(T)EE2026 DIGITAL DESIGN

TUTORIAL 4: BOOLEAN ALGEBRA AND MINIMIZATION

GU JING (DR)

ELEGUJI@NUS.EDU.SG

Summary of Lecture 5: Gate-level Design & Minimization

- **Gate-level logic design**
  1. Simplify the Boolean function
  2. Implement the simplified Boolean function using logic gates, minimizing the gate count

- **Karnaugh Map (K-Map)**

Truth Table

A	B	F
0	0	0
0	1	0
1	0	1
1	1	1

K-Map

A \ B	0	1
0	0	0
1	1	1

Three-variable K-map

A \ BC	00 $\bar{B}\bar{C}$	01 $\bar{B}C$	11 BC	10 $B\bar{C}$
$\bar{A}$ 0	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$	$\bar{A}BC$	$\bar{A}B\bar{C}$
A 1	$A\bar{B}\bar{C}$	$A\bar{B}C$	ABC	$AB\bar{C}$

Four-variable K-map

AB \ CD	00	01	11	10
00	0000	0001	0011	0010
01	0100	0101	0111	0110
11	1100	1101	1111	1110
10	1000	1001	1011	1010

Summary of Lecture 5: Gate-level Design & Minimization

- **Simplification using K-map for SOP minimization** group "1"

Truth Table

AB CD	F
0000	1

AB \ CD	00	01	11	10
00	1	0	0	1
01	X	1	X 1	1
11	X	1	X	1
10	1	0	0	1

$$F = \boxed{B} + \boxed{\overline{D}}$$

- Group the squares that only contains "1", or don't care condition "X"
- Groups must be either horizontal or vertical (diagonal is invalid)
- Group size is always 2^n, that is, 2, 4, 8, ...
- Group should be as large as possible (contains as many as squares with "1" as possible)
- Each square with "1" must be part of a group if possible
- Simplified term retains those variables that don't change value
- Variables that change value in the group are eliminated

Summary of Lecture 5: Gate-level Design & Minimization

- **Simplification using K-map for POS minimization**

AB \ CD	00	01	11	10
00	1	0	1	0
01	0	1	1	0
11	0	1	1	0
10	1	0	1	0

$\bar{C} + D$

$\bar{B} + D$

$$F = (B + C + \bar{D}) \cdot (\bar{C} + D) \cdot (\bar{B} + D)$$

- Group the squares that only contains “0”
- Form an OR term (sum) for each group, instead of a product
- Value “1”, instead of “0”, represent complement of the variable
- Follow similar grouping rules for SOP

