Computational Photonics

Seminar 03, SS 2022

Homework 0: Implementation of the Matrix Method

- calculation of the transfer matrix
- calculation of reflection and transmission characteristics of stratified media
- calculation of fields inside layers

Abbe School of Photonics | JENA
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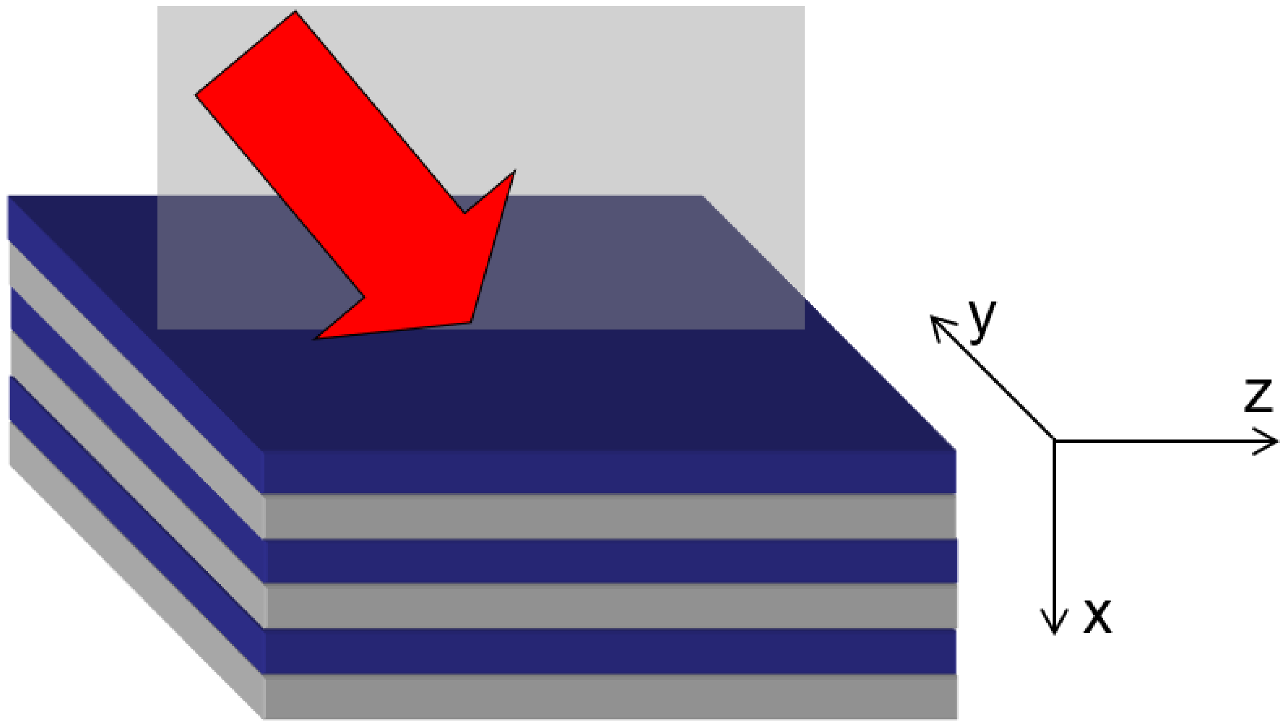
Optics in stratified media



- Bragg mirror
- mirror with chirp for compensating dispersion
- interferometer

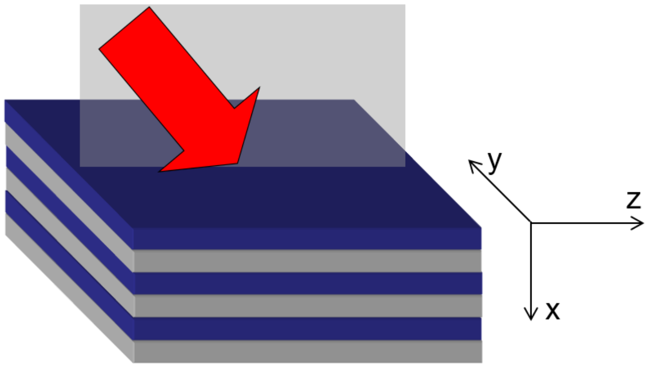
Overview of the matrix method

Assumptions:

