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The Hamilton neural network model

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Abstract

In this paper, the Dirac symbol is used to represent a neural network, and a discrete Hamilton neural network model with a 16-state ($\pm 1 \pm \mathbf{i} \pm \mathbf{j} \pm \mathbf{k}$) neuron has been presented. By using signal-to-noise theory and computer numerical simulation, the stability, the storage capacity and the error correction ability of the model are analysed. The storage capacity ratio of the presented model equals that of the Hopfield model. This 16-state neural network can be applied to recognize 16-level gray or color patterns.

1. Introduction

During the last few years, a lot of attention has been focussed on neural network models of associative or content-addressable memory. The Hopfield model is one of the best known neural network models of associative memory [1]. The properties of the Hopfield model have been investigated extensively by using statistical mechanics methods and by numerical simulations [2–6]. Most of the properties of this model are fairly well understood now. During recent years, extensive research has also been carried out on the development and analysis of associative memory models which are based on the same general principle as the Hopfield model, but go beyond the Hopfield model in different directions. There are a number of different motivations behind these efforts [7–14]. One of these is to assume each neuron to be a grey-level neuron in order to represent the distributed processing of grey-tone data [11–14]. Rieger et al. suggested a Q-state neural network model [11] in which the input signal is divided into Q intervals. The Q-state Potts-glass model of neural networks is discussed too [12–14]. By defining the Potts Hamiltonian in different ways, different representations of the Potts spins are used. One way is to define the neuron q unit vectors, pointing in q directions, which span a hypertetrahedron in $\mathbb{R}^{q-1}$. Another way is to take the neuron for the q points on the unit circle in the complex plane.

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In this paper, a 16-level neural network model is suggested by using the Hamilton number [15] in the network. This 16-state neural network can be applied to recognize 16-level gray or color patterns, where the 16-level color patterns are widely used in the computer. The paper is organized as follows. In Section 2 a simple introduction about the Hamilton algebra is presented. Section 3 is devoted to set up a Hamilton neural network after introducing the Dirac symbol representation for the Hopfield model. In the model, each neuron is assumed to be a 16-state one which has the Hamilton-number value ($\pm 1 \pm \mathbf{i} \pm \mathbf{j} \pm \mathbf{k}$). In Section 4 a signal-to-noise theory is used to analyse the stability and the storage capacity of the model. Then in the next section we discuss the computer numerical simulations about the storage capacity and the error correction ability of the model. Its application for recognizing the 16-level color patterns is also discussed simply. In the Conclusion, further generalizations are suggested.

2. The Hamilton algebra

We are familiar with the natural, integral, real and complex numbers. However, a kind of multi-dimensional numbers are defined in mathematics, i.e., 2^n-element numbers [15]. For $n = 0$ and 1, it stands for the real and complex numbers; for $n = 2, 3$ and 4, it stands for the four-element (the Hamilton), eight-element (the Cayley) and sixteen-element (the Clifford) numbers, respectively.

Assume the Hamilton number $Q(\mathbb{R}) = \{\alpha : \alpha = a + b\mathbf{i} + c\mathbf{j} + d\mathbf{k},\ a, b, c, d \in \mathbb{R}\}$, where $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are the three basis unit vectors of the Hamilton number. The addition between the Hamilton numbers is defined as usual, and also the multiplication between the real number and the Hamilton number. The multiplication between the Hamilton numbers is defined to expand the following equation by using the distribution law:

$$\begin{aligned}&(a_0 + a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k})(b_0 + b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k})\\ &\quad = (a_0b_0 - a_1b_1 - a_2b_2 - a_3b_3) + (a_0b_1 + a_1b_0 + a_2b_3 - a_3b_2)\mathbf{i}\\ &\qquad + (a_0b_2 - a_1b_3 + a_2b_0 + a_3b_1)\mathbf{j} + (a_0b_3 + a_1b_2 - a_2b_1 + a_3b_0)\mathbf{k}. \qquad (1)\end{aligned}$$