Summary of Lecture 5: Gate-level Design & Minimization

- **Minimal SOP (MSOP):** group “1”s in **most efficient way**

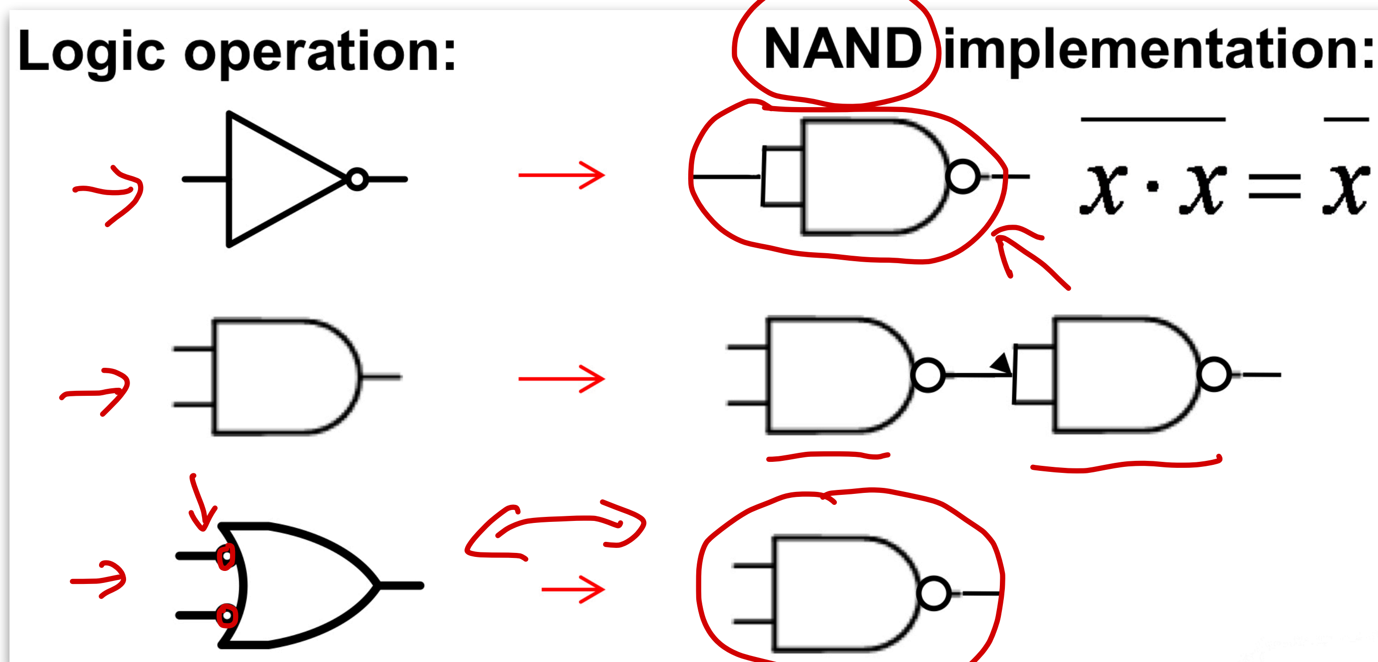
AB \ CD	00	01	11	10
00	1	0	1	1
01	1	0	1	0
11	1	1	1	1
10	0	0	0	0

AB \ CD	00	01	11	10
00	0	0	1	0
01	1	0	1	1
11	1	1	1	1
10	0	0	1	0

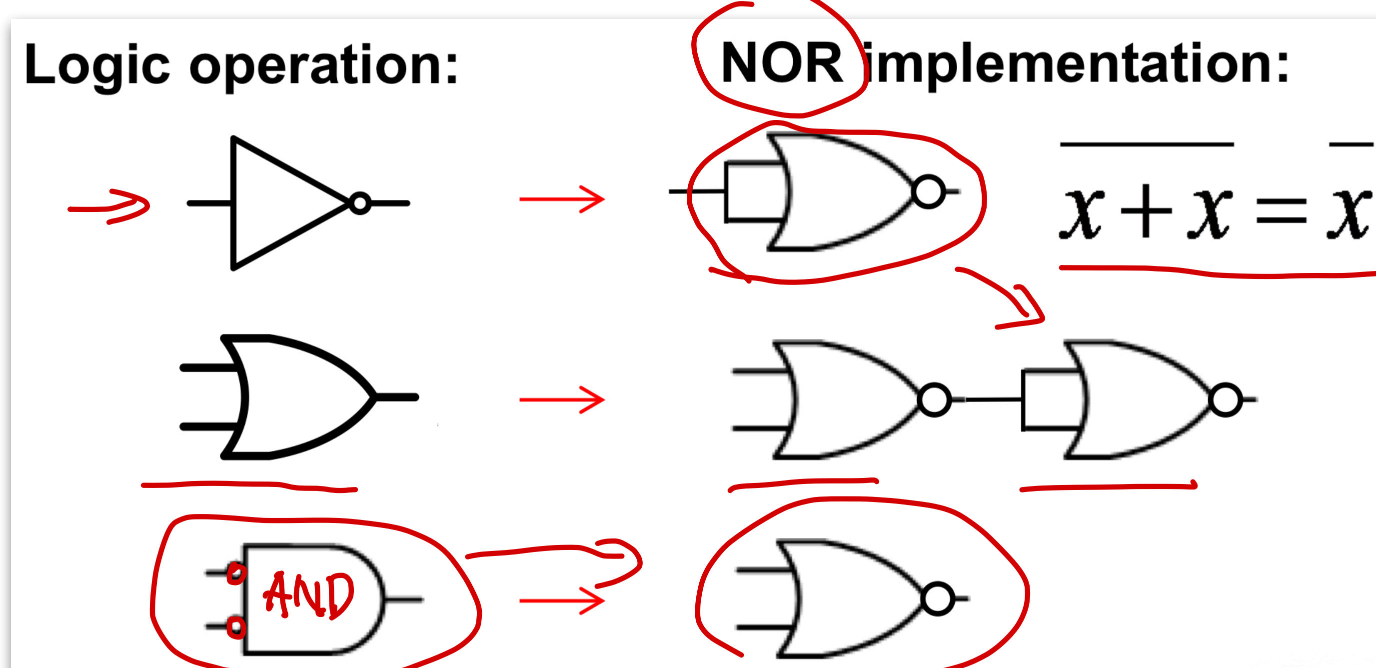
AB \ CD	00	01	11	10
00	1	0	0	1
01	1	1	1	1
11	1	0	1	0
10	1	0	1	1

