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A Chaotic Neural Network Model*

SHUAI Jian-wei, CHEN Zhen-xiang, LIU Rui-tang, WU Bo-xi
Department of Physics, Xiamen University, Xiamen 361005

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A chaotic neural network model is introduced in this letter. With the variation of a controlling parameter, the model can process the input by a content-addressable means or fall into a chaotic state. Hence the features of brain such as the clear active state and the chaotic rest state can be simply simulated.

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Recently, a lot of attention has been focused on the chaotic neural networks.[1-4] In this letter, a simple chaotic neuron model is introduced. Compared with the Hopfield neural network model,[5] only an even exponential function $\exp(-\beta x^2)$ is added, so the neuron outputs a nonmonotonic transform:

$$f(x) = \tanh(\alpha x)\exp(-\beta x^2) \ . \tag{1}$$

The function $f(x)$ is a nonmonotonic odd function. With the increasing of $|x|$ from zero, $|f(x)|$ first increases, then decreases, and finally drops to zero. The constants $\alpha > 0$ and $\beta > 0$ respectively describe the nonlinear and nonmonotonic degree of the neuron response.

When $\beta = 0$, the neuron becomes an analytic Hopfield neuron[5] and can process the input more definitely. In this case the neuron is in the clear state. With the increasing of β, the retrieval goes to the chaos through bifurcation. Figure 1(a) shows an example of the bifurcation diagram versus the nonmonotonic degree β. The initial value is positive. When choosing a negative initial value, a symmetric bifurcation diagram can be obtained on the negative axis. From Fig. 1(a), one can see that there are triple-periodic attractors in the interval [11.85, 13.77]. Figure 1(b) gives the corresponding characteristic of the Lyapunov exponent λ with respect to β. On the other hand, one can find that the nonlinearity α can also lead to chaos.

Consider a chaotic neural networks composed by N chaotic neurons. The dynamics of the ith chaotic neuron can be described by the following equation

$$S_i(t+1) = f\left[\sum_{j=1}^{N} J_{ij} S_j(t)\right] \ , \tag{2}$$

where J_{ij} is the synaptic connections. The present model becomes the analytic Hopfield model if $\beta = 0$; and the discrete Hopfield model if $\alpha \to \infty$ and $\beta = 0$. We surmise that when β increases and exceeds a threshold, the model should have the power of producing chaotic behaviors.

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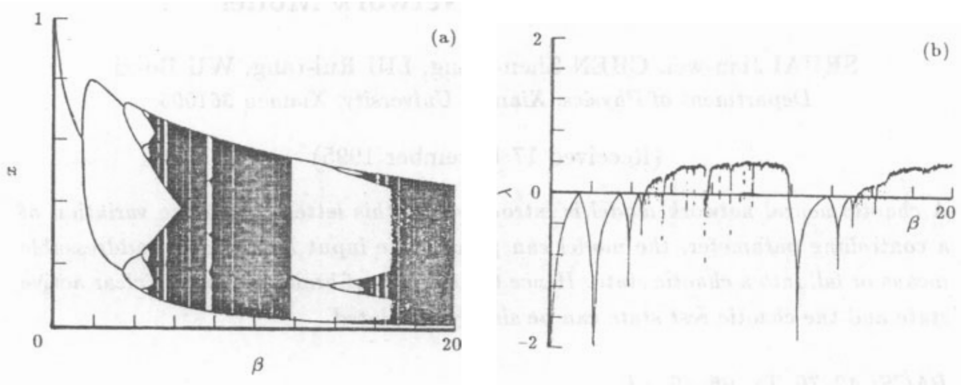

Fig. 1. Response characteristics of function (1) with β. (a) The bifurcation diagram; (b) the Lyapunov exponent λ. Here $\alpha = 6.0$, $0.0 < \beta < 20.0$ and the beginning value $x(0) = 0.5$.

