18/19 Sem 1

1. a)
$$\left(\begin{array}{ccc|c} 2 & -a & 3 & 0 \\ 4 & -2 & 5 & -a \\ -2 & a & -2 & a \end{array}\right) \xrightarrow[R_3+R_1 \checkmark]{R_2-2R_1 \checkmark} \left(\begin{array}{ccc|c} 2 & -a & 3 & 0 \\ 0 & 2a-2 & -1 & -a \\ 0 & 0 & 1 & a \end{array}\right)$$

i) If $a=1$
$$\left(\begin{array}{ccc|c} 2 & -1 & 3 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 2 & -1 & 3 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left.\begin{array}{l} z=1 \\ y=s \\ x=\frac{s-3}{2} \end{array}\right\} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\frac{3}{2} \\ 0 \\ 1 \end{pmatrix} + s\begin{pmatrix} \frac{1}{2} \\ 1 \\ 0 \end{pmatrix}, \quad s \in \mathbb{R}$$

$$s=0 \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\frac{3}{2} \\ 0 \\ 1 \end{pmatrix} \quad \checkmark$$

$$s=1 \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \quad \checkmark$$

ii) If $a \neq 1$
$$\left(\begin{array}{ccc|c} 2 & -a & 3 & 0 \\ 0 & 2a-2 & -1 & -a \\ 0 & 0 & 1 & a \end{array}\right) \xrightarrow[R_1-3R_3 \checkmark]{R_2+R_3 \checkmark} \left(\begin{array}{ccc|c} 2 & -a & 0 & -3a \\ 0 & 2a-2 & 0 & 0 \\ 0 & 0 & 1 & a \end{array}\right)$$

$$\xrightarrow{\frac{1}{2a-2}R_2 \checkmark} \left(\begin{array}{ccc|c} 2 & -a & 0 & -3a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & a \end{array}\right) \xrightarrow{R_1+aR_2 \checkmark} \left(\begin{array}{ccc|c} 2 & 0 & 0 & -3a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & a \end{array}\right) \xrightarrow{\frac{1}{2}R_1 \checkmark} \left(\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{3}{2}a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & a \end{array}\right)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\frac{3}{2}a \\ 0 \\ a \end{pmatrix} \quad \checkmark$$

i) $\begin{pmatrix} w_1 & w_2 & w_3 \end{pmatrix} = \begin{pmatrix} 1 & -3 & -5 \\ 1 & 1 & -2 \\ 1 & -3 & 1 \\ 1 & 1 & 4 \end{pmatrix} \xrightarrow[R_4-R_1]{R_2-R_1 \\ R_3-R_1} \begin{pmatrix} 1 & -3 & -5 \\ 0 & 4 & 3 \\ 0 & 0 & 6 \\ 0 & 4 & 9 \end{pmatrix}$

$\xrightarrow{R_4-R_2} \begin{pmatrix} 1 & -3 & -5 \\ 0 & 4 & 3 \\ 0 & 0 & 6 \\ 0 & 0 & 6 \end{pmatrix} \xrightarrow{R_4-R_3} \begin{pmatrix} 1 & -3 & -5 \\ 0 & 4 & 3 \\ 0 & 0 & 6 \\ 0 & 0 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ ✓

$\{w_1, w_2, w_3\}$ is linearly independent ✓ and $W = \text{span}\{w_1, w_2, w_3\}$

$\Rightarrow \{w_1, w_2, w_3\}$ is a basis for W ✓

ii) $v_1 = w_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$

$$v_2 = \begin{pmatrix} -3 \\ 1 \\ -3 \\ 1 \end{pmatrix} - \frac{\begin{pmatrix} -3 \\ 1 \\ -3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}}{4} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ -2 \\ 2 \end{pmatrix} = 2 \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}$$ ✓

$$v_3 = \begin{pmatrix} -5 \\ -2 \\ 1 \\ 4 \end{pmatrix} - \frac{\begin{pmatrix} -5 \\ -2 \\ 1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}}{4} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} - \frac{\begin{pmatrix} -5 \\ -2 \\ 1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}}{4} \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -3 \\ -3 \\ 3 \\ 3 \end{pmatrix} = 3 \begin{pmatrix} -1 \\ -1 \\ 1 \\ 1 \end{pmatrix}$$ ✓

orthogonal basis $= \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 1 \\ 1 \end{pmatrix} \right\}$ ✓

iii) $w' = A(A^TA)^{-1}A^T w$

Please state $\underset{\sim}{A}$.

$$= \begin{pmatrix} 3/2 \\ -1/2 \\ 5/2 \\ 1/2 \end{pmatrix}$$

vector orthogonal to W = $w - w' = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ 1/2 \\ -1/2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -1 \\ 1 \\ 1 \\ -1 \end{pmatrix}$ ✓

orthogonal basis for $\mathbb{R}^4$ = $\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \right\}$ ✓