Summary of Lecture 5: Gate-level Design & Minimization

- **NAND only Implementation**



- **NOR only Implementation**



Q1. A switching circuit has four inputs as shown. A and B represent the first and second bits of a binary number N1. C and D represent the first and second bits of a binary number N2. The output is to be 1 only if the product N1 x N2 is less than or equal to 2.

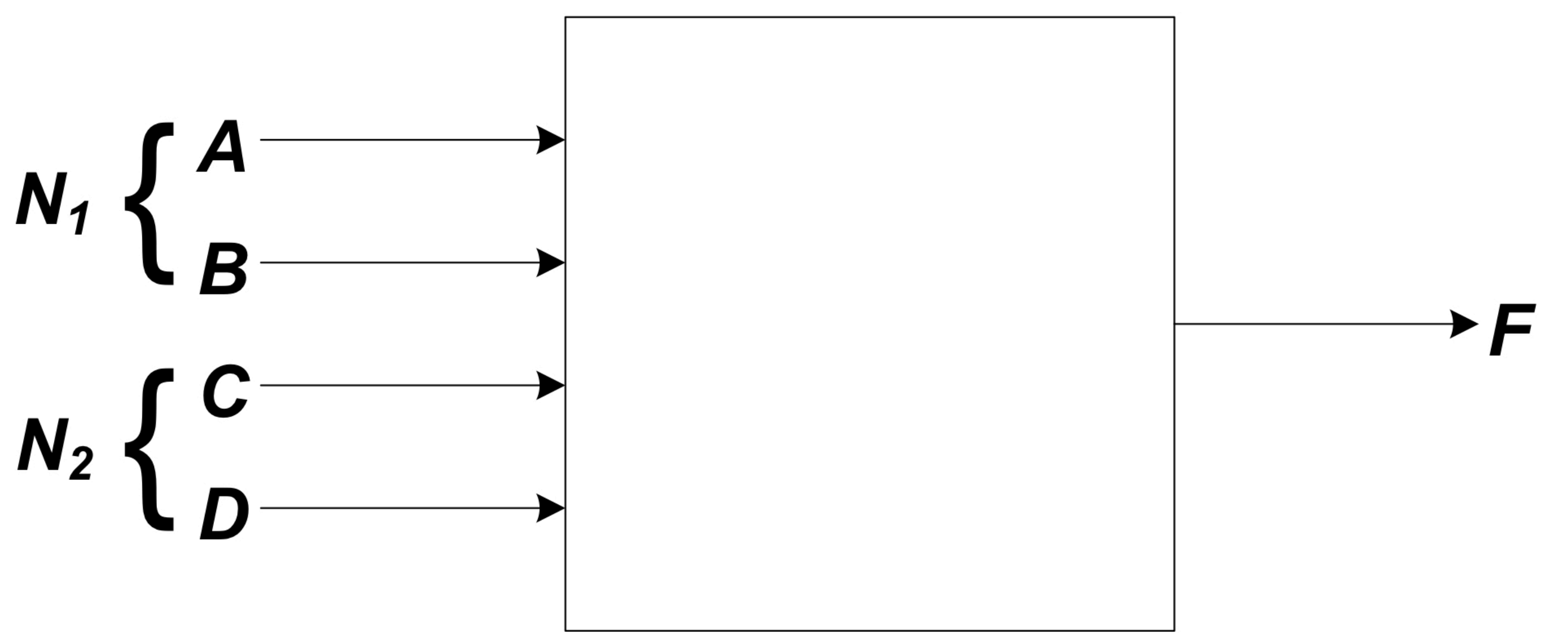
(a) Write the truth table for the system.