– infinitely extended structure along *y* and *z* directions

– Plane wave

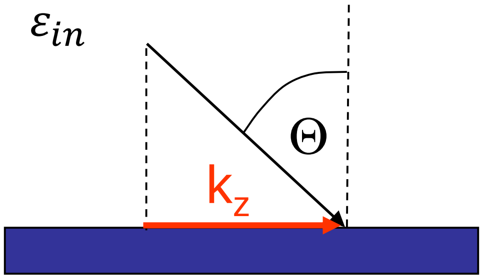
Ansatz: $$\mathbf{E}_{\mathbf{real}}(x,z,t) = \mathrm{Re}\left[\mathbf{E}(x)\exp\left(ik_z z - i\omega t\right)\right]$$
$$\mathbf{H}_{\mathbf{real}}(x,z,t) = \mathrm{Re}\left[\mathbf{H}(x)\exp\left(ik_z z - i\omega t\right)\right]$$



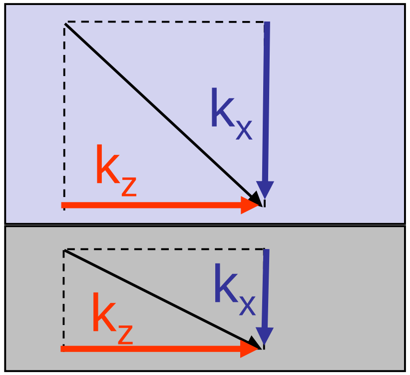
Overview of the matrix method

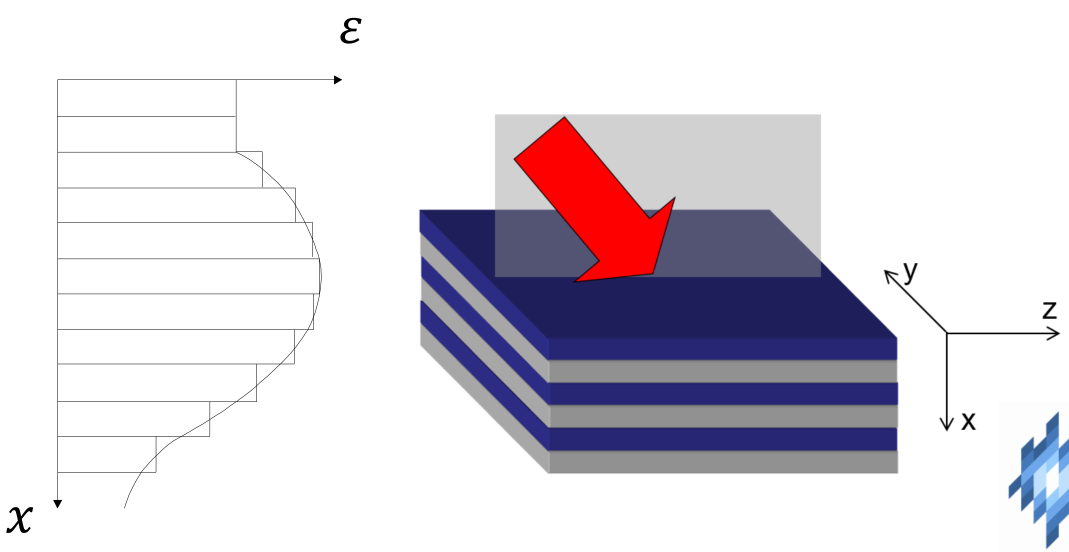
Approach:

- without y-dependence
- Decompose into TE and TM modes
- exploit the continuity of transverse dependence of field



