# [T]EE2026 DIGITAL DESIGN

# TUTORIAL 2: NUMBER SYSTEMS

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# NUMBER SYSTEMS

1. Position number system (radix 10, 2, 8 and 16)
2. Conversion among decimal, binary, octal and hex.
3. Binary arithmetic
4. Signed binary number representation and Arithmetic (S-M, 1's complement and 2's complement)
5. Binary-coded decimals (BCD) and its Decimal addition

# 4. SIGNED BINARY NUMBER REPRESENTATION & ARITHMETIC

Q1(a). What is the ordinary 8 bit binary equivalent of the decimal number 250?

2 | 250 − 0
2 | 125 − 1
2 | 62 − 0
2 | 31 − 1
15

2 | 15 − 1
2 | 7 − 1
2 | 3 − 1
1

$\therefore (1111\ 1010)_2 = (250)_{10}$

Q1(b). What decimal number does the above bit pattern correspond to if interpreted as

i) a signed magnitude number
ii) a 1's complement number and
iii) a 2's complement number?

(i) $\underline{1111}\ \underline{1010}$

"−" magnitude

$= 111\ 1010$

$= 2^6 + 2^5 + 2^4 + 2^3 + 2^1$

$= 122$

$\therefore -122$ if interpreted as S.M.

(ii) $\underline{1111}\ \underline{1010}$

↓ flip

00000101 (5)

$\therefore -5$ if interpreted as 1's.

(iii) $\underline{1111}\ \underline{1010}$

↓ flip

0000 0101

↓ +1

0000 0110 (6)

$\therefore -6$ if interpreted as 2's.

# 4. SIGNED BINARY NUMBER REPRESENTATION & ARITHMETIC

Q2. Show how the following can be added in 2's complement notation using 8-bit arithmetic

(a) (-1) + 45 (b) -128 + (-60)

the answer is wrong because of overflow.

(a) $\boxed{-1}$ 0000 0001 (1)

↓ flip

1111 1110

↓ +1

1111 1111

$\boxed{45}$

2 | 45 — 1
2 | 22 — 0
2 | 11 — 1
2 | 5 — 1
2 | 2 — 0
1

0010 1101

⇒

$$
\begin{array}{r}
1111\ 1111 \\
+\ 0010\ 1101 \\
\hline
\boxed{1}\ \underbrace{0010\ 1100}
\end{array}
$$

$2^5+2^3+2^2=44$ ※

(b) $\boxed{-128}$ 1000 0000 (128)

↓ flip

0111 1111

↓ +1

1000 0000

$\boxed{-60}$ 0011 1100

↓ flip

1100 0011

↓ +1

1100 0100

$$
\begin{array}{r}
1000\ 0000 \\
+\ 1100\ 0100 \\
\hline
\boxed{1}\ \underbrace{0100\ 0100}_{68}
\end{array}
$$

68 (wrong answer)

**Overflow** occurs when the arithmetic result is out of the range that n-bit binary number can represent.

**Discussion 1:** What is the range of 8-bit representable signed number?

| Decimal | 8-bit count |
|---|---|
| 0 | 0000 0000 |
| 1 | 0000 0001 |
| 2 | 0000 0010 |
| : | : |
| 127 | 0111 1111 |
| -128 | 1000 0000 |
| -127 | 1000 0001 |
| : | : |
| -2 | 1111 1110 |
| -1 | 1111 1111 |

RE-ORGANIZE

| Decimal | 2's Complement |
|---|---|
| -128 | 1000 0000 |
| -127 | 1000 0001 |
| : | : |
| -2 | 1111 1110 |
| -1 | 1111 1111 |
| 0 | 0000 0000 |
| 1 | 0000 0001 |
| 2 | 0000 0010 |
| : | : |
| 127 | 0111 1111 |

# DETECTION OF OVERFLOW

**Overflow** occurs when the arithmetic result is out of the range that n-bit binary number can represent.

**Discussion 2:** In what condition, Overflow may happen? OR, how can be detect the overflow?

| Decimal | 2's Complement |
|---|---|
| -128 | 1000 0000 |
| -127 | 1000 0001 |
| : | : |
| -2 | 1111 1110 |
| -1 | 1111 1111 |
| 0 | 0000 0000 |
| 1 | 0000 0001 |
| 2 | 0000 0010 |
| : | : |
| 127 | 0111 1111 |

# DETECTION OF OVERFLOW

**Conclusion:**

* **Overflow** can only happen when two operands have the same sign, e.g. MSB are the same.

