Computational Photonics

Seminar 04, May 12, 2023

Implementation of a Finite-Difference Mode Solver

- Implementation of 2nd order finite difference schemes in matrix notation
- Calculation of the guided modes in a slab waveguide system
- Calculation of the guided modes in a strip waveguide system

Abbe School of Photonics | JENA
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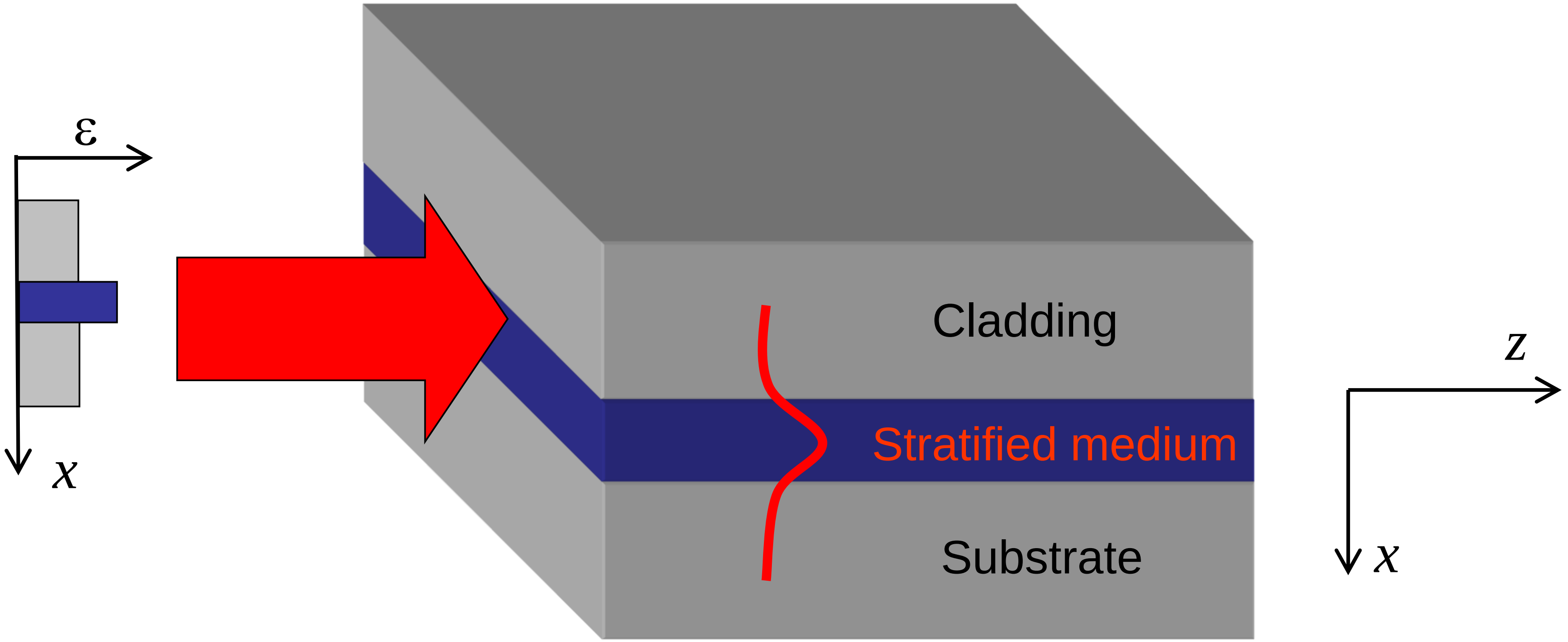
Guided modes in 1+1 (=2D) systems (stratified media)



- no y-dependence
- phase evolution in the z-direction

➔ looking for beams without diffraction in x-direction

Guided modes in 1+1D systems (TE modes)

Assuming weak guiding ($\varepsilon_0\varepsilon(\omega)\,\mathrm{div}\,\mathbf{E}(\mathbf{r},\omega)\approx 0$), we can use the

Helmholtz equation for inhomogeneous media:

$$\Delta\mathbf{E}(\mathbf{r},\omega)+\frac{\omega^2}{c^2}\varepsilon(\mathbf{r},\omega)\mathbf{E}(\mathbf{r},\omega)=0$$

Ansatz for stationary states (modes) with $\varepsilon(\mathbf{r},\omega)=\varepsilon(x,y,\omega)$:

$$\boldsymbol{E}(\boldsymbol{r})=\boldsymbol{E}_0(x,y)e^{i\beta z},\;\; \boldsymbol{H}(\boldsymbol{r})=\boldsymbol{H}_0(x,y)e^{i\beta z}$$

