1 Model

An economy: a set N regions indexed by n. Each region is endowed with an exogenous quality-adjusted supply of land H_i. $\bar{L}$ workers. Iceberg cost: $b_{ni} = b_{in} > 1 = b_{ii}$.
Preference over consumption C_n and residential land use h_n

$$U_n = \left(\frac{C_n}{\alpha}\right)^{\alpha} \left(\frac{h_n}{1-\alpha}\right)^{1-\alpha}, \quad 0 < \alpha < 1 \tag{1}$$

The consumption C_n is defined by CES preference:

$$C_n = \left[\sum_{i \in N} \int_0^{M_i} c_{ni}(j)^{\rho} dj\right]^{\frac{1}{\rho}}, \quad P_n = \left[\sum_{i \in N} \int_0^{M_i} p_{ni}(j)^{1-\sigma} \mathrm{d}j\right]^{\frac{1}{1-\sigma}} \tag{2}$$

The production of labor is

$$l_i(j) = F + \frac{x_i(j)}{A_i} \tag{3}$$

We can rewrite

$$x_i(j) = (l_i(j) - F)A_i$$

which implies the $MPL = \frac{w_i}{A_i}$ is fixed. Profit maximization and zero profit imply:

$$p_{ni}(j) = \underbrace{\frac{\sigma}{\sigma - 1}}_{\text{markup}} d_{ni} \frac{w_i}{A_i}, \tag{4}$$

and equilibrium output of each variety is equal to a constant that depends on location productivity:

$$x_i(j) = \bar{x}_i = A_i(\sigma - 1)F, \tag{5}$$

Plug in Eq (3) we have

$$l_i(j) = \bar{l} = \sigma F. \tag{6}$$

Equilibrium. Given the constant equilibrium employment for each variety, labor market clearing implies the total measure of varieties supplied

$$L_i = \sum_j l_i(j) = \bar{l} M_i \Rightarrow M_i = \frac{L_i}{\sigma F} \tag{7}$$

Price indices and expenditure shares. Using Eq (4) and labor market clearing Eq (7), one can express the price index dual to the consumption index (10) as:

$$P_n = \frac{\sigma}{\sigma - 1} \left(\frac{1}{\sigma F}\right)^{\frac{1}{1-\sigma}} \left[\sum_{i \in N} L_i \left(d_{ni} \frac{w_i}{A_i}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{8}$$

Using the equilibrium prices and labor market clearing, the share of location n's expenditure on goods produced in location i is

$$\pi_{ni} = \frac{M_i p_{ni}^{1-\sigma}}{\sum_{k \in N} M_k p_{nk}^{1-\sigma}} = \frac{L_i \left(d_{ni} \frac{w_i}{A_i}\right)^{1-\sigma}}{\sum_{k \in N} L_k \left(d_{nk} \frac{w_k}{A_k}\right)^{1-\sigma}} \tag{9}$$

Further, we can derive the gravity equation for goods trade, where the bilateral trade between locations n and i. using (8) and (9) we have

$$P_n = \frac{\sigma}{\sigma - 1}\left(\frac{L}{\sigma F \pi_{nn}}\right)^{\frac{1}{1-\sigma}} \frac{w_n}{A_n}. \tag{10}$$

Trade balance at each location implies that per capita income v_n equal w_n (wage income) plus per capita expenditure on residential land, $(1-\alpha)v_n$. namely,

$$v_n L_n = w_n L_n + (1-\alpha) v_n L_n \Rightarrow v_n = \frac{w_n}{\alpha}. \tag{11}$$

By combining land market clearing condition $L_n h_n = H_n$ with the FOC of consumer problem $r_n H_n = (1-\alpha) L_n v_n$,

$$r_n = \frac{(1-\alpha) v_n L_n}{H_n} = \frac{1-\alpha}{\alpha}\frac{w_n L_n}{H_n}. \tag{12}$$

Population mobility implies that workers receive the same *real income* in all populated locations, thus

$$V_n = \frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}} = \bar{V} \tag{13}$$

where the price index = $P_n^{\alpha} r_n^{1-\alpha}$. Using the prices Eq (12) and Eq (10) Therefore, the location

$$\bar{V} = \frac{A_n^{\alpha} H_n^{1-\alpha} \pi_{nn}^{-\alpha/(\sigma-1)} L_n^{-\frac{\sigma(1-\alpha)-1}{\sigma-1}}}{\alpha \left(\frac{\sigma}{\sigma-1}\right)^{\alpha} \left(\frac{1}{\sigma F}\right)^{\frac{\alpha}{1-\sigma}} \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}} \tag{14}$$

Gains from trade.

$$\frac{V_n^T}{V_n^A} = \left(\pi_{nn}^T\right)^{-\frac{\alpha}{\sigma-1}} \left(\frac{L_n^T}{L_n^A}\right)^{-\frac{\sigma(1-\alpha)-1}{\sigma-1}}$$

The population share of each location:

$$\lambda_n = \frac{L_n}{\bar{L}} = \frac{\left[A_n^{\alpha} H_n^{1-\alpha} \pi_{nn}^{-\alpha/(\alpha-1)}\right]^{\frac{\sigma-1}{\sigma(1-\alpha)-1}}}{\sum_{k\in N}\left[A_k^{\alpha} H_k^{1-\alpha} \pi_{kk}^{-\alpha/(\alpha-1)}\right]^{\frac{\sigma-1}{\sigma(1-\alpha)-1}}} \tag{15}$$

where each location's domestic trade share π_{nn} summarizes its market access to other locations.

1.1 General Equilibrium

Two systems of equations across locations.

