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Review–2^{nd} Quarterly–NO CALCULATOR ALLOWED

1. The area of the region between the graph of $y = 3x^2 + 2x$ and the x-axis from $x = 1$ to $x = 3$ is

(A) 36 (B) 34 (C) 31 (D) 28

2. Over a period of time in a national preserve the population of deer, P, changes at a rate based on the logistic differential equation $\frac{dP}{dt} = 0.25P(1200 - P)$, where t is given in years. For what values of P will the population of deer increase at a decreasing rate?

(A) $0 < P < 300$ (B) $300 < P < 600$ (C) $600 < P < 1200$ (D) $P > 1200$

3. A particle moves along a straight line so that its velocity is given by $v(t) = t^2$. How far does the particle travel between $t = 1$ and $t = 3$?

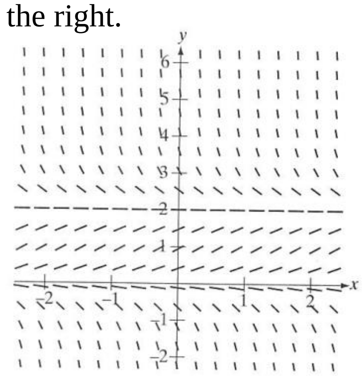
(A) $\frac{26}{3}$ (B) 8 (C) 26 (D) 27

x	0	1	2	3	4	5
$f(x)$	–8	–5	–2	0	2	1
$f'(x)$	2	4	3	2	0	–3

4. The table above gives values of a function f and its derivative at selected values of x. If f' is continuous on [0, 5], what is the value of $\int_1^4 f'(x)\,dx$?

(A) −4 (B) 2 (C) 7 (D) 9

5. The slope field for a differential equation $\frac{dy}{dx} = f(y)$ is shown in the figure at the right. Which statement is true about $y(x)$?



I. If $y(0) > 2$, then $\lim_{x\to\infty} y(x) \approx 2$.

II. If $0 < y(0) < 2$, then $\lim_{x\to\infty} y(x) \approx 2$.

III. If $y(0) < 0$, then $\lim_{x\to\infty} y(x) \approx 2$

(A) I only (B) III only (C) I and II only (D) I, II, and III

6. $\int_1^3 \frac{x}{x^2+1}\,dx =$

(A) $\frac{1}{2}\ln 5$ (B) $\ln 5$ (C) $2\ln 5$ (D) $\ln\left(\frac{5}{2}\right)$

7. A function *f* is continuous on the closed interval [4, 6] and twice differentiable on the open interval (4, 6). If $f'(5) = -3$, and *f* is concave downwards on the given interval, which of the following could be a table of values for *f*?

(A)

x	$f(x)$
4	8
5	4
6	0

(B)

x	$f(x)$
4	8
5	6
6	2

(C)

x	$f(x)$
4	8
5	6
6	5

(D)

x	$f(x)$
4	8
5	3
6	2

8. Which of the following are the coordinates of the point where the slope of the line tangent to the graph of $f(x) = \frac{kx-3}{x+2}$ equals 1 when $x = k$?

(A) $(-3,-6)$ (B) $(-1,-2)$ (C) $\left(2,\frac{1}{4}\right)$ (D) $\left(3,\frac{6}{5}\right)$

x	5	6	9	11	12
$f(x)$	10	7	11	12	8

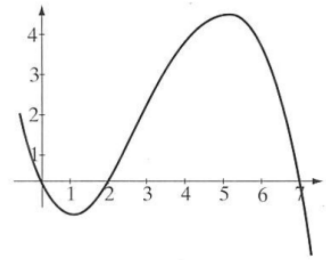
9. A function f is continuous on the closed interval [5, 12] and differentiable on the open interval (5, 12) and *f* has the values given in the table above. Using the subintervals [5, 6], [6, 9], [9, 11] and [11, 12], what is the right Riemann sum approximation to $\int_5^{12} f(x)\,dx$?

(A) 64 (B) 65 (C) 66 (D) 72

10. The following statements concerning the location of an extreme value of a twice-differentiable function, *f*, are all true. Which statement also includes the correct justification?

(A) The function has a maximum at $x = 5$ because $f'(x) < 0$ for $x < 5$ and $f'(x) > 0$ for $x > 5$.
(B) The function has a minimum at $x = 3$ because the tangent line at $x = 3$ is horizontal.
(C) The function has a minimum at $x = 3$ because $f'(x) < 0$ for $x < 3$ and $f'(x) > 0$ for $x > 3$.
(D) The function has a minimum at $x = 3$ because $f''(3) < 0$.

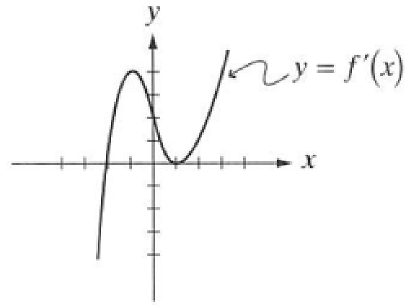
11. The graph of a differentiable function f is shown at the right. The graph has a relative minimum at $x = 1$ and a relative maximum at $x = 5$. Let g be the function defined by $g(x) = \int_0^x f(t)\,dt$. For what value of x does the graph of g change from concave up to concave down?



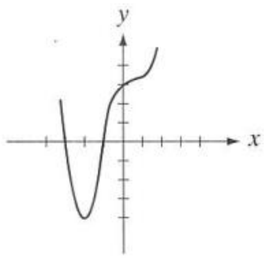
Graph of f

(A) 1 (B) 2 (C) 5 (D) 7

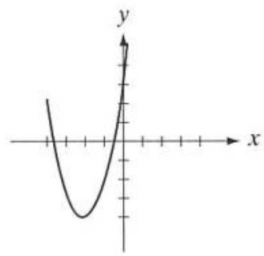
12. The graph of the <u>derivative</u> of f is shown in the figure at the right. Which of the following could be the graph of f?