Here, the following multiplication table of the basic unit vectors has been used:

$$\begin{aligned}&\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = -1, \qquad \mathbf{ij} = -\mathbf{ji} = \mathbf{k},\\ &\mathbf{jk} = -\mathbf{kj} = \mathbf{i}, \qquad \mathbf{ki} = -\mathbf{ik} = \mathbf{j}. \qquad (2)\end{aligned}$$

From Eq. (2), we know that the Hamilton number does not obey the exchange law of the multiplication, but obeys the combination law, i.e., $(\alpha\beta)\gamma = \alpha(\beta\gamma)$. It is easily demonstrated that $Q(\mathbb{R})$ is a discommutative body and $Q(\mathbb{R}) - \{0\}$ is a non-Abelian multiplicative group.

From the above multiplication table, a similar multiplication between the basis unit vectors of the Hamilton number and the Pauli matrixes can be easily looked out. By using the 2×2 unit matrix E_2 and the Pauli matrixes: σ_x, σ_y, σ_z, a matrix realization of the Hamilton algebra can be obtained:

$$\mathbf{1} = E_2, \qquad \mathbf{i} = -\mathrm{i}\sigma_x,$$
$$\mathbf{j} = -i\sigma_y, \qquad \mathbf{k} = -\mathrm{i}\sigma_z. \tag{3}$$

For a Hamilton number $\alpha = a + b\mathbf{i} + c\mathbf{j} + d\mathbf{k}$, its conjugate number is defined as $\alpha^* = a - b\mathbf{i} - c\mathbf{j} - d\mathbf{k}$. So its modulate is $\alpha\alpha^* = a^2 + b^2 + c^2 + d^2 = |\alpha|^2$.

The property of the discommutativity of the Hamilton number can be exactly used to describe the rotation of a rigid body around a fixed point. The rotation $R(n, \theta)$, with its rotation axis n, its direction cosine $(\cos\delta_1, \cos\delta_2, \cos\delta_3)$ and its rotation angle θ, can be expressed as follows:

$$Q(n, \theta) = \cos(\theta/2) + \sin(\theta/2)(\cos(\delta_1\mathbf{i}) + \cos(\delta_2\mathbf{j}) + \cos(\delta_3\mathbf{k})). \tag{4}$$

If we want to calculate the total rotation $R(n, \theta)$, i.e., the rotation axis n and angle θ, that rotated $R(n_2, \theta_2)$ after the rotation $R(n_1, \theta_1)$, we can multiply $Q(n_1, \theta_1)$ with $Q(n_2, \theta_2)$ and express it as Eq. (4). Then the angles θ and δ can be drawn out. According to the corresponding relationship between the complex number and the plane rotation, a lot of calculation rules, such as the inverse, square and square root of a complex number can be defined. Similar, according to the corresponding relationship between the Hamilton number and the rigid body rotation around a fixed point, calculation rules of the Hamilton number can be defined, too. Such as the inverse $1/\alpha = \alpha^*/|\alpha|^2$.

3. The Hamilton neural network model

We recast the well-known Hopfield model of associative memory for the sake of completeness. For theoretical convenience, the Dirac symbol is employed to symbolize the model. Consider M patterns S^μ of N bits stored in the memory of the network. If a pattern S is regarded as a state vector and S^μ the basic state vector, then the synaptis J is a matrix constructed from the basic vector. The local field JS in the dynamical equation represents the action of the matrix on the state vector. A similar case can be found in the matrix representation of quantum mechanics. Based on this similarity, the Dirac symbol can be introduced into the description of the neural network, namely, we represent the state vector with $S(t) = |t\rangle$ and the basis vector with $S^\mu = |\mu\rangle$. Thereby, the connection matrix is expressed by the sum of the projection operators

$$J = |1\rangle\langle 1| + |2\rangle\langle 2| + \cdots + |M\rangle\langle M| = \sum_{\mu=1}^{M} |\mu\rangle\langle\mu|\,. \tag{5}$$

