NATIONAL UNIVERSITY OF SINGAPORE

ST2334 Probability and Statistics

(Semester NN: AY YYYY–YYYY)

MMM YYYY — Time allowed: 2 hours

SAMPLE PAPER

Suggested solutions will be uploaded by the Wednesday of the reading week.

INSTRUCTIONS TO CANDIDATES

1. This paper contains **SIX (6)** questions and comprises **FIFTEEN (15)** printed pages.
2. Answer **ALL** questions. Marks for each question are indicated. The total marks for this paper is 60.
3. Please show workings and answers in the space provided for each question or part. Answers should be given in complete English sentences.
4. Non-programmable calculators may be used. However, candidates should lay out systematically the various steps in the calculations.
5. This is a CLOSED BOOK examination. Candidates may bring in **ONE (1)** A4-size help sheets with hand-written notes on both sides.
6. Write down your matriculation number and seat number neatly in the boxes provided below. **Do not write your name.** This booklet will be collected at the end of the examination.

Matriculation Number : | | | | | | | | | | |

Seat Number : | | | | |

Question	**1**	**2**	**3**	**4**	**5**	**6**
Score						

Total

Question 1 [10 marks]

(a) A radio show host asks listeners whether they believe that human activity is altering the global climate. Of those calling in, only 20% responded by saying yes. Is it safe to infer that only 20% of the general population believes that human activity is altering the global climate? Explain.

(b) There are 18 first year, 15 second year, 10 third year and 5 fourth year students in the course TS4332. They are allocated randomly into 4 classes of 12 each. If there are a total of 6 first year and 8 second year students in classes A and B, what is the probability that class D has 4 first year and 4 second year students?

(c) A gambler has a fair coin and a two-headed coin in his pocket. He selects one of the coins at random.

(i) When he flips the coin, it shows heads. What is the probability that it is the fair coin?

(ii) Suppose that he flips the same coin two more times, and it shows heads and tails, in that order. Now what is the probability that it is the fair coin?

Question 2 [10 marks]

(a) Let X denote the minimum of the two numbers when two fair dice are rolled.

(i) What is the probability that X is equal to 2?

(ii) What is the expected value of X?

(b) Let Y be a nonnegative random variable with $\text{var}(Y) = 7$ and $E(Y(Y-1)) = 9$. What is the value of $E(Y)$?

Question 3 [10 marks]

(a) Assume that while Larry is walking in the Gardens by the Bay, the time X, in minutes, between him seeing two people taking photographs using a camera has a density function of the form

$$f(x) = \begin{cases} cxe^{-x}, & x > 0 \\ 0, & x \leq 0 \end{cases}.$$

(i) What is the value of c?

(ii) Find the cumulative distribution function F of X and use it to compute the probability that Larry, who has just seen a person taking photographs using a camera, will see another person taking photographs using a camera in 2 to 5 minutes.

(b) The daily production of electric motors at a certain factory averaged 120 with a standard deviation of 10. Use the Chebyshev's Inequality to find an interval that contains at least 90% of the daily production levels.

Question 4 [10 marks]

Let the joint probability mass function of discrete random variables X and Y be given by

$$p(x,y) = \begin{cases} \frac{1}{25}(x^2+y^2), & \text{if } x = 1,2, y = 0,1,2 \\ 0, & \text{otherwise} \end{cases}.$$

(i) Find $P(X > Y)$ and $P(X+Y \leq 2)$.

(ii) Find the marginal distributions of X and Y. Are X and Y independent? Why or why not?

(iii) Find the conditional distribution of X given $Y = 1$.

(iv) Find $E(X|Y = 1)$.

Question 5 [10 marks]

(a) A company packages powdered soap in "6-pound" boxes. The sample mean and standard deviation of the soap in these boxes are currently 6.09 pounds and 0.02 pound, respectively. Every 0.01 pound lowered for the mean fill saves the company $14,000 per year. Adjustments were made in the filling equipment.

(i) How large a sample is needed so that the maximum error of the estimate of the new mean μ is $E = 0.001$ with 90% confidence?

(ii) A random sample of size n = 1219 yielded $\bar{x} = 6.048$ and $s = 0.022$. Calculate a 90% confidence interval for the new mean μ.

(iii) Estimate the savings per year with these new adjustments.

(b) Candidate DT believes that he can win a city election if he receives at least 55% of the votes from precinct I. Unknown to the candidate, 50% of the registered voters in the precinct favor him. If $n = 100$ voters show up to vote at precinct I, what is the probability that candidate DT will receive at least 55% of that precinct's votes?

