A02145618M

1.

majority boys $\Rightarrow$ 4, 5, 6 boys

4 boys $\Rightarrow$ ${}^6C_4 \times {}^4C_3 = 60$

5 boys $\Rightarrow$ ${}^6C_1 \times {}^4C_2 = 36$

6 boys $\Rightarrow$ ${}^6C_6 \times {}^4C_1 = 4$

$\Big\}$ 100

2.

reject H_0 | H_0 is true $\Rightarrow$ Type I error

3.

$\mu = 179$

$n = 10 \quad \bar{x} = 205 \quad s^2 = 26^2$

changes $\Rightarrow$ variance?

$$T = \frac{\bar{x} - \mu}{s/\sqrt{n}} = 3.1622$$

$\alpha = 0.01$

critical region $t < -t_{9;0.005}$ or $t > t_{9;0.005}$

$t < -3.2498$ $\quad t > 3.2498$

t does not fall in critical region do not reject H_0

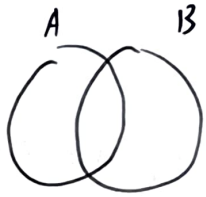
4.

A ✓

B ✓

C ✓

D X



5.

$P(A) = \frac{3}{5}$ $\quad P(A \cap B) = \frac{15}{50} = \frac{3}{10}$

$P(B) = \frac{1}{2}$ $\quad P(A) - P(B) = P(A \cap B)$

A and B are independent

6.

A $x = 0, 1 \cdots n$ ✓

B $x > 0$

C $-\infty < x < \infty$

D $x = 0, 1 \cdots \infty$

E $x = 1, 2 \cdots$

7.

$X \sim N(3, 3^2)$ $Y \sim N(3, 3^2)$

$$E\left(\frac{X+Y}{2}\right) = \frac{E(X)+E(Y)}{2} = \frac{3+3}{2} = 3$$

$$V\left(\frac{1}{2}X + \frac{1}{2}Y\right) = \frac{1}{4}V(X) + \frac{1}{4}V(Y) + 2ab\,\mathrm{cov}(X,Y) \quad (\Rightarrow 0 \text{ independent})$$

$$= \frac{1}{4}(9+9) = 4.5$$

8.

x	1	2	3	7
$f_X(x)$	0.1	0.2	0.3	0.4

$$P(1.5 < X \leq 2.5) = P(X = 2)$$

$$= \boxed{0.2}$$

9.

S I L V E R

4 consonants.

$4 \times 5! = \boxed{480}$

10. all of the given

11.

i) $f(x) \geq 0$ for all x since x has positive & negative no constant c can satisfy

ii)

$f(x) \geq 0$

$$\int_{-1}^{2} f(x)\,dx = \int_{-1}^{2} cx^2\,dx = \left[\frac{cx^3}{3}\right]_{-2}^{2}$$

$x^2 \geq 0$ for all x in range

$c \geq 0$ ✓

$$= \left(\frac{8c}{3}\right) - \left(-\frac{8c}{3}\right)$$

$$= \frac{16c}{3} = 1$$

$$c = \frac{3}{16}$$

12.

$$E(X) = \int_{2}^{4} \frac{32}{3x^2}\,dx = \frac{32}{3}\left[-\frac{1}{x}\right]_{2}^{4}$$

$$= \frac{32}{3}\left[\left(-\frac{1}{4}\right) - \left(-\frac{1}{2}\right)\right]$$

$$= 2.6667 = \boxed{2.7}$$

13.

$P(\text{system 1 works}) = 0.8 \times 0.8 + 0.8 \times 0.2 + 0.2 \times 0.8 = 0.96$

$P(\text{system 3/4 work}) = 0.8 \times 0.8 = 0.64$

$P(\text{system work}) = 0.96 \times 0.64 + 0.04 \times 0.64 + 0.96 \times 0.36$

$= 0.9856$

14.