Here the self-interaction of each neuron is taken into account. If the threshold function $\Theta\{\ \}$ is replaced with the sigmoid operator Θ, the dynamical equation is written as follows:

$$|t+1\rangle = \Theta J|t\rangle. \tag{6}$$

Now we introduce the Hamilton number into the neural network so as to form a discrete Hamilton neural network. In the model, each neuron is assumed to be a 16-state one which has one of the following values:

$$(\pm 1\ \pm \mathbf{i}\ \pm \mathbf{j}\ \pm \mathbf{k}).$$

Suppose there are N neurons and M patterns $S^{\mu} = |\mu\rangle$ $(\mu = 1, 2, \dots, M)$ stored in the network. The Hamilton connection matrix operator is also given by Eq. (5). In quantum mechanics, if $|\mu\rangle$ is a basis vector, then $\langle\mu|$ is a conjugate basic vector. In real space (e.g., the Hopfield model), there is no difference between the original space and the conjugate space; but in the Hamiltonian space, the original space and the conjugate space are not identical. As a result, it looks natural to build a conjugate-type space for the connection matrix. The matrix elements of the synaptic interconnection operator is

$$J_{mn} = \sum_{\mu=1}^{M} S_m^{\mu}(S_n^{\mu})^{*}. \tag{7}$$

Clearly, unlike the Hopfield model, the synaptic matrix is asymmetric, and because of the unexchangable property of the Hamilton number one can see that $J_{mn} \neq J_{nm}^{*}$.

Although the dynamical operator equation is the same as Eq. (6) apparently, the sigmoid operator Θ is indeed the generalized Hamilton sigmoid function. The Hamilton operating rule of the Θ is as follows: whenever a real or imaginary component of α is no-negative, a positive unit is drawn out for the corresponding component of $\Theta\alpha$; otherwise, a negative unit drawn out.

Each component of the Hamiltonian state vector $|S\rangle$ has one real part and three imaginary parts. So the modulus of $|S\rangle$ is $\langle S|S\rangle = 4N$. Due to the random nature of their storage states, the basis vectors are pseudo-orthogonal with each other, i.e., $\overline{\langle\mu|\nu\rangle} \approx 0$, and the mean square error is $\overline{\langle\mu|\nu\rangle^2} \approx \sqrt{4N}$ for $\mu \neq \nu$.

In the following section, a simple analysis of the Hamilton neural network based on signal-to-noise theory is presented.

4. The signal-to-noise theory analysis

To analyze the stability of the Hamilton dynamic system, the pattern $|t_0\rangle = |\nu\rangle$ is put into the dynamic system and the dynamic equation Eq. (6) can be expressed as

$$|\nu'\rangle = \Theta J|\nu\rangle = \Theta\left(4N|\nu\rangle + \sum_{\mu\neq\nu} |\mu\rangle(\langle\mu|\nu\rangle)\right). \tag{8}$$

Assume the stored pattern to be $|\mu\rangle = a^\mu + b^\mu + c^\mu + d^\mu$ $(a,b,c,d = \pm 1)$. Eq. (8) can be expanded in terms of each real and imaginary parts respectively as follows:

$$\begin{aligned} a_m^{\nu'} &= \Theta(4Na_m^\nu + \Delta_0), \qquad b_m^{\nu'} = \Theta(4Nb_m^\nu + \Delta_i), \\ c_m^{\nu'} &= \Theta(4Nc_m^\nu + \Delta_j), \qquad d_m^{\nu'} = \Theta(4Nd_m^\nu + \Delta_k), \end{aligned} \tag{9}$$

