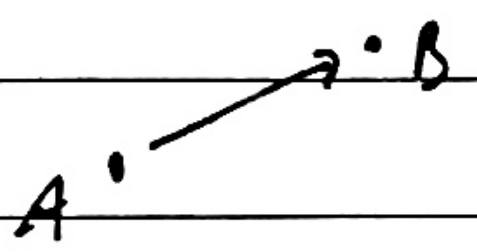
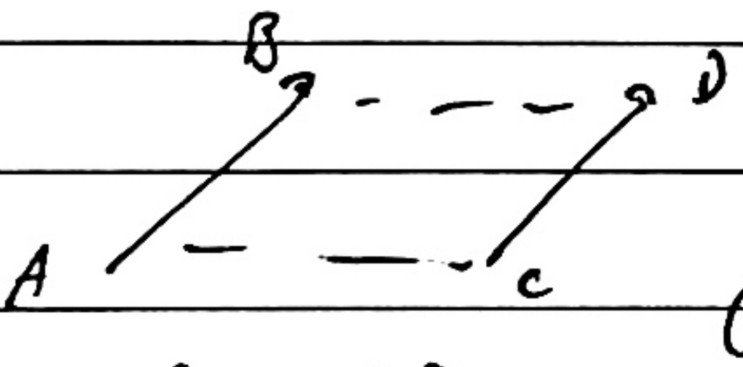
MATH H110 LECTURE 1 NOTES

Date: No.

math.berkeley.edu/~giventh MATH H110 Fall '21

Quizzes Tuesday HW Thursday

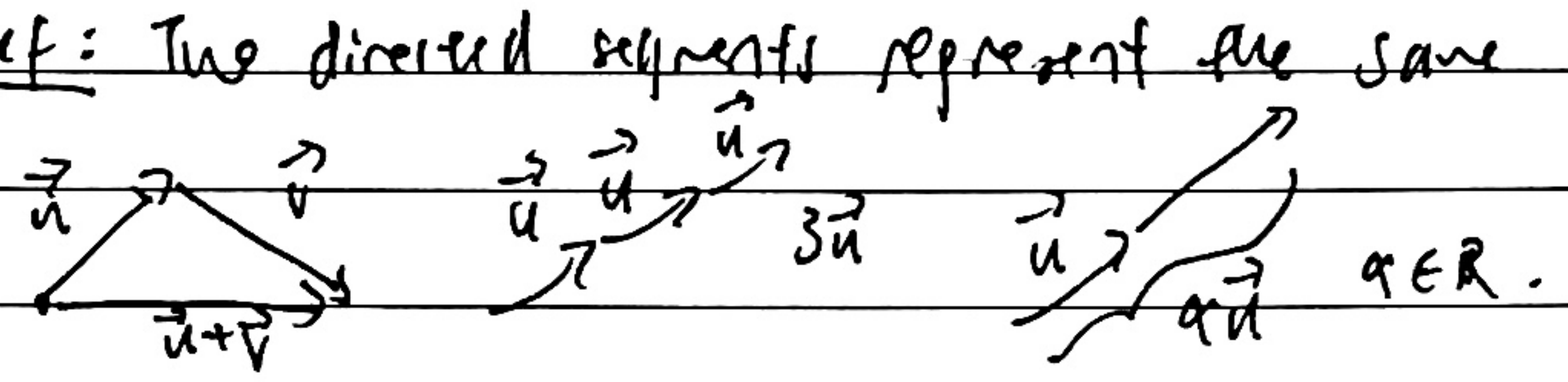




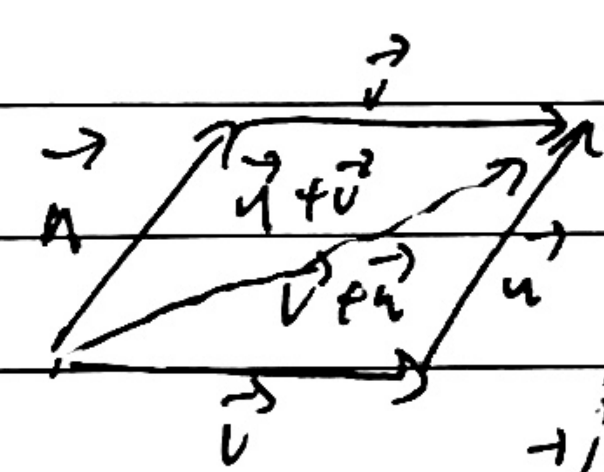
Introduction to vectors

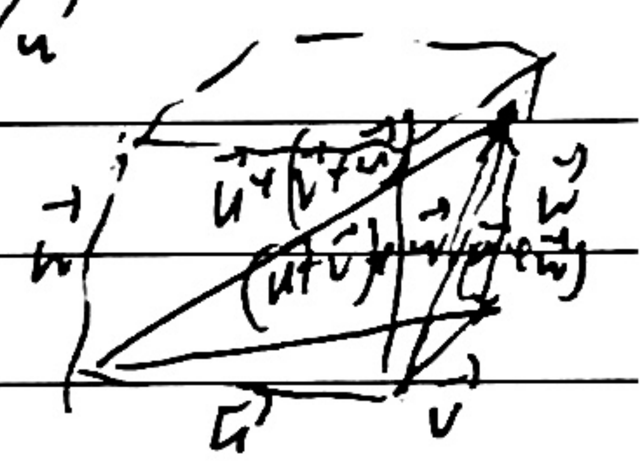
Directed segments (ordered) $\vec{AB}$

Def: Two directed segments represent the same vector if ABDC is a parallelogram (~~degenerate~~ not)



$\vec{u}$, $\vec{v}$, $\vec{u}+\vec{v}$, $3\vec{u}$, $\alpha\vec{u}$, $\alpha \in \mathbb{R}$.