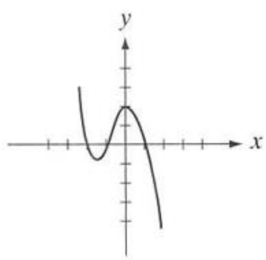
(A)



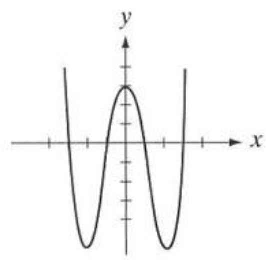
(B)



(C)



(D)



13. If $f'(x) = \dfrac{(x+2)^2(x^2-1)}{3}$ and $g(x) = f\left(\sqrt{x+1}\right)$, what is the value of $g'(3)$?

(A) 32 (B) 16 (C) 4 (D) $\dfrac{1}{4}$

14. At each point (x, y) on a certain curve, the slope of the curve is $4xy$. If the curve contains the point $(0, 4)$, then its equation is

(A) $y = 4e^{2x^2}$ (B) $y = e^{2x^2} + 3$ (C) $y = e^{2x^2} + 4$ (D) $y = 2x^2 + 4$

15. $\displaystyle\int \frac{dx}{x^2 - 9} =$

(A) $\dfrac{1}{3}\ln\left|\dfrac{x+3}{x-3}\right| + C$ (B) $\ln\left|x^2 - 9\right| + C$ (C) $\dfrac{1}{6}\ln\left|\dfrac{x+3}{x-3}\right| + C$ (D) $\dfrac{1}{6}\ln\left|\dfrac{x-3}{x+3}\right| + C$

16. Which of the following are properties of the definite integral?

I. $\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx, k \neq 0$

II. $\int_a^b x\,f(x)\,dx = x\int_a^b f(x)\,dx$

III. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

(A) I only (B) I and III only (C) II and III only (D) I, II, and III

17. If $f(x) = \sqrt{e^{2x} + 1}$, then $f'(0) =$

(A) $\frac{\sqrt{2}}{4}$ (B) $\frac{\sqrt{2}}{2}$ (C) 1 (D) $\sqrt{2}$

18. If $\lim_{x \to a} \frac{\tan^{-1}(2x) - \tan^{-1}(2a)}{x - a} = \frac{2}{3}$, then a could equal

(A) $\sqrt{2}$ (B) 1 (C) $\frac{\sqrt{2}}{2}$ (D) $\frac{\sqrt{2}}{4}$

19. The current number of bacteria in a culture is 9,000 and is increasing at the rate of $2700e^{0.3t}$ per hour. What will be the number of bacteria present in the culture in 5 hours?

(A) $2700e^{1.5}$ (B) $2700e^{3}$ (C) $9000e^{1.5}$ (D) $9000e^{3}$

20. The average value of a continuous function $f(x)$ on the closed interval [3, 7] is 12. What is the value of $\int_3^7 f(x)\,dx$?

(A) 4 (B) 12 (C) 24 (D) 48

21. Let $y(x)$ be the solution to the differential equation $\frac{dy}{dx} = y^2 - xy$ with the initial condition $y(2) = 3$. What is the approximation for $y(3)$ obtained by using Euler's Method with two steps starting with x = 2?

(A) 0 (B) 4.5 (C) 6 (D) 9

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22. If n is a positive integer, the function $f(x) = x^2 + 5\cos x$ will have how many points of inflection over the interval $[0, 2\pi n]$?

(A) 2 (B) 2π (C) n (D) $2n$

23. If $y^2 - 2xy = 21$, then $\frac{dy}{dx}$ at the point (2, −3) is

(A) $\frac{6}{5}$ (B) $\frac{3}{5}$ (C) $-\frac{2}{5}$ (D) $-\frac{3}{5}$

24. The second derivative of a function is given by $f''(x) = 0.5 + \cos x - e^{-x}$. Over the interval [−2, 4] the graph of function f changes from concave up to concave down at approximately x =

(A) −0.36 (B) 0.59 (C) 1.93 (D) 3.10

25. Let $f(x) = x^3 - 7x^2 + 25x - 39$ and let g be the inverse function of f. What is the value of $g'(0)$?

(A) $\frac{1}{25}$ (B) $\frac{1}{10}$ (C) 10 (D) 25

26. Oil flows into a concrete conical storage pit at the rate of 10 cubic feet per minute. The pit was built point down and has a depth of 15 feet and a ground level radius of 9 feet. How fast, in feet per minute, is the oil level rising when the oil is 10 feet deep?

(A) 0.05 (B) 0.09 (C) 0.13 (D) 0.44

$$f(x) = \begin{cases} e^{-x} + 2, & x < 0 \\ ax + b, & x \geq 0 \end{cases}$$

27. Let f be the function defined above, where a and b are constants. If f is differentiable at x = 0, what is the value of $a + b$?

(A) −2 (B) 0 (C) 2 (D) 4

28. Let f be a function whose derivative is given by $f'(x) = \frac{x}{15} + \sin\left(e^{0.2x}\right)$. Which of the following is the approximate x-value of a relative maximum point on the graph of f?

(A) 2.830 (B) 6.378 (C) 8.673 (D) 10.332

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29. If f is continuous for all real numbers, $\frac{dy}{dx} = f(x)$ and $y(2) = 4$, then $y(x) =$

(A) $4 + \int_2^x f'(t)\,dt$ (B) $4 + \int_2^x f(t)\,dt$ (C) $\int_2^x f(t)\,dt - 4$ (D) $4 - \int_2^x f(t)\,dt$

30. For what value(s) of x does $4x^6 - 8x^3 + 18$ have a relative minimum?

(A) −1 only (B) 0 only (C) 1 only (D) 0 and 1 only

31. The average value of $\sqrt{3x}$ on the closed interval [0, 9] is

(A) $\frac{2\sqrt{3}}{3}$ (B) $2\sqrt{3}$ (C) 6 (D) $6\sqrt{3}$

32. The position of a particle on the x-axis at time t, $t > 0$, is $\ln t$. The average velocity of the particle for $1 \le t \le e$ is

(A) $\frac{1}{e} - 1$ (B) $\frac{1}{e-1}$ (C) e (D) $e - 1$

33. An antiderivative of $2x\cos(2x)$ is

(A) $2x\cos(2x) - 2\sin(2x)$ (B) $x\sin(2x) - \frac{1}{2}\cos(2x)$

(C) $2x\sin(2x) + 2\cos(2x)$ (D) $x\sin(2x) + \frac{1}{2}\cos(2x)$

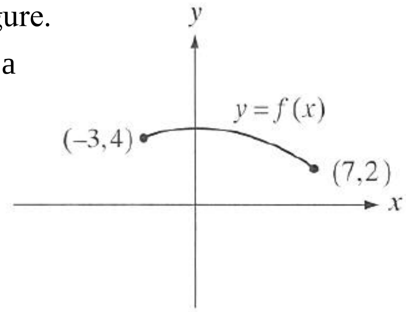
34. $\int_{-3}^{3} |x+2|\,dx =$

(A) 0 (B) 8 (C) 13 (D) 21

35. Let R be the region in the <u>fourth quadrant</u> enclosed by the x-axis and the curve $y = x^2 - 2kx$, where k is a constant. If the area of the region R is 36, then the value of k is

(A) −3 (B) 3 (C) 4 (D) 6

36. The graph of $y = f(x)$ on the closed interval [−3, 7] is shown in the figure. If f is continuous on [−3, 7] and differentiable on (−3, 7), then there exists a c, $-3 < c < 7$, such that



(A) $f'(c)$ is undefined (B) $f'(c) = \frac{1}{5}$

(C) $f'(c) = -\frac{1}{5}$ (D) $f'(c) = -5$

x	1	2	3	4	5	6
$g(x)$	0	1	3	7	2	5
$g'(x)$	4	3	1	–1	–2	–1

37. Let $f(x) = x^2 + 3x$ and let $g(x)$ and its derivative $g'(x)$ have the values shown in the table above. If $h(x) = f(g(x))$, what is $h'(2)$?