2 a)

$$A^TAx = A^Tb$$

$$\left(\begin{array}{cc|c} 24 & -12 & -4 \\ -12 & 11 & -9 \end{array}\right) \xrightarrow{R_2+\frac{1}{2}R_1} \left(\begin{array}{cc|c} 24 & -12 & -4 \\ 0 & 5 & -11 \end{array}\right)$$

$$x_2 = -\frac{11}{5}$$ ✓

$$x_1 = \frac{-4+12(-\frac{11}{5})}{24} = -\frac{19}{15}$$ ✓

projection of b onto col(A) $= A\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 4 & -3 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} -\frac{19}{15} \\ -\frac{11}{5} \end{pmatrix}$

$$= \begin{pmatrix} -\frac{1}{3} \\ \frac{23}{15} \\ -\frac{71}{15} \end{pmatrix}$$ ✓

b) i) S is not closed under linear combination.

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \in S$ $\quad \rightarrow \left\|\begin{pmatrix} 2 \\ 0 \end{pmatrix}\right\| = 2 \neq 1$

but $2\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \notin S$ ✓

ii) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = z\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$

Basis for S $= \left\{\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}\right\}$ ✓ $\quad$ dim(S) = 1 ✓

iii) S is not closed under linear combi

$\begin{pmatrix} 1 \\ 1 \\ 1 \\ 2 \end{pmatrix} \in S \quad \begin{pmatrix} 2 \\ 1 \\ 1 \\ 1 \end{pmatrix} \in S$

but $\begin{pmatrix} 1 \\ 1 \\ 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 2 \\ 3 \end{pmatrix} \notin S$ ✓

iv) $$\underbrace{\begin{pmatrix} 1 & 2 & 1 \\ 1 & 2 & -1 \\ 1 & 2 & -1 \end{pmatrix}}_{A}\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$ = solution space of A

$$A \to \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$c_3 = 0$
$c_2 = s$
$c_1 = -2s$

$$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = s\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \quad s \in \mathbb{R}$$

Basis for S = $\left\{\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}\right\}$ ✓ $\dim(S) = 1$ ✓

3 a) $$\left(\begin{array}{ccc|ccc} \multicolumn{3}{c}{A} & \multicolumn{3}{c}{I} \\ 1 & 0 & 2 & 1 & 0 & 0 \\ 3 & -1 & 1 & 0 & 1 & 0 \\ -1 & 1 & 2 & 0 & 0 & 1 \end{array}\right) \xrightarrow[R_3+R_1 \checkmark]{R_2-3R_1 \checkmark} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & -1 & -5 & -3 & 1 & 0 \\ 0 & 1 & 4 & 1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_3+R_2 \checkmark} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & -1 & -5 & -3 & 1 & 0 \\ 0 & 0 & -1 & -2 & 1 & 1 \end{array}\right) \xrightarrow[-R_3 \checkmark]{-R_2 \checkmark} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 5 & 3 & -1 & 0 \\ 0 & 0 & 1 & 2 & -1 & -1 \end{array}\right)$$

$$\xrightarrow[R_2-5R_3 \checkmark]{R_1-2R_3 \checkmark} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 2 & 2 \\ 0 & 1 & 0 & -7 & 4 & 5 \\ 0 & 0 & 1 & 2 & -1 & -1 \\ \multicolumn{3}{c}{I} & \multicolumn{3}{c}{A^{-1}} \end{array}\right) \qquad A^{-1} = \begin{pmatrix} -3 & 2 & 2 \\ -7 & 4 & 5 \\ 2 & -1 & -1 \end{pmatrix} \checkmark$$

$$x = A^{-1}\begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 & 2 & 2 \\ -7 & 4 & 5 \\ 2 & -1 & -1 \end{pmatrix}\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 10 \\ 23 \\ -6 \end{pmatrix} \checkmark$$

b) i) $$I-A = \begin{pmatrix} 0.4 & -0.3 \\ -0.4 & 0.3 \end{pmatrix}$$

$$(I-A)x = 0 \qquad x = \begin{pmatrix} 0.3 \\ 0.4 \end{pmatrix}$$ ✓

$$\Rightarrow Ax = x \qquad x \neq 0$$

$$\underset{\sim}{A}\underset{\sim}{x} = \begin{pmatrix} 0.6 & 0.3 \\ 0.4 & 0.7 \end{pmatrix}\begin{pmatrix} 0.3 \\ 0.4 \end{pmatrix} = \begin{pmatrix} 0.3 \\ 0.4 \end{pmatrix} = \underset{\sim}{x} = 1\underset{\sim}{x}$$

