

1 A Manu of QSM

1. **Preferences:** Homogeneous versus differentiated goods (love of variety).
2. **Production technology:**
 - Constant versus increasing returns
 - Exogenous productivity differences and/or endogenous productivity
 - Input-output linkages
3. Technology for trading goods
4. Technology for idea flows
5. Technology for the movement of people
 - Migration costs
 - Commuting and commuting costs
 - Agent heterogeneity
 - Congestion in transportation.
6. Endowments
 - Population and skills
 - Spatial scope and units
 - Capital and infrastructure
7. Equilibrium
 - CRS and perfect competition or IRS and monopolistic competition.
 - General versus partial equilibrium
 - Land ownership and the distribution of rents
 - Trade balance: in any spatial model one has to take a stand on the spatial unit for which trade is balanced.

2 The Conventional Model

An economy: a set N regions indexed by n . Each region is endowed with an exogenous quality-adjusted supply of land H_i . \bar{L} workers. Iceberg cost: $b_{ni} = b_{in} > 1 = b_{ii}$.

Preference over consumption C_n and residential land use h_n

$$U_n = \left(\frac{C_n}{\alpha} \right)^\alpha \left(\frac{h_n}{1-\alpha} \right)^{1-\alpha}, \quad 0 < \alpha < 1 \quad (1)$$

The consumption C_n is defined by CES preference:

$$C_n = \left[\sum_{i \in N} \int_0^{M_i} c_{ni}(j)^\rho dj \right]^{\frac{1}{\rho}}, \quad P_n = \left[\sum_{i \in N} \int_0^{M_i} p_{ni}(j)^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}} \quad (2)$$

The production of labor is

$$l_i(j) = F + \frac{x_i(j)}{A_i} \quad (3)$$

We can rewrite

$$x_i(j) = (l_i(j) - F)A_i$$

which implies the $MPL = \frac{w_i}{A_i}$ is fixed. Profit maximization and zero profit imply:

$$p_{ni}(j) = \underbrace{\frac{\sigma}{\sigma-1}}_{\text{markup}} d_{ni} \frac{w_i}{A_i}, \quad (4)$$

and equilibrium output of each variety is equal to a constant that depends on location productivity:

$$x_i(j) = \bar{x}_i = A_i(\sigma-1)F, \quad (5)$$

Plug in Eq (3) we have

$$l_i(j) = \bar{l} = \sigma F. \quad (6)$$

Equilibrium. Given the constant equilibrium employment for each variety, labor market clearing implies the total measure of varieties supplied

$$L_i = \sum_j l_i(j) = \bar{l} M_i \Rightarrow M_i = \frac{L_i}{\sigma F} \quad (7)$$

Price indices and expenditure shares. Using Eq (4) and labor market clearing Eq (7), one can express the price index dual to the consumption index (10) as:

$$P_n = \frac{\sigma}{\sigma-1} \left(\frac{1}{\sigma F} \right)^{\frac{1}{1-\sigma}} \left[\sum_{i \in N} L_i \left(d_{ni} \frac{w_i}{A_i} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad (8)$$

Using the equilibrium prices and labor market clearing, the share of location n 's expenditure on goods produced in location i is

$$\pi_{ni} = \frac{M_i p_{ni}^{1-\sigma}}{\sum_{k \in N} M_k p_{nk}^{1-\sigma}} = \frac{L_i \left(d_{ni} \frac{w_i}{A_i} \right)^{1-\sigma}}{\sum_{k \in N} L_k \left(d_{nk} \frac{w_k}{A_k} \right)^{1-\sigma}} \quad (9)$$

Further, we can derive the gravity equation for goods trade, where the bilateral trade between locations n and i . using (8) and (9) we have

$$P_n = \frac{\sigma}{\sigma-1} \left(\frac{L}{\sigma F \pi_{nn}} \right)^{\frac{1}{1-\sigma}} \frac{w_n}{A_n}. \quad (10)$$

Trade balance at each location implies that per capita income v_n equal w_n (wage income) plus per capita expenditure on residential land, $(1-\alpha)v_n$. namely,

$$v_n L_n = w_n L_n + (1-\alpha)v_n L_n \Rightarrow v_n = \frac{w_n}{\alpha}. \quad (11)$$

By combining land market clearing condition $L_n h_n = H_n$ with the FOC of consumer problem $r_n H_n = (1-\alpha)L_n v_n$,

$$r_n = \frac{(1-\alpha)v_n L_n}{H_n} = \frac{1-\alpha}{\alpha} \frac{w_n L_n}{H_n}. \quad (12)$$