Let us consider a specific example. Suppose we have a network with five chaotic neurons. The synaptic connection is reflected by a symmetric matrix J_{ij} with the diagonal elements vanishing and the other elements randomly generated:

$$J = \begin{pmatrix} 0.0 & 0.6 & -0.2 & 0.8 & -0.4 \\ 0.6 & 0.0 & 0.0 & -0.6 & 0.6 \\ -0.2 & 0.0 & 0.0 & 0.4 & 0.2 \\ 0.8 & -0.6 & 0.4 & 0.0 & 0.4 \\ -0.4 & 0.6 & 0.2 & 0.4 & 0.0 \end{pmatrix} . \tag{3}$$

In the case that $\alpha \to \infty$ and $\beta = 0$ (the discrete Hopfield model), six memories can be stably stored in the network. For instance, one of them is $S = (-1, -1, 1, 1, 1)$.

Now let $\alpha = 6.0$, $0.0 < \beta < 10.0$, and the input stimulus $S(0) = (-1, -1, 1, 1, 1)$ that is one of the memories of the Hopfield model. Figure 2(a) shows the evolution diagram of the first neuron with various values of β. When $\beta = 0$, the neuron is fixed in -1. This means that the network can recognize the input correctly and content-addressably. When β increases the neuron bifurcates from the negative fixed points into the periodic orbits. At last it falls into the zone of chaotic attractors, where there are some periodic windows. Now we analysize the evolution of the overlaps of the network, which indicates the similiarity between the evolutional pattern $S(t)$ and the input pattern $S(0)$:

$$m(t) = \frac{1}{N} \sum_{i=1}^{N} S_i(t) S_i(0) . \tag{4}$$

Figure 2(b) plots the bifurcation diagram of the overlap $m(t)$ versus the nonmonotonicity parameter β. When β goes from 0.0 to 10.0, the overlaps move from the fixed points to the periodic attractors and then to the chaotic attractors. In the chaotic region, there are also some windows of periodic attractors.

It sounds plausible to imagine that the neuron is in active state when $|S| \to 1$ ($S \to 1$ represents enhancement and $S \to -1$ inhibition); on the other hand, it is in quiescent state when $|S| \to 0$. Figure 2(c) shows the diagram of the average active rate of the whole network with respect to β. Here the average active rate ρ is defined as follows:

$$\rho = \lim_{T \leftarrow +\infty} \frac{1}{T} \sum_{t=0}^{T-1} \frac{1}{N} \sum_{i=1}^{N} |S_i(t)| \,. \tag{5}$$

From Fig. 2(c), one can find that ρ decrease when β increases; this implies that all neurons are in quiescent state. The state of the neuron is strongly dependent on β: The bigger β is, the neuron is more inert. So we can call β as the calm degree.

For a chaotic system, its characteristic exponent is the largest Lyapunov exponent, which can be calculated as follows.[4] In an N-dimensional phase space two trajectories are computed: the unperturbed $\boldsymbol{S}_0(t)$, and the perturbed $\boldsymbol{S}_\epsilon(t)$. To calculate the latter, after reaching the stationary state small values ϵu_i are added to u_i. Here ϵ is a very small decimal. The largest Lyapunov exponent is defined as

$$\lambda = \lim_{t \to \infty} \lim_{D(0) \to 0} \frac{\ln[D(t)/D(0)]}{t} \,, \tag{6}$$

where

$$D(0) = \sqrt{\sum_{i=1}^{N} [S_{i\epsilon}(0) - S_{i0}(0)]^2} \,,$$

$$D(t) = \sqrt{\sum_{i=1}^{N} [S_{i\epsilon}(t) - S_{i0}(t)]^2} \tag{7}$$

are the distances between the perturbed and unperturbed trajectories at the initial and current moments, respectively. In Fig. 2(d), the largest Lyapunov exponents versus β is drawn. In the region of $\lambda > 0$, the network reaches the chaotic state.

