Applied Statistical Analysis I/ Quantitative Methods I
POP77003/77051
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Week 9

Yao (Sara) HAN

hany3@tcd.ie

Department of Political Science

Trinity College Dublin

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Today's Agenda

(1) Lecture recap

(2) Tutorial exercises

T-TEST FOR INDIVIDUAL COEFFICIENTS

What is the t-test for individual coefficients?

T-TEST FOR INDIVIDUAL COEFFICIENTS

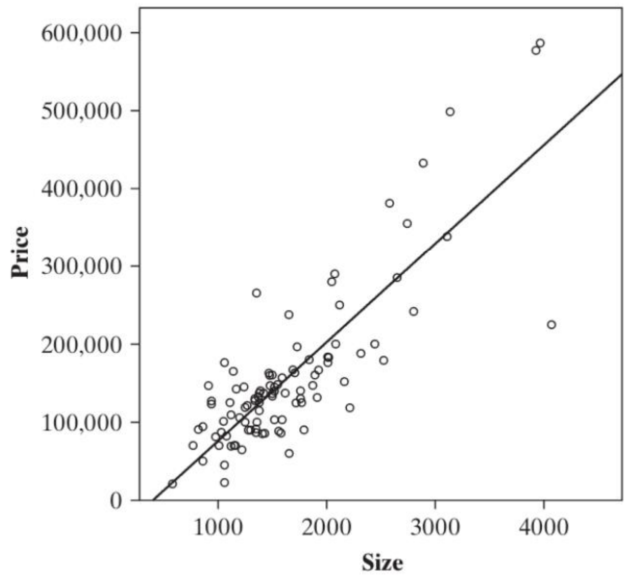
What is the t-test for individual coefficients?

- Null and alternative hypotheses:
  - there is no association between X and Y, $\beta = 0$ (H_0)
  - there is an association between X and Y, $\beta \neq 0$ (H_1)
- Test statistic: "measures the number of standard errors between the estimate and the H_0 value" (Agresti and Finlay 2009, 192).

  $t = \frac{\textit{Estimate of parameter} - \textit{Null hypothesis value of parameter}}{\textit{Standard error of estimate}}$

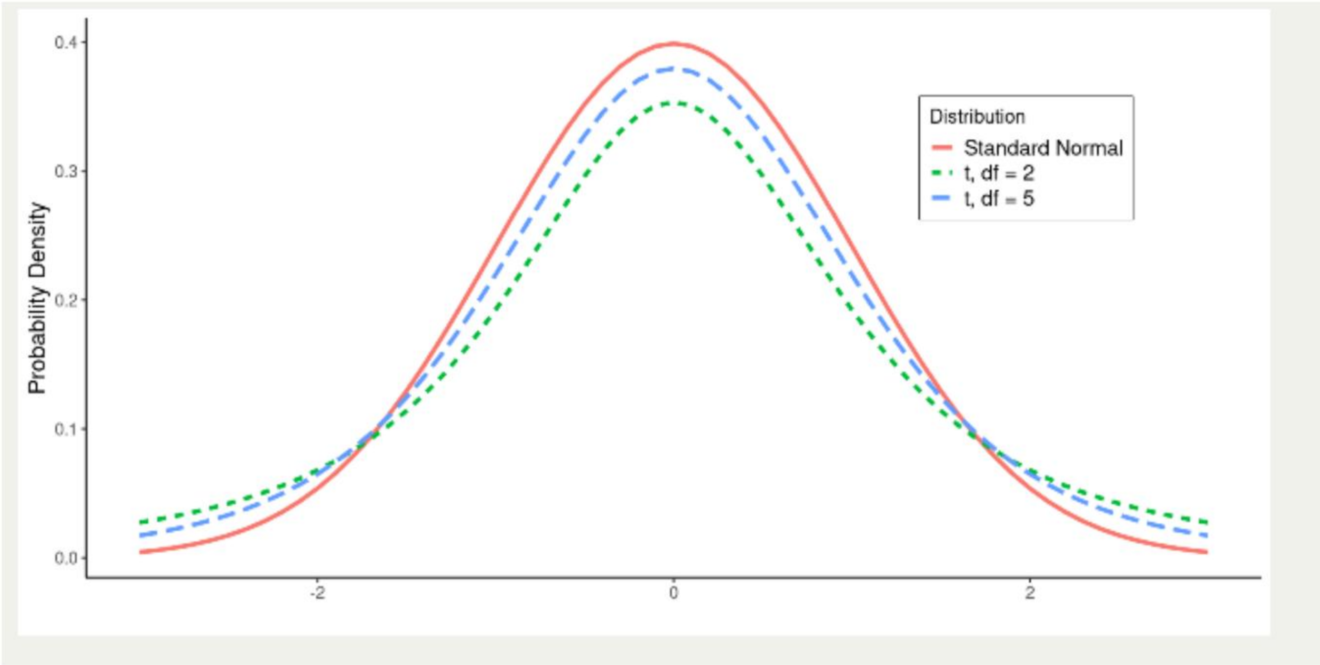
  $t = \frac{\hat{\beta} - \beta_{H_0}}{\hat{\sigma}_{\hat{\beta}}} = \frac{\hat{\beta}}{\hat{\sigma}_{\hat{\beta}}}$, H_0 assumes $\beta = 0$