where

$$\begin{aligned} \Delta_0 = \sum_{\mu\neq\nu} \Big[& a_m^\mu \sum (a_n^\mu a_n^\nu + b_n^\mu b_n^\nu + c_n^\mu c_n^\nu + d_n^\mu d_n^\nu) \\ & + b_m^\mu \sum (b_n^\mu a_n^\nu - a_n^\mu b_n^\nu + c_n^\mu d_n^\nu - d_n^\mu c_n^\nu) \\ & + c_m^\mu \sum (c_n^\mu a_n^\nu - a_n^\mu c_n^\nu + d_n^\mu b_n^\nu - b_n^\mu d_n^\nu) \\ & + d_m^\mu \sum (d_n^\mu a_n^\nu - a_n^\mu d_n^\nu + b_n^\mu c_n^\nu - c_n^\mu b_n^\nu) \Big], \\ \Delta_i = \sum_{\mu\neq\nu} \Big[& a_m^\mu \sum (a_n^\mu b_n^\nu - b_n^\mu a_n^\nu - c_n^\mu d_n^\nu + d_n^\mu c_n^\nu) \\ & + b_m^\mu \sum (b_n^\mu b_n^\nu + c_n^\mu c_n^\nu + d_n^\mu d_n^\nu + a_n^\mu a_n^\nu) \\ & + c_m^\mu \sum (c_n^\mu b_n^\nu - b_n^\mu c_n^\nu - d_n^\mu a_n^\nu + a_n^\mu d_n^\nu) \\ & + d_m^\mu \sum (d_n^\mu b_n^\nu - b_n^\mu d_n^\nu - a_n^\mu c_n^\nu + c_n^\mu a_n^\nu) \Big], \\ \Delta_j = \sum_{\mu\neq\nu} \Big[& a_m^\mu \sum (a_n^\mu c_n^\nu + b_n^\mu d_n^\nu - c_n^\mu a_n^\nu - d_n^\mu b_n^\nu) \\ & + b_m^\mu \sum (b_n^\mu c_n^\nu - a_n^\mu d_n^\nu + d_n^\mu a_n^\nu - c_n^\mu b_n^\nu) \\ & + c_m^\mu \sum (c_n^\mu c_n^\nu + d_n^\mu d_n^\nu + a_n^\mu a_n^\nu + b_n^\mu b_n^\nu) \\ & + d_m^\mu \sum (d_n^\mu c_n^\nu - c_n^\mu d_n^\nu - b_n^\mu a_n^\nu + a_n^\mu b_n^\nu) \Big], \\ \Delta_k = \sum_{\mu\neq\nu} \Big[& a_m^\mu \sum (a_n^\mu d_n^\nu - b_n^\mu c_n^\nu + c_n^\mu b_n^\nu - d_n^\mu a_n^\nu) \\ & + b_m^\mu \sum (b_n^\mu d_n^\nu + a_n^\mu c_n^\nu - d_n^\mu b_n^\nu - c_n^\mu a_n^\nu) \\ & + c_m^\mu \sum (c_n^\mu d_n^\nu - d_n^\mu c_n^\nu - a_n^\mu b_n^\nu + b_n^\mu a_n^\nu) \\ & + d_m^\mu \sum (a_n^\mu a_n^\nu + b_n^\mu b_n^\nu + c_n^\mu c_n^\nu + d_n^\mu d_n^\nu) \Big]. \end{aligned} \tag{10}$$

Clearly, for every equation in Eq. (9), the first term is the signal while the second term Δ is the noise. One can find that the value $4N$ is the signal weight, i.e., the non-normalized probability that the state S^ν is a fixed point. Each factor Δ expressed by Eq. (10) is a sum of $4(M-1)$ components of the stored patterns, each of which has a weight equal to the inside summation of $4N$ terms. Owing to the random character of the stored patterns and their independence to each other, it is reasonable to suppose that the noise weight, i.e., the term Δ, is governed by a Gaussian distribution with expectation value zero and standard deviation value $\sqrt{16N(M-1)}$. Viewed from the signal-to-noise theory, the signal-noise-ratio SNR of the real or three imaginary parts of each component of S^ν can be obtained,