- stratified layers





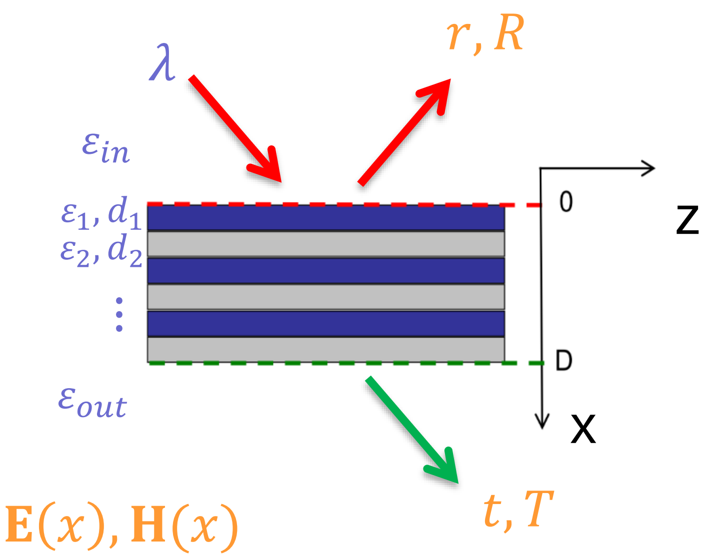
Overview of the matrix method

Inputs:

- incident wavelength λ
- dielectric function $\varepsilon(x)$, *i.e.* ε_i and thickness d_i of each layer

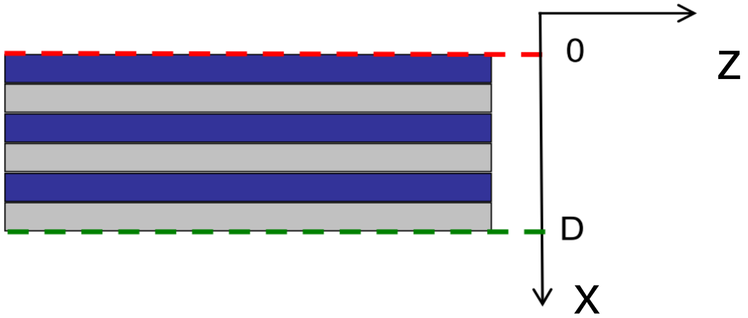
Outputs:

- reflection coefficient r, transmission coefficient t (with phase information)
- Reflectivity R, Transmissivity T (only amplitude)
- Fields across layers $\mathbf{E}(x), \mathbf{H}(x)$



Summary of the Matrix method

$$\begin{bmatrix} F(D) \\ G(D) \end{bmatrix} = \prod_i \hat{m}_i \begin{bmatrix} F(0) \\ G(0) \end{bmatrix} = \hat{M} \begin{bmatrix} F(0) \\ G(0) \end{bmatrix}$$



$$m_i = \begin{bmatrix} \cos\left(k_x^{(i)} d_i\right) & \frac{1}{q_i k_x^{(i)}} \sin\left(k_x^{(i)} d_i\right) \\ -q_i k_x^{(i)} \sin\left(k_x^{(i)} d_i\right) & \cos\left(k_x^{(i)} d_i\right) \end{bmatrix}$$

TE: $F = E_y, \quad G = \frac{\partial}{\partial x} E_y, \quad q_i = 1$

TM: $F = H_y, \quad G = q_i \frac{\partial}{\partial x} H_y, \quad q_i = 1/\varepsilon_i$

with

$$\left[k_x^{(i)}\right]^2 = \left(\frac{2\pi}{\lambda_0}\right)^2 \varepsilon_i(\omega) - k_z^2$$

Reflection and Transmission

reflection coefficient

$$r = \frac{q_{in}k_x^{in}M_{22} - q_{out}k_x^{out}M_{11} - i\left(M_{21} + q_{in}k_x^{in}q_{out}k_x^{out}M_{12}\right)}{q_{in}k_x^{in}M_{22} + q_{out}k_x^{out}M_{11} + i\left(M_{21} - q_{in}k_x^{in}q_{out}k_x^{out}M_{12}\right)}$$

transmission coefficient

$$t = \frac{2q_{in}k_x^{in}}{q_{in}k_x^{in}M_{22} + q_{out}k_x^{out}M_{11} + i\left(M_{21} - q_{in}k_x^{in}q_{out}k_x^{out}M_{12}\right)}$$