T-TEST FOR INDIVIDUAL COEFFICIENTS



- Is there an association between house selling price and size (Agresti and Finlay 2009, 278–279)? $Price = 50,926.2 + 126.6 * Size$
- $t = \frac{\hat{\beta}}{\hat{\sigma}_{\hat{\beta}}} = \frac{126.6}{8.47} = 14.95$
- How to interpret this value? How likely are we to observe data in sample (this test statistics), under the assumption that H_0 is true? $\rightarrow$ Probability distribution

T-TEST FOR INDIVIDUAL COEFFICIENTS



<u>What is the conclusion?</u> P-value < 0.05, We can reject H_0 with an error probability (p-value) of essentially 0%. $\rightarrow$ There is an association between house selling price and size

F-TEST

Table 3.5 Regression Output for Supervisor Performance Data

Variable	Coefficient	s.e.	t-Test	p-value
Constant	10.787	11.5890	0.93	0.3616
X_1	0.613	0.1610	3.81	0.0009
X_2	−0.073	0.1357	−0.54	0.5956
X_3	0.320	0.1685	1.90	0.0699
X_4	0.081	0.2215	0.37	0.7155
X_5	0.038	0.1470	0.26	0.7963
X_6	−0.217	0.1782	−1.22	0.2356
$n = 30$	$R^2 = 0.73$	$R_a^2 = 0.66$	$\hat{\sigma} = 7.068$	df = 23

Table 3.2 Description of Variables in Supervisor Performance Data

Variable	Description
Y	Overall rating of job being done by supervisor
X_1	Handles employee complaints
X_2	Does not allow special privileges
X_3	Opportunity to learn new things
X_4	Raises based on performance
X_5	Too critical of poor performance
X_6	Rate of advancing to better jobs

(Chatterjee and Hadi 2015, 59)

F-test

General set-up: Test whether reduced model (RM) is adequate (H_0) or full model (FM) is adequate (H_1).

The reduced model is nested within the full model $\rightarrow$ compare "the goodness of fit that is obtained when using the full model, to the goodness of fit that results using the reduced model".

$$F = \frac{[SSE(RM) - SSE(FM)]/(p+1-k)}{SSE(FM)/(n-p-1)}$$

(Chatterjee and Hadi 2015, 71–72)

F-test

$$F = \frac{[SSE(RM) - SSE(FM)]/(p+1-k)}{SSE(FM)/(n-p-1)}$$

* Sum of squared errors (SSE), denotes *lack of fit* $\rightarrow$ $SSE(RM) - SSE(FM)$ "represents the increase in the residual sum of squares due to fitting the reduced model".
* We use the ratio, weighted by "respective degrees of freedom to compensate for the different number of parameters involved in the two models".
* p=number of IVs full model, n=number of observations, k=number of paraneters reduced model

(Chatterjee and Hadi 2015, 71–72)

F-test

Two versions of the F-test

1. “All the regression coefficients are zero”.
2. “Some of the regression coefficients are zero”.

(Chatterjee and Hadi 2015, 71)

F-test for all coefficients

What is the F-test for all coefficients?

F-test for all coefficients

"All the regression coefficients are zero."

* Reduced model (RM): $Y = \beta_0 + \epsilon$

  all slopes are equal to zero, $\beta_k = 0$ (H_0) $\rightarrow$ the null model performs better

* Full model (FM): $Y = \beta_0 + \beta_1 X_1 + ... + \beta_p X_p \epsilon$

  at least one slope is different from zero, $\beta_p \neq 0$ (H_1) $\rightarrow$ the full model performs better

$$F = \frac{[SSE(RM) - SSE(FM)]/(p+1-k)}{SSE(FM)/(n-p-1)} = \frac{[SST - SSE]/p}{SSE/(n-p-1)} = \frac{SSR/p}{SSE/(n-p-1)}$$

"Because the least squares estimate of β_0 in the reduced model is $\bar{y}$, the residual sum of squares from the reduced model is SSE(RM)=SST." "reduced model has one regression parameter and the full model has p+1 regression parameter". "Because SST=SSR+SSE, we can replace SST-SSE by SSR"