$$\mathrm{SNR} = \sqrt{\frac{N}{M-1}}. \tag{11}$$

When the number of the neurons is much larger than that of the stored patterns, i.e., $N \gg M$, then $\mathrm{SNR} \gg 1$. Hence, the neural network converges to the stored patterns S^ν. If a pattern S, very close to the stored pattern S^ν, is put into the network, the thereinbefore conclusion basically holds true. Therefore, the pattern S automatically converges to the pattern S^ν after one or more retrieval processes. In conclusion, the stored pattern S^ν is a stable attractor of the Hamilton neural network.

The storage capacity of the network is mainly assessed by SNR. Because the SNR of the Hamilton neural network model equals to that of the Hopfield model, the storage capacity of the present model is on the same level as that of the Hopfield model.

Now we look at the storage capacity ratio $\mathrm{SCR} = M/N \approx 1/\mathrm{SNR}^2$ of the present model with the thermodynamic limit in which $N, M \to \infty$ and SCR finite. Without loss of generality, assume that the real component $\mathrm{Re}(S_m^\nu) = 1$, then the probability that the $\mathrm{Re}(S_m^{\nu'}) = 1$ can be given based on the Gaussian distribution

$$p = \frac{1}{\sqrt{2\pi}} \int_{-\mathrm{SNR}}^{\infty} \exp\left(-\frac{x^2}{2}\right) \mathrm{d}x = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{1/\mathrm{SCR}}}^{\infty} \exp\left(-\frac{x^2}{2}\right) \mathrm{d}x. \tag{12}$$

Thus, compared with the pattern S^ν, the expected number of real and imaginary error components in the pattern $S^{\nu'}$ is approximately

$$E(\mathrm{SCR}) = 4N(1 - p(\mathrm{SCR})) = \frac{4N}{\sqrt{2\pi}} \int_{\sqrt{1/\mathrm{SCR}}}^{\infty} \exp\left(-\frac{x^2}{2}\right) \mathrm{d}x. \tag{13}$$

If the number of error components in $|\nu'\rangle$ is approximately a Poisson distribution, it follows that the probability of correct components, i.e., the probability that $|\nu\rangle$ is indeed a stable attractor, is given approximately by the expression

$$P = \exp(-E(\mathrm{SCR})). \tag{14}$$

Now suppose that the probability P is a fixed number very near 1. Then inverting Eq. (14), one can obtain the following result:

$$\mathrm{SCR} \propto \frac{1}{2\ln 4N} \sim \frac{1}{2\ln N}. \tag{15}$$

Similar to the Hopfield model, if we do not consider the self-interaction term, i.e. $J_{mm} = 0$, then $\mathrm{SNR} = \sqrt{(N-1)/(M-1)}$ is obtained with the real or imaginary part of each component of the basis vectors. This ratio is different from the above ratio, i.e., Eq. (11), by a very small quantity of the order of N^{-1}, which goes to zero in the thermodynamic limit $N \to \infty$. So the above result is still correct for this case too.

So we can see that the storage capacity ratio of the present model is equal to that of the Hopfield model analyzed by Bruce [3] and Mceliece [4]. In the next section, we provide the outcomes of the numerical simulation about the storage capacity and the error correction ability of the Hamilton number neural network.

5. Numerical simulation result

In order to discuss the storage capacity of the Hamilton model, the basis state vectors $|\mu\rangle$ are put into the network and the statistical curves $M_{\max} - N$ under correct retrieval ratios CRR = 50%, 90% are obtained (see Fig. 1 and Fig. 2). The statistical curves $M_{\max} - N$ of the Hopfield model under these CRR are also plotted. From Fig.1 and Fig. 2, it can be seen that the two curves, each of the Hamilton model and the Hopfield model, are very similar, in another words, the SCRs of the two models are the same, the storage capacity of the Hamilton network is just a little smaller than that of the Hopfield network.

