

**NATIONAL UNIVERSITY OF SINGAPORE**  
**Department of Mathematics**

**AY2021, Semester 2   MA1508E Linear Algebra for Engineering   Practice 4**

1. Write your answers on A4 size papers.
2. Write down your student number, name, and tutorial group clearly on the top left corner of every page of the answer script.
3. Write the page number on the top right corner of each page of the answer script.
4. There are three questions in this worksheet with a total of 20 marks.
5. To submit your answer scripts, scan or take pictures of your work (make sure the images can be read clearly). Merge all your images into one pdf file (make sure they are in order of the page). Name the pdf file by **StudentNo\_P4** (e.g. **A123456Z\_P4**). Upload your pdf into the LumiNUS folder Practice 4 submission.

1. Let

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 3 & 1 \\ 1 & 1 & 1 & 3 \end{pmatrix}.$$

(a) [10 marks] Orthogonally diagonalize  $\mathbf{A}$ .

(b) [2 marks] Find the limit of  $\left(\frac{1}{6}\mathbf{A}\right)^n \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$  as  $n \rightarrow \infty$ , for  $a, b, c, d \in \mathbb{R}$ .

2. [5 marks] Solve the initial value problem

$$\begin{aligned} y_1' &= y_1 + y_2 - y_3 \\ y_2' &= y_3 \\ y_3' &= -2y_2 - 3y_3 \end{aligned}$$

with initial conditions  $y_1(0) = 1$ ,  $y_2(0) = 2$ ,  $y_3(0) = 3$ .

3. Let

$$\mathbf{A} = \begin{pmatrix} 1 & -3 \\ 3 & 1 \end{pmatrix}.$$

(a) [1 mark] Verify that  $\begin{pmatrix} i \\ 1 \end{pmatrix}$  is an eigenvector of  $\mathbf{A}$ .

(b) [2 marks] Find a fundamental set of real solutions for the differential system

$$\begin{aligned} y_1' &= y_1 - 3y_2 \\ y_2' &= 3y_1 + y_2 \end{aligned}$$