(Chatterjee and Hadi 2015, 73)

F-test for all coefficients

Table 3.5 Regression Output for Supervisor Performance Data

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X_1	0.613	0.1610	3.81	0.0009
X_2	−0.073	0.1357	−0.54	0.5956
X_3	0.320	0.1685	1.90	0.0699
X_4	0.081	0.2215	0.37	0.7155
X_5	0.038	0.1470	0.26	0.7963
X_6	−0.217	0.1782	−1.22	0.2356
$n = 30$	$R^2 = 0.73$	$R_a^2 = 0.66$	$\hat{\sigma} = 7.068$	df = 23

Table 3.7 Supervisor Performance Data: Analysis of Variance (ANOVA) Table

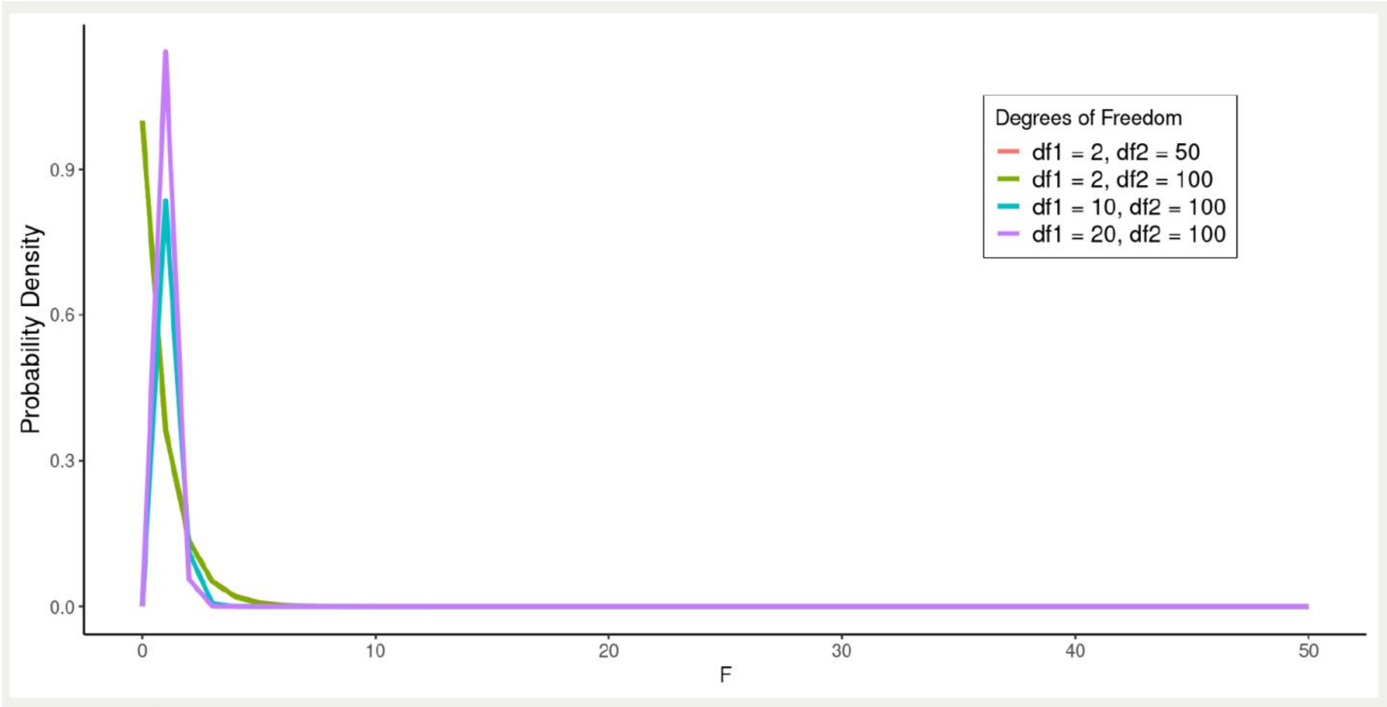
Source	Sum of Squares	df	Mean Square	F-Test
Regression	3147.97	6	524.661	10.5
Residuals	1149.00	23	49.9565	

$$F = \frac{SSR/p}{SSE/(n-p-1)} = \frac{3147.97/6}{1149.00/23} = 10.50$$

How to interpret this value? How likely are we to observe data in sample (this test statistics), under the assumption that H_0 is true? $\rightarrow$ Probability distribution

(Chatterjee and Hadi 2015, 75)

F-test for all coefficients



What is the conclusion? P-value < 0.05, We can reject H_0 with an error probability (p-value) of essentially 0%. $\rightarrow$ The full model performs better, "not all $\beta's$ can be taken as zero"

(Chatterjee and Hadi 2015, 75).

