Restuccia and Rogerson (2008) Model overview. Consumer's problem:

$$\sum_{t=0}^{\infty} p_t(C_t + K_{t+1} - (1-\delta)K_t) = \sum_{t=0}^{\infty} p_t(r_t K_t + w_t N_t + \Pi_t - T_t)$$

Using it we have the interest rate (complete market):

$$r = \frac{1}{\beta} - (1-\delta)$$

and real interest rate R:

$$R = r - \delta = \frac{1}{\beta} - 1$$

The incumbents' problem under span of control production ($\alpha + \gamma < 1$):

$$\pi(s,\tau) = \max_{n,k\geq 0} \left\{(1-\tau)sk^{\alpha}n^{\gamma} - wn - rk - c_f\right\}$$

where s and τ are the state variables representing the idiosyncratic productivity and distortion, respectively. FOCs:

$$\bar{k}(s,\tau) = \left(\frac{\alpha}{\gamma}\right)^{\frac{1-\gamma}{1-\gamma-\alpha}} \left(\frac{\gamma}{w}\right)^{\frac{\gamma}{1-\alpha-\gamma}} (s(1-\tau))^{\frac{1}{1-\alpha-\gamma}} \tag{1}$$

$$\bar{n}(s,\tau) = \left(\frac{(1-\tau)s\gamma}{w}\right)^{\frac{1}{1-\gamma}} k^{\frac{\alpha}{1-\gamma}} \tag{2}$$

Since the establishment's problem is deterministic: the value function is

$$W(s,\tau) = \frac{\pi(s,\tau)}{1-\rho}$$

where $\rho = \frac{1-\lambda}{1+R}$. λ is the exogenous existing rate.
The Entrant's problem is similar with Hopenhygn and Rogerson (1992).

$$W_e = \sum_{s,\tau} \max_{\bar{x}\in\{0,1\}} \{\bar{x}(s,\tau)W(s,\tau)g(s,\tau) - c_e\}$$

where $\bar{x}(s,\tau)$ is the optimal entry decision. $g(s,\tau) = h(s) \times \mathcal{P}(s,\tau)$ is the joint distribution of entering establishment. The twist in this paper is:

- Firms decide whether to pay entry cost c_e before they know the specific state (s,τ) they will draw.
- Only then it chooses whether to activate (i.e., actually start production).
- Some firms that draw low $W(s,\tau)$ don't produce and "fail early".

Free entering condition: $W_e = 0$.
Distribution LoM:

$$\mu'(s,\tau) = (1-\lambda)\mu(s,\tau) + \bar{x}(s,\tau)g(s,\tau)E. \tag{3}$$

Normalized the entry mass: $E = 1$ we have

$$\hat{\mu}(s,\tau) = \frac{\bar{x}(s,\tau)}{\lambda} g(s,\tau), \quad \forall s,\tau \tag{4}$$

The *equilibrium* will be defined as:

1. Consumer optimization: $r = 1/\beta - (1-\delta)$.

2. Plant optimization: given prices (w, r), the functions π, W, and W_e solve incumbent and entering establishment's problems and $\bar{k}, \bar{n}, \bar{x}$ are optimal functions.

3. Free-entry $W_e = 0$.

4. Market clearing:

$$1 = \sum_{(s,\tau)} \bar{n}(s,\tau)\mu(s,\tau), \quad K = \sum_{(s,\tau)} \bar{k}(s,\tau)\mu(s,\tau), \quad C + \delta K + c_e E = \sum_{(s,\tau)} (f(s,\bar{k},\bar{n}) - c_f)\mu(s,\tau)$$

Algorithm:

1. First, we will solve optimal capital and labor demand given wage and interest rate.
2. Calculate value function W
3. Loop for optimal entry decision ($\bar{x} = 1$ if $W > 0$)
4. Calculate and verify $|W_e| < \varepsilon$ (free entry condition)

Some statistics.

1. Distribution: $\hat{\mu}$,
2. Aggregate labor $\hat{N} = \sum_{(s,\tau)} \bar{n}(a,\tau)\hat{\mu}(s,\tau)$
3. Entry: $E = 1/\hat{N}$.
4. Aggregate output: $Y = \sum_{(s,\tau)} s\bar{k}(s,\tau)^{\alpha} n(s,\tau)^{\gamma} \mu(s,\tau)$
5. $K = \sum_{(s,\tau)} \bar{k}(s,\tau)\mu(s,\tau)$.
6. Average employment per plant (AEPP):

Distorted economy: Consider we have 5 τ level.