CS3244 Tutorial 6

Zhuang Jianning
A0214561M
T13

TM 6.3

a) FIND-G outputs maximally general consistent hypothesis

$$P(D|h) = \begin{cases} 1 & \text{if } h \text{ is consistent with } D \\ 0 & \text{otherwise} \end{cases}$$

If we choose $P(h)$ to be uniformly distributed

$$P(h) = \frac{1}{|H|} \text{ for all } h \in H$$

$$P(D) = \sum_{h \in H} P(D|h) \cdot P(h)$$

$$= \frac{1}{|H|} \sum_{h \in H} P(D|h)$$

hypotheses in $VS_{H,D}$ are consistent with D
$\Rightarrow$ $|VS_{H,D}|$ hypotheses equal to 1

$$= \frac{|VS_{H,D}|}{|H|}$$

$$P(h|D) = \frac{P(D|h)\,P(h)}{P(D)} = \begin{cases} \dfrac{1 \cdot \frac{1}{|H|}}{|VS_{H,D}|/|H|} = \dfrac{1}{|VS_{H,D}|} & \text{if } h \text{ is consistent with } D \\ 0 & \text{otherwise} \end{cases}$$

$\therefore$ every consistent hypothesis is a MAP hypothesis
since $G \subseteq VS_{H,D}$, all $g \in G$ is a MAP hypothesis

b) FIND-G may not produce a MAP hypothesis if distribution of $P(h)$ is such that $P(g) < P(h)$ for all $g, h \in H$ such that $g >_g h$

If h is consistent with D,

$$P(h|D) = \frac{P(D|h) \cdot P(h)}{P(D)} = \frac{1 \cdot P(h)}{|VS_{H,D}|/|H|} = \frac{P(h) \cdot |H|}{|VS_{H,D}|}$$

If h is not consistent with D,

$$P(h|D) = 0$$

Hence $\forall g,h \in VS_{H,D}$ $\quad g >_g h \rightarrow P(g|D) < P(h|D)$

Maximally general consistent hypothesis will not be MAP if there exists a less general hypothesis in $VS_{H,D}$

c) Distribution of $P(h)$ does not matter for ML hypothesis,

since $h_{ML} = \underset{h \in H}{\operatorname{argmax}} \; P(D|h)$

$=$ any h that is consistent with D $\quad (\because P(D|h) = 1)$

Hence FIND-G which outputs a maximally consistent hypothesis is a ML hypothesis regardless of distribution of $P(h)$

we can use the same distribution for $P(h)$ in b) which does not guarantee a MAP hypothesis

TM6.1

$P(\text{cancer}) = 0.008 \quad P(\neg\text{cancer}) = 0.992$

$P(+|\text{cancer}) = 0.98 \quad P(-|\text{cancer}) = 0.02$

$P(+|\neg\text{cancer}) = 0.03 \quad P(-|\neg\text{cancer}) = 0.97$

$$P(\text{cancer}|++) = \frac{P(++|\text{cancer}) \cdot P(\text{cancer})}{P(++)}$$

conditionally independent given h

$$= \frac{P(+|\text{cancer}) \cdot P(+|\text{cancer}) \cdot P(\text{cancer})}{P(+|\text{cancer})^2 \cdot P(\text{cancer}) + P(+|\neg\text{cancer})^2 \cdot P(\neg\text{cancer})}$$

$$= \frac{0.98^2 \cdot 0.008}{(0.98^2 \cdot 0.008) + (0.03^2 \cdot 0.992)}$$

$= 0.896 \quad = 0.9 \; (1\text{dp})$

$h_{MAP} = \text{cancer}$

vs

pg 6 where $h_{MAP} = \neg\text{cancer}$ given only 1 +ve test

$P(\neg\text{cancer}|++) = 1 - P(\text{cancer}|++) = 0.104 = 0.1 \; (1\text{dp})$

TM 6-4

x_d not fixed, but drawn from probability distribution defined over X

$$P(D|h) = \prod_{d\in D} P(x_d, t_d | h)$$

(conditional independence given h)