Reflectivity $$R = |r|^2$$

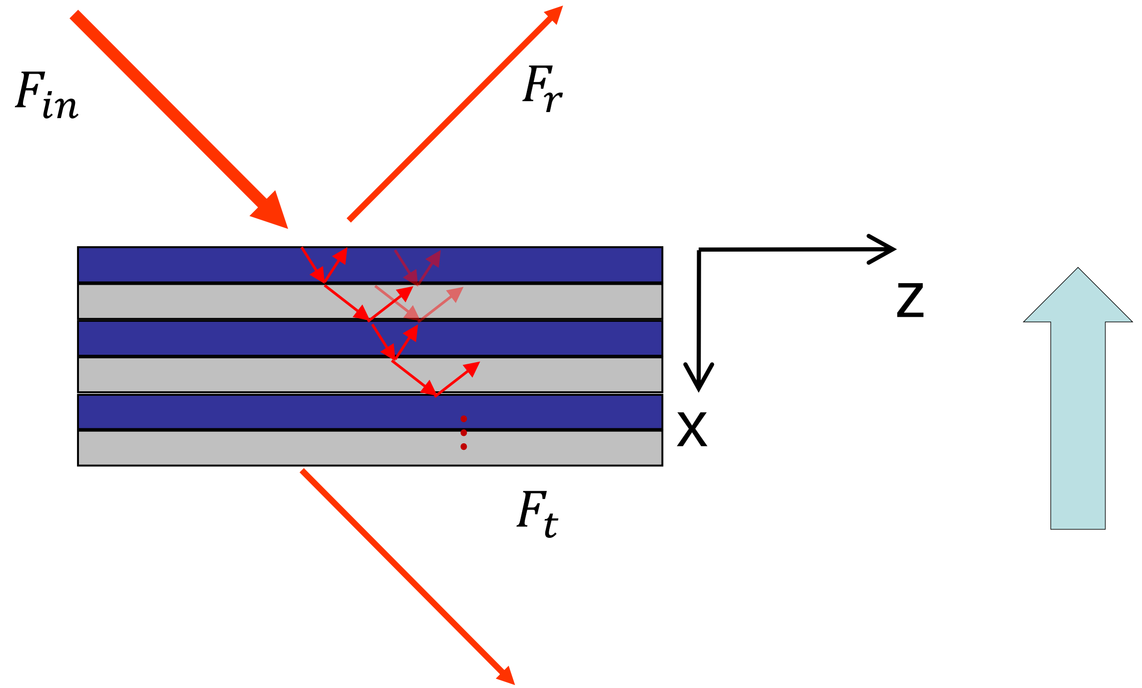
Transmissivity $$T = \frac{q_{out}\operatorname{Re}\left(k_x^{out}\right)}{q_{in}\operatorname{Re}\left(k_x^{in}\right)}|t|^2$$