* When both operands have the same sign, AND if the result's sign is altered, we say Overflow occurs.

$MSB_1 = 1$, $MSB_2 = 1$, $MSB_R = 0$: (+ve) + (+ve) = (-ve)

$MSB_1 = 0$, $MSB_2 = 0$, $MSB_R = 1$: (-ve) + (-ve) = (+ve)

*For example Q2(b): -128 + (-60) = **1**000 0000 + **1**100 0100 = **0**100 0100*

# 4. SIGNED BINARY NUMBER REPRESENTATION & ARITHMETIC

Q3. Compute and give the final answer in 2's complement notation:

$(10100)_{2's} + (00100)_{SM}$

because it's positive, $\therefore (00100)_{SM} = (00100)_{2's}$

$$\begin{array}{r} 10100 \\ +\,00100 \\ \hline 11000 \end{array}$$

$\therefore (10100)_{2's} + (00100)_{SM} = (11000)_{2's}$

Verify the result in Decimal:

10100 (−12)

↓ flip

01011

↓ +1

01100 (12)

00100 = 4

$(-12) + 4 = -8$

Verify $(11000)_{2's}$ is −8 or not:

11000

↓ flip

00111

↓ +1

01000 (8)

$\therefore (11000)_{2's}$ is −8.

# 4. SIGNED BINARY NUMBER REPRESENTATION & ARITHMETIC

A. **Signed + Magnitude**
B. 1's Complement
C. 2's Complement

* **Max magnitude: $(2^{n-1}-1)_{10}$**

* **Range: $-(2^{n-1}-1)_{10} \sim +(2^{n-1}-1)_{10}$**

* **CANNOT be used for addition of two numbers with opposite signs or subtraction when using a simple adder,**

**e.g. 011 (3) - 101 (1) = 100 (0)**

| Decimal | S-M |
|---|---|
| 3 | 011 |
| 2 | 010 |
| 1 | 001 |
| +0 | 000 |
| -0 | **100** |
| -1 | **101** |
| -2 | **110** |
| -3 | **111** |

Note: Two zeros

Negative numbers

"1" in MSB position for all negative numbers

# 5. BINARY CODED DECIMAL (BCD)

*** Represent each decimal digit with a 4-bit binary number**

Decimal → $(5\ \ 9\ \ 8)_{10}$

BCD → 0101 1001 1000

*** Six numbers (from 1010 to 1111) are not used in BCD**

*** Therefore, during addition, if sum > 9, we need to add 6 to the result.**

**Binary-Coded Decimal (BCD)**

| Decimal Symbol | BCD Digit |
|---|---|
| 0 | 0000 |
| 1 | 0001 |
| 2 | 0010 |
| 3 | 0011 |
| 4 | 0100 |
| 5 | 0101 |
| 6 | 0110 |
| 7 | 0111 |
| 8 | 1000 |
| 9 | 1001 |

**If S ≤ 9 → Sum = S and carry = 0**
(No correction is needed)

**If S > 9 → Sum = S + 6 and carry = 1**
(Need to be corrected by adding 6)

EXAMPLE

```
   8            1000
+  9     ⇨   +  1001
-----        ----------
  17         1  0001  ✗
           +    0110
           -------------
             1  0111
             1     7  ✓
```

# 5. BINARY CODED DECIMAL (BCD)

Q4. Convert the 8421 BCD number $\boxed{0100}\boxed{0110}\boxed{0010}\boxed{0011}$ into decimal.

4 6 2 3

$(4623)_{BCD}$ ※

# THE END

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