

NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics

AY2021, Semester 2 MA1508E Linear Algebra for Engineering Practice 3

1. Write your answers on A4 size papers.
2. Write down your student number, name, and tutorial group clearly on the top left corner of every page of the answer script.
3. Write the page number on the top right corner of each page of the answer script.
4. There are two questions in this worksheet with a total of 20 marks.
5. To submit your answer scripts, scan or take pictures of your work (make sure the images can be read clearly). Merge all your images into one pdf file (make sure they are in order of the page). Name the pdf file by **StudentNo_P3** (e.g. **A123456Z_P3**). Upload your pdf into the LumiNUS folder Practice 3 submission.

1. Let

$$V = \left\{ \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \mid \begin{pmatrix} 1 & 0 & -1 & 1 \\ 3 & 1 & 1 & -1 \\ 2 & 1 & 0 & 1 \\ a & b & c & d \end{pmatrix} \text{ is singular} \right\}.$$

(a) [4 marks] Show that V is a subspace of \mathbb{R}^4 .

(b) [3 marks] Find a basis for V and express V in explicit set form.

(c) [4 marks] Show that $S = \left\{ \begin{pmatrix} 3 \\ 5 \\ 5 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 4 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 2 \\ 1 \end{pmatrix} \right\}$ is a basis for V .

(d) [2 marks] Let $\mathbf{v} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$. Find $(\mathbf{v})_S$.

(e) [2 marks] Extend S to a basis for \mathbb{R}^4 .

2. Let

$$\mathbf{v}_1 = \begin{pmatrix} 4 \\ 1 \\ -2 \\ 5 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} -1 \\ 5 \\ 3 \\ 1 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 8 \\ 3 \\ 0 \\ -7 \end{pmatrix}.$$

(a) [1 mark] Find the distance between \mathbf{v}_1 and \mathbf{v}_2 . Leave your answer as surds.

(b) [1 mark] Find the norm of \mathbf{v}_3 . Leave your answer as surds.

(c) [3 marks] Check that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is an orthogonal set. Is it orthonormal?