ST2334 (2021/2022 Semester 1) Solutions to Questions in Tutorial 3

Question 1

(a) $\Pr(A \cap B \cap C) = \Pr(A)\Pr(B|A)\Pr(C|A \cap B) = 0.75(0.9)(0.8) = 0.54$.

(b) $\Pr(B) = \Pr(A \cap B) + \Pr(A' \cap B) = \Pr(A)\Pr(B|A) + \Pr(A')\Pr(B|A') = (0.75)(0.9) + (0.25)(0.8) = 0.875$.

(c) $\Pr(A|B) = \Pr(A \cap B) / \Pr(B) = [(0.75)(0.9)]/0.875 = 0.7714$.

(d) $\Pr(B \cap C) = \Pr\big(A \cap (B \cap C)\big) + \Pr(A' \cap (B \cap C))$. But $\Pr(A' \cap (B \cap C)) = \Pr(A')\Pr(B|A')\Pr(C|A' \cap B) = 0.25(0.8)(0.7) = 0.14$. Therefore $\Pr(B \cap C) = 0.54 + 0.14 = 0.68$.

(e) $\Pr(A|B \cap C) = \Pr(A \cap B \cap C) / \Pr(B \cap C) = 0.54/0.68 = 0.7941$.

Question 2

Let A = {product A profitable}, B = {product B profitable} and $C = A \cup B = \{\text{at lease one products is profitable}\}$.

$\Pr(A) = \Pr(B) = 0.18$. $\Pr(A \cap B) = 0.05$. So $\Pr(C) = \Pr(A \cup B) = 0.18 + 0.18 - 0.05 = 0.31$.

(a) $\Pr(A|B) = \Pr(A \cap B) / \Pr(B) = 0.05/0.18 = 0.2777$.

(b) $\Pr(A|C) = \Pr(A \cap C) / \Pr(C) = \Pr(A) / \Pr(C) = 0.18/0.31 = 0.5806$.
Note: $A \cap C = A \cap (A \cup B) = A$ as $A \subset (A \cup B)$

Question 3

Let A = {TQM implemented} and B = {sales increased}.

It is given that $\Pr(A) = 0.3$, $\Pr(B) = 0.6$ and $\Pr(A|B) = 20/60 = 1/3$.

(a) $\Pr(A) = 0.3$. $\Pr(B) = 0.6$.

(b) Since $\Pr(A|B) = 20/60$, therefore, $\Pr(A \cap B) = \Pr(A|B)\Pr(B) = (1/3)0.6 = 0.2$. As $\Pr(A \cap B) \neq \Pr(A)\Pr(B) = 0.18$, therefore A and B are not independent events.
Alternatively, since $\Pr(A) \neq \Pr(A|B)$, therefore, A and B are not independent events.

(c) Given that $\Pr(A|B) = 18/60$, hence, $\Pr(A \cap B) = \Pr(A|B)\Pr(B) = (0.3)0.6 = 0.18$.
As $\Pr(A \cap B) = \Pr(A)\Pr(B)$, therefore A and B are independent events.
Alternatively, since $\Pr(A) = \Pr(A|B)$, therefore, A and B are independent events.

Question 4

Let B be the event that a component needs rework. Let A_i be the event the selected component is from assembly line i.

It is given that $\Pr(B|A_1) = 0.05$, $\Pr(B|A_2) = 0.08$, $\Pr(B|A_3) = 0.1$, $\Pr(A_1) = 0.5$, $\Pr(A_2) = 0.3$ and $\Pr(A_3) = 0.2$. Then, applying the Law of Total Probability, we have
$\Pr(B) = \Pr(A_1)\Pr(B|A_1) + \Pr(A_2)\Pr(B|A_2) + \Pr(A_3)\Pr(B|A_3) = (0.5)(0.05) + (0.3)(0.08) + (0.2)(0.1) = 0.069$.

(a) $\Pr(A_1|B) = \Pr(A_1 \cap B) / \Pr(B) = \Pr(A_1)\Pr(B|A_1) / \Pr(B)$
$= [(0.5)(0.05)]/0.069 = 0.3623$.

(b) $\Pr(A_2|B) = [(0.3)(0.08)]/0.069 = 0.3478$.

(c) $\Pr(A_3|B) = [(0.2)(0.1)]/0.069 = 0.2899$.

Note:
$\Pr(A_1|B) + \Pr(A_2|B) + \Pr(A_3|B) = 1$ and
$\Pr(B|A_1) + \Pr(B|A_2) + \Pr(B|A_3) \neq 1$

Question 5

It is given that $\Pr(A_1) = \Pr(\text{draw slip 1 or 4}) = 1/2$. Similarly, $\Pr(A_2) = 1/2$ and $\Pr(A_3) = 1/2$.