(A) 5 (B) 10 (C) 15 (D) 21

38. Let $f(x)$ be a differentiable function. The table below gives the value of $f(x)$ and $f'(x)$, the derivative of $f(x)$, at selected values of x. If $g(x) = \frac{1}{f(x)}$, what is the value of $g'(2)$?

x	1	2	3	4
$f(x)$	–3	–8	–9	0
$f'(x)$	–5	–4	3	16

(A) $-\frac{1}{8}$ (B) $\frac{1}{16}$ (C) $\frac{1}{64}$ (D) 16

39. What is the 20th derivative of $y = \sin(2x)$?

(A) $-2^{20}\sin(2x)$ (B) $2^{20}\sin(2x)$ (C) $-2^{20}\cos(2x)$ (D) $2^{20}\cos(2x)$

40. If $f(x) = 15 - g(x)$ for $-2 \le x \le 2$, then $\int_{-2}^{2}\left[f(x) - g(x)\right]dx =$

(A) $2\int_{-2}^{2} g(x)\,dx - 60$ (B) $2\int_{-2}^{2} g(x)\,dx + 60$

(C) $60 - 4\int_{0}^{2} g(x)\,dx$ (D) $60 - 2\int_{-2}^{2} g(x)\,dx$

41. An equation of the line tangent to the curve $x^2 + y^2 = 169$ at the point (5, −12) is

(A) $12x - 5y = 119$ (B) $5x - 12y = 119$ (C) $5x - 12y = 169$ (D) $12x - 5y = 169$

42. Let $f(x) = \begin{cases} 1 + e^{-x}, & 0 \le x \le 5 \\ 1 + e^{x-10}, & 5 < x \le 10 \end{cases}$

Which of the following statements are true?

I. $f(x)$ is continuous for all values of x in the interval [0, 10].

II. $f'(x)$, the derivative of $f(x)$, is continuous for all values of x in the interval [0, 10].

III. The graph of $f(x)$ is concave upwards for all values of x in the interval [0, 10].

(A) I and II only (B) II and III only (C) I and III only (D) I, II, and III

43. The rate of change of the velocity of a particle moving on the x-axis is given by the function $R(t) = -\sin(t) - \sqrt{3}\cos(t)$, for $t \ge 0$. At t = 0, the velocity of the particle is 1.
At t = 0, which statement is true?

(A) The particle is moving to the right and its speed is decreasing.
(B) The particle is moving to the left and its speed is decreasing.
(C) The particle is moving to the right and its speed is increasing.
(D) The particle is moving to the left and its speed is increasing.

44. If $y = \sin^{-1}\left(\frac{3x}{4}\right)$, then $\frac{dy}{dx} =$

(A) $\frac{3}{\sqrt{16+9x^2}}$ (B) $\frac{3}{\sqrt{16-9x^2}}$ (C) $\frac{4}{\sqrt{16-9x^2}}$ (D) $\frac{12}{\sqrt{16+9x^2}}$

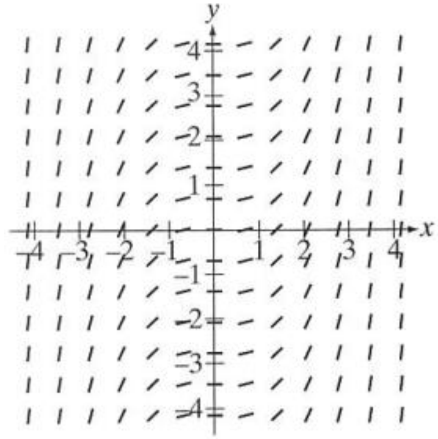
45. What are the horizontal asymptotes of all of the solutions of the logistic differential equation $\frac{dy}{dx} = y\left(8 - \frac{y}{1000}\right)$?

(A) y = 0 only (B) y = 8 only (C) y = 0 and y = 8 (D) y = 0 and y = 8000

46. A particle with velocity at any time t given by $v(t) = 2e^{2t}$ moves in a straight line. How far does the particle travel during the time interval when its velocity increases from 2 to 4?

(A) 1 (B) 2 (C) 3 (D) e^4

47. The slope field for the differential equation $\frac{dy}{dx} = f(x)$ is shown for $-4 < x < 4$ and $-4 < y < 4$. Which of the following statements is true for all possible solutions of the differential equation?

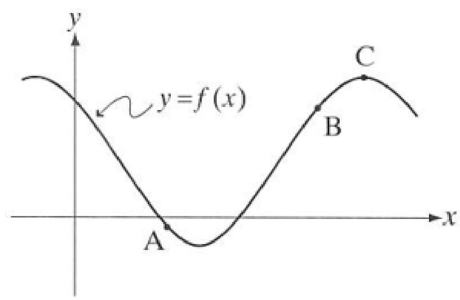


I. For $x < 0$, all solution functions are decreasing.
II. For $x > 0$, all solution functions are increasing.
III. All solution functions level off near the y-axis.

(A) I and II only (B) II and III only

(C) I and III only (D) I, II, and III

48. At which of the three points on the graph of $y = f(x)$ in the figure will $f'(x) < f''(x)$?



(A) A only (B) C only

(C) A and B only (D) A and C only

49. If the derivative of a function f is given by $f'(x) = \frac{1}{5}(x^2 - 4)^5 - x^2$, how many points of inflection will the graph of function f have?