- Population mobility.
- Gravity of trade flows

Population. Using $P_n^{\alpha} = \frac{v_n}{\bar{V} r_n^{1-\alpha}}$, $v_n = w_n/\alpha$, and land market clearing ($r_n = \frac{1-\alpha}{\alpha}\frac{w_n L_n}{H_n}$)

$$P_n = \frac{w_n}{\bar{W}}\left(\frac{H_n}{L_n}\right)^{\frac{1-\alpha}{\alpha}}, \quad \bar{W} \equiv \left[\alpha\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}\bar{V}\right]^{\frac{1}{\alpha}} \tag{16}$$

Recall

$$P_n = \frac{\sigma}{\sigma-1}\left(\frac{1}{\sigma F}\right)^{\frac{1}{1-\sigma}}\left[\sum_{i\in N} L_i\left(d_{ni}\frac{w_i}{A_i}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

Obtain a first wage equation from population mobility

$$\bar{W}^{1-\sigma}\frac{1}{\sigma F}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} = \frac{w_i^{1-\sigma}\left(\frac{H_i}{L_i}\right)^{(1-\sigma)\frac{1-\alpha}{\alpha}}}{\sum_{n\in N} L_n\left(d_{in}\frac{w_n}{A_n}\right)^{1-\sigma}} \tag{17}$$

Gravity. Gravity and income equals expenditure implies:

$$w_i L_i = \sum_{n\in N}\frac{\frac{L_i}{\sigma F}\left(\frac{\sigma}{\sigma-1}d_{ni}\frac{w_i}{A_i}\right)^{1-\sigma}}{P_n^{1-\sigma}} w_n L_n. \tag{18}$$

Recall the price index can be expressed as (10), Obtain a second wage equation from gravity

$$w_i^{\sigma}A_i^{1-\sigma} = \sum_{n\in N}\pi_{nn}d_{ni}^{1-\sigma}w_n^{\sigma}A_n^{1-\sigma}. \tag{19}$$

Using our expression for the domestic trade share on previous, slide this wage equation from gravity becomes

$$\tilde{W}_{1-\sigma}\frac{1}{\sigma F}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} = \frac{w_i^{\sigma}A_i^{1-\sigma}}{\sum_{n\in\mathcal{N}} d_{ni}^{1-\sigma}L_n(\sigma-1)^{1-\alpha}H_n^{(\sigma-1)^{1-\alpha}}w_n^{\sigma}} \tag{20}$$

Using two systems of equations (17) (20) for wages and population yields closed-form solution

$$w_n^{1-2\sigma}A_n^{\sigma-1}L_n^{(\sigma-1)\frac{1-\alpha}{\alpha}}H_n^{-(\sigma-1)\frac{1-\alpha}{\alpha}} = \phi. \tag{21}$$

After that, this system can be reduced to a single equation that determines the equilibrium population:

$$L_n^{\tilde{\sigma}\gamma_1}A_n^{-\frac{(\sigma-1)(\sigma-1)}{2\sigma-1}}H_n^{-\frac{(\sigma-1)(\sigma-1)}{\alpha(2\sigma-1)}} = \bar{W}^{1-\sigma}\sum_{i\in N}\frac{1}{\sigma F}\left(\frac{\sigma}{\sigma-1}d_{ni}\right)^{1-\sigma}\left(L_i^{\tilde{\sigma}\gamma_1}\right)^{\frac{\gamma_2}{\gamma_1}}A_i^{-\frac{\sigma(\sigma-1)}{2\sigma-1}}H_i^{\frac{(1-\alpha)(\sigma-1)(\sigma-1)}{\alpha(2\sigma-1)}} \tag{22}$$

where $\bar{W}$ is determined by the requirement that the labor market clears $\sum_{n\in N} L_n = \bar{L}$. and

$$\tilde{\sigma} \equiv \frac{\sigma-1}{2\sigma-1}, \gamma_1 \equiv \sigma\frac{1-\alpha}{\alpha}, \gamma_2 \equiv 1+\frac{\sigma}{\sigma-1}-(\sigma-1)\frac{1-\alpha}{\alpha}$$

Proposition 1. *Assume $\sigma(1-\alpha) > 1$, given the land area, productivity and amenity parameters $\{H_n, A_n, B_n\}$ and symmetric bilateral trade frictions $\{d_{ni}\}$ for all locations $n, i \in N$, there exist unique equilibrium populations (L_n^*) that solve this system of equations.*

1.2 Market access

Model provides micro-foundations for a theory-consistent measure of market access.

- Ad hoc measures of market potential following Harris (1954):

$$MP_{nt} = \sum_{i\in N}\frac{L_{it}}{dist_{ni}}$$

- Theory-based measure highlights the role of price indexes.

We now examine the predictions of the model for the equilibrium relationship between wages, population and market access.

Market access is itself an endogenous variable.

Combining profit maximization, zero profits, CES demand and market clearing, we obtain the following wage equation:

$$\left(\frac{\sigma}{\sigma-1}\frac{w_i}{A_i}\right)^{\sigma} = \frac{1}{\bar{x}_i} FMA_i$$

$$w_i = \left(\frac{\sigma-1}{\sigma}\right)^{\frac{\sigma-1}{\sigma}} A_i^{\frac{\sigma-1}{\sigma}} (\bar{I})^{-\frac{1}{\sigma}} (FMA_i)^{\frac{1}{\sigma}} \tag{23}$$

where firm market access is defined as

$$FMA_i \equiv \sum_{n \in N} d_{ni}^{1-\sigma} (w_n L_n)(P_n)^{\sigma-1}$$

- Wages increase in productivity A_i and firm market access FMA_i.
- Reductions in transport costs (d_{ni}) increase firm market access and wages w_i.

1.3 Model inversion

Estimate σ and α and parameterized symmetric bilateral trade costs d_{ni}.

1.4 Counterfactual