1 Model

Motivation: what determines the level of employment and unemployment in the economy.
Textbook: labor supply, labor demand, and unemployment as "leisure" . Alternative: labor market frictions.
Further question: how should labor market frictions be modeled? Search and matching: costly process of workers finding the right jobs.

McCall Partial Equilibrium Search Model

- The simplest model of search frictions.
- Problem of an individual getting draws from a given wage distribution.
- Decision: which jobs to accept and which to reject.
- Sequential search typically more reasonable.

Environment Risk neutral individual, discrete time. Linear preferences:

$$\sum_{t=0}^{\infty} \beta^t c_t$$

Start as unemployed, with consumption equal to b. Wage offer w drawn from a distribution $F(w)$. If she accepts, she will be employed at that job forever. Net Present Value of accepting a wage offer w is $w/(1-\beta)$ since

$$\frac{w}{1-\beta} = w + \beta w + \beta^2 w + \cdots .$$

We could use probability to describe decision rule

$$a_t : \mathcal{W} \to [0,1]$$

where $\mathcal{W}$ is the set of wage offers. Dynamic programming problem:

$$V(w) = \max\left\{\frac{w}{1-\beta}, b + \beta \int V(w')\mathrm{d}F(w')\right\}$$

(Indifference condition) Reservation wage given by

$$\frac{R}{1-\beta} = b + \int_{\mathcal{W}} V(w')\mathrm{d}F(w')$$

For all $w \geq R$, $V(w) = \frac{w}{1-\beta}$. Therefore,

$$\int_{\mathcal{W}} V(w)\mathrm{d}F(w) = \frac{RF(R)}{1-\beta} + \int_{w\geq R} \frac{w}{1-\beta}\mathrm{d}F(w)$$

Combining with reservation wage equation,

$$\frac{R}{1-\beta} = b + \beta\left[\frac{RF(R)}{1-\beta} + \int_{w\geq R}\frac{w}{1-\beta}\mathrm{d}F(w)\right]$$

Rewriting,

$$\int_{w<R}\frac{R}{1-\beta}\mathrm{d}F(w) + \int_{w\geq R}\frac{R}{1-\beta}\mathrm{d}F(w) = b + \beta\left[\frac{RF(R)}{1-\beta} + \int_{w\geq R}\frac{w}{1-\beta}\mathrm{d}F(w)\right]$$

Subtracting $\beta R\int_{w\geq R}\mathrm{d}F(w)/(1-\beta) + \beta R\int_{w<R}\mathrm{d}F(w)/(1-\beta)$ from both sides,

$$\begin{aligned}\int_{w<R}\frac{R}{1-\beta}\mathrm{d}F(w) + \int_{w\geq R}\frac{R}{1-\beta}\mathrm{d}F(w) - \beta R\int_{w\geq R}\mathrm{d}F(w)/(1-\beta) + \beta R\int_{w<R}\mathrm{d}F(w)/(1-\beta)\\ = b + \beta\left[\int_{w\geq R}\frac{w-R}{1-\beta}dF(w)\right]\end{aligned}$$

Collecting terms, we obtain

$$R - b = \frac{\beta}{1-\beta}\left[\int_{w\geq R}(w-R)dF(w)\right] \tag{1}$$

The LHS is the cost of foregoing the current job. The RHS is the expected benefit of one more search.

Define the $g(R)$ as

$$g(R) \equiv \frac{\beta}{1-\beta}\left[\int_{w\geq R}(w-R)\mathrm{d}F(w)\right]$$

First order derivative of $g(R)$ is

$$g'(R) = -\frac{\beta}{1-\beta}(R-R)f(R) - \frac{\beta}{1-\beta}\left[\int_{w\geq R}dF(w)\right] = -\frac{\beta}{1-\beta}[1-F(R)] < 0$$

We have the unique solution to Eq (7). Moreover, by the implicit function theorem,

$$\frac{dR}{db} = \frac{1}{1-g'(R)} > 0$$

1.1 Continuous time model with layoff

The value function

$$U = b\Delta + e^{-r\Delta}[(1-e^{-f\Delta})\int\max\{V(w),U\}\mathrm{d}F(w) + e^{-f\Delta}U \tag{2}$$

and

$$V(w) = \frac{w}{r} + e^{-r\Delta}[e^{s\Delta}V(w) + (1-e^{s\Delta})U] \tag{3}$$

Take the continuous time limit $\Delta \to 0$:

$$rU = b + f\int\max\{V(w)-U,0\}\mathrm{d}F(w) \quad \text{and } rV(w) = w + s[V(w)-U] \tag{4}$$

Combining these two equations, we have

$$rU = b + f \int \max\{\frac{w + sU}{r + s} - U, 0\} \mathrm{d}F(w) \tag{5}$$

The reservation wage:

$$\frac{R + sU}{r + s} = U \tag{6}$$

Combining the previous two equations to eliminate U, we have

$$R - b = f \int_{w \geq R} \frac{w - R}{r + s} \mathrm{d}F(w) \tag{7}$$

LHS: benefit of accepting a wage offer R. **RHS**: cost of accepting an offer R = foregoing future better offer. At the optimum, two should be equated.

