SUPPLEMENTAL MATERIAL: COMPRESSIVE SIGNAL RECOVERY UNDER SENSING MATRIX ERRORS COMBINED WITH UNKNOWN MEASUREMENT GAINS

Jian Vora[1], Ajit Rajwade[2]

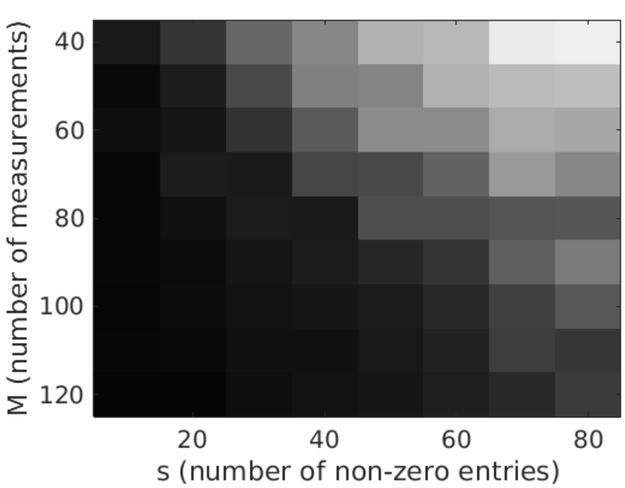
Department of Electrical Engg., IIT Bombay[1]
Department of Computer Science and Engg., IIT Bombay[2]

1. EMPIRICAL COMPARISONS WITH DIFFERENT BASELINES

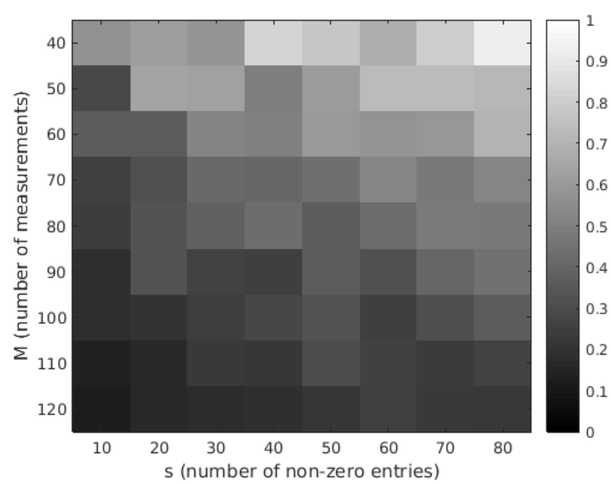
We compare the results of joint calibration of calibration errors with three baselines namely:

- **Baseline 1:** Calibration of only the frequency perturbations $\boldsymbol{\delta}$ by ignoring the gains
- **Baseline 2:** Calibration of only the gain perturbations $\boldsymbol{g}$ by ignoring the frequency perturbations
- **Baseline 3:** Signal reconstruction without taking any perturbations into consideration. *This is the only baseline shown un the main paper.*

The above baselines are implemented using the same SQ-LASSO estimator proposed but by alternating over only the signal and the frequency perturbations or gains in case of Baseline 1 and Baseline 2 respectively. The results presented here are for the *third experiment in the main paper*, i.e. for the case when the signal $\boldsymbol{x} \in \mathbb{R}^{128}$ is sparse in the Haar wavelet basis, with $P = 5, Q = M, r_\beta = 0.1, r_g = 0.2, f_n = 0.02$. This is in accordance with the same notation as introduced in the main paper. We clearly perform better than the naive baselines using the three-way alternating minimisation algorithm. Fig. 1 corroborates our claim that both the sources of errors are important to be calibrated jointly for a faithful reconstruction.