Population mobility implies that workers receive the same *real income* in all populated locations, thus

$$V_n = \frac{v_n}{P_n^\alpha r_n^{1-\alpha}} = \bar{V} \quad (13)$$

where the price index = $P_n^\alpha r_n^{1-\alpha}$. Using the prices Eq (12) and Eq (10) Therefore, the location

$$\bar{V} = \frac{A_n^\alpha H_n^{1-\alpha} \pi_{nn}^{-\alpha/(\sigma-1)} L_n^{-\frac{\sigma(1-\alpha)-1}{\sigma-1}}}{\alpha \left(\frac{\sigma}{\sigma-1}\right)^\alpha \left(\frac{1}{\sigma F}\right)^{\frac{\alpha}{1-\sigma}} \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}} \quad (14)$$

Gains from trade.

$$\frac{V_n^T}{V_n^A} = \left(\pi_{nn}^T\right)^{-\frac{\alpha}{\sigma-1}} \left(\frac{L_n^T}{L_n^A}\right)^{-\frac{\sigma(1-\alpha)-1}{\sigma-1}}$$

The population share of each location:

$$\lambda_n = \frac{L_n}{\bar{L}} = \frac{\left[A_n^\alpha H_n^{1-\alpha} \pi_{nn}^{-\alpha/(\alpha-1)}\right]^{\frac{\sigma-1}{\sigma(1-\alpha)-1}}}{\sum_{k \in N} \left[A_k^\alpha H_k^{1-\alpha} \pi_{kk}^{-\alpha/(\alpha-1)}\right]^{\frac{\sigma-1}{\sigma(1-\alpha)-1}}} \quad (15)$$

where each location's domestic trade share π_{nn} summarizes its market access to other locations.

2.1 General Equilibrium

Two systems of equations across locations.

- Population mobility.
- Gravity of trade flows

Population. Using $P_n^\alpha = \frac{v_n}{\bar{V} r_n^{1-\alpha}}$, $v_n = w_n/\alpha$, and land market clearing ($r_n = \frac{1-\alpha}{\alpha} \frac{w_n L_n}{H_n}$)

$$P_n = \frac{w_n}{\bar{W}} \left(\frac{H_n}{L_n}\right)^{\frac{1-\alpha}{\alpha}}, \quad \bar{W} \equiv \left[\alpha \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} \bar{V}\right]^{\frac{1}{\alpha}} \quad (16)$$

Recall

$$P_n = \frac{\sigma}{\sigma-1} \left(\frac{1}{\sigma F}\right)^{\frac{1}{1-\sigma}} \left[\sum_{i \in N} L_i \left(d_{ni} \frac{w_i}{A_i}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

Obtain a first wage equation from population mobility

$$\bar{W}^{1-\sigma} \frac{1}{\sigma F} \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} = \frac{w_i^{1-\sigma} \left(\frac{H_i}{L_i}\right)^{(1-\sigma)\frac{1-\alpha}{\alpha}}}{\sum_{n \in N} L_n \left(d_{in} \frac{w_n}{A_n}\right)^{1-\sigma}} \quad (17)$$

Gravity. Gravity and income equals expenditure implies:

$$w_i L_i = \sum_{n \in N} \frac{\frac{L_i}{\sigma F} \left(\frac{\sigma}{\sigma-1} d_{ni} \frac{w_i}{A_i}\right)^{1-\sigma}}{P_n^{1-\sigma}} w_n L_n. \quad (18)$$

Recall the price index can be expressed as (10), Obtain a second wage equation from gravity

$$w_i^\sigma A_i^{1-\sigma} = \sum_{n \in N} \pi_{nn} d_{ni}^{1-\sigma} w_n^\sigma A_n^{1-\sigma}. \quad (19)$$

Using our expression for the domestic trade share on previous, slide this wage equation from gravity becomes

$$\tilde{W}^{1-\sigma} \frac{1}{\sigma F} \left(\frac{\sigma}{\sigma-1} \right)^{1-\sigma} = \frac{w_i^\sigma A_i^{1-\sigma}}{\sum_{n \in N} d_{ni}^{1-\sigma} L_n (\sigma-1)^{1-\alpha} H_n^{(\sigma-1)^{1-\alpha}} w_n^\sigma} \quad (20)$$

Using two systems of equations (17) (20) for wages and population yields closed-form solution

$$w_n^{1-2\sigma} A_n^{\sigma-1} L_n^{(\sigma-1)\frac{1-\alpha}{\alpha}} H_n^{-(\sigma-1)\frac{1-\alpha}{\alpha}} = \phi. \quad (21)$$