Partial F-test

What is the F-test for some coefficients?

Partial F-test

"Some of the regression coefficients are zero".

* Reduced model (RM): $Y = \beta_0 + \beta_1 X_1 + \beta_3 X_3 + \epsilon$
  subset of slopes is equal to zero, $\beta_k = 0$ (H_0) $\rightarrow$ the reduced model performs better
* Full model (FM): $Y = \beta_0 + \beta_1 X_1 + ... + \beta_p X_p \epsilon$
  at least one slope in the subset is different from zero, $\beta_p \neq 0$ (H_1) $\rightarrow$ the full model performs better

$$F = \frac{[SSE(RM) - SSE(FM)]/(p+1-k)}{SSE(FM)/(n-p-1)}$$

(Chatterjee and Hadi 2015, 77)

Partial F-test

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X_2	−0.073	0.1357	−0.54	0.5956
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$n = 30$	$R^2 = 0.73$	$R_a^2 = 0.66$	$\hat{\sigma} = 7.068$	df = 23

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(Chatterjee and Hadi 2015, 75)

Partial F-test

Table 3.8 Regression Output from the Regression of Y on X_1 and X_3

ANOVA Table				
Source	Sum of Squares	df	Mean Square	F-Test
Regression	3042.32	2	1521.1600	32.7
Residuals	1254.65	27	46.4685	

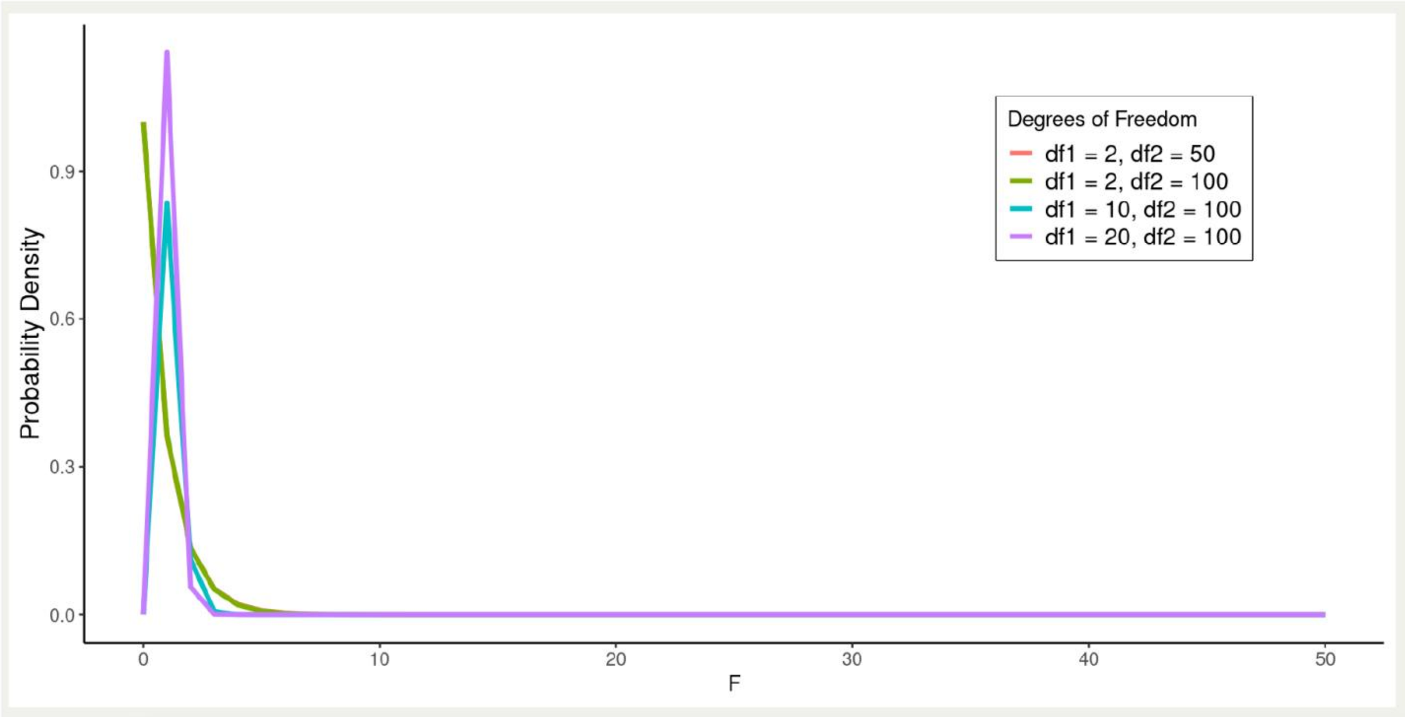
Coefficients Table				
Variable	Coefficient	s.e.	t-Test	p-value
Constant	9.8709	7.0610	1.40	0.1735
X_1	0.6435	0.1185	5.43	< 0.0001
X_3	0.2112	0.1344	1.57	0.1278
$n = 30$	$R^2 = 0.708$	$R_a^2 = 0.686$	$\hat{\sigma} = 6.817$	df = 27