minterms maxterms (A+B+C+D)

(b) Write the canonical SOP and POS expressions for F.

(c) Draw a Karnaugh-map (K-map) for the function F.

(d) From the K-map, derive a simplified SOP expression for F. (Hint: Use groups of only size 4.)



Q1 Ans: (a) b-d

A	B	C	D	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

K-Map.

CD \ AB	00	01	11	10
00	1	1	1	1
01	1	1	0	1
11	1	0	0	0
10	1	1	0	0

SOP (check all "1"s)

$$F = \bar{A}\bar{B} + \bar{C}\bar{D} + \bar{A}\bar{C} + \bar{B}\bar{C} + \bar{A}\bar{D}$$ ※

POS (check all "0"s)

$$F = (\bar{A} + \bar{C}) \cdot (\bar{A} + \bar{B} + \bar{D}) \cdot (\bar{B} + \bar{C} + \bar{D})$$ ※

Q1 Ans: (a)

A	B	C	D	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

(b)

Canonical SOP (check “1”s):

$$F = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}CD + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}BC\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D + AB\bar{C}\bar{D}$$

Canonical POS (check “0”s):

$$F = (A+\bar{B}+\bar{C}+\bar{D})\cdot(\bar{A}+B+\bar{C}+D)\cdot(\bar{A}+B+\bar{C}+\bar{D})\cdot(\bar{A}+\bar{B}+C+\bar{D})\cdot(\bar{A}+\bar{B}+\bar{C}+D)\cdot(\bar{A}+\bar{B}+\bar{C}+\bar{D})$$

Q1 Ans: (c)

A	B	C	D	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

CD \ AB	00	01	11	10
00	1	1	1	1
01	1	1	0	1
11	1	0	0	0
10	1	1	0	0

Q1 Ans: (d)

CD \ AB	00	01	11	10
00	1	1	1	1
01	1	1	0	1
11	1	0	0	0
10	1	1	0	0

CD \ AB	00	01	11	10
00	1	1	1	1
01	1	1	0	1
11	1	0	0	0
10	1	1	0	0

SOP (check all “1”s):

POS (check all “0”s):

Q1 Ans: (d)

CD \ AB	00	01	11	10
00	1	1	1	1
01	1	1		1
11	1			
10	1	1		

CD \ AB	00	01	11	10
00				
01			0	
11		0	0	0
10			0	0

SOP (check all "1"s):

POS (check all "0"s):

$$Z_{MSOP} = \bar{A}\bar{B} + \bar{C}\bar{D} + \bar{A}\bar{C} + \bar{A}\bar{D} + \bar{B}\bar{C}$$

$$Z_{MPOS} = (\bar{A}+\bar{C})(\bar{A}+\bar{B}+\bar{D})(\bar{B}+\bar{C}+\bar{D})$$

Q2. A bank vault has three locks with a different key for each lock. Each key is owned by a different person. To open the door, at least two people must insert their keys into the assigned locks. The signal lines A, B and C are 1 if there is a key inserted into lock 1, 2 or 3, respectively. Write an equation for the variable Z which is 1 if and only if the door should open.

(a) Write the truth table for the system.

(b) Write the canonical SOP and POS expressions for Z.

(c) Draw a Karnaugh-map (K-map) for the function Z.

