Definitions

List: A list is either null or a pair whose tail is a list.
Tree: A tree of a certain data type is a list whose elements are of that data type or trees of that data type. (No tree of null or tree of pairs)
Binary tree: A binary tree is the empty tree, or it has an entry, a left branch or left subtree, a right branch or right subtree.
BST: A BST is a binary tree where all entries in the left subtree are smaller than the entry, and all entries in the right subtree are larger than the entry.
Arrays: An array is a data structure that stores a sequence of data elements. Arrays can be read and written in constant time. *Exception: Assigning to an array element A[i], where index I >= array_length(A), takes Θ(i – array_length(A)) time.*
Streams: A stream is either the empty list, or a pair whose tail is a nullary function that returns a stream.
Recursive vs iterative: Recursive means there are increasing deferred operations with respect to n, iterative means there are constant deferred operations. (Can be 0)

Environment model

3 things to check for when creating a frame for a function call

1. Extend frame from where function is created. (unless the function comes from the global env)
2. Are there function params? If so, extend frame for params. If not, don't.
3. Are there **new** name declarations in the function body? If so, extend another frame. If not, don't.

If function has no params and no new name declarations in the body, no new frame should be extended.
An environment is a sequence of frames. Each frame contains bindings of values to names.

Searching & Sorting Algorithms

Linear search: O(n) time
Binary search: O(log n) time, requires the input to be sorted, which probably takes another O(n log n) time (Merge sort).

Selection sort: ***Θ*** **(n^2)**
Lists: Find the smallest element x and remove it from the list. Sort the remaining list and put the x in front.
Arrays: Build the sorted array from left to right. For each remaining unsorted portion to the right of position i, find the smallest element and swap it into position i.
Insertion sort: ***Θ*** **(n^2)**
Lists: Sort the tail of the given list using wishful thinking. Insert the head into the right place.
Arrays: Move a pointer i from left to right. The array to the left of i is sorted already. Swap the value at i with its neighbor to the left, until the neighbor is smaller.
Merge sort: ***Θ*** **(n log n)**
Lists: Split the list in half, sort each half using wishful thinking. Merge the sorted lists together.
Arrays: Sort the halves. Merge the halves (using temporary arrays).
Quicksort: ***Θ*** **(n log n)**
Partition the list about a pivot. Append left and right of the pivot. Left portion should have numbers smaller than the pivot, vice versa for right portion.

Common Recurrence Relations

T(n) = T(n-1) + O(1) = O(n)
T(n) = T(n/2) + O(1) = O(log n)
T(n) = T(n-1) + O(log n) = O(n log n)
T(n) = T(n-1) + O(n) = O(n^2)
Generally, T(n) = T(n-1) + O(n^k) = O(n^k+1)
T(n) = 2 T(n/2) + O(n) = O(n log n) → Merge sort
T(n) = T(n/2) + O(n) = O(n)
T(n) = 2 T(n/2) + O(1) = O(n)
T(n) = 2 T(n-1) + O(1) = O(2^n)

The space complexity of a program refers to the additional space required by the execution of the program. You need time to create space. But taking time, does not necessarily mean you create space. Theta(g(n)) time → O(g(n)) space. Theta(g(n)) space → Omega(g(n)) time

Continuous Passing Style

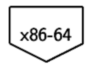
1. Look at the recursive solution and duplicate it
2. Add a parameter, c.
3. Slightly modify the base case so that it actually calls c.
4. Initially, start with wish => c(wish).
5. Treat wish as the result of map_cps(f, tail(xs), x => x) which contains the list we want to return except the mapped head of the list. Then, we just need to pair f(head(xs)) with wish to get the result.

```
function map(f, xs) {
  return is_null(xs)
             ? null
              : pair(f(head(xs)), map(f, tail(xs)));
}
function map_cps(f, xs, c) { // step 2
  return is_null(xs)
        ? c(null) // step 3
//      : map_cps(f, tail(xs), wish => c(wish)) // step 4
        : map_cps(f, tail(xs), wish => c(pair(f(head(xs)), wish))); // step 5
}
```

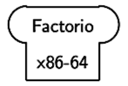
Metacircular evaluator

The evaluator, which determines the meaning of statements and expressions in a programming language, is just another program.
Parse → Evaluate → Apply

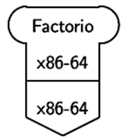
Programming Language Processing



x86-64 Processor

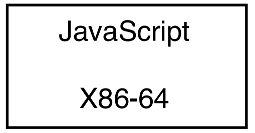


Program "Factorio" (x86-64 code)

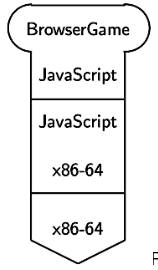


"Factorio" running on x86-64

Interpreter (Represented as a square): Interpreter is a program that executes another program. The interpreter's source language is the language in which the interpreter is written. The interpreter's target language is the languages in which the programs are written which the interpreter can execute.

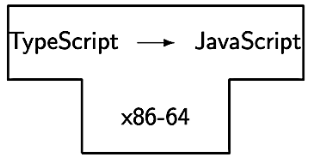


Chrome browser for PC, seen as interpreter for JavaScript, written in x86-64 machine code.

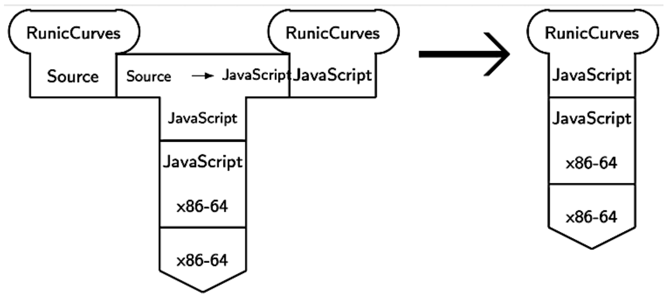


Program BrowserGame is written in JavaScript, which is then put through a JavaScript to x86-64 interpreter. This x86-64 program is then run by the x86-64 processor.