$$F = \frac{[1254.65 - 1149]/4}{1149/23} = 0.0528$$

How to interpret this value? How likely are we to observe data in sample (this test statistics), under the assumption that H_0 is true?
$\rightarrow$ Probability distribution

(Chatterjee and Hadi 2015, 76)

Partial F-test

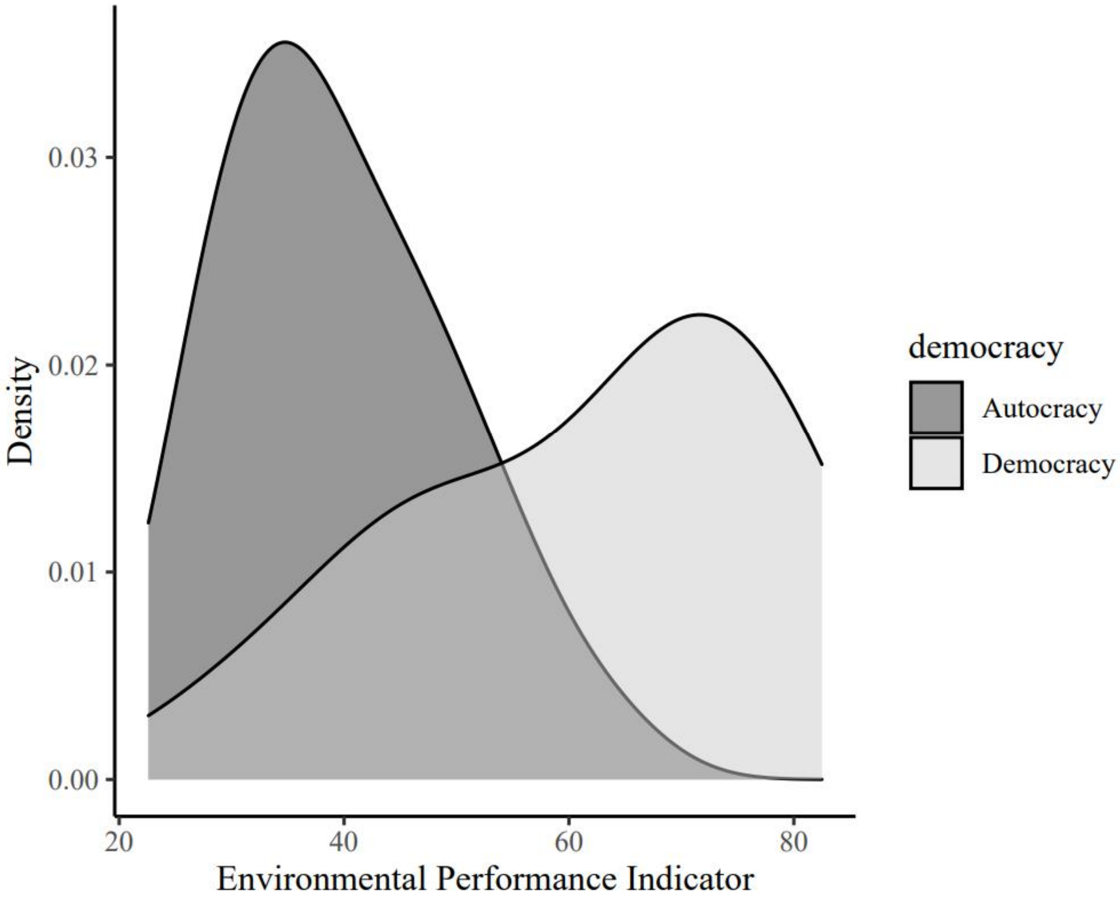


What is the conclusion? P-value > 0.05, We cannot reject H_0. $\rightarrow$ The reduced model performs better. "The variables X_1 and X_3 together explain the variation in Y as adequately as the full set of six variables" (Chatterjee and Hadi 2015, 77).

Categorical explanatory variables

What is the reference category?

What is the reference category?



$$\textit{Environmental Performance}_i = \alpha + \beta_1 * \textit{Regime Type}_i$$

What is the reference category?

