NATIONAL UNIVERSITY OF SINGAPORE

ST2334 Probability and Statistics

(SEMESTER NN: AY YYYY–YYYY)

MMM YYYY — Time allowed: 2 hours

SAMPLE PAPER

Suggested solutions will be uploaded by the Wednesday of the reading week.

INSTRUCTIONS TO CANDIDATES

1. This paper contains **SIX (6)** questions and comprises **FIFTEEN (15)** printed pages.
2. Answer **ALL** questions. Marks for each question are indicated. The total marks for this paper is 60.
3. Please show workings and answers in the space provided for each question or part. Answers should be given in complete English sentences.
4. Non-programmable calculators may be used. However, candidates should lay out systematically the various steps in the calculations.
5. This is a CLOSED BOOK examination. Candidates may bring in **ONE (1)** A4-size help sheets with hand-written notes on both sides.
6. Write down your matriculation number and seat number neatly in the boxes provided below. **Do not write your name.** This booklet will be collected at the end of the examination.

Matriculation Number : | | | | | | | | | | |

Seat Number : | | | | |

Question	**1**	**2**	**3**	**4**	**5**	**6**
Score						

Total

Question 1 [10 marks]

(a) We want to know the mean winning score at the US Open golf championship. An internet search gives us all the scores for the history of that tournament, and we create a 95% confidence interval based on a t-distribution.

Is this procedure appropriate? Explain in a sentence or two.

Solution

This procedure is not appropriate. The entire population of scores was gathered so there is no reason to do inference. The mean can be obtained by taking the average of the scores collected.

(b) 8 speakers are to speak one after another in a conference. Dr Cooper's lecture is related to Dr Hofstadter's and should not precede it. How many schedules could be arranged?

Solution

There are 8! schedule possibilities.

By symmetry, in $8!/2$ of them Dr Cooper's lecture precedes Dr Hofstadter's and in $8!/2$ of them Dr Hofstadter's lecture precedes Dr Cooper's. So the answer is

$$8!/2 = 20160.$$

ALTERNATIVE SOLUTION

We can select 2 positions out of 8 to place Hofstadter and Cooper (in that order). The rest of the speakers can be arranged in 6! ways.

The answer is then $\binom{8}{2} \times 6! = 20160$.

(c) A headline in a local newspaper announced "Video game playing can lead to better spatial reasoning abilities." The article reported that a study found "statistically significant differences" between teens who play video games and teens who do not, with teens who play video games testing better in spatial reasoning.

Do you think the headline was appropriate? Explain in a sentence or two.

Solution

No, this was not a controlled experiment, so no determination of cause and effect can be made. Perhaps there is something about teens who play video games that make them good at video games and at spatial reasoning, or maybe teens with good spatial reasoning enjoy games more.

(d) It was discovered that 25% of the paintings of a certain gallery are not original. A collector in 15% of the cases makes a mistake in judging if a painting is authentic or a copy. If she buys a piece thinking that it is original, what is the probability that it is not?

Solution

Let A denote the event that the painting bought is authentic and O the event that the collector deems the painting bought to be authentic.

We then know that

$$P(A) = 3/4, \quad P(O|A) = 17/20, \quad P(O|A^c) = 3/20.$$

The required probability will be given as

$$\begin{aligned} P(A^c|O) &= \frac{P(O|A^c)P(A^c)}{P(O|A^c)P(A^c) + P(O|A)P(A)} \\ &= \frac{3/20 \times 1/4}{3/20 \times 1/4 + 17/20 \times 3/4} \\ &= 3/54 = 1/18 \\ &\approx 0.05556. \end{aligned}$$

Question 2 [10 marks]

(a) There are 5 couples (husbands and wives) in a party. We assume in what follows that all in the party have birthday months that are independent, and that any month is equally likely to be the birthday month of a particular person.

(i) What is the probability that at least two of the wives share the same birthday month?

Solution

This is the birthday problem with 12 "days" and $n = 5$ persons.