Field Distribution

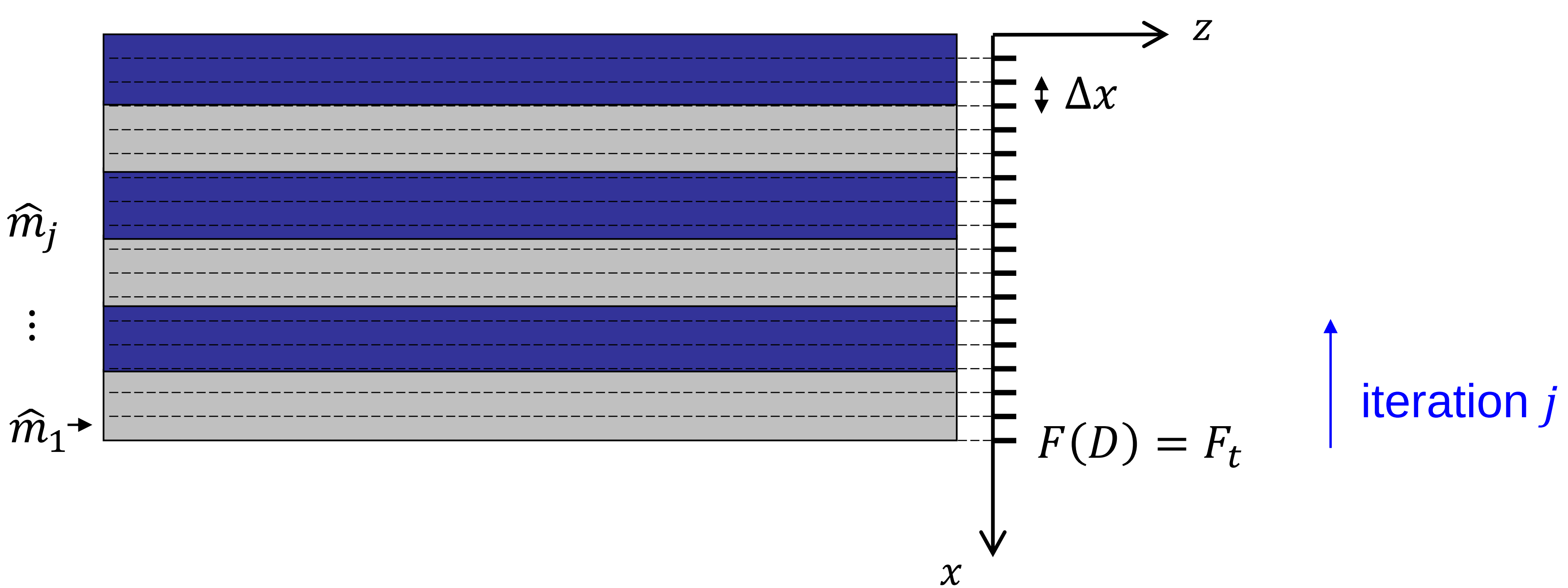
- Field at the incident side and within layers consists of incidence and reflected field.
- In contrast, field at the transmitted side consists only of transmitted field.

$$F(x < D) = F_{in} + F_r$$

$$F(x > D) = F_t$$



Approach: iterates backwards along x from the transmitted side



1st iteration:

$$\overbrace{\begin{bmatrix} F(D) \\ G(D) \end{bmatrix} = F_t \begin{bmatrix} 1 \\ iq_{out}k_x^{out} \end{bmatrix}}^{\text{known}} = \hat{m}_1 \overbrace{\begin{bmatrix} F(D-\Delta x) \\ G(D-\Delta x) \end{bmatrix}}^{\text{unknown}}$$

$$\hat{m}_1 = \begin{bmatrix} \cos\left(k_x^{(1)}\Delta x\right) & \dfrac{1}{q_1 k_x^{(1)}}\sin\left(k_x^{(1)}\Delta x\right) \\ -q_1 k_x^{(1)}\sin\left(k_x^{(1)}\Delta x\right) & \cos\left(k_x^{(1)}\Delta x\right) \end{bmatrix}$$

$$\overbrace{\begin{bmatrix} F(D-\Delta x) \\ G(D-\Delta x) \end{bmatrix}}^{\text{unknown}} = \overbrace{F_t[\hat{m}_1]^{-1} \begin{bmatrix} 1 \\ iq_{out}k_x^{out} \end{bmatrix}}^{\text{known}}$$

$$[\hat{m}_N]^{-1} = \begin{bmatrix} \cos\left(k_x^{(1)}\Delta x\right) & -\dfrac{1}{q_1 k_x^{(1)}}\sin\left(k_x^{(1)}\Delta x\right) \\ q_1 k_x^{(1)}\sin\left(k_x^{(1)}\Delta x\right) & \cos\left(k_x^{(1)}\Delta x\right) \end{bmatrix}$$

j^th iteration:

$$\begin{bmatrix} F(D-j\Delta x) \\ G(D-j\Delta x) \end{bmatrix} = F_t [\widehat{M}_j]^{-1} \begin{bmatrix} 1 \\ iq_{out} k_x^{out} \end{bmatrix}$$

$$[\widehat{M}_j]^{-1} = [\widehat{m}_j]^{-1} \cdots [\widehat{m}_2]^{-1} [\widehat{m}_1]^{-1}$$