Dummy variables should take value 0 and 1 for easy interpretation $\rightarrow$ Re-code existing variables.

```
# Import data from Quality of Government dataset
qog_data <- read.csv("qog_bas_cs_jan21.csv")

# Generate dummy variable for regime type as factor variable - democracy
# vdem_polyarchy ranges between 0 and 1; cutoff at 0.7
# Countries with score equal or above 0.7 are democracies, those below autocracies
qog_data$democracy <- factor(ifelse(qog_data$vdem_polyarchy >= 0.7, 1, 0))

# Define levels of democracy in factor variable
levels(qog_data$democracy) <- c("Autocracy", "Democracy")

# Summarize generated dummy variable
summary(qog_data$democracy)
```

```
## Autocracy Democracy      NA's
##       119        54        21
```

What is the reference category?

```r
# Generate dummy variable for regime type as factor variable — autocracy
qog_data$autocracy <- factor(ifelse(qog_data$vdem_polyarchy < 0.7, 1, 0))

# Define levels of autocracy in factor variable
levels(qog_data$autocracy) <- c("Democracy", "Autocracy")

# Print first 10 rows in dataset
head(qog_data[c("democracy", "autocracy")], 10)
```

```
     democracy      autocracy
1    0 Autocracy    1 Autocracy
2    0 Autocracy    1 Autocracy
3    0 Autocracy    1 Autocracy
4    <NA>           <NA>
5    0 Autocracy    1 Autocracy
6    <NA>           <NA>
7    0 Autocracy    1 Autocracy
8    1 Democracy    0 Democracy
9    1 Democracy    0 Democracy
10   1 Democracy    0 Democracy
```

What happens if we run:

$$Environmental\ Performance_i = \alpha + \beta_1 Democracy_i + \beta_2 Autocracy_i + \epsilon_i$$

What is the reference category?

$$\textit{Environmental Performance}_i = \alpha + \beta_1 \textit{Democracy}_i + \beta_2 \textit{Auocracy}_i + \epsilon_i$$

```r
# Fit regression model
m1_trap <- lm(epi_epi ~ democracy + autocracy, data = qog_data)

# Print results
summary(m1_trap)
```

```
lm(formula = epi_epi ~ democracy + autocracy, data = qog_data)

Residuals:
    Min      1Q  Median      3Q     Max
-34.107  -8.860  -0.610   9.293  26.190

Coefficients: (1 not defined because of singularities)
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   39.610      1.138   34.80   <2e-16 ***
democracy1    22.098      2.002   11.04   <2e-16 ***
autocracy1        NA         NA      NA       NA
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Violates assumption of no perfect multicollinearity (essentially a data problem) $\rightarrow$ One category needs to be excluded = reference category. Interpretation of the model is relative to the reference category.

Binary Explanatory Variables

How to include binary explanatory variables in multiple linear regression?

$$Environmental\ Performance_i = \alpha + \beta_1 * Regime\ Type_i + \beta_2 * Income_i$$

```
## Call:
## lm(epi_epi ~ democracy + income, data = qog_data)

## Residuals:
##     Min      1Q  Median      3Q     Max
## -53.563  -6.502   0.498   6.773  20.198

## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)         35.3027     1.1269  31.327  < 2e-16 ***
## democracy           16.5270     1.8409   8.978 9.08e-16 ***
## income               3.5793     0.4266   8.390 2.92e-14 ***
## ---
## Signif. codes:
## 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