(d) From the K-map, derive a simplified SOP expression for Z.

Q2 Ans:

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Canonical SOP (check "1")

$$F = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$$

Canonical POS (check "0"s)

$$F = (A+B+C)\cdot(A+B+\bar{C})\cdot(A+\bar{B}+C)\cdot(\bar{A}+B+C)$$

K-Map

BC \ A	0	1
00	0	0
01	0	1
11	1	1
10	0	1

minimal SOP:

$$F = AB + AC + BC$$

Q2 Ans:

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

BC \ A	0	1
00	0	0
01	0	1
11	1	1
10	0	1

$$Z_{SOP} = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$$

$$Z_{POS} = (A+B+C)\,(A+B+\bar{C})\,(A+\bar{B}+C)\,(\bar{A}+B+C)$$

$$F = AC + BC + AB$$

Q3. Use K-maps to obtain an MSOP and an MPOS for each of the following functions:

(a) $Z = \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}B\overline{C}D + AB\overline{C}D + A\overline{B}\,\overline{C}D + A\overline{B}\,\overline{C}\,\overline{D}$ with don't care for ABCD = 1010

(b) $Z = \left(\overline{A} + B + \overline{C}\right)\left(A + B + \overline{C}\right)$ with don't cares for ABC = 111 and 110

(c) $f(x_1, \ldots, x_4) = \sum m(0,4,5,6,7) + D(1,12,13,14,15)$, where m() is the set of minterms for which f=1 and D() is the set of don't cares. For example, m(2) is the minterm corresponding to $x_1x_2x_3x_4$ = 0010 (this alternate shorthand notation is often used to express SOPs).

Canonical format

ABCD=0001 0101 1101 1001 1000 SOP

Q3(a). $Z = \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}B\overline{C}D + AB\overline{C}D + A\overline{B}\,\overline{C}D + A\overline{B}\,\overline{C}\,\overline{D}$ with don't care for ABCD = 1010

MPOS MSOP

CD \ AB	00	01	11	10
00	0	0	0	1
01	1	1	1	1
11	0	0	0	0
10	0	0	0	X 0

MPOS

CD \ AB	00	01	11	10
00	0	0	0	
01				
11	0	0	0	0
10	0	0	0	X 0

MSOP: ("1"s)

$Z = \overline{C}D + A\overline{B}\,\overline{C}$ ※

MPOS:

$Z = \overline{C} \cdot (A+D) \cdot (\overline{B}+D)$ #

Q3(a). $Z = \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}B\overline{C}D + AB\overline{C}D + A\overline{B}\,\overline{C}D + A\overline{B}\,\overline{C}\,\overline{D}$ with don't care for ABCD = 1010

CD \ AB	00	01	11	10
00				1
01	1	1	1	1
11				
10				X

CD \ AB	00	01	11	10
00	0	0	0	
01				
11	0	0	0	0
10	0	0	0	X

$$Z = A\overline{B}\,\overline{C} + \overline{C}D$$

$$Z = \overline{C}(\overline{B}+D)\,(A+D)$$

→ Canonical POS

Q3(b). $Z = \left(\overline{A} + B + \overline{C}\right)\left(A + B + \overline{C}\right)$ with don't cares for ABC = 111 and 110

0 A=1 B=0 C=1 ABC=001

BC \ A	0	1
00	1	1
01	0	0
11	1	X
10	1	X

BC \ A	0	1
00		
01	0	0
11		X
10		X

MSOP: (check "1"s)

$Z = B + \overline{C}$ #

MPOS: (check "0"s)

$Z = (B + \overline{C})$ #

Q3(b). $Z = \left(\overline{A} + B + \overline{C}\right)\left(A + B + \overline{C}\right)$ with don't cares for ABC = 111 and 110

BC \ A	0	1
00		
01	0	0
11		X
10		X

BC \ A	0	1
00	1	1
01		
11	1	X
10	1	X

$$Z = B + \overline{C}$$

$$Z = B + \overline{C}$$

m(0) m(4) = 0100 m(5) = 0101 m(7) = 0111
m(6) = 0110 m(12) = 1100
m(13) = 1101
0001 1110 1111