The required probability is

$$1 - \frac{12 \times 11 \times \cdots \times 8}{12^5} = \frac{89}{144} \approx 0.6181.$$

(ii) What is the probability that there are at least two couples where both husbands share the same birthday month and both wives share the same birthday month?

Solution

For couples, a possible birthday month configuration is (m_m, w_m), where m_m (respectively w_m) denotes the birthday month of the man (respectively, women) for that couple. Thus the number of possible birthday month configuration is $12^2 = 144$.

This then corresponds to the birthday problem with 144 "days" and $n = 5$ "persons".

The required probability is

$$1 - \frac{144 \times 143 \times \cdots \times 140}{144^5} \approx 0.0678.$$

(iii) One of the 5 couples is the couple hosting the party. What is the chance that there exists at least one other couple in which the husband has the same birthday month as the host husband and the wife has the same birthday month as the host wife?

Solution

The probability that all 4 other couples have a different birthday configuration is $(143/144)^4$. Thus the required probability is

$$1-\left(\frac{143}{144}\right)^4 \approx 0.0275.$$

(b) Let A and B be events. Suppose $P(A) = 1$. Determine if A and B are independent.

Solution

Note that $P(A \cup B) \geq P(A) = 1$. Thus $P(A \cup B) = 1$, which gives

$$P(A)+P(B)-P(AB) = P(A \cup B) = 1 \Longleftrightarrow P(AB) = P(B) = P(A)P(B).$$

Thus A and B are independent.

Question 3 [10 marks]

Tom and Jerry gamble against one other by rolling dice. Tom's die has an 8 on one of the face and 2's on the other five faces. Jerry's die has four 3's and two 1's on the six faces.

(i) They each roll their own die once, and the player with the higher score wins. Which player has a greater probability of winning?

Solution

Let T and J denote the outcome of Tom's and Jerry's dice, respectively.

The probability that Tom wins is

$$P(T > J) = P(\{T = 8\}) + P(\{T = 2, J = 1\}) = 1/6 \times 6/6 + 5/6 \times 2/6 = 16/36,$$

while that of Jerry winning is

$$P(T < J) = P(\{T = 2, J = 3\}) = 5/6 \times 4/6 = 20/36.$$

So Jerry has a greater probability of winning.

(ii) If Tom wins, Jerry pays him \$10. How much should Tom pay Jerry if Jerry wins in order to make the game fair?

Solution

Let X denote the winnings of Tom. Then the probability mass function of X is given as

x	\$10	\$$y$
$P(X = x)$	$16/36$	$20/36$

For a fair game,

$$E(X) = 0 = \$10 \times 16/36 + \$y \times 20/36.$$

Thus $y = -8$ and Tom should pay Jerry \$8 should Jerry win.

(iii) After playing the game for a while, both got bored of it and decided to change the rules. In the new game, the person who wins will collect the number of dollars shown on his die. As an illustration, if Tom obtains a 8 and Jerry gets a 1, Jerry will have to pay Tom \$8. Write down the probability mass function of Y, if Y denotes Tom's winnings.

Solution

The probability mass function of Y is given as

y	\$8	\$2	$-$\$3
$P(Y=y)$	$6/36$	$10/36$	$20/36$

(iv) Find the expected value and standard deviation of Y.

Solution

Note that

$$E(Y) = 8 \times 6/36 + 2 \times 10/36 - 3 \times 20/36 = 8/36 \approx 0.22,$$

and

$$E(Y^2) = 64 \times 6/36 + 4 \times 10/36 + 9 \times 20/36 = 604/36.$$

Thus

$$SD(Y) = \sqrt{604/36 - (8/36)^2} \approx 4.09.$$

(v) If they play this new game repeatedly which player has the advantage? Explain.

Solution

Tom has the advantage, because his expected value is positive. He expects to win an average of \$0.22 each time they play the game.

Question 4 [10 marks]

(a) A manufacturer claims that the lifetime of an appliance costing \$300 follows an exponential distribution with mean 3 years. Customers will be given a full refund if the appliance fails to last a year following its purchase. A 50% refund will be given if the appliance lasts between 1 year to 3 years. How much should the manufacturer expect to pay in refunds if it sells 200 such appliances?