One can see from Fig. 2 that the present model is able to simulate not only chaos but also more biological features. When the calm degree $\beta = 0$ or very small, the network falls into the fixed points; in another word, the network is able to associate uniquely the input to one of its storage memories. While from Fig. 2(c) it is shown that the network is more active in these states. In this case the network is like a brain in the head-clear states: it is active and can calculate correctly. As β increases, successive bifurcations occur while the overlaps and the average active rates decrease. That is to say, the network becomes a little restful. So the association accuracy lowers down. The network periodically wanders among some patterns. The number of the periodic orbits gets bigger with the increasing of β. This is similar to the brain in rest or in pre-sleep state: the active rate is a little low and the imagination is rich but somewhat random. When β increases to a greater extent, the network goes to the chaotic states, while the average active rates become smaller. The states of the low activity and chaotic wander can be regarded as a deep rest state and can simply simulate the sleeping state of the brain. In these states, any input stimulus is suppressed by the nonmonotonic function and the whole network remains asleep. Actually, the phenomenon of periodical wander also appears intermittently in the chaotic region with big β. We may call these states as the dreaming time during sleep.

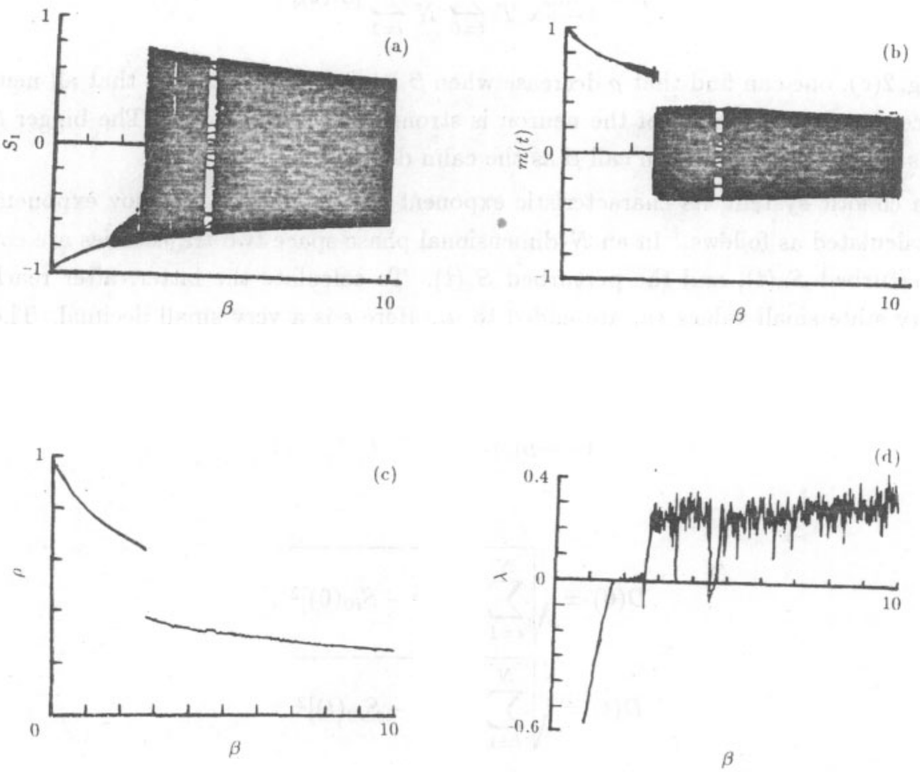

Fig. 2. Response characteristics of the chaotic neural networks with β. (a) The evolution of the first neuron; (b) the overlap between the pattern $S(t)$ and $S(0)$; (c) the average active rate of the whole network; (d) the largest Lyapunov exponent of the network.

In conclusion, we have proposed a simple model for the chaotic neural network. The clear active state, the rest or pre-sleep state, the sleeping state, the dreaming state of the brain can be simply imitated by the present model.

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