In order to compare the storage capacity of two models, the correct retrieval ratios CRR via the pattern number M with constant neuron number N are tested too. In the

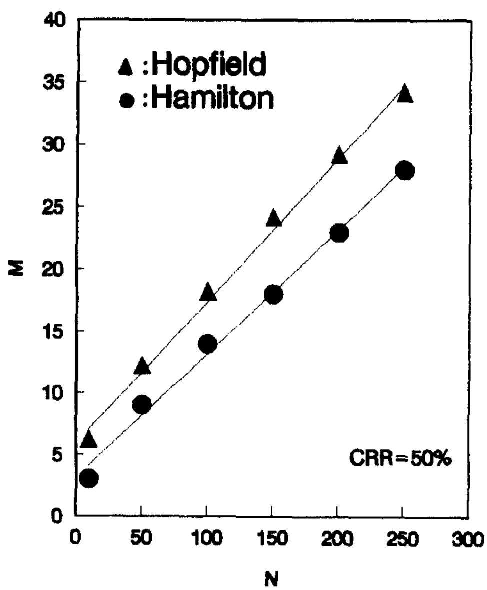

Fig. 1. The statistical curves M–N under the correct retrieval ratio CRR = 50%. Here, ▲ for the Hopfield model and ● for the Hamilton model.

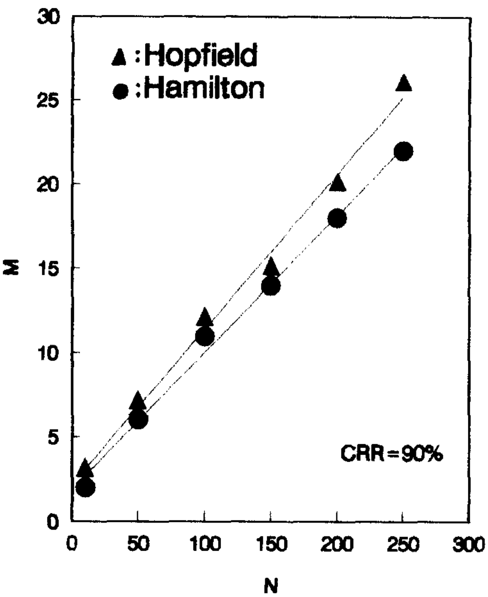

Fig. 2. The statistical curves M–N under the correct retrieval ratio CRR = 90%. Here, ▲ for the Hopfield model and ● for the Hamilton model.

Fig. 3 the CRR–M curves with $N = 100, 200$ are drawn out for two models. From the figure, one can also come to the above conclusion.

Now we discuss the error-correction ability of the model by numerical simulation. A distance function D is defined to describe the difference between two patterns: $|\alpha\rangle = a_1^\mu + b_1^\mu + c_1^\mu + d_1^\mu$ and $|\beta\rangle = a_2^\mu + b_2^\mu + c_2^\mu + d_2^\mu$, here $a, b, c, d = \pm 1$,

$$D = |\alpha - \beta| = |a_1^\mu - a_2^\mu| + |b_1^\mu - b_2^\mu|\,|c_1^\mu - c_2^\mu| + |d_1^\mu - d_2^\mu|. \tag{16}$$

The distance D is a generalization of the Hamming distance. So if a state $|S\rangle$ has a distance D compared to the basis vector $|\mu\rangle$, one can say that $|S\rangle$ has a noise fraction $\mathrm{NF} = D/4N$ compared with the $|\mu\rangle$. The error-correction radius D via N with the correct association ratio 85% for $M = 3, 5, 7$ is represented in Fig. 4. From the figure, one can see that error-correction radius D is mainly determined by the neural number N when $N \gg M$.