(A) 2 (B) 3 (C) 4 (D) 5

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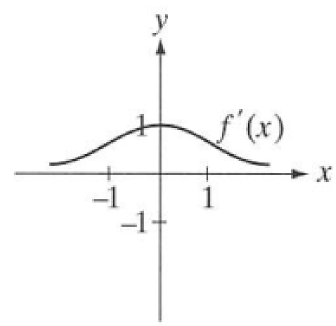
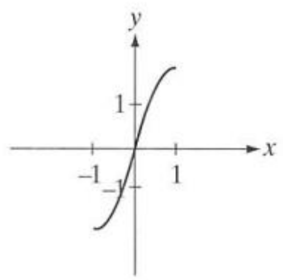
50. What is an equation of the line tangent to the graph of $f(x) = 7x - x^2$ at the point where $f'(x) = 3$?

(A) $y = 3x + 4$ (B) $y = 3x + 8$ (C) $y = 3x - 10$ (D) $y = 3x - 16$

51. Suppose the derivative of f has the graph shown at the right. Which of the following could be the graph of f?



(A)



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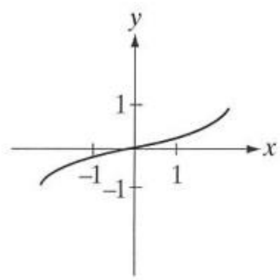

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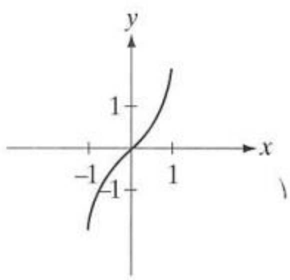

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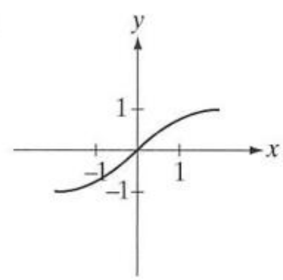


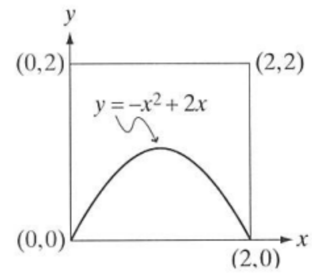
52. Let f be the function given by $f(t) = \int_0^t e^{x\cos(x)}\left(\cos(x) - x\sin(x)\right)dx, 0 \le t \le 10$. At which of the following values of t does f attain its absolute maximum value?

(A) 0.860 (B) 3.426 (C) 6.437 (D) 9.529

53. If $\dfrac{dy}{dt} = \dfrac{2y}{t(t+2)}$ for $t > 0$ and $y = 1$ when $t = 1$, then when $t = 2$, $y =$

(A) $\dfrac{1}{2}$ (B) $\dfrac{2}{3}$ (C) 1 (D) $\dfrac{3}{2}$

54. As shown in the figure, a square with vertices (0, 0), (2, 0), (2, 2), and (0, 2) is divided into two regions by the graph of $y = -x^2 + 2x$. If a point is picked at random from inside the square, what is the probability that the point lies in the region <u>above</u> the parabola?



(A) $\frac{3}{4}$ (B) $\frac{2}{3}$ (C) $\frac{5}{8}$ (D) $\frac{3}{5}$

55. As $h \to 0$ let f be a differentiable function such that for all x, $f(x+h) - f(x) = 6xh + 4h^2$. If $f(2) = 5$, what is the value of $f(1)$?

(A) −4 (B) −1 (C) 3 (D) 10

56. The circumference of a circle is increasing at the rate of 0.5 meters/minute. What is the rate of change of the area of the circle when the radius is 4 meters?

(A) 8π m²/min (B) 4π m²/min (C) 4 m²/min (D) 2 m²/min

57. Let g be the function defined by $g(x) = \int_3^x \left((5 + 4t - t^2)(2^{-t})\right) dt$. Which of the following statements about g must be true?

I. g is increasing on [3, 5].
II. g is increasing on [5, 7].
III. $g(7) < 0$

(A) I only (B) III only (C) I and III only (D) I, II, and III

58. The solution of the differential equation $\frac{dy}{dx} = -\frac{x^2}{y}$ contains the point (3, −2). Using Euler's Method with $\Delta x = -0.3$, what is the approximate value of y when $x = 2.7$?

(A) −2.98 (B) −3.08 (C) −3.25 (D) −3.35

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t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

1. The height of a tree at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table at the right.

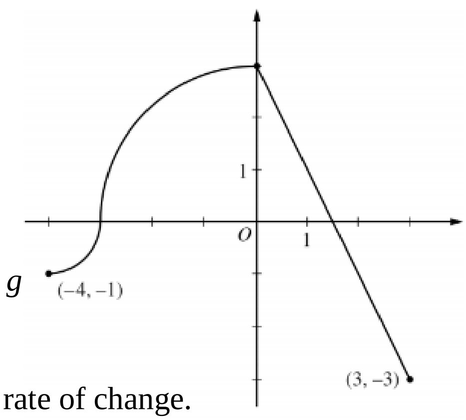
(a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.

(b) Explain why there must be at least one time t, for $2 < t < 10$, such that $H'(t) = 2$.

(c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the average height of the tree over the time interval $2 \le t \le 10$.

(d) The height of the tree, in meters, can also be modeled by the function G, given by $G(x) = \dfrac{100x}{1+x}$, where x is the diameter of the base of the tree, in meters. When the tree is 50 meters tall, the diameter of the base of the tree is increasing at a rate of 0.03 meter per year. According to this model, what is the rate of change of the height of the tree with respect to time, in meters per year, when the tree is 50 meters tall?

2. The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure at the right. Let $g(x) = 2x + \int_0^x f(t)\,dt$.



Graph of f

(a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

(b) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \le x \le 3$. Justify your answer.

(c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.

(d) Find the average rate of change of f on the interval $-4 \le x \le 3$. There is no point c, $-4 < c < 3$, for which $f'(c)$ is equal to the average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

3. Consider the curve given by $y^2 = 2 + xy$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{y}{2y - x}$.

(b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\dfrac{1}{2}$.

(c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

(d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At time $t = 5$, the value of y is 3 and $\dfrac{dy}{dt} = 6$. Find the value of $\dfrac{dx}{dt}$ at time $t = 5$.

4. Consider the differential equation $\dfrac{dy}{dx} = y^2(2x + 2)$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = -1$.

(a) Find $\displaystyle\lim_{x \to 0} \frac{f(x) + 1}{\sin x}$. Show the work that leads to your answer.

(b) Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f\left(\dfrac{1}{2}\right)$.

(c) Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = -1$.