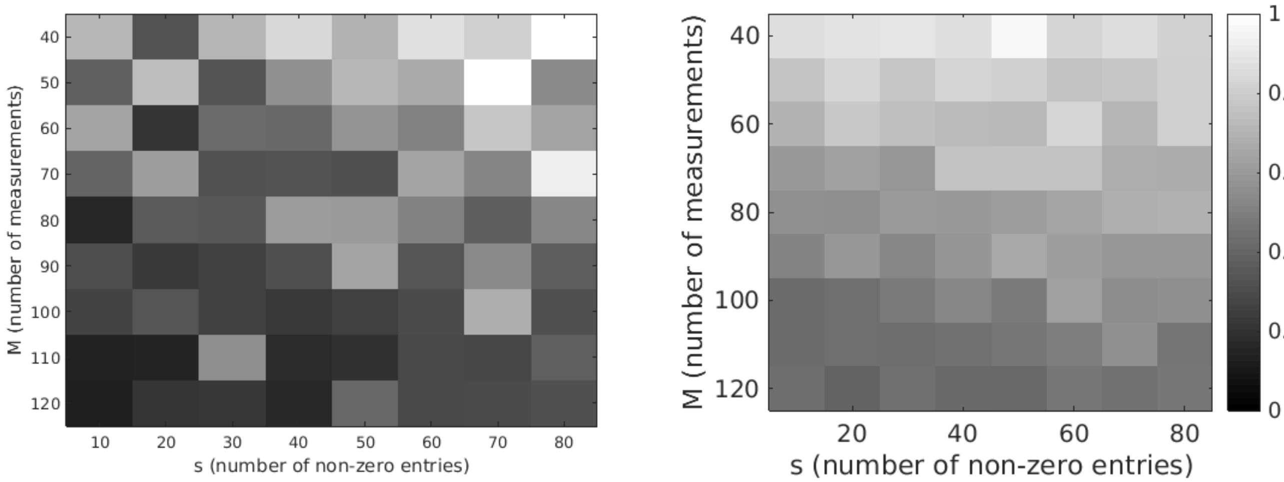


(a)



(b)

Email ids: jianvora@iitb.ac.in , ajitvr@cse.iitb.ac.in. AR thanks SERB Grant #10013890.



(c) (d)

Fig. 1. NE values vs. $(M, \|\boldsymbol{x}\|_0)$ for CS recon. with $P = 5, Q = M$ using (a) Algorithm 1, (b) Baseline 1, (c) Baseline 2, (d) Baseline 3. The signal is sparse in Haar wavelet basis.

2. VARIATION OF ϵ WITH FREQUENCY PERTURBATION VALUES

In this section, we analyze how $\epsilon_1 \triangleq \max_k \frac{\|\boldsymbol{E_k}\|_2}{\|\boldsymbol{A_k}\|_2}$ varies with the value of r_β where $\boldsymbol{E_k} \triangleq \boldsymbol{F^{(k)}}\boldsymbol{\Psi}(\boldsymbol{\Psi}^H\boldsymbol{\Delta_k}\boldsymbol{\Psi}\boldsymbol{\Delta_k}^H - \boldsymbol{I}))$ and $\boldsymbol{A_k} \triangleq \boldsymbol{F^{(k)}}\boldsymbol{\Psi}$. Thus we can make the following comments for ϵ_1,

$$\epsilon_1 = \max_k \frac{\|\boldsymbol{E_k}\|_2}{\|\boldsymbol{A_k}\|_2} = \max_k \frac{\|\boldsymbol{F^{(k)}}\boldsymbol{\Psi}(\boldsymbol{\Psi^H}\boldsymbol{\Delta_k}\boldsymbol{\Psi}\boldsymbol{\Delta_k}^{\boldsymbol{H}} - \boldsymbol{I}))\|_2}{\|\boldsymbol{F^{(k)}}\boldsymbol{\Psi}\|_2} \leq \max_k \|\boldsymbol{\Psi}^H\boldsymbol{\Delta_k}\boldsymbol{\Psi}\boldsymbol{\Delta_k}^H - \boldsymbol{I}\|_2 \tag{1}$$

If we assume that the frequency perturbations are not large enough, then we can apply Taylor series as the first order approximation to the quantity $\boldsymbol{\Delta_k}$. We use the following approximation that $e^{-j2\pi\delta l/N} \approx 1 - jsin(2\pi\delta l/N)$ where $j \triangleq \sqrt{-1}$. Thus using this, we can rewrite $\boldsymbol{\Delta_k} \approx \boldsymbol{I} - \boldsymbol{Z_k}$ where $\boldsymbol{Z_k} \triangleq \text{diag}(-jsin(2\pi\delta_k l/N))$ where δ_k is the frequency perturbation for those set of measurements which are indexed by k. Similary, we can say that $\boldsymbol{\Delta_k^H} \approx \boldsymbol{I} + \boldsymbol{Z_k}$ where $\boldsymbol{A}^H$ is the complex conjugate transpose of any matrix $\boldsymbol{A}$. Thus on substituting these approximations in eq. 1, we get -

$$\epsilon_1 \leq \max_k \|\boldsymbol{\Psi}^H(\boldsymbol{I} - \boldsymbol{Z_k})\boldsymbol{\Psi}(\boldsymbol{I} + \boldsymbol{Z_k}) - \boldsymbol{I}\|_2 \leq \max_k \|\boldsymbol{\Psi}^H\boldsymbol{Z_k}\boldsymbol{\Psi}\boldsymbol{Z_k}\|_2 \leq \max_k \sin^2(2\pi\delta_k) = \sin^2(2\pi r_\beta) \tag{2}$$

Empirically, we see how ϵ_1 varies with r_β. For this we take as a Haar wavelet basis and $\boldsymbol{F}$ has 70 rows for a signal $\boldsymbol{x}$ of length 128. The value of r_β is varied from 0 to 0.9 and the value of ϵ_1 is plotted on the y-axis.