(a) $\Pr(A_1 \cap A_2) = \Pr(\text{draw slip 4}) = 1/4$, Similarly, $\Pr(A_1 \cap A_3) = 1/4$ and $\Pr(A_2 \cap A_3) = 1/4$.
Since $\Pr(A_1 \cap A_2) = \Pr(A_1)\Pr(A_2)$, $\Pr(A_1 \cap A_3) = \Pr(A_1)\Pr(A_3)$ and $\Pr(A_2 \cap A_3) = \Pr(A_2)\Pr(A_3)$, therefore the events A_1, A_2 and A_3 are pairwise independent.

(b) $\Pr(A_1 \cap A_2 \cap A_3) = \Pr(\text{draw slip 4}) = 1/4$. But $\Pr(A_1)\Pr(A_2)\Pr(A_3) = 1/8 \neq 1/4$, therefore the events A_1, A_2 and A_3 are not mutually independent.

Note: A_1, A_2 and A_3 are mutually independent if and only if $\Pr(A_1 \cap A_2 \cap A_3) = \Pr(A_1)\Pr(A_2)\Pr(A_3)$, $\Pr(A_1 \cap A_2) = \Pr(A_1)\Pr(A_2)$, $\Pr(A_1 \cap A_3) = \Pr(A_1)\Pr(A_3)$ and $\Pr(A_2 \cap A_3) = \Pr(A_2)\Pr(A_3)$.

Question 6

Let A, B, C and D be the events that components A, B, C and D work respectively.
It is given that $\Pr(A) = 0.95$, $\Pr(B) = 0.7$, $\Pr(C) = 0.8$ nad $\Pr(D) = 0.9$.
The system works when both A and D work and at least one of B or C works. (That is, system works $= A \cap (B \cup C) \cap D$

(a) Since all the four components work independently, therefore $\Pr(B \cup C) = \Pr(B) + \Pr(C) - \Pr(B)\Pr(C)$ and $\Pr(A \cap (B \cup C) \cap D) = \Pr(A)\Pr(B \cup C)\Pr(D)$.
$\Pr(\text{system works}) = \Pr(A \cap (B \cup C) \cap D) = \Pr(A)\Pr(B \cup C)\Pr(D) = (0.95)[0.7 + 0.8 - (0.7)(0.8)](0.9) = 0.8037$.

(b) System works while C does not work $= A \cap (B \cap C') \cap D$
$\Pr(\text{C does not work} \mid \text{system works}) =$
$\Pr(\text{system works but C does not work}) / \Pr(\text{System works}) = \Pr(A \cap B \cap C' \cap D) / \Pr(\text{system works}) = [(0.95)(0.7)(0.2)(0.9)]/0.8037 = 0.1489$.

Question 7

Let $A_i = \{i\text{th vehicle passes the inspection}\}$. $\Pr(A_1) = \Pr(A_2) = \Pr(A_3) = 0.6$.

(a) $\Pr(A_1 \cap A_2 \cap A_3) = \Pr(A_1)\Pr(A_2)\Pr(A_3) = (0.6)^3 = 0.216$ since A_i's are independent.

(b) $\Pr(\text{At least one failures}) = 1 - \Pr(\text{All pass}) = 1 - 0.216 = 0.784$.
Or $\Pr(A_1' \cup A_2' \cup A_3') = \Pr((A_1 \cap A_2 \cap A_3)') = 1 - \Pr(A_1 \cap A_2 \cap A_3) = 1 - 0.216 = 0.784$.

(c) $\Pr(A_1 \cap A_2' \cap A_3') + \Pr(A_1' \cap A_2 \cap A_3') + \Pr(A_1' \cap A_2' \cap A_3) = (0.6)(0.4)(0.4) + (0.4)(0.6)(0.4) + (0.4)(0.4)(0.6) = 0.288$.

(d) $\Pr(\text{At least one pass}) = 1 - \Pr(\text{All fail}) = 1 - (0.4)^3 = 0.936$.
$\Pr(\#\text{pass} = 3 \mid \#\text{pass} \geq 1) = \Pr(\#\text{pass} = 3 \cap \#\text{pass} \geq 1) / \Pr(\#\text{pass} \geq 1) = \Pr(\#\text{pass} = 3) / \Pr(\#\text{pass} \geq 1) = 0.216/0.936 = 0.2308$.

Question 8

Let A = {Get into a house}, B = {the house is unlocked} and C = {Agent gets the correct key}
It is given that $\Pr(B) = 0.3$.
$\Pr(C) = 1/8 + (7/8)(1/7) + (7/8)(6/7)(1/6) = 3/8$.
Alternatively, $\Pr(C) = ({}_1C_1)({}_7C_2)/({}_8C_3) = 3/8$.
$\Pr(A) = \Pr(B)\Pr(A|B) + \Pr(B')\Pr(A|B') = 0.3(1) + 0.7(3/8) = 0.5625$.
Note: If the house is locked, then the agent needs the correct key to get into the house.