NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics

AY2021, Semester 2 MA1508E Linear Algebra for Engineering Practice 1

1. Write your answers on A4 size papers.
2. Write down your student number, name, and tutorial group clearly on the top left corner of every page of the answer script.
3. Write the page number on the top right corner of each page of the answer script.
4. There are three questions in this worksheet with a total of 20 marks.
5. To submit your answer scripts, scan or take pictures of your work (make sure the images can be read clearly). Merge all your images into one pdf file (make sure they are in order of the page). Name the pdf file by **StudentNo_P1** (e.g. **A123456Z_P1**). Upload your pdf into the LumiNUS folder Practice 1 submission.

1. Consider the following augmented matrix

$$\left(\begin{array}{ccc|c} 1 & a & 2 & a \\ 1 & 1 & 1 & a \\ 1 & 1 & a+1 & 2a \end{array}\right).$$

(a) [2 marks] Write down the original linear system that correspond to the augmented matrix above. Use variables x_1, x_2, x_3.

(b) [8 marks]

(i) Find the conditions on a such that the system has no solution.

(ii) Find the conditions on a such that the system has a unique solution, and write down the unique solution.

(iii) Find the conditions on a such that the system has infinitely many solutions, and write down a general solution and a particular solution.

Write down the elementary row operation that you used in each step clearly.

2. (a) Consider the following linear system

$$\begin{array}{rcrcrcr} x_1 & + & x_2 & & & = & 1 \\ x_1 & + & 2x_2 & + & x_3 & = & 0 \\ x_1 & - & x_2 & + & x_3 & = & 1 \\ x_1 & + & x_2 & & & = & -1 \\ x_1 & + & x_2 & + & x_3 & = & -1 \end{array}$$

(i) [1 mark] Write the corresponding matrix equation $\mathbf{Ax} = \mathbf{b}$.

(ii) [3 marks] Compute $\mathbf{A}^T\mathbf{A}$ and $\mathbf{A}^T\mathbf{b}$ in part (i).

(b) [1 mark] Write down a linear system that has the following general solution.

$$\begin{aligned} x_1 &= -s + 2t \\ x_2 &= s - t \\ x_3 &= s \\ x_4 &= t \end{aligned}$$

3. [5 marks] Let $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}$. Find all the matrices $\mathbf{B} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ such that $\mathbf{AB} = \mathbf{BA}$.