NATIONAL UNIVERSITY OF SINGAPORE

ST2334 Probability and Statistics

(SEMESTER NN: AY YYYY–YYYY)

MMM YYYY — Time allowed: 2 hours

SAMPLE PAPER

Suggested solutions will be uploaded by the Wednesday of the reading week.

INSTRUCTIONS TO CANDIDATES

1. This paper contains **SIX (6)** questions and comprises **FIFTEEN (15)** printed pages.
2. Answer **ALL** questions. Marks for each question are indicated. The total marks for this paper is 60.
3. Please show workings and answers in the space provided for each question or part. Answers should be given in complete English sentences.
4. Non-programmable calculators may be used. However, candidates should lay out systematically the various steps in the calculations.
5. This is a CLOSED BOOK examination. Candidates may bring in **ONE (1)** A4-size help sheets with hand-written notes on both sides.
6. Write down your matriculation number and seat number neatly in the boxes provided below. **Do not write your name.** This booklet will be collected at the end of the examination.

Matriculation Number :

Seat Number :

Question	**1**	**2**	**3**	**4**	**5**	**6**
Score						

Total

Question 1 [10 marks]

In a certain population of fish, the length of individual fish follows a normal distribution with mean μ and standard deviation 4.5mm. It is known that 84.13% of the fish are less than 58.5mm long.

(i) What is the value of μ?

(ii) A random sample of four fish is chosen from the population. Find the probability that all four fish are between 51 and 60 mm long.

(iii) Continuing from the previous part, find the probability that the mean length of the four fish in the sample is between 51 and 60 mm long.

Question 2 [10 marks]

(a) In a study, the mean CAP (cumulative average point) of a random sample of 49 final year students is calculated to be 4.5. The standard deviation for this sample is given as 0.75.

(i) Find a 95% confidence interval for the mean CAP of the entire final year class.

(ii) The university administration claims that the mean CAP for the entire final year class is 4.3. Does our study offer evidence against this claim? Explain.

(iii) How many more students should be included in the study if we want to be 95% confident that our estimate of the mean CAP of the entire final year class is off by at most 0.1?

(b) 500 students are enrolled in the course TS4332. To estimate the average score of the recently concluded mid term examination, Xiao Ming took a random sample of 50 scores from students enrolled in the course. The mean and standard deviation of these 50 scores were 40 and 15 respectively. Say whether each statement below is true or false and give justification in a sentence or two. If there is not enough information to decide, explain what else you need to know.

(i) The sample mean, 40 is a good estimate of the population mean score.

(ii) The histogram for the 50 scores approximately follows the normal curve.

Question 3 [10 marks]

(a) A plant physiologist conducted an experiment to determine whether mechanical stress can retard the growth of soybean plants. Young plants were allocated to two groups of 13 plants each. Plants in one group were mechanically agitated by shaking for 20 minutes twice daily, while plants in the other group were not agitated. After 16 days of growth, the total stem length (cm) of each plant was measured, with results summarized as follows:

$$\bar{x} = 30.59, \quad s_x = 2.13, \quad \bar{y} = 27.78, \quad s_y = 1.73.$$

Here $\bar{x}$ and s_x denote the mean length and standard deviation of plants that were not agitated, while $\bar{y}$ and s_y denote the mean length and standard deviation of plants that were.

(i) Is there evidence to show that the standard deviation of plant length is different for plants subjected to agitation compared to those which were not?

(ii) Conduct a suitable test at $\alpha = 0.01$ level to check the claim that mechanical stress can retard the growth of soybean plants. State also any assumptions made.

(iii) Write down the (approximate) p-value of your test in the previous part.

(b) Suppose we wish to test the hypothesis

$$H_0 : \mu = 2 \text{ vs } H_1 : \mu \neq 2$$

and found a two-sided p-value of 0.03. Separately, a 95% confidence interval for μ is computed to be $(1.5, 4.0)$. Are these two results compatible? Why or why not?

Question 4 [10 marks]

Sweetie is a store that sells rose and chocolate with free delivery service. Below is the list of available items this week.

- white rose
- red rose
- pink rose
- orange rose
- yellow rose
- white chocolate
- milk chocolate
- dark chocolate

Mr. Brown, who has no preference on the colours of the rose and flavours of the chocolate, is one of Sweetie's regular customers.

(i) If Mr. Brown orders an item randomly for his wife, what is the probability that Mrs. Brown will receive a stalk of yellow rose?

(ii) If Mr. Brown orders two different items randomly for his wife, what is the probability that Mrs. Brown will receive a stalk of red rose and a box of dark chocolate?

(iii) If Mr. Brown orders a stalk of rose and a box of chocolate randomly for his wife, what is the probability that Mrs. Brown will receive a stalk of red rose and a box of dark chocolate?