∴ 1 is an eigenvalue of $\underset{\sim}{A}$

ii) $$(0.3I - A) = \begin{pmatrix} -0.3 & -0.3 \\ -0.4 & -0.4 \end{pmatrix} \qquad v_{0.3} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

A is a square matrix of order 2 with 2 distinct eigenvalues

$\Rightarrow$ A is diagonalizable ✓

$\underset{\sim}{A}\underset{\sim}{v}_{0.3} = \dots = 0.3\,\underset{\sim}{v}_{0.3}$

∴ 0.3 is another eigenvalue of $\underset{\sim}{A}$

iii) $$a_{i+1} = 0.6a_i + 0.3b_i$$

$$b_{i+1} = 0.4a_i + 0.7b_i$$

$$x_{i+1} = \begin{pmatrix} 0.6 & 0.3 \\ 0.4 & 0.7 \end{pmatrix}\begin{pmatrix} a_i \\ b_i \end{pmatrix} \Rightarrow B = \begin{pmatrix} 0.6 & 0.3 \\ 0.4 & 0.7 \end{pmatrix}$$ ✓

$$B = PDP^{-1} \quad \text{where } P = \begin{pmatrix} 0.3 & -1 \\ 0.4 & 1 \end{pmatrix} \qquad D = \begin{pmatrix} 1 & 0 \\ 0 & 0.3 \end{pmatrix}$$ ✓ ✓

In the long run, $n \to \infty$

$$x_n = B^n \begin{pmatrix} 5000 \\ 5000 \end{pmatrix}$$

$$= PD^nP^{-1}\begin{pmatrix} 5000 \\ 5000 \end{pmatrix}$$

$$= P\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}P^{-1}\begin{pmatrix} 5000 \\ 5000 \end{pmatrix}$$

$$= \begin{pmatrix} 30000/7 \\ 40000/7 \end{pmatrix}$$

$\frac{3}{7}$ of residents support A in the long run ✓

4. a)

$$\begin{pmatrix} -1 & 1 & 2 & 0 \\ 6 & -2 & -6 & 2 \\ -5 & 3 & 7 & -1 \\ 3 & 1 & 0 & 2 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

first 2 col form basis for V $= \left\{ \begin{pmatrix} -1 \\ 6 \\ -5 \\ 3 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \\ 3 \\ 1 \end{pmatrix} \right\}$ ✓

vectors orthogonal to V lie in null (A^T)

$$\begin{pmatrix} -1 & 6 & -5 & 3 \\ 1 & -2 & 3 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & -\frac{1}{2} & 1 \end{pmatrix}$$

$d = s$

$c = t$

$b = -s + \frac{1}{2}t$

$a = -3s - 2t$

$$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = s \begin{pmatrix} -3 \\ -1 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} -2 \\ \frac{1}{2} \\ 1 \\ 0 \end{pmatrix}$$ ✓

Basis for null (A^T) $= \left\{ \begin{pmatrix} -3 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ \frac{1}{2} \\ 1 \\ 0 \end{pmatrix} \right\}$ ✓

b) i) Basis for column space of X = pivot columns in X

= pivot columns in A & pivot columns in B

$\dim(\mathrm{col}(X)) = \dim(\mathrm{col}(A)) + \dim(\mathrm{col}(B))$

$\mathrm{rank}(X) = \mathrm{rank}(A) + \mathrm{rank}(B)$ ✓

ii) Yes. ✗ column operations preserve column space

$$\begin{pmatrix} A & C \\ 0 & B \end{pmatrix} \longrightarrow \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix} \quad ?$$

No.

Take $\underset{\sim}{A} = \underset{\sim}{B} = \underset{\sim}{0}$

and $\underset{\sim}{C} = \underset{\sim}{I}$.

$\mathrm{rank}(\underset{\sim}{Y}) > 0 = \mathrm{rank}(\underset{\sim}{A}) + \mathrm{rank}(\underset{\sim}{B})$

5.

$$a'(t) = -0.16\, a(t) + 0.04\, b(t)$$
$$b'(t) = 0.16\, a(t) - 0.16\, b(t)$$

$$A = \begin{pmatrix} -0.16 & 0.04 \\ 0.16 & -0.16 \end{pmatrix}$$ ✓

$$\det(xI - A) = \begin{vmatrix} x+0.16 & -0.04 \\ -0.16 & x+0.16 \end{vmatrix} = (x+0.08)(x+0.24)$$

$\lambda = -0.08$ ✓ : $\begin{pmatrix} 0.08 & -0.04 \\ -0.16 & 0.08 \end{pmatrix} \to \begin{pmatrix} 1 & -\frac{1}{2} \\ 0 & 0 \end{pmatrix}$ $\quad V_{\underset{-0.08}{\cancel{0.42}}} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ ✓