To compare the error-correction ability of the Hamilton model with that of the Hopfield model, the maximum noise fraction NF_0 via N with the correct association percentage CAP = 85% and $M = 5$ are simulated in Fig. 5. In Fig. 6, the correct association percentages CAP via the random noise fraction NF with $N = 100$, $M = 5$, 10, 15 are also compared for the two models. From Fig. 5 and Fig. 6, one can see that the error correction ability of the Hamilton model is lower than that of the Hopfield model.

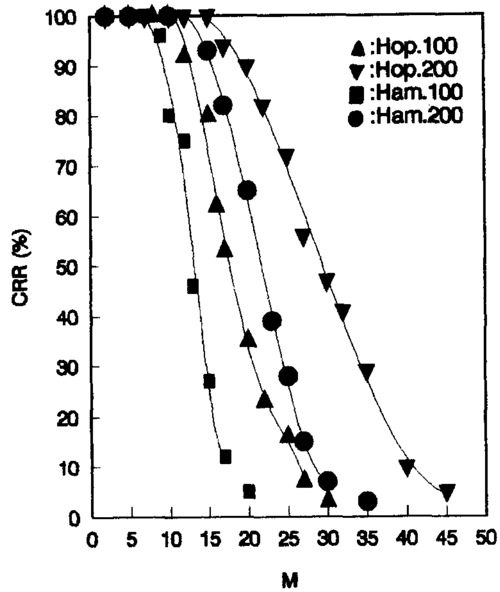

Fig. 3. The statistical curves of the correct retrieval ratio CRR via the pattern number M. Here, (1) ▲ for the Hopfield model with $N = 100$; (2) ▼ for the Hopfield model with $N = 200$; (3) ■ for the Hamilton model with $N = 100$; (4) ● for the Hamilton model with $N = 200$.

Since the Hamilton neuron is a 16-state neuron, the Hamilton neural network can store 16-level gray or color patterns. For the application of recognizing 16-level color patterns that are widely used in the computer, the three imaginary parts of the Hamilton number can be treated as the three basis colors (i.e., red, blue, green) in the color monitor screen and the real part of the Hamilton number indicates the color saturation degree. Then the corresponding relationship between the computer code and the Hamilton neuron code of the 16-level colors can be set up naturally. Using the Hamilton neural network, we process some numerical simulation to recognize the color English letters that are composed by a 7×10 dots matrix. For example, we store five letters A with colors: cyan, magenta, gray, light red and yellow. Numerical simulation results show that these five letters A with different colors are all the stable patterns stored in the Hamilton neural network. While, for input patterns added with about 11% random noise (i.e., $D = 30$), the correct association percentage is more than 90%. If the noise is 18% (i.e., $D = 50$), the correct association percentage is about 60%.

6. Conclusion

In this paper, a 16-state discrete Hamilton neural network is suggested. The stability and the storage capacity are analysed by using signal-to-noise theory. The

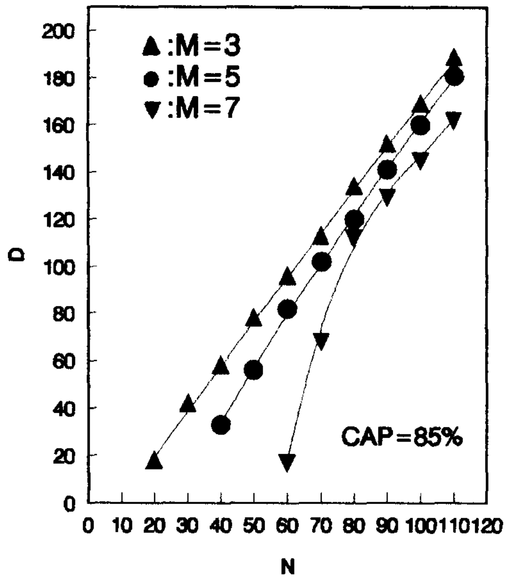

Fig. 4. The error correction ability statistical curves of the error correction radius D via neuron number N with the correct association percent 85%. Here, (1) ▲ for $M = 3$; (2) ● for $M = 5$; (3) ▼ for $M = 7$.