The real field

The observable (real) field

$$\mathbf{E}_\mathbf{r}(x,z,t) = \mathrm{Re}\left[\mathbf{E}(x)\exp\left(ik_z z - i\omega t\right)\right]$$
$$\mathbf{H}_\mathbf{r}(x,z,t) = \mathrm{Re}\left[\mathbf{H}(x)\exp\left(ik_z z - i\omega t\right)\right]$$

What you have actually calculated is the complex value of a certain component:

TE: $\mathbf{E}(x) = F(x)\,\mathbf{e}_\mathbf{y}$

TM: $\mathbf{H}(x) = F(x)\,\mathbf{e}_\mathbf{y}$

Task I : Transfer matrix

Goal : calculation of $\hat{M}$

```
import numpy as np
from matplotlib import pyplot as plt

def transfermatrix(thickness, epsilon, polarisation, wavelength, kz):
    '''Computes the transfer matrix for a given stratified medium.

    Parameters
    ----------
    thickness : 1d-array
        Thicknesses of the layers in µm.
    epsilon : 1d-array
        Relative dielectric permittivity of the layers.
    polarisation : str
        Polarisation of the computed field, either 'TE' or 'TM'.
    wavelength : float
        The wavelength of the incident light in µm.
    kz : float
        Transverse wavevector in 1/µm.

    Returns
    -------
    M : 2d-array
        The transfer matrix of the medium.
    '''
    pass
```

Task II: Reflection and transmission coefficients (1/2)

Goal: computation of r, t, R, t as a function of the wavelength

```
def spectrum(thickness, epsilon, polarisation, wavelength, angle_inc, n_in, n_out):

    '''Computes the reflection and transmission of a stratified medium.

    Parameters
    ----------
    thickness : 1d-array
        Thicknesses of the layers in µm.
    epsilon : 1d-array
        Relative dielectric permittivity of the layers.
    polarisation : str
        Polarisation of the computed field, either 'TE' or 'TM'.
    wavelength : 1d-array
        The wavelength of the incident light in µm.
    angle_inc : float
        The angle of incidence in degree (not radian!).
    n_in, n_out : float
        The refractive indices of the input and output layers.
```

Task II: Reflection and transmission coefficients (2/2)

Goal: computation of r, t, R, t as a function of the wavelength

```
Returns
-------
t : 1d-array
    Transmitted amplitude
r : 1d-array
    Reflected amplitude
T : 1d-array
    Transmitted energy
R : 1d-array
    Reflected energy
'''
pass
```

Task III*: Field distribution (1/2)

Goal: Computation of the complex field f at predefined values of x

```
def field(thickness, epsilon, polarisation, wavelength, kz,
          n_in, n_out, Nx, l_in, l_out):
    '''Computes the field inside a stratified medium.

    The medium starts at x = 0 on the entrance side. The transmitted field
    has a magnitude of unity.

    Parameters
    ----------
    thickness : 1d-array
        Thicknesses of the layers in µm.
    epsilon : 1d-array
        Relative dielectric permittivity of the layers.
    polarisation : str
        Polarisation of the computed field, either 'TE' or 'TM'.
    wavelength : float
        The wavelength of the incident light in µm.
    kz : float
        Transverse wavevector in 1/µm.
```

Task III*: Field distribution (2/2)

Goal: Computation of the complex field f at predefined values of x

```
    n_in, n_out : float
        The refractive indices of the input and output layers.
    Nx : int
        Number of points where the field will be computed.
    l_in, l_out : float
        Additional thickness of the input and output layers where the field will
be computed.

    Returns
    -------
    f : 1d-array
        Field structure
    index : 1d-array
        Refractive index distribution
    x : 1d-array
        Spatial coordinates
    '''
    pass
```

