NATIONAL UNIVERSITY OF SINGAPORE

ST2334   Probability and Statistics

(Semester NN: AY YYYY–YYYY)

MMM YYYY — Time allowed: 2 hours

SAMPLE PAPER

INSTRUCTIONS TO CANDIDATES

1. This paper contains **SIX (6)** questions and comprises **FIFTEEN (15)** printed pages.

2. Answer **ALL** questions. Marks for each question are indicated. The total marks for this paper is 60.

3. Please show workings and answers in the space provided for each question or part. Answers should be given in complete English sentences.

4. Non-programmable calculators may be used. However, candidates should lay out systematically the various steps in the calculations.

5. This is a CLOSED BOOK examination. Candidates may bring in **ONE (1)** A4-size help sheets with hand-written notes on both sides.

6. Write down your matriculation number and seat number neatly in the boxes provided below. **Do not write your name.** This booklet will be collected at the end of the examination.

Matriculation Number   :

Question	1	2	3	4	5	6	Total
Score							

Question 1 [10 marks]

(a) We want to know the mean winning score at the US Open golf championship. An internet search gives us all the scores for the history of that tournament, and we create a 95% confidence interval based on a t-distribution.

Is this procedure appropriate? Explain in a sentence or two.

(b) 8 speakers are to speak one after another in a conference. Dr Cooper's lecture is related to Dr Hofstadter's and should not precede it. How many schedules could be arranged?

(c) A headline in a local newspaper announced "Video game playing can lead to better spatial reasoning abilities." The article reported that a study found "statistically significant differences" between teens who play video games and teens who do not, with teens who play video games testing better in spatial reasoning.

Do you think the headline was appropriate? Explain in a sentence or two.

(d) It was discovered that 25% of the paintings of a certain gallery are not original. A collector in 15% of the cases makes a mistake in judging if a painting is authentic or a copy. If she buys a piece thinking that it is original, what is the probability that it is not?

Question 2 [10 marks]

(a) There are 5 couples (husbands and wives) in a party. We assume in what follows that all in the party have birthday months that are independent, and that any month is equally likely to be the birthday month of a particular person.

   (i) What is the probability that at least two of the wives share the same birthday month?

   (ii) What is the probability that there are at least two couples where both husbands share the same birthday month and both wives share the same birthday month?

(iii) One of the 5 couples is the couple hosting the party. What is the chance that there exists at least one other couple in which the husband has the same birthday month as the host husband and the wife has the same birthday month as the host wife?

(b) Let A and B be events. Suppose $P(A) = 1$. Determine if A and B are independent.

Question 3 [10 marks]

Tom and Jerry gamble against one other by rolling dice. Tom's die has an 8 on one of the face and 2's on the other five faces. Jerry's die has four 3's and two 1's on the six faces.

(i) They each roll their own die once, and the player with the higher score wins. Which player has a greater probability of winning?

(ii) If Tom wins, Jerry pays him $10. How much should Tom pay Jerry if Jerry wins in order to make the game fair?

(iii) After playing the game for a while, both got bored of it and decided to change the rules. In the new game, the person who wins will collect the number of dollars shown on his die. As an illustration, if Tom obtains a 8 and Jerry gets a 1, Jerry will have to pay Tom $8. Write down the probability mass function of Y, if Y denotes Tom's winnings.

(iv) Find the expected value and standard deviation of Y.

(v) If they play this new game repeatedly which player has the advantage? Explain.

Question 4 [10 marks]

(a) A manufacturer claims that the lifetime of an appliance costing $300 follows an exponential distribution with mean 3 years. Customers will be given a full refund if the appliance fails to last a year following its purchase. A 50% refund will be given if the appliance lasts between 1 year to 3 years. How much should the manufacturer expect to pay in refunds if it sells 200 such appliances?

(b) The score of students taking the final examination is a random variable with mean 75 and variance 25. How many students would have to take the examination to ensure, with probability at least 0.9, that the class average would be within 5 of 75?

Hint: Use the Chebyshev Inequality.

(c) Assume that yield per acre for a particular variety of soybeans is $N(\mu, \sigma^2)$. For a random sample of $n = 5$ plots, the yields in bushels per acre were given to be

$$37.4, 48.8, 46.9, 55.0, 44.0.$$

(i) Give a point estimate for σ^2.

(ii) Give a 90% confidence interval for μ.

Question 5 [10 marks]

(a) The joint probability density function of X and Y is given by

$$f(x,y) = k(x^2 + xy/2), \quad 0 < x < 1, \, 0 < y < 2.$$

(i) Find the value of k.

(ii) Find the marginal density function of X.

(iii) Find $E(Y|X = 0.5)$.

(b) Based on data from two very large independent samples, two students tested a hypothesis about equality of population means using $\alpha = 0.01$. One student used a one-tail test and rejected the null hypothesis, but the other used a two-tail test and failed to reject the null. If the calculated value of the test statistics c is positive, what are the maximum possible value and minimum possible value of c?

Question 6 [10 marks]

(a) Plants convert carbon dioxide (CO2) in the atmosphere, along with water and energy from sunlight, into the energy they need for growth and reproduction. Experiments were performed under normal atmospheric air conditions and in air with enriched CO2 concentrations to determine the effect on plant growth. The plants were given the same amount of water and light for a four-week period.

