National University of Singapore
School of Computing
CS3244 Machine Learning

Tutorial 4A: Concept Learning (Revision)

Issue: March 17, 2022 Due: March 21, 2022

Important Instructions:

- *Your solutions for this tutorial must be TYPE-WRITTEN.*
- *Make TWO copies of your solutions: one for you and one to be SUBMITTED TO THE TUTOR IN CLASS. Your submission in your respective tutorial class will be used to indicate your CLASS ATTENDANCE. Late submission will NOT be entertained.*
- *Indicate your NAME, STUDENT NUMBER, and TUTORIAL GROUP in your submitted solution.*
- *YOUR SOLUTION TO QUESTION* **BL 8a** *will be GRADED for this tutorial.*
- *You may discuss the content of the questions with your classmates. But everyone should work out and write up ALL the solutions by yourself.*

BL 8a (Midterm Exam AY2019/20) Let $Z = \{0, 1, \ldots, 10\}$. Consider the input instance space $X = \{(x_1, x_2)\}_{x_1, x_2 \in Z}$ consisting of integer points in the x_1, x_2 plane, and the hypothesis space H such that each hypothesis $h \in H$ is defined as

$$h(x_1, x_2) = \begin{cases} 1 & \text{if } a \leq x_1 \leq b \text{ and } c \leq x_2 \leq d, \\ 0 & \text{otherwise;} \end{cases}$$

where $a, b, c, d \in Z$. We represent hypothesis h in the form (a, b, c, d). For example, a typical hypothesis in H is $(3, 5, 2, 9)$. Note that for any $h = (a, b, c, d) \in H$, if $a > b$ or $c > d$, then **no** input instance $(x_1, x_2) \in X$ satisfies h.

Trace the CANDIDATE-ELIMINATION algorithm for the hypothesis space H given the sequence of positive ($c(x_1, x_2) = 1$) and negative ($c(x_1, x_2) = 0$) training examples from Table 1 below (i.e., show the sequence of S and G boundary sets). You only need to show the **semantically distinct**[1]

[1] The notion of **semantically distinct** hypotheses was mentioned and explained informally on page 10 of the "Concept Learning" lecture slides. For a formal definition, continue reading. Let hypotheses h and h' be in the same hypothesis space H, i.e., $h, h' \in H$. We know that a hypothesis represents a set of input instances in X such that every input instance in this set satisfies this hypothesis.

The **difference** $h \setminus h'$ of the hypotheses h and h' is defined as $h \setminus h'(x) = ((h(x) = 1) \wedge (h'(x) = 0))$ for all $x \in X$ and therefore represents the set difference of the sets of input instances represented by h and h'.

The hypotheses h and h' are **semantically distinct** iff there exists some $x \in X$ satisfying $h \setminus h'$ or $h' \setminus h$, that is, $\exists x \in X \ (h \setminus h'(x) = 1) \vee (h' \setminus h(x) = 1)$.

hypotheses in each boundary set.

Example	Input Instance		Target Concept
	x_1	x_2	$c(x_1, x_2)$
1	6	3	1
2	8	7	0
3	4	7	1
4	2	1	0
5	3	9	0

Table 1: Positive ($c(x_1, x_2) = 1$) and negative ($c(x_1, x_2) = 0$) training examples for target concept c.

Solution.

$$
\begin{aligned}
G_0 &= \{(0, 10, 0, 10)\} \\
S_0 &= \{(6, 5, 3, 2), (7, 4, 4, 1), (10, 0, 10, 0), \ldots\} = \{(6, 5, 3, 2)\} \\
&\ \text{The above hypotheses in } S_0 \text{ are } \textbf{not semantically distinct.} \\
G_1 &= G_0 \\
S_1 &= \{(6, 6, 3, 3)\} \\
S_2 &= S_1 \\
G_2 &= \{(0, 10, 0, 6), (0, 7, 0, 10)\} \\
G_3 &= \{(0, 7, 0, 10)\} \\
S_3 &= \{(4, 6, 3, 7)\} \\
S_4 &= S_3 \\
G_4 &= \{(3, 7, 0, 10), (0, 7, 2, 10)\} \\
S_5 &= S_4 \\
G_5 &= \{(3, 7, 0, 8), (4, 7, 0, 10), (0, 7, 2, 8)\}
\end{aligned}
$$

BL 8b (Midterm Exam AY2019/20) Let G and S be the general and specific boundaries of the version space $VS_{H,D}$, respectively.

Prove formally or disprove that for any $h \in H$, if none of $g \in G$ is *more general than or equal to* h, then h is not *more general than or equal to* any $s \in S$, that is,

$$\forall h \in H \quad (\forall g \in G \;\; g \not\geq_g h) \longrightarrow (\forall s \in S \;\; h \not\geq_g s) \; .$$

Hint: You may wish to consider using the content of question BL 8a to help you establish an informal intuition for solving this question.

Solution.

1. Let $x^+, x^- \in X$ denote +ve and −ve training instances, respectively. For example, from Table 1, let $x^+ = (6, 3)$ and $x^- = (8, 7)$.
2. Define an inconsistent hypothesis h s.t. $h(x^+) = h(x^-) = 1$. For example, let $h = (6, 8, 3, 7)$.
3. Since any $g \in G_2$ is consistent with D, $g(x^-) = 0$.
4. $\forall g \in G_2 \;\; h(x^-) = 1 \wedge g(x^-) = 0$, by steps 2 and 3.
5. Therefore, $\forall g \in G_2 \;\; g \not\geq_g h$, by definition of $\geq_g$.
6. Define hypothesis s s.t. $s(x) = 1$ if $x = x^+$, and $s(x) = 0$ otherwise. For example, $s = (6, 6, 3, 3)$.
7. Therefore, $s \in S_2$.
8. Since $s(x^+) = h(x^+) = 1$ and $\forall x \in X \setminus \{x^+\} \;\; s(x) = 0$, $\forall x \in X \;\; s(x) = 1 \longrightarrow h(x) = 1$.
9. Therefore, $h \geq_g s$.
10. Therefore, $(\forall g \in G_2 \;\; g \not\geq_g h) \wedge (h \geq_g s)$, by steps 5 and 9, hence contradicting the above claim.