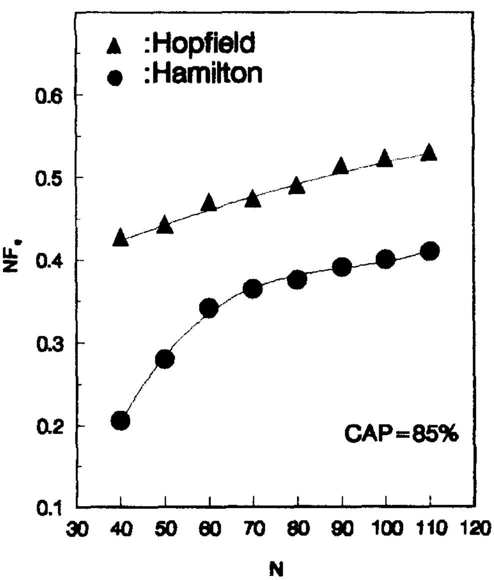

Fig. 5. The statistical curves of the maximum noise fraction NF_0 via the neuron number N with the correct association percentage 85% and $M = 5$. Here, (1) ▲ for the Hopfield model; (2) ● for the Hamilton model.

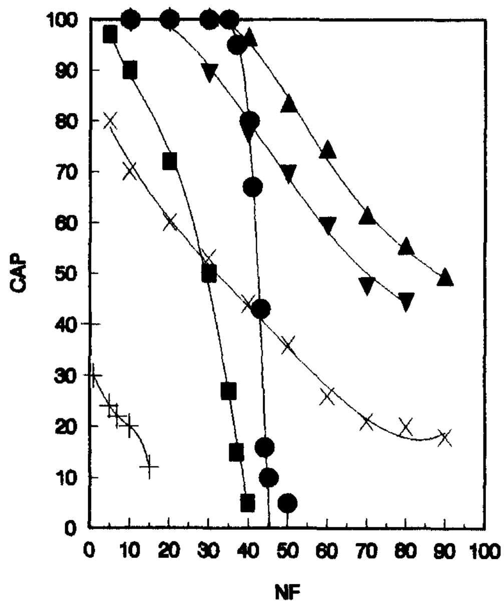

Fig. 6. The statistical curves of the correct association percent CAP via the random noise fraction NF with $N = 100$, $M = 5$, 10. Here, (1) ▲ for the Hopfield model with $M = 5$; (2) ▼ for the Hopfield model with $M = 10$; (3) × for the Hopfield model with $M = 15$; (4) ● for the Hamilton model with $M = 5$; (5) ■ for the Hamilton model with $M = 10$; (6) + for the Hamilton model with $M = 15$.

storage capacity ratio SCR equals that of the Hopfield model. The storage capacity and the error correction ability of the model are also simulated. The storage capacity and the error correction ability of the present model is a little lower than that of the Hopfield model. Due to the feature of one real part and three imaginary parts of the Hamilton number, the 16-state discrete Hamilton neural network can be applied to recognize 16-level gray or color patterns.

Naturally, not only can the Hamilton number be introduced into the neural network, but also the other 2^n-element numbers, such as the complex number and Cayley number. For the latter two kinds of numbers, the 4-level (i.e., $\pm\mathbf{1} \pm\mathbf{i}$) and 256-level (i.e., $\pm\mathbf{1}\ \pm\mathbf{i}_1\ \pm\mathbf{i}_2\ \pm\mathbf{i}_3\ \pm\mathbf{i}_4\ \pm\mathbf{i}_5\ \pm\mathbf{i}_6\ \pm\mathbf{i}_7$) neural networks can be set up. On the theoretical side, further generalization seems possible. The discrete state phasor neural network [13] is set up based on the plane rotation property of the complex number. Similar, based on the property of the rigid rotation of the Hamilton number, a Hamilton phasor neural network can also be suggested. The details of this work will be discussed in other papers.

Acknowledgements

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