   (i) BK is investigating if CO2-enriched atmosphere increases plant growth using a suitable hypothesis test. Write down the null and alternative hypotheses for his test.

   (ii) The mean and standard deviation for a sample of 12 plants grown in normal air are given to be 4.163 and 0.9562, while the mean and standard deviation for a sample of 8 plants grown in enriched air are given to be 5.105 and 1.6098. On the basis of these data, determine whether CO2-enriched atmosphere increases plant growth at $\alpha = 0.05$ level. State any assumptions you are making.

(iii) Write down the (approximate) p-value of your test in the previous part.

(b) Researchers developing new drugs must be concerned about possible side effects. They must check a new medication to be sure that it does not cause an unsafe increase in blood pressure. They measure the blood pressures of a group of 12 subjects, then administer the drug and recheck the blood pressures one hour later. The drug will be approved for use unless there is evidence that blood pressure has increased an average of more than 20 points. They will test a hypothesis using $\alpha = 0.05$.

In this context, which do you consider to be more serious — a type I or a type II error? Explain briefly.

APPENDIX A: DISTRIBUTION FUNCTION OF THE NORMAL DISTRIBUTION
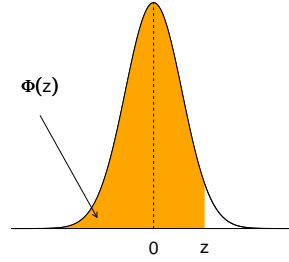
The function tabulated is $\Phi(z) = \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{-\infty}^{z} e^{-\frac{1}{2}u^2} \, du.$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999822	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967

APPENDIX B: CRITICAL VALUES FOR STUDENT'S *t* DISTRIBUTION

Two-tail area

One-tail area

$1 - \alpha$ *confidence level*
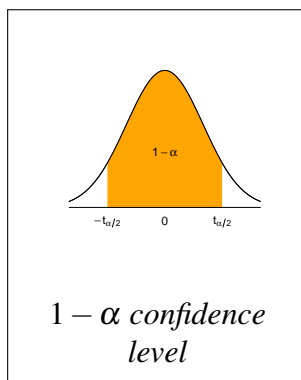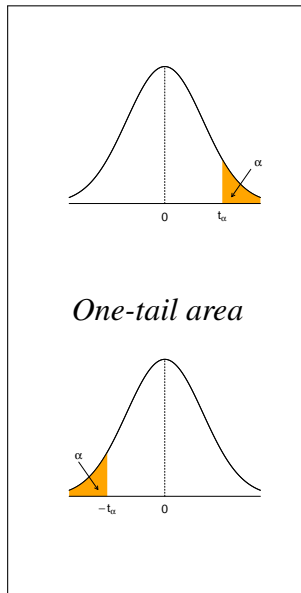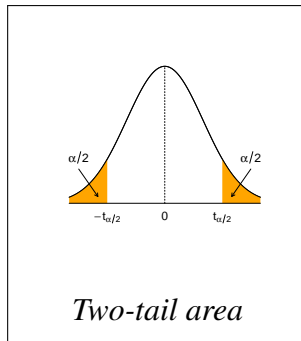
two-tail	0.5	0.2	0.1	0.05	0.02	0.01	0.005	0.002	0.001
one-tail	0.25	0.1	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
df = 1	1.000	3.078	6.314	12.706	31.821	63.657	127.321	318.309	636.619
2	0.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.599
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.215	12.924
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
32	0.682	1.309	1.694	2.037	2.449	2.738	3.015	3.365	3.622
34	0.682	1.307	1.691	2.032	2.441	2.728	3.002	3.348	3.601
36	0.681	1.306	1.688	2.028	2.434	2.719	2.990	3.333	3.582
38	0.681	1.304	1.686	2.024	2.429	2.712	2.980	3.319	3.566
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
42	0.680	1.302	1.682	2.018	2.418	2.698	2.963	3.296	3.538
44	0.680	1.301	1.680	2.015	2.414	2.692	2.956	3.286	3.526
46	0.680	1.300	1.679	2.013	2.410	2.687	2.949	3.277	3.515
48	0.680	1.299	1.677	2.011	2.407	2.682	2.943	3.269	3.505
50	0.679	1.299	1.676	2.009	2.403	2.678	2.937	3.261	3.496
60	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
70	0.678	1.294	1.667	1.994	2.381	2.648	2.899	3.211	3.435
80	0.678	1.292	1.664	1.990	2.374	2.639	2.887	3.195	3.416
90	0.677	1.291	1.662	1.987	2.368	2.632	2.878	3.183	3.402
100	0.677	1.290	1.660	1.984	2.364	2.626	2.871	3.174	3.390
120	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
140	0.676	1.288	1.656	1.977	2.353	2.611	2.852	3.149	3.361
160	0.676	1.287	1.654	1.975	2.350	2.607	2.846	3.142	3.352
180	0.676	1.286	1.653	1.973	2.347	2.603	2.842	3.136	3.345
200	0.676	1.286	1.653	1.972	2.345	2.601	2.839	3.131	3.340
∞	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291
confidence level	0.5	0.8	0.9	0.95	0.98	0.99	0.995	0.998	0.999

END OF PAPER