## Residual standard error: 9.982 on 154 degrees of freedom
##   (37 observations deleted due to missingness)
## Multiple R-squared:  0.6175,Adjusted ## R-squared:  0.6126
## F-statistic: 124.3 on 2 and 154 DF,  p-value: < 2.2e-16
```

In comparison to autocracies (= reference category), democracies have a 16.5270 scale point higher score on the Environmental Performance Index, under control of income.

$$\hat{Y}_i = \alpha + \beta_1 * \textit{Regime Type}_i + \beta_2 * \textit{Income}_i$$

Model for Autocracies:

$$\hat{Y}_i = 35.303 + (16.527 * \textit{Regime Type}_i) + (3.579 * \textit{Income}_i)$$

$$\hat{Y}_i = 35.303 + (16.527 * 0) + (3.579 * \textit{Income}_i)$$

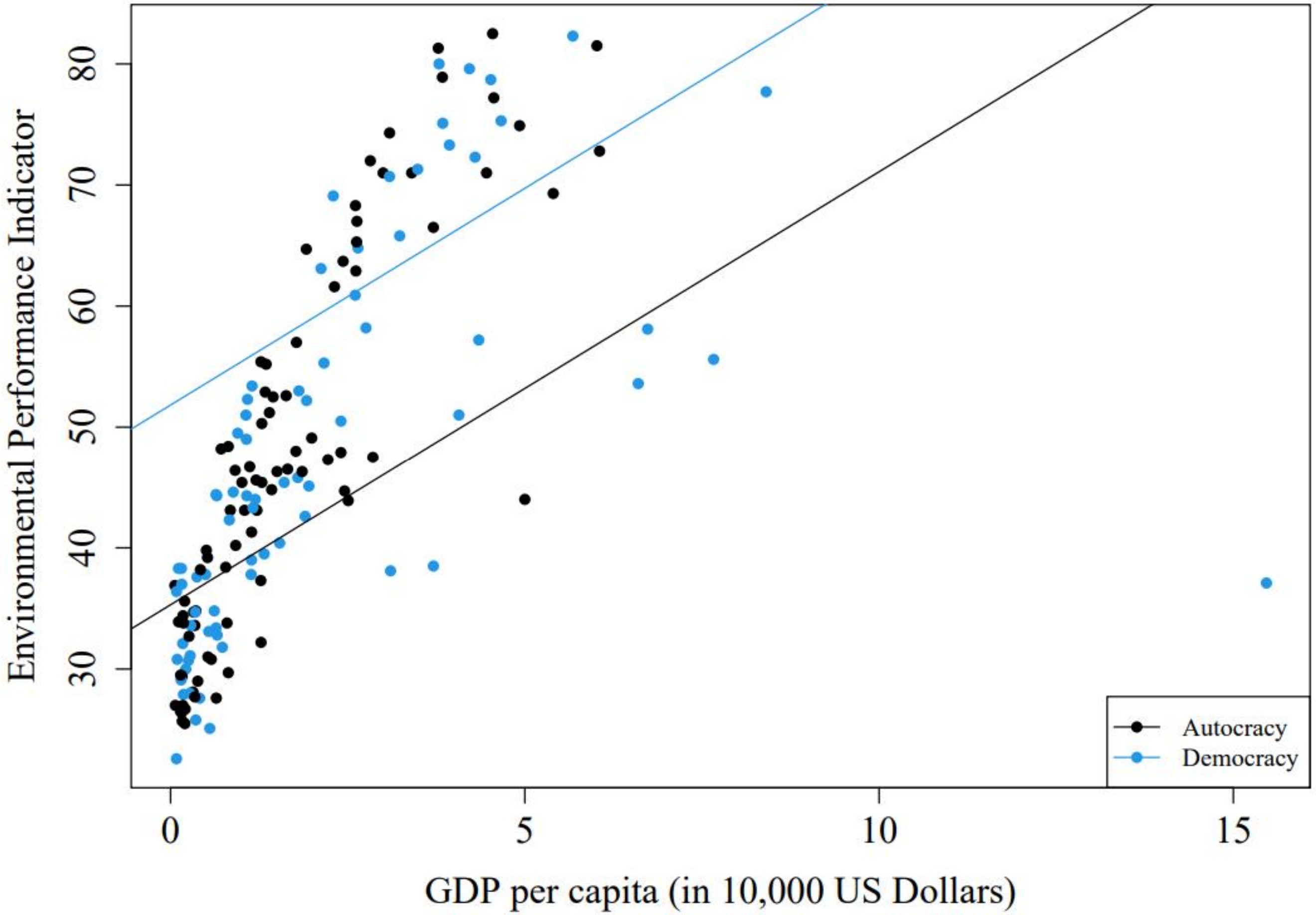
$$\hat{Y}_i = 35.303 + (3.579 * \textit{Income}_i)$$

Model for Democracies:

$$\hat{Y}_i = 35.303 + (16.527 * \textit{Regime Type}_i) + (3.579 * \textit{Income}_i)$$

$$\hat{Y}_i = 35.303 + (16.527 * 1) + (3.579 * \textit{Income}_i)$$

$$\hat{Y}_i = 51.83 + (3.579 * \textit{Income}_i)$$



Environmental Performance Indicator
30
40
50
60
70
80
0
5
10
15
GDP per capita (in 10,000 US Dollars)
Autocracy
Democracy

Categorical Explanatory Variables

How to select the reference category?

How to select the reference category?

