NATIONAL UNIVERSITY OF SINGAPORE

ST2334 Probability and Statistics

(SEMESTER NN: AY YYYY–YYYY)

MMM YYYY — Time allowed: 2 hours

SAMPLE PAPER

Suggested solutions will be uploaded by the Wednesday of the reading week.

INSTRUCTIONS TO CANDIDATES

1. This paper contains **SIX (6)** questions and comprises **FIFTEEN (15)** printed pages.
2. Answer **ALL** questions. Marks for each question are indicated. The total marks for this paper is 60.
3. Please show workings and answers in the space provided for each question or part. Answers should be given in complete English sentences.
4. Non-programmable calculators may be used. However, candidates should lay out systematically the various steps in the calculations.
5. This is a CLOSED BOOK examination. Candidates may bring in **ONE (1)** A4-size help sheets with hand-written notes on both sides.
6. Write down your matriculation number and seat number neatly in the boxes provided below. **Do not write your name.** This booklet will be collected at the end of the examination.

Matriculation Number :

Seat Number :

Question	**1**	**2**	**3**	**4**	**5**	**6**
Score						

Total

Question 1 [10 marks]

(a) A radio show host asks listeners whether they believe that human activity is altering the global climate. Of those calling in, only 20% responded by saying yes. Is it safe to infer that only 20% of the general population believes that human activity is altering the global climate? Explain.

Solution

No. Those who respond to this question are unlikely to be representative of the general population for two reasons.

First, listeners to the show are unlikely to be representative of the general population, as they are likely to share the views of the radio show host, and may not represent a wide spectrum of views. This is a case of a self-select sample.

Second, those listeners who respond voluntarily to the question are likely to be those who feel the most strongly about the issue. Their views are unlikely to be representative of all listeners to the show let alone of the general population. This is a case of non-response bias.

(b) There are 18 first year, 15 second year, 10 third year and 5 fourth year students in the course TS4332. They are allocated randomly into 4 classes of 12 each. If there are a total of 6 first year and 8 second year students in classes A and B, what is the probability that class D has 4 first year and 4 second year students?

Solution

Given that there are 6 first year and 8 second year students in classes A and B, there are 12 first year, 7 second year and 5 third or fourth year students to be allocated to classes C and D.

The required probability is then given as

$$\frac{\binom{12}{4}\binom{7}{4}\binom{5}{4}}{\binom{24}{12}} \approx 0.032.$$

(c) A gambler has a fair coin and a two-headed coin in his pocket. He selects one of the coins at random.

(i) When he flips the coin, it shows heads. What is the probability that it is the fair coin?

Solution

Let F denote that the fair coin is chosen, and T that the two-headed coin is chosen. Let h denote that the coin shows a head.
So we know that

$$P(F) = P(T) = 1/2, \quad P(h|F) = 1/2, \quad P(h|T) = 1.$$

What we need is

$$P(F|h) = \frac{P(h|F)P(F)}{P(h|F)P(F) + P(h|T)P(T)} = \frac{1/2 \times 1/2}{1/2 \times 1/2 + 1 \times 1/2} = 1/3.$$

(ii) Suppose that he flips the same coin two more times, and it shows heads and tails, in that order. Now what is the probability that it is the fair coin?

Solution

What we need is

$$P(F|hht) = \frac{P(hht|F)P(F)}{P(hht|F)P(F) + P(hht|T)P(T)} = \frac{1/2 \times 1/2 \times 1/2 \times 1/2}{1/2 \times 1/2 \times 1/2 \times 1/2 + 1 \times 1 \times 0 \times 1/2} = 1.$$

Question 2 [10 marks]

(a) Let X denote the minimum of the two numbers when two fair dice are rolled.

(i) What is the probability that X is equal to 2?

Solution

When two balanced dice are rolled, 36 equally likely outcomes are possible as shown below.

```
(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)
```

The number of possible ways to get $X = 2$ is the number of points to the right and below $(2,2)$, inclusive. So the required probability is $9/36 = 0.25$.

(ii) What is the expected value of X?

Solution

The probability mass function of X is given as

x	1	2	3	4	5	6
$P(X = x)$	$11/36$	$9/36$	$7/36$	$5/36$	$3/36$	$1/36$

This means that

$$E(X) = 1 \times 11/36 + 2 \times 9/36 + 3 \times 7/36 + 4 \times 5/36 + 5 \times 3/36 + 6 \times 1/36 = 91/36 \approx 2.53.$$

(b) Let Y be a nonnegative random variable with $\text{var}(Y) = 7$ and $E(Y(Y-1)) = 9$. What is the value of $E(Y)$?