NAME ANSWER KEY AP CALC BC

Review−2nd Quarterly

1. $\int_1^3 \left(3x^2+2x\right)dx = \left[x^3+x^2\right]_1^3 = (27+9)-(1+1)=34$ $\boxed{B}$

2. In a logistic differential equation $\frac{dP}{dt}=kP\left(L-P\right)$, L = 1200, which is the carrying capacity. The point of inflection occurs at the point where $P=\frac{L}{2}=600$. The graph is concave up for 0 < P < 600 and concave down for 600 < P < 1200. If a graph is increasing at a decreasing rate, then that graph is concave down. $\boxed{C}$

3. If the velocity is positive on [1, 3], then distance = $\int_1^3 v(t)\,dt = \int_1^3 t^2\,dt = \left[\frac{t^3}{3}\right]_1^3 = 9-\frac{1}{3}=\frac{26}{3}$ $\boxed{A}$

4. According to the FTC, $\int_1^4 f'(x)\,dx = f(4)-f(1)=2--5=7$ $\boxed{C}$

5. Based on the slope field, there is a horizontal asymptote at y = 2 and y = 0. Any particular solution where $y(x)>0$, the function is decreasing towards y = 2, so I is true. Any particular solution where $0<y(x)<2$, the function is increasing towards y = 2 and away from y = 0, so II is true. Any particular solution where $y(x)<0$, the function is decreasing away from y = 0, so III is false. $\boxed{C}$

6. $u=x^2+1 \rightarrow du=2xdx$ Upper $u=(3)^2+1=10$ Lower $u=(1)^2+1=2$

$\frac{1}{2}\int_2^{10}\frac{du}{u}=\frac{1}{2}[\ln u]_2^{10}=\frac{1}{2}(\ln 10-\ln 2)=\frac{1}{2}\ln\left(\frac{10}{2}\right)=\frac{1}{2}\ln(5)$ $\boxed{A}$

7. If f is concave down, then f' is decreasing and the Mean Value Theorem applies, which implies that the average rate of change on [4, 5] > −3 and the average rate of change on [5, 6] < −3. $\boxed{B}$

8. $f'(x)=\frac{k(x+2)-(kx-3)}{(x+2)^2} \rightarrow f'(x)\big|_{x=k}=\frac{k(k+2)-(k^2-3)}{(k+2)^2}=\frac{2k+3}{(k+2)^2}=1 \rightarrow k^2+4k+4=2k+3$

$k^2+2k+1=0 \rightarrow (k+1)^2=0 \rightarrow k=-1$ $\therefore x=k=-1$ and $f(-1)=\frac{1-3}{-1+2}=-2$ $\boxed{B}$

9. $\int_5^{12} f(x)\,dx \approx 1\cdot f(6)+3\cdot f(9)+2\cdot f(11)+1\cdot f(8)=1\cdot 7+3\cdot 11+2\cdot 12+1\cdot 8=7+33+24+8=72$ $\boxed{D}$

10. A function has a minimum at a point, because the derivative changes from negative to positive. $\boxed{C}$

11. According to the FTC, $g'(x)=f(x)$ and $g''(x)=f'(x)$. g is concave up where $g''(x)=f'(x)>0$, and g is concave down where $g''(x)=f'(x)<0$, so f changes from increasing to decreasing at x = 5. $\boxed{C}$

12. $f'(x) \xleftarrow[\;-2\qquad 1\;]{\text{NEG} \mid \text{POS} \mid \text{POS}} x$

If $f'(x)$ is increasing, then $f''(x)>0$, and if $f'(x)$ is decreasing, then $f''(x)<0$

$f''(x) \xleftarrow[\;-1\qquad 1\;]{\text{POS} \mid \text{NEG} \mid \text{POS}} x$ $\boxed{A}$

13. $g'(x)=f'\left(\sqrt{x+1}\right)\bullet\frac{d}{dx}\sqrt{x+1}=f'\left(\sqrt{x+1}\right)\bullet\frac{1}{2\sqrt{x+1}}$

$g'(3)=f'(2)\bullet\frac{1}{4}=\frac{1}{4}\bullet\frac{(4)^2(3)}{3}=4$ $\boxed{\text{C}}$

14. $\frac{dy}{dx}=4xy\rightarrow\frac{dy}{y}=4xdx\rightarrow\ln|y|=2x^2+C\rightarrow|y|=e^{2x^2+C}=ke^{2x^2}$

If $y=4>0$, then $y=ke^{2x^2}$. At the point (0, 4), $4=ke^0\rightarrow k=4$. Therefore, $y=4e^{2x^2}$. $\boxed{\text{A}}$

15. $\frac{1}{x^2-9}=\frac{A}{x+3}+\frac{B}{x-3}\rightarrow 1=A(x-3)+B(x+3)$

If $x=3$, then $1=6B\rightarrow B=\frac{1}{6}$, and if $x=-3$, then $1=-6A\rightarrow A=-\frac{1}{6}$.

$\int\frac{dx}{x^2-9}=-\frac{1}{6}\int\frac{dx}{x+3}+\frac{1}{6}\int\frac{dx}{x-3}=-\frac{1}{6}\ln|x+3|+\frac{1}{6}\ln|x-3|+C=\frac{1}{6}\ln\left|\frac{x-3}{x+3}\right|+C$ $\boxed{\text{D}}$

16. I and III are properties of integrals. Only a constant multiple can be factored out of an integral, so II is false. $\boxed{\text{B}}$

17. $f'(x)=\frac{1}{2\sqrt{e^{2x}+1}}\bullet 2e^{2x}=\frac{e^{2x}}{\sqrt{e^{2x}+1}}$ $f'(0)=\frac{e^0}{\sqrt{e^0+1}}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$ $\boxed{\text{B}}$

18. $f'(a)=\lim\limits_{x\to a}\frac{f(x)-f(a)}{x-a}$

$f'\left(\tan^{-1}(2a)\right)=\lim\limits_{x\to a}\frac{\tan^{-1}(2x)-\tan^{-1}(2a)}{x-a}$ $\frac{d}{dx}\left[\tan^{-1}(2x)\right]=\frac{1}{1+(2x)^2}\bullet 2=\frac{2}{1+(2x)^2}$

$\frac{2}{1+(2a)^2}=\frac{2}{3}\rightarrow 1+(2a)^2=3\rightarrow(2a)^2=2\rightarrow 4a^2=2\rightarrow a^2=\frac{1}{2}\rightarrow a=\pm\frac{1}{\sqrt{2}}=\pm\frac{\sqrt{2}}{2}$ $\boxed{\text{C}}$

19. $9000+\int_0^5 2700e^{0.3t}dt=9000+2700\left[\frac{e^{0.3t}}{0.3}\right]_0^5=9000+9000\left[e^{0.3t}\right]_0^5=9000+9000\left(e^{1.5}-1\right)=9000e^{1.5}$ $\boxed{\text{C}}$

20. Average value of $f(x)$ on $[a,b]=\frac{1}{b-a}\int_a^b f(x)dx$.

$\frac{1}{7-3}\int_3^7 f(x)dx=12\rightarrow\frac{1}{4}\int_3^7 f(x)dx=12\rightarrow\int_3^7 f(x)dx=48$ $\boxed{\text{D}}$

21.