Q3(c). $f(x_1,\ldots,x_4) = \sum m(0,4,5,6,7) + D(1,12,13,14,15)$, where m() is the set of minterms for which f=1 and D() is the set of don't cares. For example, m(2) is the minterm corresponding to $x_1x_2x_3x_4$ = 0010 (this alternate shorthand notation is often used to express SOPs). m(2)

x_3x_4 \ x_1x_2	00	01	11	10
00	1	1	X	0
01	X	1	X	0
11	0	1	X	0
10	0	1	X	0

$\Rightarrow$

x_3x_4 \ x_1x_2	00	01	11	10
00			X	0
01	X		X	0
11	0		X	0
10	0		X	0

MSOP: (check "1"s)

MPOS: (check "0"s)

$f = X_2 + \overline{X_3}\cdot\overline{X_1}$ ※ $\Longleftrightarrow$ $f = (\overline{X_1})\cdot(x_2+\overline{X_3})$

Q3(c). $f(x_1,L\ ,x_4)=\sum m(0,4,5,6,7)+D(1,12,13,14,15)$, where m() is the set of minterms for which f=1 and D() is the set of don't cares. For example, m(2) is the minterm corresponding to $X_1X_2X_3X_4$ = 0010 (this alternate shorthand notation is often used to express SOPs).

X_3X_4 \ X_1X_2	00	01	11	10
00	1	1	X	
01	X	1	X	
11		1	X	
10		1	X	

$$Z = X_2 + \overline{X_1}\,\overline{X_3}$$

X_3X_4 \ X_1X_2	00	01	11	10
00			X	0
01	X		X	0
11	0		X	0
10	0		X	0

$$Z = \overline{X_1}\ (X_2+\overline{X_3})$$

3 or more "1's $\rightarrow Z=1$

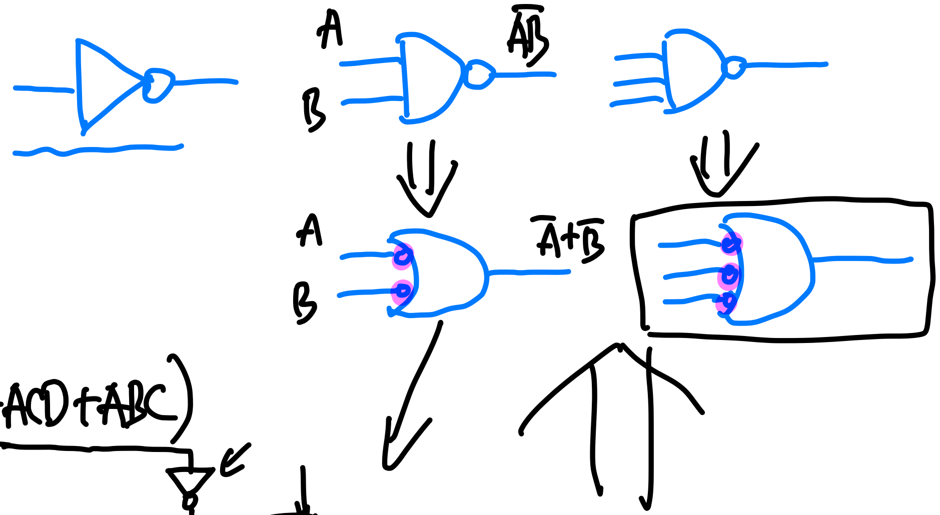
Q4. A combinational circuit has four inputs A, B, C and D and an output Z. The output is asserted whenever three or more of the inputs are asserted, otherwise the output is de-asserted. Find an MSOP expression for Z. Design combinational circuits using only 74'04 inverters, 74'00 2-input NAND gates and 74'10 3-input NAND gates. Assume that A, B and Z are active high signals, while C and D are active low signals. Use alternate gate representations for clarity of circuit diagrams.