Solution

$$\begin{aligned}
\text{var}(Y) &= E(Y^2) - [E(Y)]^2 \\
7 &= E[Y(Y-1)] + E(Y) - [E(Y)]^2 \\
7 &= 9 + E(Y) - [E(Y)]^2
\end{aligned}$$

Hence,

$$\begin{aligned}
[E(Y)]^2 - E(Y) - 2 &= 0 \\
[E(Y) - 2]\,[E(Y) + 1] &= 0.
\end{aligned}$$

Note that $E(Y) \neq -1$ as Y is nonnegative, implying $E(Y) \geq 0$. Thus $E(Y) = 2$.

Question 3 [10 marks]

(a) Assume that while Larry is walking in the Gardens by the Bay, the time X, in minutes, between him seeing two people taking photographs using a camera has a density function of the form

$$f(x) = \begin{cases} cxe^{-x}, & x > 0 \\ 0, & x \leq 0 \end{cases}.$$

(i) What is the value of c?

Solution

We know that $\int_{\infty}^{\infty} f(x)\,\mathrm{dx} = 1$. This gives

$$\begin{aligned} 1 &= \int_0^{\infty} cxe^{-x}\,\mathrm{dx} \\ &= c\left[-xe^{-x}\right]_0^{\infty} + c\int_0^{\infty} e^{-x}\,\mathrm{dx} \\ &= c\left[-xe^{-x} - e^{-x}\right]_0^{\infty} \\ &= c[0+1] \end{aligned}$$

Thus $c = 1$.

(ii) Find the cumulative distribution function F of X and use it to compute the probability that Larry, who has just seen a person taking photographs using a camera, will see another person taking photographs using a camera in 2 to 5 minutes.

Solution

When $a \leq 0$,

$$F(a) = P(X \leq a) = 0.$$

When $a > 0$,

$$\begin{aligned} F(a) &= P(X \leq a) \\ &= \int_0^{a} xe^{-x}\,\mathrm{dx} \\ &= \left[-xe^{-x} - e^{-x}\right]_0^{a} \\ &= 1 - (a+1)e^{-a}. \end{aligned}$$

Thus the cumulative distribution function F of X is given as

$$F(a) = \begin{cases} 1-(a+1)e^{-a}, & a > 0 \\ 0, & a \leq 0 \end{cases}.$$

The required probability is given as

$$P(2 < X < 5) = F(5) - F(2) = [1-6e^{-5}] - [1-3e^{-2}] = 3e^{-2} - 6e^{-5} \approx 0.37.$$

(b) The daily production of electric motors at a certain factory averaged 120 with a standard deviation of 10. Use the Chebyshev's Inequality to find an interval that contains at least 90% of the daily production levels.

Solution

Let X be the daily production of electric motors at that factory. Chebyshev's Inequality gives

$$P(|X-\mu|>k\sigma)<\frac{1}{k^2}\Longleftrightarrow P(|X-\mu|<k\sigma)\geq 1-\frac{1}{k^2}.$$

This gives

$$P(\mu-k\sigma<X<\mu+k\sigma)\geq 1-\frac{1}{k^2}.$$

Setting $1-\frac{1}{k^2}=0.90$ we obtain $k=\sqrt{10}=3.16$, so

$$P\left(\mu-\sqrt{10}\sigma<X<\mu+\sqrt{10}\sigma\right)\geq 0.90.$$

The interval we seek is

$$\left(120-\sqrt{10}\times 10, 120+\sqrt{10}\times 10\right)=(88.4, 151.6).$$

Question 4 [10 marks]

Let the joint probability mass function of discrete random variables X and Y be given by

$$p(x,y) = \begin{cases} \frac{1}{25}(x^2+y^2), & \text{if } x = 1,2, y = 0,1,2 \\ 0, & \text{otherwise} \end{cases}.$$

(i) Find $P(X > Y)$ and $P(X+Y \leq 2)$.

Solution

$$P(X > Y) = p(1,0) + p(2,0) + p(2,1) = \frac{1+4+5}{25} = \frac{2}{5}.$$

while

$$P(X+Y \leq 2) = p(1,0) + p(1,1) + p(2,0) = \frac{1+2+4}{25} = \frac{7}{25}.$$

(ii) Find the marginal distributions of X and Y. Are X and Y independent? Why or why not?

Solution

The marginal distributions of X and Y are given as

x	1	2
$p_X(x)$	$8/25$	$17/25$

y	0	1	2
$p_Y(x)$	$5/25$	$7/25$	$13/25$

We note that

$$p_X(1) \times p_Y(0) = 8/25 \times 5/25 \neq 1/25 = p(1,0).$$

Thus X and Y are not independent.

