NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics

AY2021, Semester 2   MA1508E Linear Algebra for Engineering   Practice 2

1. Write your answers on A4 size papers.

2. Write down your student number, name, and tutorial group clearly on the top left corner of every page of the answer script.

3. Write the page number on the top right corner of each page of the answer script.

4. There are three questions in this worksheet with a total of 20 marks.

5. To submit your answer scripts, scan or take pictures of your work (make sure the images can be read clearly). Merge all your images into one pdf file (make sure they are in order of the page). Name the pdf file by **StudentNo_P2** (e.g. **A123456Z_P2**). Upload your pdf into the LumiNUS folder Practice 2 submission.

1. [3 marks] Solve the following linear system

$$
\begin{array}{ccccccccc}
x_1 & + & x_2 & + & 2x_3 & - & x_4 & = & a \\
x_1 & & & & & + & 2x_4 & = & b \\
& & x_2 & + & 2x_3 & - & 2x_4 & = & c
\end{array}
$$

for

(i) $a = 1, b = 1, c = 2$,

(ii) $a = 0, b = -1, c = 1$.

2. (a) [4 marks] Let

$$
\mathbf{A} = \begin{pmatrix} 1 & -2 & 0 \\ 3 & -6 & -3 \\ 1 & 0 & 2 \end{pmatrix}.
$$

Compute the inverse of $\mathbf{A}$ by performing elementary row operations. Write down the elementary row operation that you used in each step clearly.

(b) [3 marks] Suppose

$$
\mathbf{A} \xrightarrow{R_1 + 2R_3} \xrightarrow{R_1 \leftrightarrow R_2} \xrightarrow{R_2 - R_3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1/2 \\ 0 & 3 & 0 \end{pmatrix}.
$$

Write $\mathbf{A}$ as a product of 6 elementary matrices, $\mathbf{A} = \mathbf{E}_1\mathbf{E}_2\mathbf{E}_3\mathbf{E}_4\mathbf{E}_5\mathbf{E}_6$.

(c) [2 marks] Compute the determinant of $\mathbf{A}$ from 2b.

3. Let $\mathbf{A} = \begin{pmatrix} 1 & 2 & -1 & 0 \\ 2 & 2 & 3 & 1 \\ 0 & 2 & 0 & 0 \\ 1 & 2 & 5 & 7 \end{pmatrix}$.

(a) [3 marks] Compute the determinant of $\mathbf{A}$ by cofactor expansion along the first row.

(b) [2 marks] Let $\mathbf{b} = \begin{pmatrix} 1 \\ a \\ 0 \\ -2 \end{pmatrix}$. For which value of a is $\mathbf{Ax} = \mathbf{b}$ consistent? Why?

(c) [3 marks] Suppose $\mathbf{B}$ is an order 4 square matrix such that $\det(B) = 3$. Find

(i) $\det(\frac{1}{2}\mathbf{A}^T)$,

(ii) $\det(\mathbf{AB}^{-1})$,

(iii) $\det((3\mathbf{B})^{-1})$.