```r
# Run regression model with democracy variable
m1_dem <- lm(epi_epi ~ income + democracy, data = qog_data)

# Run regression model with autocracy variable
m1_aut <- lm(epi_epi ~ income + autocracy, data = qog_data)

# Get regression table with stargazer
stargazer(m1_dem, m1_aut)
```

	Dependent variable:	
	epi_epi	
	(1)	(2)
income	3.579^{***}	3.579^{***}
	(0.427)	(0.427)
democracy1	16.527^{***}	
	(1.841)	
autocracy1		-16.527^{***}
		(1.841)
Constant	35.303^{***}	51.830^{***}
	(1.127)	(1.892)
Observations	157	157
R^2	0.618	0.618
Adjusted R^2	0.613	0.613
F Statistic (df = 2; 154)	124.331^{***}	124.331^{***}
Note:	$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$	

How to select the reference category?

Model 1 for Autocracies:

$\hat{Y}_i = 35.303 + (16.527 * \mathit{Regime\ Type}_i) + (3.579 * \mathit{Income}_i)$

$\hat{Y}_i = 35.303 + (16.527 * 0) + (3.579 * \mathit{Income}_i)$

$\hat{Y}_i = 35.303 + (3.579 * \mathit{Income}_i)$

Model 2 for Autocracies:

$\hat{Y}_i = 51.830 + (-16.527 * \mathit{Regime\ Type}_i) + (3.579 * \mathit{Income}_i)$

$\hat{Y}_i = 51.830 + (-16.527 * 1) + (3.579 * \mathit{Income}_i)$

$\hat{Y}_i = 35.303 + (3.579 * \mathit{Income}_i)$

$\rightarrow$ Mathematically identical models.

How do we select the reference category?

How to select the reference category?

```r
# Run regression model with democracy variable
m1 <- lm(epi_epi ~ income + democracy, data = qog_data)

# Get regression table with stargazer
stargazer(m1)
```

	Dependent variable:
	epi_epi
democracy1	16.527***
	(1.841)
income	3.579***
	(0.427)
Constant	35.303***
	(1.127)
Observations	157
R^2	0.618
Adjusted R^2	0.613
F Statistic (df = 2; 154)	124.331***
Note:	*p<0.1; **p<0.05; ***p<0.01

In comparison to autocracies (= reference category), democracies have a 16.5270 scale point higher score on the Environmental Performance Index, under control of income.

Categorical Explanatory Variables

How to include categorical explanatory variables with more than two levels?

Country	X_{region}
Afghanistan	Asia
Albania	EE
Algeria	MENA
Argentina	LA
Australia	Advanced
⋮	⋮

Country	X_{region}
Afghanistan	2
Albania	3
Algeria	5
Argentina	4
Australia	1
⋮	⋮

Country	X_{Asia}	X_{EE}	X_{LA}	X_{MENA}	$X_{Sub-Saharan}$
Afghanistan	1	0	0	0	0
Albania	0	1	0	0	0
Algeria	0	0	0	1	0
Argentina	0	0	1	0	0
Australia	0	0	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮

$$School\ enrollment\ rate = \alpha + \beta_1 Democracy_i + \beta_2 Region_{EE} + \beta_3 Region_{LA} + \beta_4 Region_{MENA} + \beta_5 Region_{Sub-Saharan} + \epsilon_i$$

$\rightarrow$ Include binary/dummy variables for all levels minus one (=reference category).

- α (intercept): expected value of Y when $X_k = 0$
- β (coefficient): expected change in Y for $X = 1$, in comparison to reference category

→ Convert into factor variable, then R automatically generates dummy variables, with first level as reference category (or change with relevel-function).

```r
# Code dummy variables on the fly
# specify region Sub-Saharan Africa = reference category
lm <- lm(primary_ser ~ democracy + relevel(as.factor(region), ref="Sub-Saharan Africa"), data = paglayan2021)

# Print model output
summary(lm)
```

```
Call:
lm(formula = primary_ser ~ democracy + relevel(as.factor(region),
    ref = "Sub-Saharan Africa"), data = paglayan2021)

Coefficients:
                                                            Estimate Std. Error t value Pr(>|t|)
(Intercept)                                                   48.060      1.796  26.754  < 2e-16 ***
democracy                                                     41.291      1.351  30.557  < 2e-16 ***
ref = "Sub-Saharan Africa")Advanced Economies                  3.063      2.143   1.429 0.153007
ref = "Sub-Saharan Africa")Asia and the Pacific               -9.101      2.437  -3.734 0.000192 ***
ref = "Sub-Saharan Africa")Eastern Europe                     12.991      2.825   4.599 4.46e-06 ***
ref = "Sub-Saharan Africa")Latin America and the Caribbean   -13.090      2.073  -6.315 3.20e-10 ***
ref = "Sub-Saharan Africa")Middle East and North Africa        4.389      2.695   1.629 0.103515
```

Under control of regime type, Eastern Europe has a student enrollment rate of 12.991 percentage points higher than Sub-Saharan Africa.

References I

Agresti, Alan, and Barbara Finlay. 2009. *Statistical methods for the social sciences.* Essex: Pearson Prentice Hall.

Chatterjee, Samprit, and Ali S. Hadi. 2015. *Regression analysis by example.* Somerset: Wiley.