(iii) Find the conditional distribution of X given $Y = 1$.

Solution

We know that $P(Y = 1) = 7/25$.
Thus the conditional distribution of X given $Y = 1$ is

x	1	2
$p_{X\|Y}(x\|y = 1)$	$\frac{2/25}{7/25} = 2/7$	$\frac{5/25}{7/25} = 5/7$

(iv) Find $E(X|Y = 1)$.

Solution

$$E(X|Y = 1) = 1 \times p_{1|Y}(x|y = 1) + 2 \times p_{1|Y}(x|y = 1) = 1 \times 2/7 + 2 \times 5/7 = 12/7.$$

Question 5 [10 marks]

(a) A company packages powdered soap in "6-pound" boxes. The sample mean and standard deviation of the soap in these boxes are currently 6.09 pounds and 0.02 pound, respectively. Every 0.01 pound lowered for the mean fill saves the company \$14,000 per year. Adjustments were made in the filling equipment.

(i) How large a sample is needed so that the maximum error of the estimate of the new mean μ is $E = 0.001$ with 90% confidence?

Solution

Note that $\alpha = 0.1$.

The sample size n needed is given by

$$n \geq \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2 = \left(\frac{1.645 \times 0.02}{0.001}\right)^2 = 1082.41 \approx 1083.$$

So we need 1083 samples.

(ii) A random sample of size n = 1219 yielded $\bar{x} = 6.048$ and $s = 0.022$. Calculate a 90% confidence interval for the new mean μ.

Solution

A 90% confidence interval for μ is given as

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{s}{\sqrt{n}} = 6.048 \pm 1.645 \times \frac{0.022}{\sqrt{1219}} = (6.047, 6.049).$$

(iii) Estimate the savings per year with these new adjustments.

Solution

The savings is given as

$$\frac{6.09-6.048}{0.01}\times \$14,000 = \$58,800.$$

(b) Candidate DT believes that he can win a city election if he receives at least 55% of the votes from precinct I. Unknown to the candidate, 50% of the registered voters in the precinct favor him. If $n = 100$ voters show up to vote at precinct I, what is the probability that candidate DT will receive at least 55% of that precinct's votes?

Solution

Let X denote the number of votes DT will receive in precinct I. Then $X \sim \text{Bin}(100, 0.5)$, with

$$E(X) = 100\times 0.5 = 50 \text{ and } \text{var}(X) = 100\times 0.5\times 0.5 = 5^2.$$

Since $n \geq 30$, we shall use the Central Limit Theorem. Let $Y \sim N(50, 5^2)$. Then the required probability is

$$\begin{aligned} P(X \geq 55) &\approx P(Y \geq 54.5) \\ &= P\left(\frac{Y-50}{5} \geq \frac{54.5-50}{5}\right) \\ &= P(Z \geq 0.9) \\ &= 1 - 0.8159 = 0.1841. \end{aligned}$$

Question 6 [10 marks]

(a) An investigator suspects that the mean concentration of suspended particles, measured in $\mu g/m^3$, in the city center of City A is lower than that in City B. To verify that, $n = 13$ observations are collected from City A and $m = 16$ observations are collected from City B. The following summary statistics based on the samples are obtained.

$$\bar{x} = 72.9, \quad s_x = 25.6, \quad \bar{y} = 81.7, \quad s_y = 28.3.$$

(i) Conduct a suitable test at $\alpha = 0.05$ level to determine if there is evidence to support the investigator's claim. State any assumptions made.

Solution

We assume that the measurements come from populations that follow normal distributions.

Let μ_1 be the mean concentration level for City A and μ_2 be that for City B.

Step 1. We test

$$H_0 : \mu_1 = \mu_2 \quad \text{vs} \quad H_1 : \mu_1 < \mu_2.$$

Step 2. Level of significance: $\alpha = 0.05$.

Step 3. Note that

$$1/2 < s_x/s_y < 2,$$

so we can assume that the two normal populations have the same variance.

We shall use the two-sample t statistic

$$t = \frac{\overline{X} - \overline{Y}}{\sqrt{(n-1)S_x^2 + (m-1)S_y^2}} \sqrt{\frac{nm(n+m-2)}{n+m}}.$$

We shall reject the null hypothesis when $t < -t_{27,0.05} = -1.703$ for $n + n - 2 = 27$ degrees of freedom.

Step 4. We obtain, by substituting the values given,

$$t = -0.869.$$

Step 5. Decision

Since $t = -0.869 > -t_{0.05} = -1.703$, we do not reject the null hypothesis at level of significance $\alpha = 0.05$.