$\boxed{\textbf{Solution}}$

Let T denote the lifetime of such an appliance. Then $T \sim \exp(1/3)$.

The refund Y is a random variable defined as

$$Y = g(T) = \begin{cases} 300, & \text{if } T < 1 \\ 150, & \text{if } 1 < T < 3 \end{cases}.$$

The expected value of Y is given as

$$\begin{aligned} E(Y) &= \int g(t)f(t)\,\mathrm{dt} \\ &= \int_0^1 300 \times \frac{1}{3}e^{-t/3}\,\mathrm{dt} + \int_1^3 150 \times \frac{1}{3}e^{-t/3}\,\mathrm{dt} \\ &= 300(1 - e^{-1/3}) + 150(e^{-1/3} - e^{-1}). \end{aligned}$$

Thus the total refund expected is

$$200 \times (300 - 150e^{-1/3} - 150e^{-1}) \approx 27467.68.$$

(b) The score of students taking the final examination is a random variable with mean 75 and variance 25. How many students would have to take the examination to ensure, with probability at least 0.9, that the class average would be within 5 of 75?

Hint: Use the Chebyshev Inequality.

$\boxed{\textbf{Solution}}$

Note that if the class size is n, then $E(\overline{X}) = 75$, and $\text{var}(\overline{X}) = 5^2/n$.
Using the Chebyshev Inequality, we have

$$\begin{aligned} P(|\overline{X} - 75| \le 5) &= P(|\overline{X} - 75| \le \sqrt{n} \times 5/\sqrt{n}) \\ &\ge 1 - 1/n. \end{aligned}$$

Since we want to probability to be at least 0.9, we solve for

$$1 - 1/n \ge 0.9$$

to get $n \ge 10$.

(c) Assume that yield per acre for a particular variety of soybeans is $N(\mu, \sigma^2)$. For a random sample of $n = 5$ plots, the yields in bushels per acre were given to be

$$37.4, 48.8, 46.9, 55.0, 44.0.$$

(i) Give a point estimate for σ^2.

Solution

The sample mean is given as

$$\bar{x} = \frac{1}{5}\sum x_i = \frac{37.4 + 48.8 + 46.9 + 55.0 + 44.0}{5} = 46.42.$$

Sum of squares is given as

$$\sum x_i^2 = 10940.81.$$

Thus the sample variance is

$$\frac{1}{4}\left(\sum x_i^2 - 5\bar{x}^2\right) = 41.682.$$

(ii) Give a 90% confidence interval for μ.

Solution

A 90% confidence interval for μ is given as

$$\bar{x} \pm t_{0.95,4} \times \frac{s}{\sqrt{5}} = 46.42 \pm 2.132 \times \frac{\sqrt{41.682}}{\sqrt{5}} = (40.264, 52.576).$$

Question 5 [10 marks]

(a) The joint probability density function of X and Y is given by

$$f(x,y) = k(x^2 + xy/2), \quad 0 < x < 1,\ 0 < y < 2.$$

(i) Find the value of k.

Solution

We know that

$$\begin{aligned}
1 &= \int_0^1 \int_0^2 f(x,y)\,\mathrm{dy}\,\mathrm{dx} \\
&= k \int_0^1 \int_0^2 (x^2 + xy/2)\,\mathrm{dy}\,\mathrm{dx} \\
&= k \int_0^1 \left[(x^2 y + xy^2/4)\right]_0^2 \mathrm{dx} \\
&= k \int_0^1 (2x^2 + x)\,\mathrm{dx} \\
&= k \left[2x^3/3 + x^2/2\right]_0^1 \\
&= 7k/6.
\end{aligned}$$

Thus $k = 6/7$.

(ii) Find the marginal density function of X.

Solution

The marginal density function of X is given as

$$f_X(x) = \int_0^2 f(x,y)\,\mathrm{dy} = 6/7(2x^2 + x), \quad 0 < x < 1.$$

(iii) Find $E(Y|X=0.5)$.