(x,y)	$\frac{dy}{dx}\bullet\Delta x=\Delta y$	$(x+\Delta x,y+\Delta y)$
$(2,3)$	$3\bullet\frac{1}{2}=\frac{3}{2}$	$\left(\frac{5}{2},\frac{9}{2}\right)$
$\left(\frac{5}{2},\frac{9}{2}\right)$	$\left(\frac{81}{4}-\frac{45}{4}\right)\bullet\frac{1}{2}=\frac{9}{2}$	$(3,9)$

$\therefore f(3)\approx 9$ $\boxed{\text{D}}$

22. $f'(x)=2x-5\sin x \to f''(x)=2-5\cos x$

$f''(x)$ changes signs where $2-5\cos x=0$, which means $\cos x=\frac{2}{5}$.

In every interval [a, 2π + a], there are two solutions, so in the interval [0, 2π*n*], there are 2*n* solutions. $\boxed{\text{D}}$

23. Implicit differentiation: $2y\frac{dy}{dx}-2x\frac{dy}{dx}-2y=0$

Substitute (2, −3): $-6\frac{dy}{dx}-4\frac{dy}{dx}+6=0 \to -10\frac{dy}{dx}+6=0 \to \frac{dy}{dx}=\frac{6}{10}=\frac{3}{5}$ $\boxed{\text{B}}$

24. If *f* changes from concave up to concave down, then f'' changes from positive to negative. $\boxed{\text{C}}$

25. If (*a*, *b*) is on $f(x)$, and $g(x)=f^{-1}(x)$, then $g'(b)=\frac{1}{f'(a)}$

$x^3-7x^2+25x-39=0 \to x=3$, and $f'(x)=3x^2-14x+25 \to f'(3)=27-42+25=10$

$\therefore g'(0)=\frac{1}{f'(3)}=\frac{1}{10}$ $\boxed{\text{B}}$

26. If the cone is point down, the radius and the height are in proportion.

$\frac{r}{h}=\frac{9}{15}=\frac{3}{5} \to r=\frac{3}{5}h$ $\qquad V_{\text{cone}}=\frac{1}{3}\pi r^2 h=\frac{1}{3}\pi\left(\frac{3}{5}h\right)^2 \bullet h=\frac{3}{25}\pi h^3$

$\frac{dV}{dt}=\frac{9}{25}\pi h^2 \bullet \frac{dh}{dt} \to 10=\frac{9}{25}\pi(10)^2 \bullet \frac{dh}{dt} \to \frac{dh}{dt}=\frac{10}{36\pi}\approx 0.088$ $\boxed{\text{B}}$

27. If *f* is differentiable at *x* = 0, then *f* is also continuous at *x* = 0.

$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)=f(0)$

$\lim_{x\to 0^-}\left(e^{-x}+2\right)=\lim_{x\to 0^+}(ax+b)=b \to 3=b$

If *f* is differentiable at *x* = 0, then $\lim_{x\to 0^-} f'(x)=\lim_{x\to 0^+} f'(x)$.

$\lim_{x\to 0^-}\left(-e^{-x}\right)=\lim_{x\to 0^+}(a) \to -1=a$ $\qquad \therefore a+b=-1+3=2$ $\boxed{\text{C}}$

28. *f* has a relative maximum at a point where f' changes from positive to negative. $\boxed{\text{B}}$

29. According to the FTC, $y(x)-y(2)=\int_2^x y'(t)\,dt$, so $y(x)=y(2)+\int_2^x f(t)\,dt=4+\int_2^x f(t)\,dt$ $\boxed{\text{B}}$

30. $f'(x)=24x^5-24x^2=24x^2\left(x^3-1\right)=24x^2(x-1)\left(x^2+x+1\right)$

$24x^2(x-1)\left(x^2+x+1\right)=0 \to x=0,1$ $\qquad f'(x)$ ←NEG | NEG | POS→ x (0, 1) $\boxed{\text{C}}$

31. Average value of $f(x)$ on $[a,b]=\frac{1}{b-a}\int_a^b f(x)\,dx$.

$\frac{1}{9-0}\int_0^9 \sqrt{3x}\,dx=\frac{\sqrt{3}}{9}\int_0^9 \sqrt{x}\,dx=\frac{\sqrt{3}}{9}\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^9=\frac{2\sqrt{3}}{27}(27-0)=2\sqrt{3}$ $\boxed{\text{B}}$

32. Average velocity on $[1,e] = \dfrac{x(e)-x(1)}{e-1} = \dfrac{\ln e - \ln 1}{e-1} = \dfrac{1-0}{e-1} = \dfrac{1}{e-1}$ $\boxed{\text{B}}$

33. Integration by Parts:

$u = 2x$	$dv = \cos(2x)$
2	$\frac{1}{2}\sin(2x)$
0	$-\frac{1}{4}\cos(2x)$

$$2x \bullet \frac{1}{2}\sin(2x) - 2 \bullet -\frac{1}{4}\cos(2x) = x\sin(2x) + \frac{1}{2}\cos(2x) + C$$ $\boxed{\text{D}}$

34. $|x+2| = \begin{cases} -(x+2), & x < -2 \\ x+2, & x \geq -2 \end{cases}$ $\quad \int_{-3}^{3} |x+2|\,dx = -\int_{-3}^{-2}(x+2)\,dx + \int_{-2}^{3}(x+2)\,dx$

$$= -\left[\frac{x^2}{2} + 2x\right]_{-3}^{-2} + \left[\frac{x^2}{2} + 2x\right]_{-2}^{3} = -\left(-2 - -\frac{3}{2}\right) + \left(\frac{21}{2} - -2\right) = \frac{1}{2} + \frac{25}{2} = 13$$ $\boxed{\text{C}}$

OR Graph the function and find the sum of the areas of the two triangles: $\frac{1}{2}(1)(1) + \frac{1}{2}(5)(5) = \frac{1}{2} + \frac{25}{2} = 13$