$P(U) = 0.25$

$P(J) = 0.3$

$P(U' \cap J') = 0.55$

$P((U \cup J)') = 0.55 \quad P(U \cup J) = 0.45 = P(U) + P(J) - P(U \cap J)$

$P(U \cap J) = 0.3 + 0.25 - 0.45$

$= 0.1$

15. large n $\quad \bar{x} = 3.22 \quad s^2 = 1.99^2 \quad n = 678$

$95\%\ CI = \bar{x} \pm z_{0.025} \frac{s}{\sqrt{n}}$ ($z_{0.025} = 1.96$)

$= (3.07, 3.37)$

16. False

17. width of interval $= 2 \times z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$V(X) \uparrow \Rightarrow \sigma^2 \uparrow \Rightarrow \sigma \uparrow \Rightarrow$ width increase

18.

True

19.

A ✓ 2x min vs just draft

B ✓

C ✗ CO1 only used to compare

D ✓

E

20.

$\mu = 3262$

Test H_0: $\mu = 3262$ vs H_1: $\mu < 3262$

known variance

$n = 50$ $\sigma^2 = 1100^2$ $\bar{x} = 2995$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = -1.71674$$

$$\text{prob} = P(Z < -1.71674) = 0.043041$$

21.

$p = \frac{1}{10}$ Let X be # time 1 appears

$X \sim B(6, \frac{1}{10})$

$$P(X = 3) = \binom{6}{3} \left(\frac{1}{10}\right)^3 \left(\frac{9}{10}\right)^3 = 0.01458$$

22.

$P(X=1) = 0.3 \times 0.4 + 0.2 \times 0.5 + 0.5 \times 0.3$

$= 0.33$

$P(X \neq Y) = 0.67$

23.

Urn 1	2 Black	8 white	$P(\text{urn}) = \frac{1}{3}$
Urn 2	4 Black	6 white	
Urn 3	6 Black	4 white	

$$P(\text{Urn1} \mid \text{white}) = \frac{P(\text{Urn1} \cap \text{white})}{P(\text{white})}$$

$$= \frac{\frac{1}{3} \times \frac{8}{10}}{\frac{1}{3}\left[\frac{8}{10} + \frac{6}{15} + \frac{4}{10}\right]} = \frac{4}{9}$$

$= 0.4444$

24.

$$\bar{d} = \frac{(3.2) + (-1.5) + (0.7) + (5.9) + (-3.7) + (0.9) + (17) + (1)}{8}$$

$= 4.8125$

$$s_n^2 = \frac{1}{7}\left(\sum d_i^2 - 8\bar{d}^2\right) = \frac{1}{7}\left(493.69 - 8 \cdot (4.8125)^2\right)$$

$= 44.05839$

$$95\%\ CI = \bar{d} \pm t_{7;0.025} \frac{s_n}{\sqrt{n}} \quad (t = 2.3646)$$

$= (-0.7367, 10.3616)$

25.

Y	100	200	300	350	400
$P_Y(y)$	$\frac{1}{3}$	$\frac{4}{15}$	$\frac{1}{5}$	$\frac{2}{15}$	$\frac{1}{15}$