Solution

Note that

$$f_{Y|X=x}(y|x) = \frac{f(x,y)}{f_X(x)} = \frac{(x^2 + xy/2)}{2x^2 + x}, \quad 0 < y < 2.$$

When $x = 0.5$,

$$f_{Y|X=0.5}(y|0.5) = \frac{1+y}{4}, \quad 0 < y < 2.$$

Thus

$$E(Y|X=0.5) = \int_0^2 \frac{y(1+y)}{4}\,\mathrm{dy} = 7/6.$$

(b) Based on data from two very large independent samples, two students tested a hypothesis about equality of population means using $\alpha = 0.01$. One student used a one-tail test and rejected the null hypothesis, but the other used a two-tail test and failed to reject the null. If the calculated value of the test statistics c is positive, what are the maximum possible value and minimum possible value of c?

Solution

Note that the test statistics would follow a standard normal distribution since both sample sizes are large.

For a two-sided test, the acceptance region will be given as $Z \in (z_{0.005}, z_{0.995}) = (-2.576, 2.576)$. Since the null failed to be rejected for the two-tailed test, $c < 2.576$.

For a one-sided test, the rejection region will be given as $Z > z_{0.99} = 2.326$. Since the null is rejected for the one-tailed test, $c > 2.326$.

Thus the minimum and maximum values of c are 2.326 and 2.576 respectively.

Question 6 [10 marks]

(a) Plants convert carbon dioxide (CO2) in the atmosphere, along with water and energy from sunlight, into the energy they need for growth and reproduction. Experiments were performed under normal atmospheric air conditions and in air with enriched CO2 concentrations to determine the effect on plant growth. The plants were given the same amount of water and light for a four-week period.

(i) BK is investigating if CO2-enriched atmosphere increases plant growth using a suitable hypothesis test. Write down the null and alternative hypotheses for his test.

Solution

Let μ_1 be the mean plant growth for normal air and μ_2 be the mean plant growth for enriched air.

BK should test

$$H_0 : \mu_1 = \mu_2 \quad \text{vs} \quad H_1 : \mu_1 < \mu_2.$$

(ii) The mean and standard deviation for a sample of 12 plants grown in normal air are given to be 4.163 and 0.9562, while the mean and standard deviation for a sample of 8 plants grown in enriched air are given to be 5.105 and 1.6098. On the basis of these data, determine whether CO2-enriched atmosphere increases plant growth at $\alpha = 0.05$ level. State any assumptions you are making.

Solution

We shall assume that both samples come from populations that follow normal distributions.

We are given that

$$\bar{x} = 4.163, \quad s_x = 0.9562, \quad \bar{y} = 5.105, \quad s_y = 1.6098.$$

Since

$$1/2 < s_x/s_y < 2,$$

we can assume that the two normal populations have the same variance.

We shall use the two-sample t statistic

$$t = \frac{\overline{X} - \overline{Y}}{\sqrt{(n-1)S_x^2 + (m-1)S_y^2}} \sqrt{\frac{nm(n+m-2)}{n+m}}.$$

The null hypothesis will be rejected when $t < -t_{18,0.05} = -1.734$.

We obtain, by substituting the values given,

$$t = -1.6489.$$

Since $t = -1.6489 > -t_{0.05} = -1.734$, we do not reject the null hypothesis at $\alpha = 0.05$.

There is thus no evidence to say that CO2-enriched atmosphere increases plant growth.

(iii) Write down the (approximate) p-value of your test in the previous part.

Solution

From the t-table, we see that

$$1.330 < 1.6489 < 1.734$$

on the row with 18 degrees of freedom. This means that

$$0.1 = P(t > 1.330) > P(t > 1.6489) > P(t > 1.734) = 0.05.$$

Thus

$$0.1 > P(t > 1.6489) > 0.05.$$

Since $P(t > 1.6489) = P(t < -1.6489)$, the p-value lies between 0.05 and 0.1.