Thus there is no evidence to say that City A has a lower mean concentration level compared to City B.

(ii) Write down the (approximate) p-value of your test in the previous part.

Solution

From the t-table, we see that

$$0.706 < 0.869 < 1.397$$

on the row with 27 degrees of freedom. This means that

$$0.25 = P(t > 0.706) > P(t > 0.869) > P(t > 1.397) = 0.10.$$

Thus

$$0.25 > P(t > 0.869) > 0.10.$$

Since $P(t > 0.869) = P(t < -0.869)$, the p-value lies between 0.10 and 0.25.

(b) A random sample of size 25 gives $\bar{x} = 104$. We are interested to test

$$H_0 : \mu = 100 \quad \text{vs} \quad H_1 : \mu \neq 100.$$

The significance level of the test is $\alpha = 0.05$ and the p-value of the test is 0.057.

Consider the following statement:

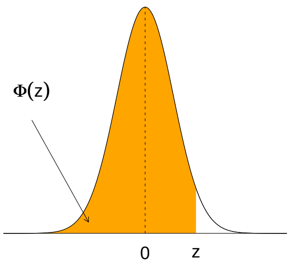
"The probability that $\bar{x} = 104$ if H_0 is true equals to 0.057."

Do you agree with it? Why or why not?

Solution

No, $\bar{x}$ is the sample mean. The probability that the sample mean equals 104, regardless of whether the null hypothesis is true, is 100%. The probability that we would obtain a sample mean of at least as extreme as 104 if the null hypothesis is true is 0.057.

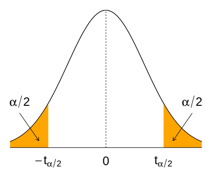
APPENDIX A: DISTRIBUTION FUNCTION OF THE NORMAL DISTRIBUTION



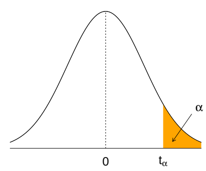
The function tabulated is $\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}u^2}\,\mathrm{du}.$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999822	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967

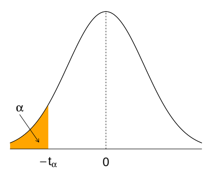
APPENDIX B: CRITICAL VALUES FOR STUDENT'S t DISTRIBUTION

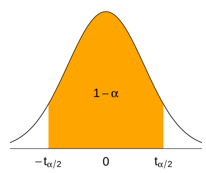


Two-tail area



One-tail area





$1-\alpha$ confidence level

two-tail	0.5	0.2	0.1	0.05	0.02	0.01	0.005	0.002	0.001
one-tail	0.25	0.1	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
df = 1	1.000	3.078	6.314	12.706	31.821	63.657	127.321	318.309	636.619
2	0.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.599
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.215	12.924
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
32	0.682	1.309	1.694	2.037	2.449	2.738	3.015	3.365	3.622
34	0.682	1.307	1.691	2.032	2.441	2.728	3.002	3.348	3.601
36	0.681	1.306	1.688	2.028	2.434	2.719	2.990	3.333	3.582
38	0.681	1.304	1.686	2.024	2.429	2.712	2.980	3.319	3.566
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
42	0.680	1.302	1.682	2.018	2.418	2.698	2.963	3.296	3.538
44	0.680	1.301	1.680	2.015	2.414	2.692	2.956	3.286	3.526
46	0.680	1.300	1.679	2.013	2.410	2.687	2.949	3.277	3.515
48	0.680	1.299	1.677	2.011	2.407	2.682	2.943	3.269	3.505
50	0.679	1.299	1.676	2.009	2.403	2.678	2.937	3.261	3.496
60	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
70	0.678	1.294	1.667	1.994	2.381	2.648	2.899	3.211	3.435
80	0.678	1.292	1.664	1.990	2.374	2.639	2.887	3.195	3.416
90	0.677	1.291	1.662	1.987	2.368	2.632	2.878	3.183	3.402
100	0.677	1.290	1.660	1.984	2.364	2.626	2.871	3.174	3.390
120	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
140	0.676	1.288	1.656	1.977	2.353	2.611	2.852	3.149	3.361
160	0.676	1.287	1.654	1.975	2.350	2.607	2.846	3.142	3.352
180	0.676	1.286	1.653	1.973	2.347	2.603	2.842	3.136	3.345
200	0.676	1.286	1.653	1.972	2.345	2.601	2.839	3.131	3.340
∞	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291
confidence level	0.5	0.8	0.9	0.95	0.98	0.99	0.995	0.998	0.999

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