$E(Y) = 230$

26. $X \sim N(2, 1^2)$ $Y \sim N(3, 1^2)$

$E(X+Y) = E(X) + E(Y) = 5$

$V(Z) = V(X+Y) = V(X) + V(Y) + 2\,Cov(X,Y)$

$= 1 + 1 + 2 \cdot \frac{1}{2} = 3$

$Z \sim N(5, 3)$

$P(Z \le 7) = P\left(\frac{Z-5}{\sqrt{3}} \le \frac{7-5}{\sqrt{3}}\right)$

$= 0.87589$

27. $P(T) = 0.3$ $P(L|T) = \frac{1}{4}$

$P(B) = 0.2$ $P(L|B) = \frac{1}{3}$

$P(S) = 0.1$ $P(L|S) = \frac{1}{12}$

$P(O) = 0.4$ $P(L|O) = 0$

$P(B|L) = \frac{P(B \cap L)}{P(L)}$

$= \frac{\frac{1}{15}}{\frac{3}{20}}$

$= \frac{4}{9}$

≈ 0.444

28. $2\times(1,2,3,4,5,6)$ $P(\frac{1}{2})$

$1\times(2,2,4,4,6,6)$

$1\times(6,6,6,6,6,6)$

$P(6 \text{ both times}) = \frac{1}{4}\times 1\times 1 + \frac{1}{4}\times\frac{1}{3}\times\frac{1}{3} + \frac{2}{4}\times\frac{1}{3}\times\frac{1}{3}$

$= 0.2916$

29. $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2$

A

B ✓

C

D ✗

$\frac{(n-1)s^2}{\sigma^2} \sim \chi^2(n-1)$

$\frac{\chi_1^2/1}{\chi_{n-1}^2/n-1} \sim F(1,n-1)$

$n\bar{x} = \sum_{i=1}^{n} x_i \sim \chi^2(n)$

$E(n\bar{x})$

$E(x) =$

30. $X\sim(0,1^2)$ $Y\sim(0,\sqrt{2}^2)$

$P(X \geq 2\sigma) \leq \frac{1}{4}$

$P(Y \geq 2\sqrt{2}\sigma) \leq \frac{1}{(2\sqrt{2})^2} \leq \frac{1}{8}$

A ✓

B

C

D

31. $P(N \geq 1 \mid N \leq 4) = \dfrac{P(1 \leq N \leq 4)}{P(N \leq 4)}$

$= \dfrac{2}{5}$

$=$ $N=1 \Rightarrow \frac{1}{2}$, $N=2 \Rightarrow \frac{1}{6}$, $N=3 \Rightarrow \frac{1}{20}$, $N=4 \Rightarrow \frac{1}{30}$, $N=0 \Rightarrow \frac{1}{2}$ } $\frac{5}{6}$

32.

A	X
B	✓
C	X
D	✓

X = not symmetric

33. $X \sim t(n)$ $\quad \dfrac{Z}{\sqrt{U/n}}$ ($Z \sim N(0,1)$, $\sqrt{U/n}$: $\chi^2(n)$) $\quad X^2 = \dfrac{Z^2}{U/n}$ ($Z^2 \sim \chi^2(1)$)

$\dfrac{U/n}{\chi^2(1)} = F(n,1)$

34. $P(A \cap B' \cap C') = 0.1$ $\quad P(A \cap B \cap C') = 0.12$

$P(A' \cap B \cap C') = 0.1$ $\quad P(A \cap B' \cap C) = 0.1$

$P(A' \cap B' \cap C) = 0.1$ $\quad P(A' \cap B \cap C) = 0.12$

$P(A \cap B \cap C \mid A \cap B) = \frac{1}{3}$ $\quad \dfrac{P(A \cap B \cap C)}{P(A \cap B)} = \frac{1}{3}$

$P(A' \cap B' \cap C') = 1 - 0.3 - 0.36 - 0.06$

$= 0.28$

$P(A \cap B \cap C) + P(A \cap B \cap C')$

$\dfrac{a}{a + 0.12} = \frac{1}{3}$

$a = 0.06$

35.

$P(X \le 1) = 1 - e^{-\lambda}$

$P(X > 1) = e^{-\lambda}$

$1 - e^{-\lambda} = e^{-\lambda}$

$1 = 2e^{-\lambda}$

$\frac{1}{2} = e^{-\lambda}$

$\ln \frac{1}{2} = -\lambda$

$\lambda = -\ln \frac{1}{2} = \ln 2$

$V(X) = \frac{1}{\lambda^2} = \frac{1}{(\ln 2)^2}$

$\doteq 2.09$