Compiler: A compiler is a program that translates from one language to another language.



TypeScript-to-JavaScript compiler written in x86-64 machine code.



Program RunicCurves is written in Source. It is then compiled by a Source-to-JavaScript compiler. This gives a JavaScript program. This compiler program is run on a JavaScript interpreter written in x86-64 machine code, which is run on the x86-64 processor. The program RunicCurves in JavaScript in interpreted by the JavaScript interpreter written in x86-64 machine code, which is run on the x86-64 processor.

Some useful functions

```
function mutable_reverse(xs) {
 // version 1
 if (is_null(xs)) {
   return xs;
 } else if (is_null(tail(xs))) {
   return xs;
 } else {
   let temp = mutable_reverse(tail(xs));
   set_tail(tail(xs), xs);
   set_tail(xs, null);
   return temp;
 }
 // version 2
 function helper(prev, xs) {
   if (is_null(xs)) {
     return prev;
   } else {
     let rest = tail(xs);
     set_tail(xs, prev);
     return helper(xs, rest);
   }
 }
 return helper(null, xs);
}
// merge two ordered lists using recursion without creating new pairs
function mergeB(xs, ys) {
 if (is_null(xs) && is_null(ys)) {
   return null;
 } else if (is_null(xs)) {
   set_tail(ys, mergeB(xs, tail(ys))); return ys;
 } else if (is_null(ys)) {
   set_tail(xs, mergeB(tail(xs), ys)); return xs;
 } else if (head(xs) <= head(ys)) { set_tail(xs, mergeB(tail(xs), ys));
   return xs;
 } else {
   set_tail(ys, mergeB(xs, tail(ys))); return ys;
 }
}
// accumulate_tree
function accumulate_tree(f, op, initial, tree) {
 return accumulate((x, y) => is_list(x)
                          ? op(accumulate_tree(f, op, null, x), y)
                           : op(f(x), y),
                   null, tree);
}
// flatten tree to list
function flatten_tree(T) {
   return accumulate((x, y) => is_list(x)
                          ? append(flatten_tree(x), y)
                          : append(list(x), y),
                   null,
                   T);
}
OR
function flatten_tree(t) {
  return accumulate_tree(x => list(x), append, null, tree);
}
// reverse tree
function transform_tree(t) {
  return reverse(map(x => is_list(x)
                       ? transform_tree(x)
                        : x,
                   t));
}
// d_append
function d_append(xs, ys) {
  if (is_null(xs)) {
    return null;
  } else {
    set_tail(xs, d_append(tail(xs), ys));
    return xs;
  }
}
// d_map
function d_map(f, xs) {
  if (!is_null(xs)) {
    set_head(xs, f(head(xs)));
    d_map(f, tail(xs));
  }
}
// d_filter (note the first element is never filtered)
function d_filter(pred, xs) {
    if (is_null(xs)) {
        return null;
    } else if (pred(head(xs))) {
        set_tail(xs, d_filter(pred, tail(xs)));
        return xs;
    } else {
        return d_filter(pred, tail(xs));
    }
}
// count looped pairs
function correct_count_pairs(x) {
  let pairs = null;
  function check(y) {
    if (!is_pair(y)) {
      return undefined;
    } else if (!is_null(member(y, pairs)) {
      return undefined;
    } else {
      pairs = pair(y, pairs);
      check(head(x));
      check(head(y));
    }
    check(x);
    return length(pairs);
}
// rotate_matrix (transpose and then swap items)
function rotate_matrix(M) {
    const n = array_length(M);  // M is assumed n x n.

    function swap(r1, c1, r2, c2) {
        const temp = M[r1][c1];
        M[r1][c1] = M[r2][c2];
        M[r2][c2] = temp;
    }
    // transpose matrix first
    for (let i = 0; i < n; i = i + 1) {
        for (let j = i + 1; j < n; j = j + 1) {
            swap(i, j, j, i);
        }
    }
    // swap the items in the same row
    for (let i = 0; i < n; i = i + 1) {
        for (let j = 0; j < math_floor(n / 2); j = j + 1) {
            swap(i, j, i, n - j - 1);
        }
    }
}
// hold_stream(list(2,4,6)) returns a stream 2,4,6,6,6,6…
function hold_stream(xs) {
  return is_null(tail(xs))
           ? pair(head(xs), () => hold_stream(xs))
            : pair(head(xs), () => hold_stream(tail(xs)));
}
//search_stream
function search_stream(xs, pos, x) {
  return is_null(xs) || pos < 0
           ? false
            : head(xs) === x
           ? true
            : search_stream(stream_tail(xs), pos – 1, x);
}
// memoized streams (may not actually memoize)
function memo(fun) {
  let already_run = false;
  let result = undefined;
  return () => {
                if (!already_run) {
                  result = fun();
                  already_run = true;
                  return result;
                } else {
                  return result;
                }
  };
}
// fast_and_furious (iterative, using cps)
function fast_and_furious_expt(b, n) {
  function ffe(bb, nn, cont) {
    return nn === 0
             ? cont(1)
             : is_even(nn)
             ? ffe(bb, nn / 2, x => cont(square(x)))
              : ffe(bb, nn – 1, x => cont(b * x)); } return ffe(b, n, x => x);
```