$\boxed{\text{Trick 2}}$ see notes

$$= \prod_{d\in D} P(t_d | h, x_d) \cdot P(x_d|h)$$

assume x_d independent of h

$$= \prod_{d\in D} P(t_d|h, x_d) \cdot P(x_d)$$

$$= \begin{cases} \prod_{d\in D} P(x_d) & \text{if } h \text{ consistent with } D \\ 0 & \text{otherwise} \end{cases}$$

$$P(t_d|h, x_d) = \begin{cases} h(x_d) & \text{if } t_d = 1 \\ 1 - h(x_d) & \text{if } t_d = 0 \end{cases}$$

$$= \begin{cases} 1 & \text{if } h \text{ consistent with } d \\ 0 & \text{otherwise} \end{cases}$$

if h consistent with D

$$P(h|D) = \frac{P(D|h) \cdot P(h)}{P(D)}$$

uniform prior

$$= \frac{\prod_{d\in D} P(x_d) \cdot \frac{1}{|H|}}{P(D)}$$

$$= \frac{1}{|VS_{H,D}|}$$

$$P(D) = \sum_{h\in H} P(D|h) \cdot P(h)$$

$$= \frac{1}{|H|} \sum_{h \in VS_{H,D}} \prod_{d\in D} P(x_d)$$

$$= \frac{|VS_{H,D}|}{|H|} \cdot \prod_{d\in D} P(x_d)$$

If h inconsistent with D

$$P(h|D) = \frac{\cancel{P(D|h)}^{\,0} \cdot P(h)}{P(D)} = 0$$

$$P(h|D) = \begin{cases} \frac{1}{|VS_{H,D}|} & \text{if } h \text{ is consistent with } D \\ 0 & \text{otherwise} \end{cases}$$

$\Rightarrow$ same posterior belief even in more general condition

BL9

Suppose $P(D|h) = \begin{cases} 1 & \text{if } h \text{ is consistent with } D \\ 0 & \text{otherwise} \end{cases}$

and $\forall s, h \in H \quad h >_g s \rightarrow P(s) > P(h)$

more specific hypothesis is more probable a priori

$$P(h|D) = \frac{P(D|h) \cdot P(h)}{P(D)}$$

$$= \begin{cases} \frac{P(h)}{P(D)} & \text{if } h \text{ is consistent with } D \\ 0 & \text{otherwise} \end{cases}$$

since s is maximally specific hypothesis

$\frac{P(s)}{P(D)} \geq \frac{P(h)}{P(D)} \quad \forall h \in VS_{H,D} \quad \therefore \underset{h \in H}{\text{argmax}} \; P(h|D)$

$= s \Rightarrow h_{MAP} = s$

since s is also consistent with D, it is also a ML hypothesis

$\therefore$ since $h_{MAP} = h_{ML}$, but $P(h) \neq P(h')$ for all $h, h' \in H$

$T \rightarrow F$ is a false statement / implication

BL10

a) $P(\text{carrier}) = 0.02$ $\quad P(\sim\text{carrier}) = 0.98$

$P(+|\text{carrier}) = 0.998$ $\quad P(-|\text{carrier}) = 0.002$

$P(+|\sim\text{carrier}) = 0.004$ $\quad P(-|\sim\text{carrier}) = 0.996$

$$P(\text{carrier}|+) = \frac{P(+|\text{carrier}) \cdot P(\text{carrier})}{P(+)}$$

$$= \frac{0.998 \cdot 0.02}{0.998 \cdot 0.02 + 0.004 \cdot 0.98}$$

$$= 0.836$$

$$P(\sim\text{carrier}|+) = 1 - P(\text{carrier}|+)$$

$$= 0.164$$

b) $$P(\sim\text{carrier}|++) = \frac{P(++|\sim\text{carrier}) \cdot P(\sim\text{carrier})}{P(++)}$$

$$= \frac{P(+|\sim\text{carrier})^2 \cdot P(\sim\text{carrier})}{P(+|\sim\text{carrier})^2 \cdot P(\sim\text{carrier}) + P(+|\text{carrier})^2 \cdot P(\text{carrier})}$$

$$= \frac{0.004^2 \cdot 0.98}{0.004^2 \cdot 0.98 + 0.998^2 \cdot 0.02}$$

$$= 0.000787$$

very likely is carrier