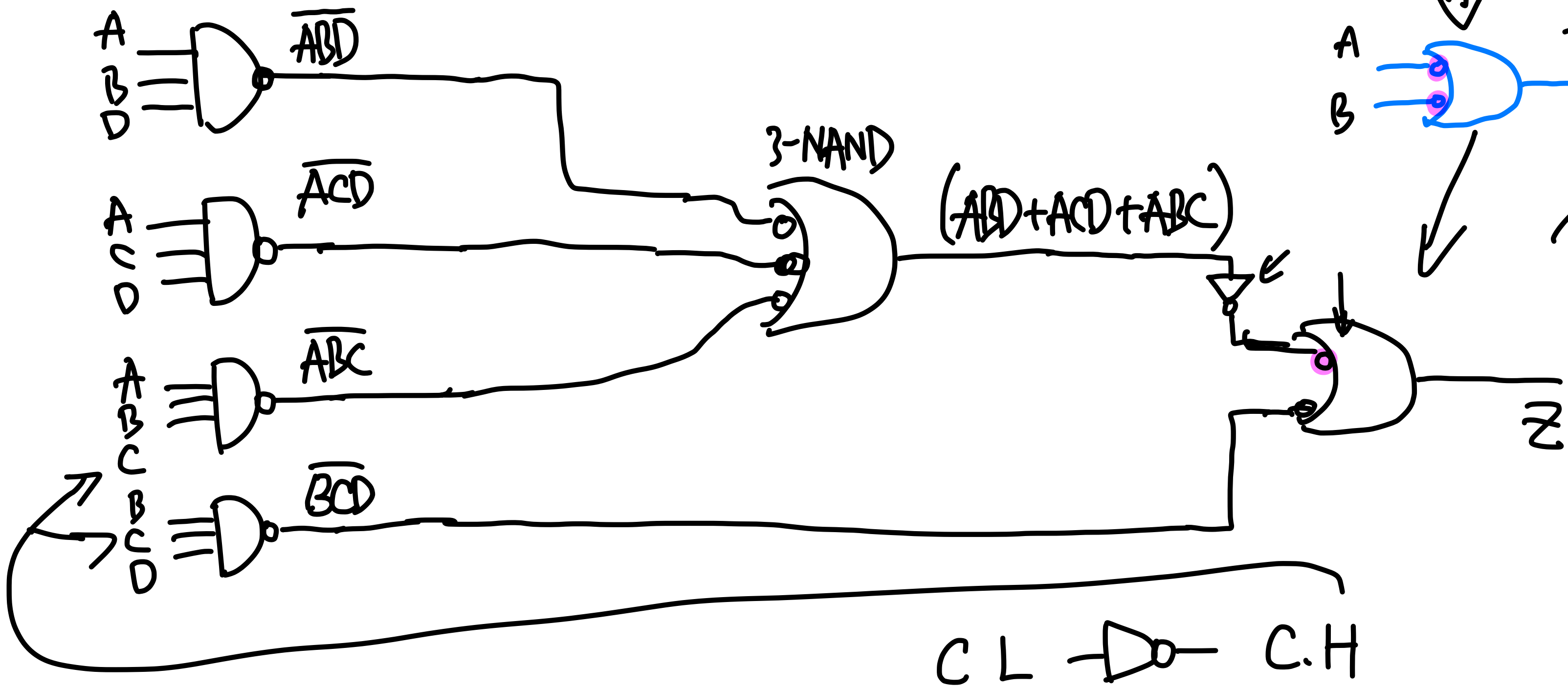
CD \ AB	00	01	11	10
00				
01			1	
11		1	1	1
10			1	

MSOP:

$Z = ABD + ACD + ABC + BCD$

$$Z = \boxed{ABD} + \boxed{ACD} + \boxed{ABC} + \boxed{BCD}$$





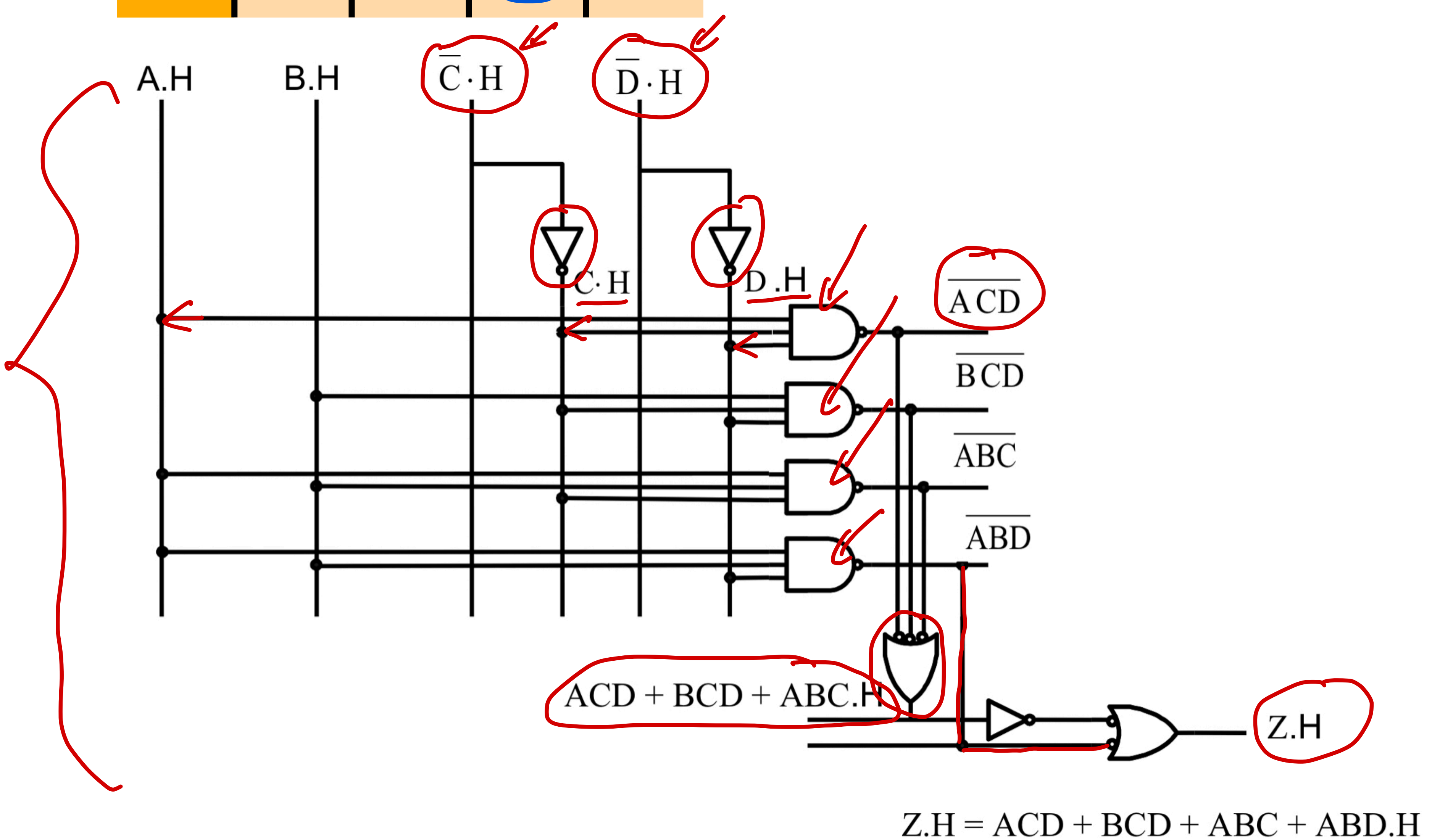
C L —▷o— C.H

C & D active <u>low</u>

Q4 Ans.

CD \ AB	00	01	11	10
00				
01			1	
11		1	1	1
10			1	

$$Z = ABD + ACD + ABC + BCD$$



THE END

For Consultation: eleguji@nus.edu.sg
Office: E4-03-10