$\lambda = -0.24$ ✓ : $\begin{pmatrix} -0.08 & -0.04 \\ -0.16 & -0.08 \end{pmatrix} \to \begin{pmatrix} 1 & \frac{1}{2} \\ 0 & 0 \end{pmatrix}$ $\quad V_{\underset{-0.24}{\cancel{0.76}}} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ ✓

general solution $\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = c_1 e^{-0.08t} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 e^{-0.24t} \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ ✓

$$\left.\begin{aligned} a(0) &= 40 = c_1 + c_2 \\ b(0) &= 20 = 2c_1 - 2c_2 \end{aligned}\right\} \quad \begin{aligned} c_1 &= 25 \\ c_2 &= 15 \end{aligned}$$ ✓

$$a(t) = 25 e^{-0.08t} + 15 e^{-0.24t}$$
$$b(t) = 50 e^{-0.08t} - 30 e^{-0.24t}$$ ✓

~~As $t \to \infty$, dominant factor is $e^{-0.08t}$~~

~~$b(t) \to 50 e^{-0.08t}$~~ ~~$b(t) < 2a(t)$~~

For all t, ~~$a(t) \to 25 e^{-0.08t}$~~

$$\begin{aligned} 2a(t) &= 2(25e^{-0.08t} + 15e^{-0.24t}) \\ &= 50e^{-0.08t} + 30e^{-0.24t} \\ &> 50e^{-0.08t} - 30e^{-0.24t} \\ &= b(t). \end{aligned}$$

6. a)

$$\begin{pmatrix}\cos\theta\\ \sin\theta\\ 0\end{pmatrix}\cdot\begin{pmatrix}-\sin\theta\\ \cos\theta\\ 0\end{pmatrix}=0$$

$$\begin{pmatrix}\cos\theta\\ \sin\theta\\ 0\end{pmatrix}\cdot\begin{pmatrix}0\\ 0\\ 1\end{pmatrix}=0$$

$$\begin{pmatrix}-\sin\theta\\ \cos\theta\\ 0\end{pmatrix}\cdot\begin{pmatrix}0\\ 0\\ 1\end{pmatrix}=0$$

all columns are ⊥

R_θ is orthogonal

✗

Need to show: $\underset{\sim}{R}_\theta^{T}\underset{\sim}{R}_\theta=\underset{\sim}{I}$

b) when $\theta=\frac{\pi}{4}$

$$R_{\frac{\pi}{4}}=\begin{pmatrix}\sqrt{2}/2 & -\sqrt{2}/2 & 0\\ \sqrt{2}/2 & \sqrt{2}/2 & 0\\ 0 & 0 & 1\end{pmatrix}$$

The vectors in B are precisely the columns in $\underset{\sim}{R}_{\frac{\pi}{4}}$ which is orthogonal.
∴ B is an orthonormal basis for $\mathbb{R}^3$.

norm of each column $=1$ ⇒ orthonormal set

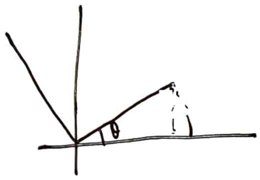
columns of $R_{\frac{\pi}{4}}$ form an orthonormal basis for $\mathbb{R}^3$

c)

$x'=x\cos\theta-y\sin\theta$

$y'=x\sin\theta+y\cos\theta$

$z'=z$



rotation matrix about z-axis

rotate counterclockwise by θ angle $=\frac{\pi}{4}$

✓

d) i) $m=20\begin{pmatrix}\sqrt{2}/2\\ \sqrt{2}/2\\ 0\end{pmatrix}+20\begin{pmatrix}-\sqrt{2}/2\\ \sqrt{2}/2\\ 0\end{pmatrix}+10\begin{pmatrix}0\\ 0\\ 1\end{pmatrix}$

$=\begin{pmatrix}0\\ 20\sqrt{2}\\ 10\end{pmatrix}$ $\quad (m)_E=m=\begin{pmatrix}0\\ 20\sqrt{2}\\ 10\end{pmatrix}$

✓

ii)

coordinate when P closest to M

$x = y$
$y = y$
$z = 0$

Basis = $\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right\}$

$$A = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$v = A(A^TA)^{-1}A^T \begin{pmatrix} 0 \\ 20\sqrt{2} \\ 10 \end{pmatrix}$$

$$= \begin{pmatrix} 10\sqrt{2} \\ 10\sqrt{2} \\ 0 \end{pmatrix}$$ ✓

$$(v)_B = \begin{pmatrix} 20 \\ 0 \\ 0 \end{pmatrix}$$ ✓

$$\|m - v\| = \left\| \begin{pmatrix} -10\sqrt{2} \\ 10\sqrt{2} \\ 10 \end{pmatrix} \right\| = \sqrt{500}$$ ✓