1 Model

Motivation: what determines the level of employment and unemployment in the economy. Textbook: labor supply, labor demand, and unemployment as "leisure" . Alternative: labor market frictions.

Further question: how should labor market frictions be modeled? Search and matching: costly process of workers finding the right jobs.

McCall Partial Equilibrium Search Model

- The simplest model of search frictions.
- Problem of an individual getting draws from a given wage distribution.
- Decision: which jobs to accept and which to reject.
- Sequential search typically more reasonable.

Environment Risk neutral individual, discrete time. Linear preferences:

$$\sum_{t=0}^{\infty} \beta^t c_t$$

Start as unemployed, with consumption equal to b. Wage offer w drawn from a distribution $F(w)$. If she accepts, she will be employed at that job forever. Net Present Value of accepting a wage offer w is $w/(1-\beta)$ since $\frac{w}{1-\beta} = w + \beta w + \beta^2 w + \cdots$.

We could use probability to describe decision rule

$$a_t : \mathcal{W} \to [0, 1]$$

where $\mathcal{W}$ is the set of wage offers. Dynamic programming problem:

$$V(w) = \max \left\{ \frac{w}{1-\beta}, b + \beta \int V(w')\mathrm{d}F(w') \right\}$$

Reservation wage given by

$$\frac{R}{1-\beta} = b + \int_{\mathcal{W}} V(w')\mathrm{d}F(w')$$

For all $w \geq R$, $V(w) = \frac{w}{1-\beta}$. Therefore,

$$\int_{\mathcal{W}} V(w)\mathrm{d}F(w) = \frac{RF(R)}{1-\beta} + \int_{w \geq R} \frac{w}{1-\beta}\mathrm{d}F(w)$$

Combining with reservation wage equation,

$$\frac{R}{1-\beta} = b + \beta\left[\frac{RF(R)}{1-\beta} + \int_{w\geq R}\frac{w}{1-\beta}\mathrm{d}F(w)\right]$$

Rewriting,

$$\int_{w<R}\frac{R}{1-\beta}\mathrm{d}F(w) + \int_{w\geq R}\frac{R}{1-\beta}\mathrm{d}F(w) = b + \beta\left[\frac{RF(R)}{1-\beta} + \int_{w\geq R}\frac{w}{1-\beta}\mathrm{d}F(w)\right]$$

Subtracting $\beta R\int_{w\geq R}\mathrm{d}F(w)/(1-\beta) + \beta R\int_{w<R}\mathrm{d}F(w)/(1-\beta)$ from both sides,

$$\int_{w<R}\frac{R}{1-\beta}\mathrm{d}F(w) + \int_{w\geq R}\frac{R}{1-\beta}\mathrm{d}F(w) - \beta R\int_{w\geq R}\mathrm{d}F(w)/(1-\beta) + \beta R\int_{w<R}\mathrm{d}F(w)/(1-\beta)$$
$$= b + \beta\left[\int_{w\geq R}\frac{w-R}{1-\beta}dF(w)\right]$$

Collecting terms, we obtain

$$R - b = \frac{\beta}{1-\beta}\left[\int_{w\geq R}(w-R)dF(w)\right] \tag{1}$$

The LHS is the cost of foregoing the current job. The RHS is the expected benefit of one more search.

Define the $g(R)$ as

$$g(R) \equiv \frac{\beta}{1-\beta}\left[\int_{w\geq R}(w-R)\mathrm{d}F(w)\right]$$

First order derivative of $g(R)$ is

$$g'(R) = -\frac{\beta}{1-\beta}(R-R)f(R) - \frac{\beta}{1-\beta}\left[\int_{w\geq R}dF(w)\right] = -\frac{\beta}{1-\beta}[1-F(R)] < 0$$

We have the unique solution to Eq (1). Moreover, by the implicit function theorem,

$$\frac{dR}{db} = \frac{1}{1-g'(R)} > 0$$