Alternatively, the exact p-value is given to be 0.058 by way of a graphical calculator.

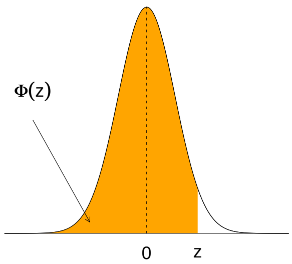
(b) Researchers developing new drugs must be concerned about possible side effects. They must check a new medication to be sure that it does not cause an unsafe increase in blood pressure. They measure the blood pressures of a group of 12 subjects, then administer the drug and recheck the blood pressures one hour later. The drug will be approved for use unless there is evidence that blood pressure has increased an average of more than 20 points. They will test a hypothesis using $\alpha = 0.05$.

In this context, which do you consider to be more serious — a type I or a type II error? Explain briefly.

Solution

A type II error is dangerous; the medication is approved even though blood pressure increases too much. A type I error means that an acceptable medication is not approved; that's too bad, but not dangerous.

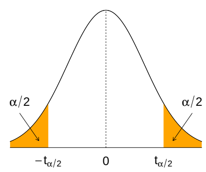
APPENDIX A: DISTRIBUTION FUNCTION OF THE NORMAL DISTRIBUTION



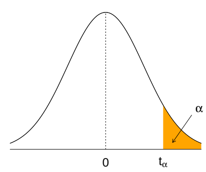
The function tabulated is $\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}u^2}\,\mathrm{du}.$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999822	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967

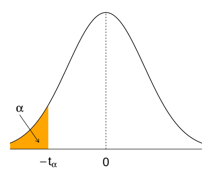
APPENDIX B: CRITICAL VALUES FOR STUDENT'S t DISTRIBUTION

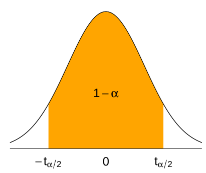


Two-tail area



One-tail area





$1-\alpha$ *confidence level*

two-tail	0.5	0.2	0.1	0.05	0.02	0.01	0.005	0.002	0.001
one-tail	0.25	0.1	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
df = 1	1.000	3.078	6.314	12.706	31.821	63.657	127.321	318.309	636.619
2	0.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.599
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.215	12.924
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
32	0.682	1.309	1.694	2.037	2.449	2.738	3.015	3.365	3.622
34	0.682	1.307	1.691	2.032	2.441	2.728	3.002	3.348	3.601
36	0.681	1.306	1.688	2.028	2.434	2.719	2.990	3.333	3.582
38	0.681	1.304	1.686	2.024	2.429	2.712	2.980	3.319	3.566
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
42	0.680	1.302	1.682	2.018	2.418	2.698	2.963	3.296	3.538
44	0.680	1.301	1.680	2.015	2.414	2.692	2.956	3.286	3.526
46	0.680	1.300	1.679	2.013	2.410	2.687	2.949	3.277	3.515
48	0.680	1.299	1.677	2.011	2.407	2.682	2.943	3.269	3.505
50	0.679	1.299	1.676	2.009	2.403	2.678	2.937	3.261	3.496
60	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
70	0.678	1.294	1.667	1.994	2.381	2.648	2.899	3.211	3.435
80	0.678	1.292	1.664	1.990	2.374	2.639	2.887	3.195	3.416
90	0.677	1.291	1.662	1.987	2.368	2.632	2.878	3.183	3.402
100	0.677	1.290	1.660	1.984	2.364	2.626	2.871	3.174	3.390
120	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
140	0.676	1.288	1.656	1.977	2.353	2.611	2.852	3.149	3.361
160	0.676	1.287	1.654	1.975	2.350	2.607	2.846	3.142	3.352
180	0.676	1.286	1.653	1.973	2.347	2.603	2.842	3.136	3.345
200	0.676	1.286	1.653	1.972	2.345	2.601	2.839	3.131	3.340
∞	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291
confidence level	0.5	0.8	0.9	0.95	0.98	0.99	0.995	0.998	0.999

END OF PAPER