35. Solve for the x-intercepts of the parabola: $x^2 - 2kx = 0 \to x(x-2k) = 0 \to x = 0,\ 2k$

R is below the x-axis in Quadrant IV, which means $\int_0^{2k}\left(x^2 - 2kx\right)dx = -36$

$$-36 = \left[\frac{x^3}{3} - kx^2\right]_0^{2k} = \left(\frac{8k^3}{3} - 4k^3\right) - (0-0) = -\frac{4k^3}{3}$$

$$\frac{4k^3}{3} = 36 \to k^3 = 27 \to k = 3$$ $\boxed{\text{B}}$

36. If f is continuous on [−3, 7] and differentiable on (−3, 7), then the Mean Value Theorem states that there exists a c, $-3 < c < 7$, such that $f'(c) = \dfrac{f(7) - f(-3)}{7 - -3} = \dfrac{2-4}{10} = -\dfrac{1}{5}$ $\boxed{\text{C}}$

37. $f'(x) = 2x + 3$

$$h'(x) = f'(g(x)) \bullet g'(x) \to h'(2) = f'(g(2)) \bullet g'(2) = f'(1) \bullet g'(2) = 5 \bullet 3 = 15$$ $\boxed{\text{C}}$

38. $g'(x) = -\dfrac{1}{f^2(x)} \bullet f'(x) = -\dfrac{f'(x)}{f^2(x)} \to g'(2) = -\dfrac{f'(2)}{f^2(2)} = -\dfrac{-4}{(-8)^2} = \dfrac{4}{64} = \dfrac{1}{16}$ $\boxed{\text{B}}$

39. $y' = 2\cos(2x) \to y'' = -4\sin(2x) \to y''' = -8\cos(2x) \to y^{(4)} = 16\sin(2x) = 2^4\sin(2x)$

This cyclic pattern repeats itself every 4 derivatives, so $y^{(20)} = 2^{20}\sin(2x)$ $\boxed{\text{B}}$

40. If $f(x) = 15 - g(x)$ for $-2 \leq x \leq 2$, then

$$\int_{-2}^{2}\left[f(x) - g(x)\right]dx = \int_{-2}^{2}\left[15 - 2g(x)\right]dx = \int_{-2}^{2} 15\,dx - 2\int_{-2}^{2} g(x)\,dx$$

$$= 15(2 - -2) - 2\int_{-2}^{2} g(x)\,dx = 60 - 2\int_{-2}^{2} g(x)\,dx$$ $\boxed{\text{D}}$

41. $2x+2y\frac{dy}{dx}=0\to\frac{dy}{dx}=-\frac{x}{y}$ $\quad m_{\tan}=\left.\frac{dy}{dx}\right|_{(5,-12)}=-\frac{5}{-12}=\frac{5}{12}$

Equation$_{\tan}$: $y--12=\frac{5}{12}(x-5)\to y+12=\frac{5}{12}(x-5)\to 12y+144=5(x-5)\to 5x-12y=169$ $\boxed{\text{C}}$

42. $f(x)=\begin{cases}1+e^{-x}, & 0\le x\le 5\\ 1+e^{x-10}, & 5<x\le 10\end{cases}$ $\quad\lim_{x\to 5^-}\left(1+e^{-x}\right)=1+e^{-5}$ $\quad\lim_{x\to 5^+}\left(1+e^{x-10}\right)=1+e^{-5}$ $\quad f(5)=1+e^{-5}$

$f(x)$ is continuous on [0, 10], because $\lim_{x\to 5^-}f(x)=\lim_{x\to 5^+}f(x)=f(5)$, and both functions are continuous at all other values of *x*, so I is true.

$f'(x)=\begin{cases}-e^{-x}, & 0\le x\le 5\\ e^{x-10}, & 5<x\le 10\end{cases}$ $\quad\lim_{x\to 5^-}\left(-e^{-x}\right)=-e^{-5}$ $\quad\lim_{x\to 5^+}\left(e^{x-10}\right)=e^{-5}$ $\quad\lim_{x\to 5}f'(x)\,\text{DNE}$, so II is false.

$f''(x)=\begin{cases}e^{-x}, & 0\le x\le 5\\ e^{x-10}, & 5<x\le 10\end{cases}$ III is true.

$f''(x)>0$ for all values of *x*, which means that *f* is concave up for all values of *x*. $\boxed{\text{C}}$

43. $v(0)=1>0$, which means the particle is moving to the right.

$a(t)=R(t)=-\sin(t)-\sqrt{3}\cos(t)\to a(0)=R(0)=-\sin(0)-\sqrt{3}\cos(0)=-\sqrt{3}<0$

The speed of the particle is decreasing at t = 0, because $v(0)$ and $a(0)$ have different signs. $\boxed{\text{A}}$

44. $\frac{d}{dx}\left[\sin^{-1}(u)\right]=\frac{u'}{\sqrt{1-u^2}}$

$$\frac{d}{dx}\left[\sin^{-1}\left(\frac{3x}{4}\right)\right]=\frac{\frac{3}{4}}{\sqrt{1-\left(\frac{3x}{4}\right)^2}}=\frac{\frac{3}{4}}{\sqrt{1-\frac{9x^2}{16}}}=\frac{\frac{3}{4}}{\sqrt{\frac{16-9x^2}{16}}}=\frac{\frac{3}{4}}{\frac{\sqrt{16-9x^2}}{4}}=\frac{3}{\sqrt{16-9x^2}}$$ $\boxed{\text{B}}$

45. In the logistic differential equation, the horizontal asymptotes occur where $\frac{dy}{dx}=0$.

$y\left(8-\frac{y}{1000}\right)=0\to y=0, y=8000$ $\boxed{\text{D}}$

46. $v(t)=2\to 2e^{2t}=2\to e^{2t}=1\to 2t=0\to t=0$ $\quad v(t)=4\to 2e^{2t}=4\to e^{2t}=2\to 2t=\ln 2\to t=\frac{1}{2}\ln 2$

If $v(t)>0$, then distance on [a, b] $=\int_a^b v(t)\,dt=\int_0^{\frac{1}{2}\ln 2}2e^{2t}\,dt=\left[e^{2t}\right]_0^{\frac{1}{2}\ln 2}=e^{\ln 2}-e^0=2-1=1$ $\boxed{\text{A}}$

47. According to the slope field, $\frac{dy}{dx}>0$ for $x<0$, which means $f(x)$ is increasing, so I is false.

$\frac{dy}{dx}>0$ for $x>0$, which means $f(x)$ is increasing, so II is true.