Task IV**: Time animation of the field

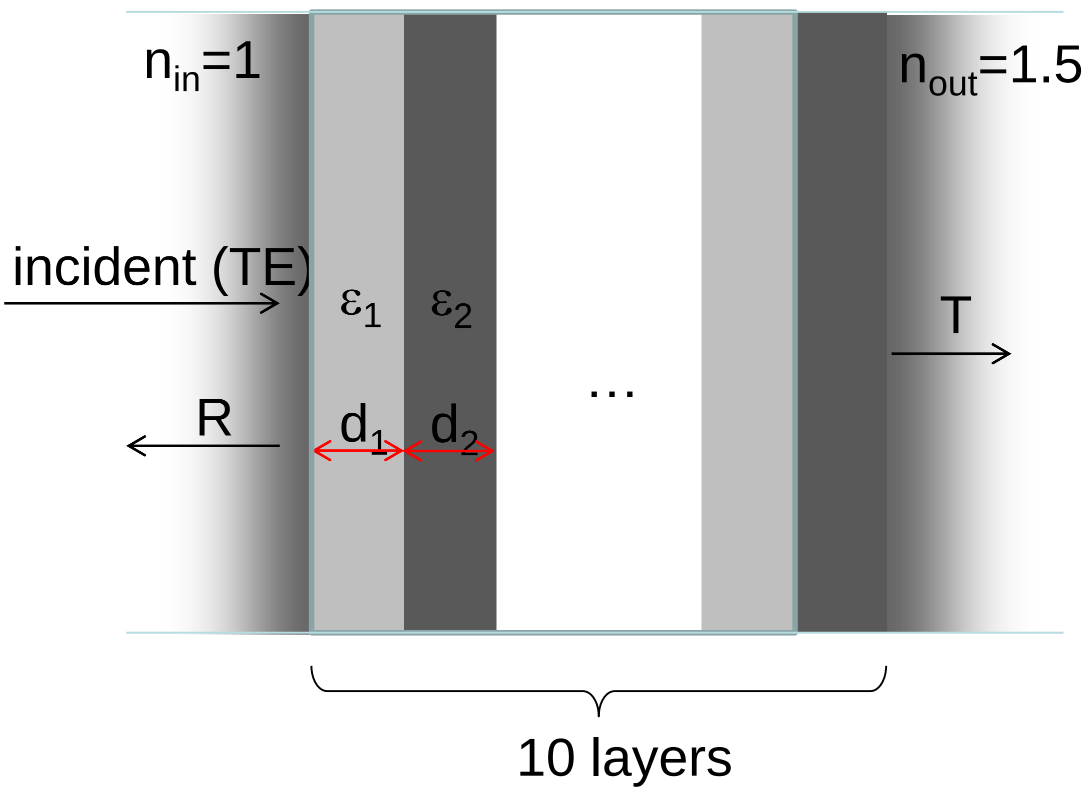
Goal: Visualization of the temporal evolution of the field

```
def timeanimation(x, f, index, steps, periods):
    ''' Animation of a quasi-stationary field.

    Parameters
    ----------
    x : 1d-array
        Spatial coordinates
    f : 1d-array
        Field
    index : 1d-array
        Refractive index
    steps : int
        Total number of time points
    periods : int
        Number of the oscillation periods.

    '''
    pass
```

Example parameters

Define a Bragg mirror at 780nm:

```
>> eps1 = 2.25;
>> eps2 = 15.21;
>> d1 = 0.13;    %[µm]
>> d2 = 0.05;    %[µm]
>> N  = 5;
>> polarisation ='TE';
>> angle_inc = 0.0;
>> n_in = 1.0;
>> n_out = 1.5;
```



Create the arrays

```
>> epsilon = zeros(1, 2*N);
>> epsilon(1:2:2*N)  = eps1;
>> epsilon(2:2:2*N)  = eps2;
>> thickness = zeros(1, 2*N);
>> thickness(1:2:2*N)= d1;
>> thickness(2:2:2*N) = d2;
>> lambda = linspace(0.5, 1.5, 100); %[µm]
```

Now, e.g. calculate the transmission/reflection spectrum:

```
>> [t, r, T, R] = spectrum(thickness, epsilon,
                            lambda_vector, ang]
```

Voluntary Homework

- These tasks are **voluntary,** but it is strongly encouraged to solve at least the first two (I and II).
- We will assign you a group if you indicate in the poll on Moodle.
- Since this homework is voluntary, you should **not** send us your solution.
- The standard solution will be discussed in the next seminar on 20th May 2022.