For a 2D geometry & TE $\mathbf{E}_0(x)=\begin{pmatrix}0\\ E_0\\ 0\end{pmatrix}$

$$\boxed{\left(\frac{1}{k_0^2}\frac{\partial^2}{\partial_x^2}+\varepsilon(x,\omega)\right)E_0(x)=\varepsilon_{eff}E_0(x)}$$

With: $\varepsilon_{\mathrm{eff}}=\left(\frac{\beta}{k_0}\right)^2$

Guided modes in 1+1D systems (TE modes)

Numerical solution by discretizing functions and operators:

1. Assume Perfectly electric conducting boundaries ($E_0(x_{\min}) = E_0(x_{\max})=0$).
2. Discretizing transverse space: $x_j = x_{\min}, x_{\min} + h, \ldots, x_{\max}$
3. Discretizing the E-fields and ε : $E_j(x) = E_0\,(x_j),\ \varepsilon_j(x,\omega) = \varepsilon(x_j,\omega)$
4. Discretizing 2nd order derivative: $\left.\dfrac{\partial^2 f}{\partial x^2}\right|_{x_j} \approx \dfrac{f(x_{j+1}) - 2f(x_j) + f(x_{j-1})}{h^2}$

➔ matrix diagonal: $-2/h^2 + k_0^2\varepsilon$
adjacent diagonals: $1/h^2$

Guided modes in 1+1D systems (matrix form)

$$\frac{1}{k_0^2}\begin{pmatrix} -\frac{2}{h^2}+k_0^2\varepsilon_1 & 1/h^2 & 0 & 0 & 0 & 0 & \cdots \\ 1/h^2 & -\frac{2}{h^2}+k_0^2\varepsilon_2 & 1/h^2 & 0 & 0 & 0 & \cdots \\ 0 & 1/h^2 & -\frac{2}{h^2}+k_0^2\varepsilon_3 & 1/h^2 & 0 & 0 & \cdots \\ 0 & 0 & \ddots & \ddots & \ddots & 0 & \cdots \\ & & & & & & \end{pmatrix}\begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ \vdots \\ \\ \end{pmatrix} = \varepsilon_{\text{eff}}\begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ \vdots \\ \\ \end{pmatrix}$$

Hint: np.diag and np.linalg.eig might be useful

Task I: Calculate TE eigenmodes of a film waveguide

```
def guided_modes_1DTE(prm, k0, h):
    """Computes the effective permittivity of a TE polarized guided eigenmode.
    All dimensions are in µm.
    Note that modes are filtered to match the requirement that
    their effective permittivity is larger than the substrate (cladding).

    Parameters
    ----------
    prm : 1d-array
        Dielectric permittivity in the x-direction
    k0 : float
        Free space wavenumber
    h : float
        Spatial discretization

    Returns
    -------
    eff_eps : 1d-array
        Effective permittivity vector of calculated modes
    guided : 2d-array
        Field distributions of the guided eigenmodes
    """
    pass
```

Task I: Calculate TE eigenmodes of a film waveguide

Input variables: $\varepsilon(x,\omega)$, h, k_0

Gaussian waveguide profile:

$$\varepsilon(x,\omega) = \varepsilon_{\text{Substrate}} + \Delta\varepsilon \mathrm{e}^{-(x/W)^2}$$

Select guided modes according to their eigenvalue:

$$\varepsilon_{\text{Substrate}} < \varepsilon_{\text{eff}} < \varepsilon(x)_{\max}$$

Use the following parameters for testing:

$$\varepsilon_{Substrate} = 2.25, \Delta\varepsilon = 0.015,\ W = 15\mu m,\ \lambda = 780nm$$

Guided modes in 2+1 (=3D) systems (strip waveguide)
in scalar approximation

scalar Helmholtz equation ➔ eigenvalue problem for scalar fields

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]u(x,y)+\left[k_0^2\varepsilon(x,y,\omega)-\beta^2(\omega)\right]u(x,y)=0$$

transform to standard notation

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k_0^2\varepsilon(x,y,\omega)\right]u(x,y)=\beta^2(\omega)u(x,y)$$

This eigenvalue problem is to be solved by a finite difference scheme using electric conducting boundaries.