1.2 Mean-Preserving Spread

Rewrite

$$R - b = f \int_{w \geq R} \frac{w - R}{r + s} \mathrm{d}F(w) - f \int_{w \leq R} \frac{w - R}{r + s} \mathrm{d}F(w) \tag{8}$$

Applying intergration by parts,

$$\int_{w \leq R} w \mathrm{d}F(w) = RF(R) - \int_{w \geq R} F(w) \mathrm{d}w \tag{9}$$

Plugging back,

$$R - b = f \frac{w - R}{r + s} \mathrm{d}F(w) + f \frac{1}{r + s} \int_{w \leq R} F(w) \mathrm{d}w \tag{10}$$

We say distribution $\tilde{F}$ is a mean-preserving spread of F **iff** $\mathbb{E}_{\tilde{F}}[w] = \mathbb{E}[w]$ and $\int^{\bar{w}} \tilde{F}(w) \mathrm{d}w > \int^{\bar{w}} F(w) \mathrm{d}w$ for all $\bar{w}$. It says that the mean is the same but the variance is higher.

Reservation wage is now

$$\frac{r + s + f}{f} R - \frac{r + s}{f} b = \mathbb{E}[w] + \underbrace{\int^{R} F(w) \mathrm{d}w}_{\equiv h(R)} \tag{11}$$

note that

$$h(0) = 0, h'(R) = F(R) \in [0, 1] \tag{12}$$

when F shift from F to $\tilde{F}$, the reservation wage will increase. It is very similar to the American option, which says that accept the job offer only if the wage is high enough ("option value").

Therefore, you only care about the right tail of the wage distribution.
More variance/risk $\rightarrow$ more chances of a very good wage offer $\rightarrow$ search more.

The rate at which workers transition from unemployed to employed is

$$UE = f(1 - F(R)) \tag{13}$$

If f increases,

$$\frac{\mathrm{d}\ln UE}{\mathrm{d}\ln f} = 1 - \frac{F'(R)R}{1-F(R)}\frac{\mathrm{d}\ln R}{\mathrm{d}\ln f} \tag{14}$$

Under what condition $\frac{\mathrm{d}\ln UE}{\mathrm{d}\ln f} = 0$. consider that case: wage distribution follows Pareto distribution, $F(w) = 1 - (w/\underline{w})^{-\alpha}$ and outside option b is proportional to the average wage in the economy,

$$b = \bar{b}E[w \mid w \geq R] = \frac{\bar{b}}{\alpha - 1}R \tag{15}$$

Plugging the condition into the equation (7), we have

$$R - b = \frac{f}{r+s}\frac{1}{\alpha-1}\underline{w}^{\alpha}(R)^{1-\alpha} \tag{16}$$

Solving for R:

$$R = \left(\frac{f}{r+s}\frac{1}{\alpha-1}\underline{w}^{\alpha}\right)^{\frac{1}{\alpha}} \tag{17}$$

The UE rate is

$$UE = (\alpha-1)(r+s)(1-\bar{b}\frac{\alpha}{\alpha-1}) \tag{18}$$

Diamond Paradox For all $\beta < 1$ the unique equilibrium in the above economy is $R = 0$, and all workers accept the first wage offer.

Implication: starting from an allocation where all firm offer R, any firm can deviate and offer a wage $R - \varepsilon$ and increase its profits. This proves that no wage $R > 0$ can be an equilibrium.

Solutions of Diamond Paradox:

- Assume $F(w)$ is not the distribution of wages, but the distribution of "fruits" exogenously offered by "trees".
- Introduce other dimensions of heterogeneity
- Modify the wage determination assumptions

DMP model Matching function: $x(U,V)$ is CRS.

$$xL = x(uL, vL) \Rightarrow x = x(u,v)$$

Under this assumption, we can express everything as a function of the *tightness* of the labor market.

$$q(\theta) = \frac{x}{v} = x\left(\frac{u}{v}, 1\right)$$

where θ is the tightness of the labor market.

- $q(\theta)$ is the job filling rate.
- $1/q(\theta)$ is the expected duration of unemployment.

- $\theta q(\theta)$ is the job finding rate for the unemployed.
- Job creation is $u\theta q(\theta)L$.
- Job destruction: exogenously given by $\delta(1-u)$.

$$\delta(1-u) = u\theta q(\theta)$$

Therefore,

$$u = \frac{\delta}{\delta + \theta q(\theta)}$$

is called Beveridge curve. Equilibrium: unemployed flow equals to the job destruction flow. First, we need to use Bellman equation of unemployed worker:

$$rU = b + p(\theta)E[W(w) - U]$$

Employed flow value:

$$rW(w) = w + \delta[U - W(w)]$$

We could solve these equations. However, we cannot pin down the wage function since we have 3 unknowns. Introduce the wage bargaining assumption: Nash bargaining. Before that, we need to consider the value function of firms. Assume each firm only has one position. Vacant flow value:

$$rV = -\kappa + q(\theta)E[J(w) - V]$$

Matched flow value:

$$rJ(w) = (p - w) + \delta[V - J(w)]$$

Free entry equilibrium condition:

$$rV = 0 \Rightarrow \frac{\kappa}{E[J(w)]} = q(\theta)$$

This is just a market clearing condition!

Steady State:

$$u = \frac{s}{s + q\theta(q)} \tag{19}$$

$$0 = Af(k) - (r+\delta)k - w - \frac{(r+s)}{q(\theta)}\gamma_0 \tag{20}$$

$$w = (1-\beta)z + \beta[Af(k) - (r+\delta)k + \theta\gamma_0] \tag{21}$$

$$Af'(k) = r + \delta \tag{22}$$