Firm Dynamics

This two papers Hopenhayn (1992) and Hopenhayn and Rogerson (1993) are important papers in firm dynamics.

- Workhorse model of industry dynamics
- Output price p endogenous
- Endogenous measure of heterogeneous firms
  - **DRS** production function
  - Perfect competition in product and labor market
  - No aggregate shocks
  - Idiosyncratic risk
  - Entry-exit dynamics
  - fixed cost to enter
  - fixed cost to operate each period.
- Partial equilibrium: exogenous industry demand

Heterogeneous firms productivity z and the output

$$f(z, n) = zn^{\alpha}, \alpha \in (0, 1) \tag{1}$$

- Static firm-level profits

$$\pi(z; p) = \max p \cdot f(z, n) - n - p \cdot c_o \tag{2}$$

  c_o operating costs per period.
- optimal employment

$$n(z; p) = (p \cdot \alpha \cdot z)^{\frac{1}{1-\alpha}} \tag{3}$$

- Incumbents enter the period with states z_{-1}
- Exit decision: if stay, firms draw new productivity level $z \sim$

Dynamic problem of the firm There is a large pool of identical potential entrants deciding whether to become active or not.

1. Model I

- General equilibrium version of (Hopenhayn 1992)
- Non-convex adjustment cost (firing costs) $\to$ firm-level employment additional state variable
- No aggregate shocks
- Optimal employment policy characterized by *inaction* region.
- Endogenous *Misallocation* across heterogeneous plants.

Overview time is discrete, the wage is the model numeraire, and output price p is endogenous. representative household. Endogenous measure of heterogeneous firms.

2. Model II

A stochastic production function:

$$p_t f(n_t, s_t) - n_t - p_t f_t - g(n_t, n_{t-1}) \tag{4}$$

- c_f: fixed operating cost.
- n_t, p_t: hire n_t and output p_t.
- s_t: firm-specific shock to production opportunities.
- $F(s, s')$: each current value of the shock.
- $g(n_t, n_{t-1}) = \tau \cdot \max(0, n_{t-1} - n_t)$ captures the presence of adjustment costs.
- c_e one-time enter cost.

Each new entrant receives its current value of s as a draw from the dist. ν.

Preference

$$\sum_{t=1}^{\infty} \beta^t [u(c_t) - v(n_t)] \tag{5}$$

They have idiosyncratic risks:

$$\sum_{t=1}^{\infty} \beta^t [u(c_t) - aN_t] \tag{6}$$

- N_t is the fraction of individuals employed in period t.
- discount factor $\beta = \frac{1}{1+r}$

Equilibrium The Bellman equation is

$$\begin{aligned} W(s, n; p) = \max_{n' \geq 0} \{ p f(n', s) - n' - p c_f - g(n', n) \} \\ + \beta \max[E_s W(s', n'; p) - g(0, n')] \end{aligned} \tag{7}$$

- list p as a stationary price level.

Value function the value of entering gross of entry costs, W^e can be computed by given $W(s, 0; p)$

$$W^e(p) = \int W(s, 0; p) \, \mathrm{d}\nu(s) \tag{8}$$

- state variable pairs (s, n) we denote $\mu(s, n)$

Output Y it takes

$$
\begin{aligned}
Y(\mu, M; p) = \int \left[f(N(s,n;p),s) - c_f\right] \mathrm{d}\mu(s,n) \\
+M \int f(N(s,0;p),s) \, \mathrm{d}\nu(s).
\end{aligned} \tag{9}
$$

The first integral, output for a firm with state variable (s, n) is computed using the optimal employment rule N.

Labor demand and profits

$$
\begin{aligned}
L^{d(\mu,M,p)} &= \int N(s,n;p) \, \mathrm{d}\mu(s,n) + M \int N(s,0;p) \, \mathrm{d}\nu(s), \\
\Pi(\mu, M, p) &= pY(\mu, M, p) - L^{d(\mu,M,p)} - R(\mu, M, p) - Mpc_e.
\end{aligned} \tag{10}
$$

Individual optimization problem

$$
\begin{aligned}
\max u(c) - aN \\
s.t. \quad pc \leq N + \Pi + R(w = 1 \text{ is the numeraire})
\end{aligned} \tag{11}
$$

solution:

$$
N = L^s(p, \Pi + R) \tag{12}
$$

Equilibrium output price $p^* \geq 0$ a mass of entrants $M^* \geq 0$, and a measure of incumbents μ^*, such that

- Labor market clearing: $L^d(\mu^*, M^*, p^*) = L^s(p^*, \Pi(\mu^*, M^*, p^*) + R(\mu^*, M^*, p^*))$
- $T = \mu^*$
- Free-entry: $\nu^e = c_e$

3. Algorithm

The stationary equilibrium can be characterized by

- production function: concave and differentiable.
- L_2^s and income effect: negative.
- F: continuous and decreasing in the first argument.