(iv) Mr. Brown orders a stalk of rose and a box of chocolate randomly to be delivered separately on two days for his wife, with either item on the first day. What is the probability that Mrs. Brown will receive a stalk of red rose on day 1 and a box of dark chocolate on day 2?

(v) Mr. Brown orders two different items randomly to be delivered separately on two days for his wife, with either item on the first day. What is the probability that Mrs. Brown will receive a stalk of red rose on day 1 and a box of dark chocolate on day 2?

Question 5 [10 marks]

Consider the random variable X that has the probability density function given as

$$f(x) = \frac{3}{(1+x)^3}, \quad \text{for } 0 \leq x \leq c.$$

(i) What is the value of c?

(ii) Compute $E(X)$.

Question 6 [10 marks]

A fast food restaurant operates a drive-up facility and a walk-up window. On a randomly selected day, let $X =$ proportion of time that the drive-up facility is in use (at least one customer is being served or waiting to be served) and $Y =$ the proportion of the time that the walk-up window is in use. Suppose that the joint probability density function of (X,Y) is given by

$$f(x,y) = \begin{cases} \frac{2}{3}(x+2y), & 0 \le x \le 1, \quad 0 \le y \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

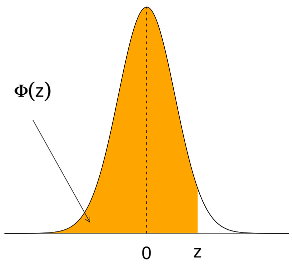
(i) Find the probability that neither facility is busy more than one-quarter of the time.

(ii) Find the probability that the drive-up facility is busy more than one-quarter of the time but less than three quarters of the time.

(iii) Given that the drive-up facility is busy 80% of the time, what is the probability that the walk-in facility is busy at most half the time?

(iv) Given that the drive-up facility is busy 80% of the time, what is the expected proportion of time that the walk-in facility is busy?

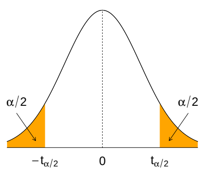
APPENDIX A: DISTRIBUTION FUNCTION OF THE NORMAL DISTRIBUTION



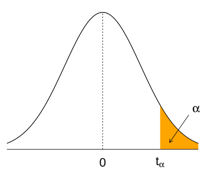
The function tabulated is $\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}u^2}\,\mathrm{du}$.

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999822	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967

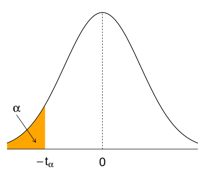
APPENDIX B: CRITICAL VALUES FOR STUDENT'S t DISTRIBUTION

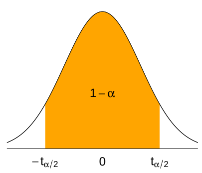


Two-tail area



One-tail area





$1-\alpha$ confidence level

two-tail	0.5	0.2	0.1	0.05	0.02	0.01	0.005	0.002	0.001
one-tail	0.25	0.1	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
df = 1	1.000	3.078	6.314	12.706	31.821	63.657	127.321	318.309	636.619
2	0.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.599
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.215	12.924
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
32	0.682	1.309	1.694	2.037	2.449	2.738	3.015	3.365	3.622
34	0.682	1.307	1.691	2.032	2.441	2.728	3.002	3.348	3.601
36	0.681	1.306	1.688	2.028	2.434	2.719	2.990	3.333	3.582
38	0.681	1.304	1.686	2.024	2.429	2.712	2.980	3.319	3.566
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
42	0.680	1.302	1.682	2.018	2.418	2.698	2.963	3.296	3.538
44	0.680	1.301	1.680	2.015	2.414	2.692	2.956	3.286	3.526
46	0.680	1.300	1.679	2.013	2.410	2.687	2.949	3.277	3.515
48	0.680	1.299	1.677	2.011	2.407	2.682	2.943	3.269	3.505
50	0.679	1.299	1.676	2.009	2.403	2.678	2.937	3.261	3.496
60	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
70	0.678	1.294	1.667	1.994	2.381	2.648	2.899	3.211	3.435
80	0.678	1.292	1.664	1.990	2.374	2.639	2.887	3.195	3.416
90	0.677	1.291	1.662	1.987	2.368	2.632	2.878	3.183	3.402
100	0.677	1.290	1.660	1.984	2.364	2.626	2.871	3.174	3.390
120	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
140	0.676	1.288	1.656	1.977	2.353	2.611	2.852	3.149	3.361
160	0.676	1.287	1.654	1.975	2.350	2.607	2.846	3.142	3.352
180	0.676	1.286	1.653	1.973	2.347	2.603	2.842	3.136	3.345
200	0.676	1.286	1.653	1.972	2.345	2.601	2.839	3.131	3.340
∞	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291
confidence level	0.5	0.8	0.9	0.95	0.98	0.99	0.995	0.998	0.999

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