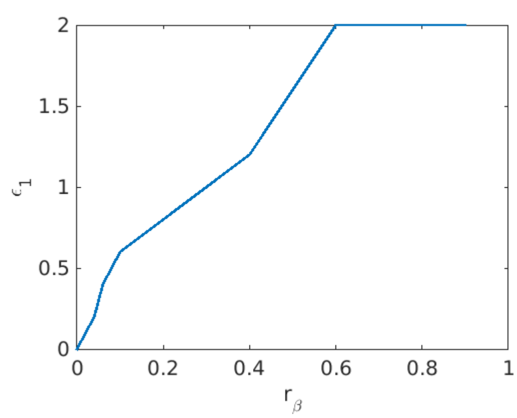


Fig. 2. Variation of ϵ_1 with r_β

3. GMMV RECOVERY UNDER NOISE

After modelling our framework as the perturbed GMMVproblem, we invoke bounds from [1] and [2] to prove theoretical bounds on the recovery of $\boldsymbol{z_k}$s. Specifically, [1] consider the following model for signal acquisition -

$$\boldsymbol{y} = (\boldsymbol{A} + \boldsymbol{E})\boldsymbol{x} + \boldsymbol{\eta} \tag{3}$$

In 3, $\boldsymbol{E}$ and $\boldsymbol{\eta}$ is unknown and that if $\frac{\boldsymbol{E^{(k)}}}{\boldsymbol{A}^{(k)}} \leq \epsilon_{\boldsymbol{A}}^{(K)}$, it is shown that the bounds on the signal recovery are same as that of [3] if $\boldsymbol{A}$ follows RIP with a constant $\delta_{2K}^{'}$ as opposed to the unperturbed CS recovery which needs $\delta_{2K} < \sqrt{2} - 1$ where

$$\delta_{2K}^{'} < \frac{\sqrt{2}}{(1 + \epsilon_{\boldsymbol{A}}^{(K)})^2} - 1 \tag{4}$$

We use the above idea in the GMMV framework, where we have d signals $\boldsymbol{x_i}, 1 \leq i \leq d$ where each signal is observed through different sensing matrices $\boldsymbol{A_i}$. Theorem 24 in [2] states that for all these signals $\boldsymbol{x_i}$ stacked together as a single vector $\boldsymbol{x} \in \mathbb{R}^{dn}$ and with the effective sensing matrix $\boldsymbol{A} \in \mathbb{R}^{dm \times dn}$ to get the forward sensing system as $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{e}$, then, if $\boldsymbol{A}$ satisfies RIP with a constant δ and $\|\epsilon\|$ is of the order of $\mathcal{O}(\sqrt{d}(1-\delta))$, then with the signal recovery follows the following bound with large probabiility exponential in the number of signals d using the block lasso estimator:

$$\|\boldsymbol{x} - \boldsymbol{x}^{*}\|_2 \leq \epsilon \frac{\delta + 4c_1\sqrt{s}}{1 - \delta} \tag{5}$$

where c_1 is a constant dependent on measurement matrix $\boldsymbol{A}$. Our case is a combination of the above two problems where we have signal dependent noise in the GMMV framework. In our case, we have $d = T$ and for each signal $\boldsymbol{z_k}, 1 \leq k \leq T$, we take around M/T measurements for each signal, thus our matrix $\boldsymbol{A} \in \mathbb{R}^{M \times TN}$. Define $\boldsymbol{z} \triangleq [z_1; z_2; ...z_T]$From [1], we can find the 'effective' RIC of the matrix $\boldsymbol{A}$ with perturbations as $\delta^{'} = (1 + \delta)(1 + \epsilon_1)^2 - 1$ where ϵ_1 was defined in Section 2. We define $\epsilon^{'} = \|Ez + \eta\|_2$, then with high probability,

$$\|\boldsymbol{z} - \boldsymbol{z}^{*}\|_2 \leq \epsilon^{'} \frac{\delta^{'} + 4c_1\sqrt{s}}{1 - \delta^{'}} \tag{6}$$

4. REFERENCES

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