1. compute p, any given p can be used for computing $W(s, n; p)$ and $W^e(p)$
2. determine whether an equilibrium: if not, finds μ^* to compute transition function T. The fixed point exists if the firm's inflow equals outflow.
3. determines the scale factor M^*. (from market clearing condition)

4. Benchmark model

$$
\begin{gathered}
f(n,s) = sn^{\theta}, \quad 0 \leq \theta \leq 1 \\
g(n_t, n_{t-1}) = 0, \\
\log(s_t) = a + \rho \log(s_{t-1}) + \varepsilon_t, \quad \varepsilon_t \\
u(c) = \ln(c), \quad \nu(n) = An, \quad A > 0.
\end{gathered}
\tag{13}
$$

Two decision rules

$$
\begin{gathered}
\log(n_{t+1}) = \frac{1-\rho}{1-\theta}\left(\log\theta + \log p + \frac{a}{1-\rho}\right) \\
+\rho(\log(n_{t-1})) + \left(\frac{1}{1-\theta}\right)\varepsilon_t.
\end{gathered}
\tag{14}
$$

MPL

$$
\frac{\partial f(z,n)}{\partial n} = \frac{1}{p} \tag{15}
$$

optimal employment decision given by:

$$
n' = (\alpha p z)^{\frac{1}{1-\alpha}} \tag{16}
$$

Labor is a fully flexible input production. MPL is the solution of two necessary conditions:

$$
\begin{gathered}
p\frac{\partial f(z,n)}{\partial n} + \frac{1}{1+r}\frac{\partial \tilde{\nu}(z,n)}{\partial n} = 1 \quad \text{if } n > n_{-1} \\
p\frac{\partial f(z,n)}{\partial n} + \frac{1}{1+r}\frac{\partial \tilde{\nu}(z,n)}{\partial n} = \left[1 + \frac{\partial g(n,n_{-1})}{\partial n}\right] \quad \text{if } n < n_{-1}
\end{gathered}
\tag{17}
$$

4.1. Role of volatility σ_ε

1. When shocks are less volatile, efficient employment does not change often.
   - Adj. costs are more important.
2. More vol. $\rightarrow$ efficient employment changes much more often.
   - Adj. costs are less important.
3. While reducing dispersion of MPL, lower vol. also reduces selection.

5. Embedded Searching and Match

- Elsby M. and Michaels R. 2013. "Marginal Jobs, Heterogeneous Firms, and Unemployment Flows." American Economic Journal: Macro, Vol. 5, No. 1, pp. 1-48

- Multi-worker heterogeneous firms $\rightarrow$ well-defined firm-size.
- Idiosyncratic productivity shocks $\rightarrow$ endogenous job.
- Search frictions $\rightarrow$ unemployment
- Wage bargaining $\rightarrow$ wage dispersion.

6. Financial Markets and Firm Dynamics (Cooley and Quadrini 2001)

- financial frictions can account for the simultaneous dependence of firm dynamics on size.
- In each period, firms have access to a production technology.
- $y = (z + \varepsilon)G(k, l)$ where ε is the idiosyncratic shock.
- As in (Hopenhayn 1992), assume there exists a fixed cost of production.
- Maximization of expected profits,

$$\begin{aligned} &\max_k \left\{ \int_\varepsilon (z+\varepsilon)F(k)f(\mathrm{d}\varepsilon) - (r+\phi)k \right\} \\ &= \max_k \{ zF(k) - (r+\phi)k \}. \end{aligned} \tag{18}$$

  and ϕ is the cost of labor and capital. $\phi k = \left[\delta + w\left(\frac{l}{k}\right)\right]k$, where w is the wage rate.
- In partial eqbm, the mass of new entrant firms is nondegenerate only if the surplus from creating new firms is nonpositive, $V(z) - \kappa \leq 0$.

The firm's problem each period, after the realization of the revenues and the observation of the new z, but before issuring new shares or paying dividends, the firm decides whether to default on its debt.

$$\pi(e, b, z+\varepsilon) = (1-\phi)(e+b) + (z+\varepsilon)F(e+b) - (1+\tilde{r})b \tag{19}$$

- $\tilde{r}$: interest rate charged by intermediary.
- z: tech level,
- e: assets held by firms. b: firm's debt. $k = e + b$ is the input capital.

Firm needs to borrow from intermediary, it takes λ peri unit of funds. If firm choose to default, it will charge a cost ξ.

- Denote $\underline{e}(z')$ the value of net worth if defaults. And denote $\underline{\varepsilon}(z, e, b, z')$ as threhold shock.

$$\pi(e, b, z+\underline{e}) = (1-\phi)(e+b) + (z+\underline{\varepsilon})F(e+b) - (1+\tilde{r})b = \underline{e}(z) \tag{20}$$

As the lender, she needs to consider default probability. Using the distribution to describe the risk.

$$(1+r)b = \underbrace{\begin{matrix}\int_{-\infty}^{\underline{\varepsilon}} [(1-\phi)(e+b) \\ +(z+\varepsilon)F(e+b)-\xi] f(\mathrm{d}\varepsilon)\end{matrix}}_{\text{Default}} + \underbrace{(1+\tilde{r})b \int_{\underline{\varepsilon}}^{\infty} f(\mathrm{d}\varepsilon)}_{\text{pay back}} \quad (21)$$

Eliminate $\tilde{r}$ in

Proposition 3: There exists a unique function $\Omega^*(z, e)$ that satisfies the functional equation.

Bibliography

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