After that, this system can be reduced to a single equation that determines the equilibrium population:

$$L_n^{\tilde{\sigma}\gamma_1} A_n^{-\frac{(\sigma-1)(\sigma-1)}{2\sigma-1}} H_n^{-\frac{(\sigma-1)(\sigma-1)}{\alpha(2\sigma-1)}} = \tilde{W}^{1-\sigma} \sum_{i \in N} \frac{1}{\sigma F} \left(\frac{\sigma}{\sigma-1} d_{ni} \right)^{1-\sigma} \left(L_i^{\tilde{\sigma}\gamma_1} \right)^{\frac{\gamma_2}{\gamma_1}} A_i^{-\frac{\sigma(\sigma-1)}{2\sigma-1}} H_i^{\frac{(1-\alpha)(\sigma-1)(\sigma-1)}{\alpha(2\sigma-1)}} \quad (22)$$

where \tilde{W} is determined by the requirement that the labor market clears $\sum_{n \in N} L_n = \bar{L}$. and

$$\tilde{\sigma} \equiv \frac{\sigma-1}{2\sigma-1}, \gamma_1 \equiv \sigma \frac{1-\alpha}{\alpha}, \gamma_2 \equiv 1 + \frac{\sigma}{\sigma-1} - (\sigma-1) \frac{1-\alpha}{\alpha}$$

Proposition 1. Assume $\sigma(1-\alpha) > 1$, given the land area, productivity and amenity parameters $\{H_n, A_n, B_n\}$ and symmetric bilateral trade frictions $\{d_{ni}\}$ for all locations $n, i \in N$, there exist unique equilibrium populations (L_n^*) that solve this system of equations.

2.2 Market access

Model provides micro-foundations for a theory-consistent measure of market access.

- Ad hoc measures of market potential following Harris (1954):

$$MP_{nt} = \sum_{i \in N} \frac{L_{it}}{dist_{ni}}$$

- Theory-based measure highlights the role of price indexes.

We now examine the predictions of the model for the equilibrium relationship between wages, population and market access.

Market access is itself an endogenous variable.

Combining profit maximization, zero profits, CES demand and market clearing, we obtain the following wage equation:

$$\left(\frac{\sigma}{\sigma-1} \frac{w_i}{A_i} \right)^\sigma = \frac{1}{\tilde{x}_i} FMA_i$$

$$w_i = \left(\frac{\sigma-1}{\sigma} \right)^{\frac{\sigma-1}{\sigma}} A_i^{\frac{\sigma-1}{\sigma}} (\bar{I})^{-\frac{1}{\sigma}} (FMA_i)^{\frac{1}{\sigma}} \quad (23)$$

where firm market access is defined as

$$FMA_i \equiv \sum_{n \in N} d_{ni}^{1-\sigma} (w_n L_n) (P_n)^{\sigma-1}$$

- Wages increase in productivity A_i and firm market access FMA_i .
- Reductions in transport costs (d_{ni}) increase firm market access and wages w_i .

2.3 Model inversion

- Rationalize data: has estimates σ and α
- parameterized symmetric bilateral trade costs $\{d_{ni}\}$ and observed endogenous population $\{L_n\}$ and nominal wages $\{w_n\}$.
- The model can be inverted to recover the unique values of unobserved quality-adjusted land $\{H_n\}$ and productivities $\{A_n\}$. More specific, inverting the model amounts to using Eq (21) and Eq (22).
- After that, using Eq (9) to determine unobs. π_{ni} .

2.4 Counterfactual

2.5 Welfare

Change trade costs can be expressed solely in terms of empirically observable sufficient statistics.

Perfect population mobility implies that the transport infrastructure improvement leads to reallocations of population across locations, until real wages are equalized. Hence

$$\frac{\bar{V}^1}{\bar{V}^0} = \left(\frac{\pi_{nn}^0}{\pi_{nn}^1} \right)^{\frac{\alpha}{\sigma-1}} \left(\frac{\lambda_n^0}{\lambda_n^1} \right)^{\frac{\sigma(1-\alpha)-1}{\sigma-1}} \quad (24)$$

a larger reduction in a locations domestic trade share must be offset by a larger increase in its population to preserve real wage equalization

3 Quantitative Illustration

An economy on a 30×30 latitude and longitude grid. Two countries: one is West and another is East. Assume labor is perfectly mobile across locations within country but perfectly immobile across countries.

4 Dynamic QSM