Theory for Rod impacting the Ground

Direction Definitions

The +y axis is defined to be the upward direction.

The +x axis is defined to be the right direction.

The +z axis is defined to be the out of the page direction.

Anticlockwise is taken to be the positive angular direction.

Variable Definitions

Let e be the coefficient of restitution.

Let $\dot{y}'$ be the initial y velocity and $\dot{y}$ be the final y velocity.

Let ω' be the initial angular velocity and ω be the final angular velocity.

l is the length of the stick.

$\int N\,dt$ is the normal impulse by the floor, note we take this to be always positive.

ϕ is the angle between the stick and the floor.

Let $\dot{y}_0'$ be the initial y velocity of the point of contact with the ground and $\dot{y}_0$ be the final y velocity.

Actual Theory

Ok we will start with what happen if the left point of stick touches the ground first.

We can find $\dot{y}_0'$ by the following equation:

$$\dot{y}' - r\omega' \cos\theta = \dot{y}_0'$$

We get the above equation because if ω' is positive, (i.e. the rod is spinning anticlockwisely), then $\dot{y}_0'$ will be more negative.

Also, coefficient of restitution gives us:

$$-e\,\dot{y}_0' = \dot{y}_0$$

By considering impulses, we have the following expression:

$$\int N\,dt = m(\dot{y} - \dot{y}')$$

$$\frac{l}{2}\cos\theta\int N\,dt = I(\omega' - \omega) \Rightarrow 6\cos\theta\int N\,dt = ml(\omega' - \omega)$$

The last equation:

$$\dot{y} - r\omega\cos\theta = \dot{y}_0$$

Here we have 5 equations and 5 unknowns:

$\int N\,dt, \dot{y}_0, \dot{y}, \omega, \dot{y}_0'$.

Hence we can solve.

The solutions are as given before:

$$\omega = \frac{l\omega' + 6\,(1+e)\,\dot{y}'\cos\theta - 3\,e\,l\omega'\cos^2\theta}{(1+3\cos^2\theta)l}$$

$$\dot{y} = \frac{(6\dot{y}'\cos^2\theta + l(1+e)\omega'\cos\theta - \ 2\,e\,\dot{y}')}{2(1+3\cos^2\theta)}$$

Now we will go on to what happens if the right stick reaches ground first.

We can find $\dot{y}_0'$ by the following equation:

$$\dot{y}' + r\omega'\cos\theta = \dot{y}_0'$$

We get the above equation because if ω' is positive, (i.e. the rod is spinning anticlockwisely), then $\dot{y}_0'$ will be less negative.

Also, coefficient of restitution gives us:

$$-e\,\dot{y}_0' = \dot{y}_0$$

By considering impulses, we have the following expression:

$$\int N\,dt = m(\dot{y} - \dot{y}')$$

$$\frac{l}{2}\cos\theta\int N\,dt = I(\omega - \omega') \Rightarrow 6\cos\theta\int N\,dt = ml(\omega - \omega')$$

The last equation:

$$\dot{y} + r\omega\cos\theta = \dot{y}_0$$

Here we have 5 equations and 5 unknowns:

$\int N\,dt, \dot{y}_0, \dot{y}, \omega, \dot{y}_0'$.

Hence we can solve.

The solutions are as given before:

$$\omega = \frac{l\omega' - 6\,(1+e)\,\dot{y}'\cos\theta - 3\,e\,l\omega'\cos^2\theta}{(1+3\cos^2\theta)l}$$

$$\dot{y} = \frac{(6\dot{y}'\cos^2\theta - l(1+e)\omega'\cos\theta - \ 2\,e\,\dot{y}')}{2(1+3\cos^2\theta)}$$