$\frac{dy}{dx}=0$ for $x=0$, which means $f(x)$ levels off, so III is true. $\boxed{\text{B}}$

48. Point A, f is decreasing and concave up: $f'(x)<0$ and $f''(x)>0 \rightarrow f'(x)<f''(x)$

Point B, f is increasing and concave down: $f'(x)>0$ and $f''(x)<0 \rightarrow f'(x)>f''(x)$

Point C, f has a relative maximum : $f'(x)=0$ and $f''(x)<0 \rightarrow f'(x)>f''(x)$ $\boxed{\text{A}}$

49. f has a point of inflection at an x-value where f'' changes signs.

$$f''(x)=\left(x^2-4\right)^4 \bullet 2x-2x=2x\left[\left(x^2-4\right)^4-1\right]=2x\left[\left(x^2-4\right)^2+1\right]\left[\left(x^2-4\right)^2-1\right]$$

$$=2x\left[\left(x^2-4\right)^2+1\right]\left[\left(x^2-4\right)+1\right]\left[\left(x^2-4\right)-1\right]$$

$2x\left[\left(x^2-4\right)^2+1\right]\left[\left(x^2-4\right)+1\right]\left[\left(x^2-4\right)-1\right]=0 \rightarrow x=0,\pm\sqrt{3},\pm\sqrt{5}$ (or use your graphing calculator)

f'' changes signs at all 5 points, so there are 5 points of inflection. $\boxed{\text{D}}$

50. $f'(x)=7-2x \rightarrow 7-2x=3 \rightarrow x=2$ $\qquad f(2)=7(2)-(2)^2=10$

$\text{Equ}_{\tan}: y-10=3(x-2) \rightarrow y=3x+4$ $\boxed{\text{A}}$

51. $f'(x)>0$ for all x, which means that $f(x)$ is increasing for all x. $f'(0)=1$, which eliminates A and B. $f'(x)$ changes from increasing to decreasing at $x=0$, which means that $f(x)$ is concave up, and then concave down with a point of inflection at $x=0$, which eliminates C. $\boxed{\text{D}}$

52. f attain its absolute maximum value either at a critical point where f' changes from positive to negative or at an endpoint of the interval [0, 10].

According to the FTC, $f'(t)=e^{t\cos(t)}\left(\cos(t)-t\sin(t)\right)$. Using a graphing calculator (ZoomFit helps), there are two values of t where f' changes from positive to negative: $t=0.860$ and $t=6.437$.

The area under the curve from $t=0$ to $t=6.437$ is much larger than the area from $t=0$ to $t=0.860$, so the absolute maximum occurs at $t=6.437$. Neither of the two endpoints are possible solutions. $\boxed{\text{C}}$

53. $\dfrac{dy}{y}=\dfrac{2dt}{t(t+2)}$

$$\frac{2}{t(t+2)}=\frac{A}{t}+\frac{B}{t+2} \rightarrow 2=A(t+2)+Bt=At+2A+Bt$$

$$2A=2 \rightarrow A=1 \qquad A+B=0 \rightarrow B=-A=-1$$

$$\int\frac{dy}{y}=\int\frac{2dt}{t(t+2)}=\int\frac{dt}{t}-\int\frac{dt}{t+2}$$

$$\ln|y|=\ln|t|-\ln|t+2|+C$$

$$(1,1): \ln|1|=\ln|1|-\ln|3|+C \rightarrow C=\ln 3$$

$$\ln|y|=\ln|t|-\ln|t+2|+\ln 3$$

$$|y|=e^{\ln|t|-\ln|t+2|+\ln 3}=\frac{e^{\ln|3t|}}{e^{\ln|t+2|}}=\left|\frac{3t}{t+2}\right|$$

$$y=1>0, t=1>0 \rightarrow y=\frac{3t}{t+2} \rightarrow y(2)=\frac{6}{4}=\frac{3}{2} \qquad \boxed{\text{D}}$$

54. The area of the square is 4.

The area under the graph of $y = -x^2 + 2x$ on [0, 2] is $\int_0^2 \left(-x^2 + 2x\right) dx = \left[-\frac{x^3}{3} + x^2 \right]_0^2 = \left(-\frac{8}{3} + 4 \right) - (0 + 0) = \frac{4}{3}$

The area above the parabola is $4 - \frac{4}{3} = \frac{8}{3} \rightarrow \frac{8/3}{4} = \frac{2}{3}$ $\boxed{\text{B}}$

55. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{6xh + 4h^2}{h} = \lim_{h \to 0} (6x + 4h) = 6x$

$f(x) = f(2) + \int_2^x 6t\, dt \rightarrow f(1) = 5 + \int_2^1 6x\, dx = 5 - \int_1^2 6x\, dx = 5 - \left[3x^2\right]_1^2 = 5 - (12 - 3) = -4$ $\boxed{\text{A}}$

56. $\frac{dC}{dt} = +0.5$ $\qquad C = 2\pi r \rightarrow \frac{dC}{dt} = 2\pi \bullet \frac{dr}{dt} \rightarrow 0.5 = 2\pi \bullet \frac{dr}{dt} \rightarrow \frac{dr}{dt} = \frac{0.5}{2\pi}$

$A = \pi r^2 \rightarrow \frac{dA}{dt} = 2\pi r \bullet \frac{dr}{dt} = 2\pi(4) \bullet \frac{0.5}{2\pi} = 2$ $\boxed{\text{D}}$

57. According to the FTC, $g'(x) = \frac{d}{dx} \int_3^x \left(\left(5 + 4t - t^2\right)\left(2^{-t}\right) \right) dt = \left(5 + 4x - x^2\right)\left(2^{-x}\right) = -(x-5)(x+1)\left(2^{-x}\right)$

$g' > 0$ on $(3, 5)$, which means that g is increasing on [3, 5], so I is true.

$g' < 0$ on $(5, 7)$, which means that g is decreasing on [5, 7], so II is false.

$g(7) = \int_3^7 \left(\left(5 + 4x - x^2\right)\left(2^{-x}\right) \right) dx = 0.562$, so III is false. $\boxed{\text{A}}$

58. Euler's Method with one step is the same as the tangent line approximation.

$y(2.7) \approx -2 + \frac{9}{2} \bullet -0.3 = -2 - 1.35 = -3.35$ $\boxed{\text{D}}$

Section II−ANSWER KEY: Scoring Guidelines can be found at
https://apcentral.collegeboard.org/courses/ap-calculus-bc/exam/past-exam-questions?course=ap-calculus-bc

1. 2018 AP Calculus BC Exam #4

2. 2011 AP Calculus BC Exam #4

3. 2005 (Form B) AP Calculus BC Exam #5

4. 2013 AP Calculus BC Exam #5