Discretizing 2D 2$^{\text{nd}}$ order derivative:

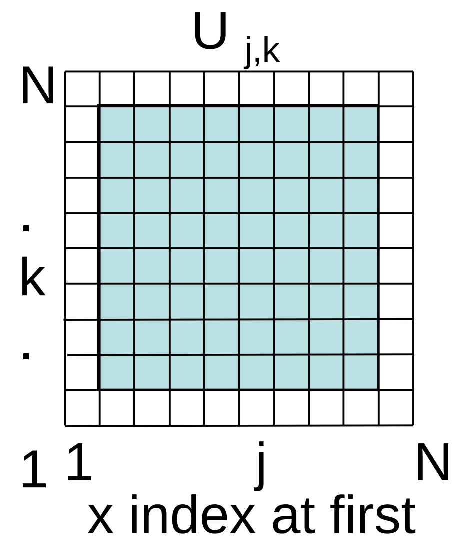
$$\left.\frac{\partial^2 f}{\partial x^2}\right|_{x_j,y_k}+\left.\frac{\partial^2 f}{\partial y^2}\right|_{x_j,y_k}\approx\frac{f(x_{j+1},y_k)+f(x_{j-1},y_k)+f(x_j,y_{k+1})+f(x_j,y_{k-1})-4f(x_j,y_k)}{h^2}$$

➔ matrix diagonal: $-4/h^2$ & adjacent diagonals: $1/h^2$
➔ 2 more bands with $1/h^2$

Numerical implementation of the 2D Laplace operator

$$
\begin{pmatrix}
-4 & 1 & 0 & 0 & .. & 0 & 1 & 0 & 0 & 0 & 0 & \dots & & & & \\
1 & -4 & 1 & 0 & .. & 0 & 0 & 1 & 0 & 0 & 0 & \dots & & & & \\
 & & & & & & & & & & & & & & & \\
1 & 0 & 0 & 0 & .. & 0 & -4 & 1 & 0 & 0 & 0 & .. & 0 & 1 & 0 & 0 & 0 & \dots \\
0 & 1 & 0 & 0 & .. & 0 & 1 & -4 & 1 & 0 & 0 & .. & 0 & 0 & 1 & 0 & 0 & \dots \\
0 & 0 & 1 & 0 & .. & 0 & 0 & 1 & -4 & 1 & 0 & .. & 0 & 0 & 0 & 1 & 0 & \dots
\end{pmatrix}
\begin{pmatrix}
U_{2,2} \\ U_{3,2} \\ \dots \\ U_{N-2,2} \\ U_{N-1,2} \\ U_{2,3} \\ U_{3,3} \\ U_{4,3} \\ \dots \\ U_{N-2,3} \\ U_{N-1,3} \\ U_{2,4} \\ U_{3,4} \\ U_{4,4} \\ \dots
\end{pmatrix}
$$

discrete Laplace operator



Hint:
scipy.sparse.linalg.eigs
scipy.sparse.spdiags

Differentiation with respect to x (same as 1D)

Differentiation with respect to y

Use the concept of sparse matrices since the matrix will be mainly filled with zeros.

Task II: Quasi-TE modes of a strip waveguide

```
def guided_modes_2D(prm, k0, h, numb):
    """Computes the effective permittivity of a quasi-TE polarized guided
    eigenmode. All dimensions are in µm.

    Parameters
    ----------
    prm  : 2d-array
        Dielectric permittivity in the xy-plane
    k0 : float
        Free space wavenumber
    h : float
        Spatial discretization
    numb : int
        Number of eigenmodes to be calculated

    Returns
    -------
    eff_eps : 1d-array
        Effective permittivity vector of calculated eigenmodes
    guided : 3d-array
        Field distributions of the guided eigenmodes
    """
    pass
```

Task II: Quasi-TE modes of a strip waveguide

Input variables: $\varepsilon(x, y, \omega)$, h, k_0

$$\varepsilon(x, y, \omega) = \varepsilon_{\text{Substrate}} + \Delta\varepsilon e^{-\frac{x^2+y^2}{W^2}}$$

Use the following parameters for testing:

$$\varepsilon_{Substrate} = 2.25, \Delta\varepsilon = 0.015,\ W = 15\mu m,\ \lambda = 780nm$$

Homework 1

- Solve tasks I & II.

- For each task we require that each group implements a program that solves the problem and documents the code and its result
(e.g. with an iPython Notebook)

- Submission via email to: shiu.hei.lam@uni-jena.de

- Due 3 a.m.Thursday, May 25, 2023

- The subject line of the email should have the following format:
  – CPho23 - solution to the homework 1: group [Number]; [family_name1, family_name2, family_name3]:
  – gather files in a single zip archive
  (no rar, tar, 7z, gz or any other compression format)