Question 6 [10 marks]

(a) An investigator suspects that the mean concentration of suspended particles, measured in $\mu g/m^3$, in the city center of City A is lower than that in City B. To verify that, $n = 13$ observations are collected from City A and $m = 16$ observations are collected from City B. The following summary statistics based on the samples are obtained.

$$\bar{x} = 72.9, \quad s_x = 25.6, \quad \bar{y} = 81.7, \quad s_y = 28.3.$$

(i) Conduct a suitable test at $\alpha = 0.05$ level to determine if there is evidence to support the investigator's claim. State any assumptions made.

(ii) Write down the (approximate) p-value of your test in the previous part.

(b) A random sample of size 25 gives $\bar{x} = 104$. We are interested to test

$$H_0 : \mu = 100 \quad \text{vs} \quad H_1 : \mu \neq 100.$$

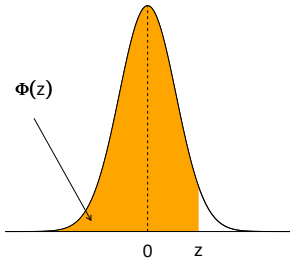
The significance level of the test is $\alpha = 0.05$ and the p-value of the test is 0.057.

Consider the following statement:

"The probability that $\bar{x} = 104$ if H_0 is true equals to 0.057."

Do you agree with it? Why or why not?

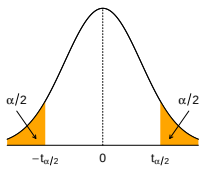
APPENDIX A: DISTRIBUTION FUNCTION OF THE NORMAL DISTRIBUTION



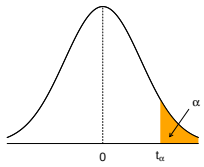
The function tabulated is $\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}u^2}\,\mathrm{du}$.

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999822	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967

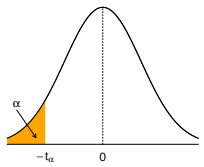
APPENDIX B: CRITICAL VALUES FOR STUDENT'S *t* DISTRIBUTION

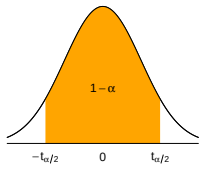


Two-tail area



One-tail area





$1-\alpha$ *confidence level*

two-tail	0.5	0.2	0.1	0.05	0.02	0.01	0.005	0.002	0.001
one-tail	0.25	0.1	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
df = 1	1.000	3.078	6.314	12.706	31.821	63.657	127.321	318.309	636.619
2	0.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.599
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.215	12.924
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
32	0.682	1.309	1.694	2.037	2.449	2.738	3.015	3.365	3.622
34	0.682	1.307	1.691	2.032	2.441	2.728	3.002	3.348	3.601
36	0.681	1.306	1.688	2.028	2.434	2.719	2.990	3.333	3.582
38	0.681	1.304	1.686	2.024	2.429	2.712	2.980	3.319	3.566
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
42	0.680	1.302	1.682	2.018	2.418	2.698	2.963	3.296	3.538
44	0.680	1.301	1.680	2.015	2.414	2.692	2.956	3.286	3.526
46	0.680	1.300	1.679	2.013	2.410	2.687	2.949	3.277	3.515
48	0.680	1.299	1.677	2.011	2.407	2.682	2.943	3.269	3.505
50	0.679	1.299	1.676	2.009	2.403	2.678	2.937	3.261	3.496
60	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
70	0.678	1.294	1.667	1.994	2.381	2.648	2.899	3.211	3.435
80	0.678	1.292	1.664	1.990	2.374	2.639	2.887	3.195	3.416
90	0.677	1.291	1.662	1.987	2.368	2.632	2.878	3.183	3.402
100	0.677	1.290	1.660	1.984	2.364	2.626	2.871	3.174	3.390
120	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
140	0.676	1.288	1.656	1.977	2.353	2.611	2.852	3.149	3.361
160	0.676	1.287	1.654	1.975	2.350	2.607	2.846	3.142	3.352
180	0.676	1.286	1.653	1.973	2.347	2.603	2.842	3.136	3.345
200	0.676	1.286	1.653	1.972	2.345	2.601	2.839	3.131	3.340
∞	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291
confidence level	0.5	0.8	0.9	0.95	0.98	0.99	0.995	0.998	0.999

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