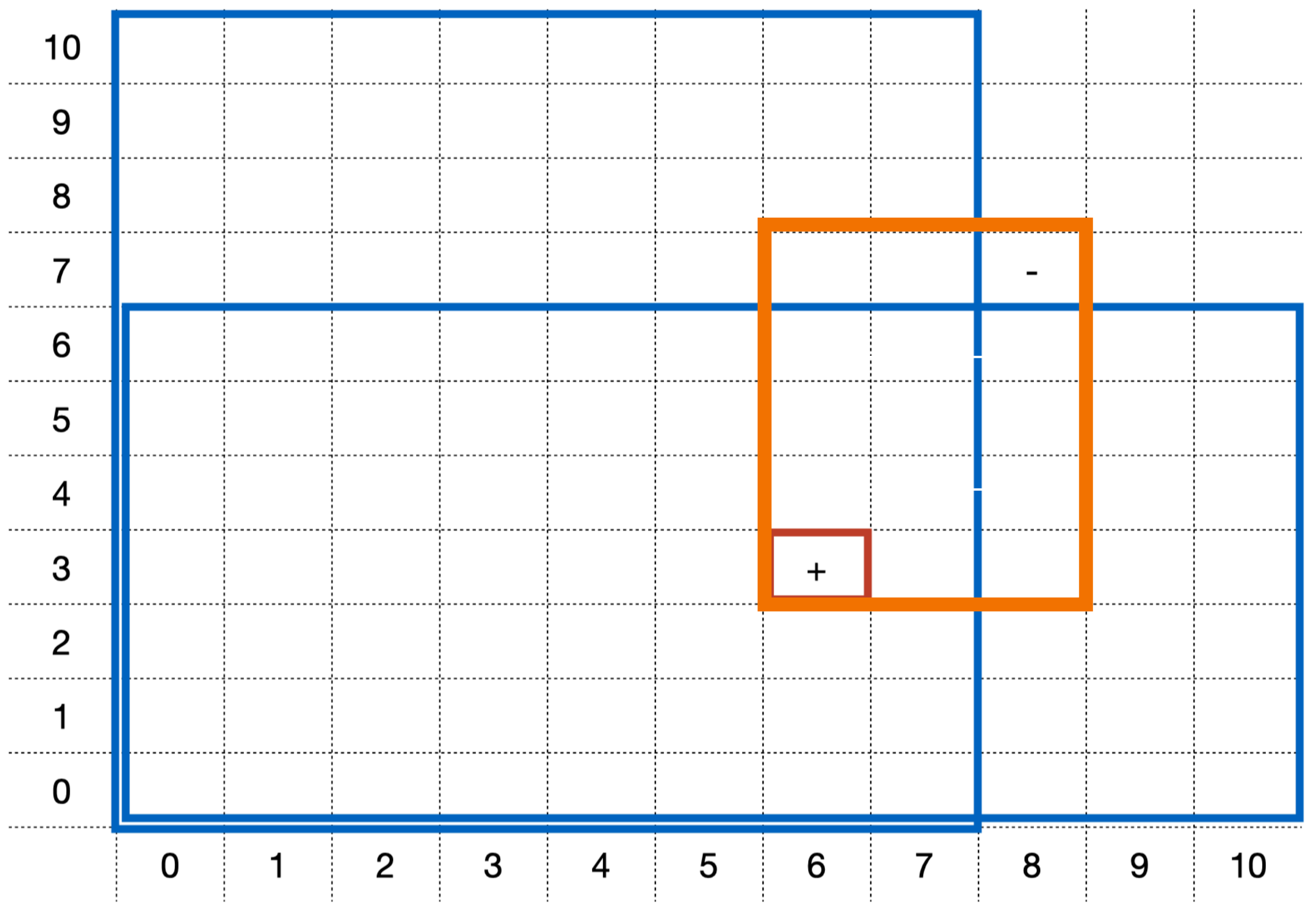
- ***Is BLUE more general or equal to ORANGE?***
- ***Is ORANGE more general or equal to RED?***



BL 8b

$$\forall h \in H \quad (\forall g \in G \quad g \ngeq_g h) \rightarrow (\forall s \in S \quad h \ngeq_g s)$$

1. Let $x^+, x^- \in X$ denote +ve and -ve training instances, respectively. For example, let $x^+ = (6{,}3)$ and $x^- = (8{,}7)$.
2. Define an inconsistent hypothesis h s.t. $h(x^+) = h(x^-) = 1$. For example, let $h = (6{,}8{,}3{,}7)$.
3. Since any $g \in G_2$ is consistent with D, $g(x^-) = 0$.
4. $\forall g \in G_2 \quad h(x^-) = 1 \wedge g(x^-) = 0$, by step 2 and 3.
5. Therefore, $\forall g \in G_2 \quad g \ngeq_g h$, by definition of $\ngeq_g$.
6. Define hypothesis s s.t. $s(x) = 1$ if $x = x^+$, and $s(x) = 0$ otherwise. For example, $s = (6{,}6{,}3{,}3)$.
7. Therefore, $s \in S_2$.



8. Since $s(x^+) = h(x^+) = 1$ and $\forall x \in X \backslash \{x^+\} \quad s(x) = 0$, we have $\forall x \in X \quad s(x) = 1 \rightarrow h(x) = 1$.
9. Therefore, $h \geq_g s$.
10. Therefore, $(\forall g \in G_2 \quad g \ngeq_g h) \wedge (h \geq_g s)$, by step 5 and 9, hence contradicting the above claim.

Thank you!

- Any questions?