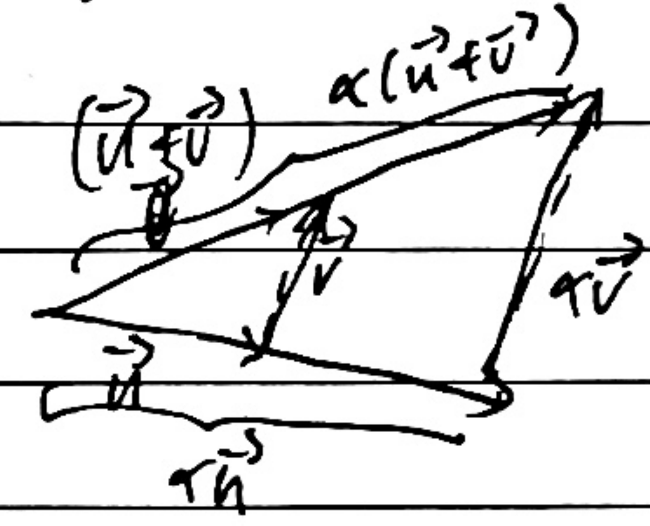


linear combinations $\alpha\vec{u} + \beta\vec{v} + \cdots + \gamma\vec{w}$

non ambiguous because associative and commutative

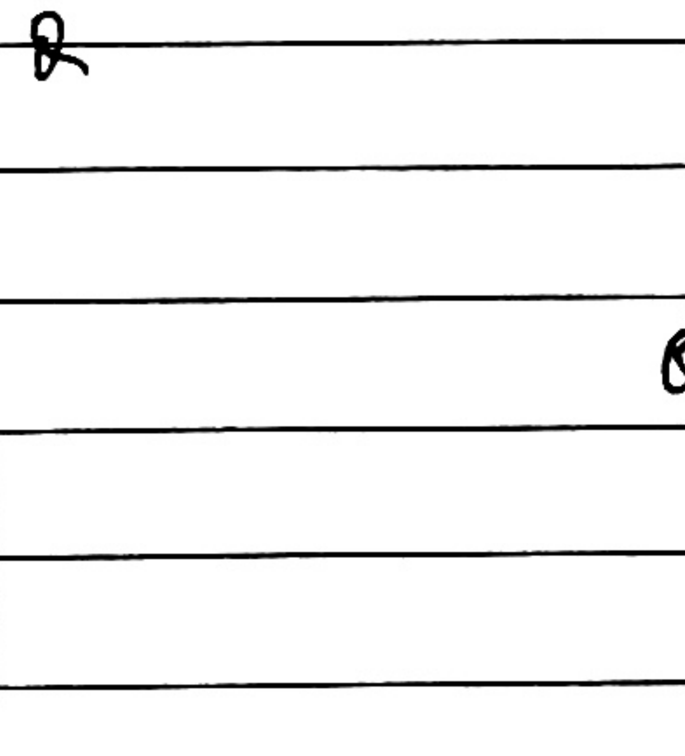
$(\vec{u}+\vec{w})+\vec{v} = \vec{u}+(\vec{w}+\vec{v})$ $\vec{u}+\vec{w} = \vec{w}+\vec{u}$

$\alpha(\vec{u}+\vec{v}) = \alpha\vec{u} + \alpha\vec{v}$



$(\alpha+\beta)\vec{u} = \alpha\vec{u} + \beta\vec{u}$ geometric interpretation is to take $\vec{u}$ as the number line. Then α, β are just numbers on this line.

Example.



Claim: medians are concurrent and intersect at M where $\frac{AM}{MA'} = 2$.

Pick arbitrary point Q as the origin.

$\vec{QA} = \vec{QA}$

$\vec{QA'} = \vec{QC} + \frac{1}{2}(\vec{QB} - \vec{QC})$

$\vec{QM} = \frac{2}{3}(QA' - QA) + QA = \frac{2}{3}\left(\frac{1}{2}\vec{QC} + \frac{1}{2}\vec{QB}\right) + \frac{1}{3}\vec{QA}$

$= \frac{1}{3}(\vec{QA} + \vec{QB} + \vec{QC})$

this expression is symmetric w.r.t. A, B, C.

will get same answer regardless which starting pt we chose

Coordinates

requires a reference frame. $\vec{OU} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k} = \vec{u}$ $\lambda\vec{u} = (\lambda\alpha, \lambda\beta, \lambda\gamma)$

(α, β, γ) is the coordinate $= \lambda(\alpha, \beta, \gamma)$

$(\alpha', \beta', \gamma') = \vec{u}'$

Then $\vec{u}+\vec{u}' = (\alpha+\alpha', \beta+\beta', \gamma+\gamma')$

AZONE

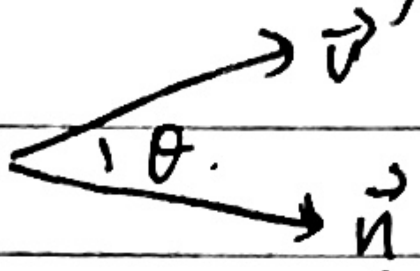
Question: What is a vector?

Vectors are translations of space.

Addition of vectors are composition of translations

Metric Concepts (such as distances and angles)

Definition. $\langle \vec{u}, \vec{v} \rangle := |\vec{u}||\vec{v}| \cos\theta$



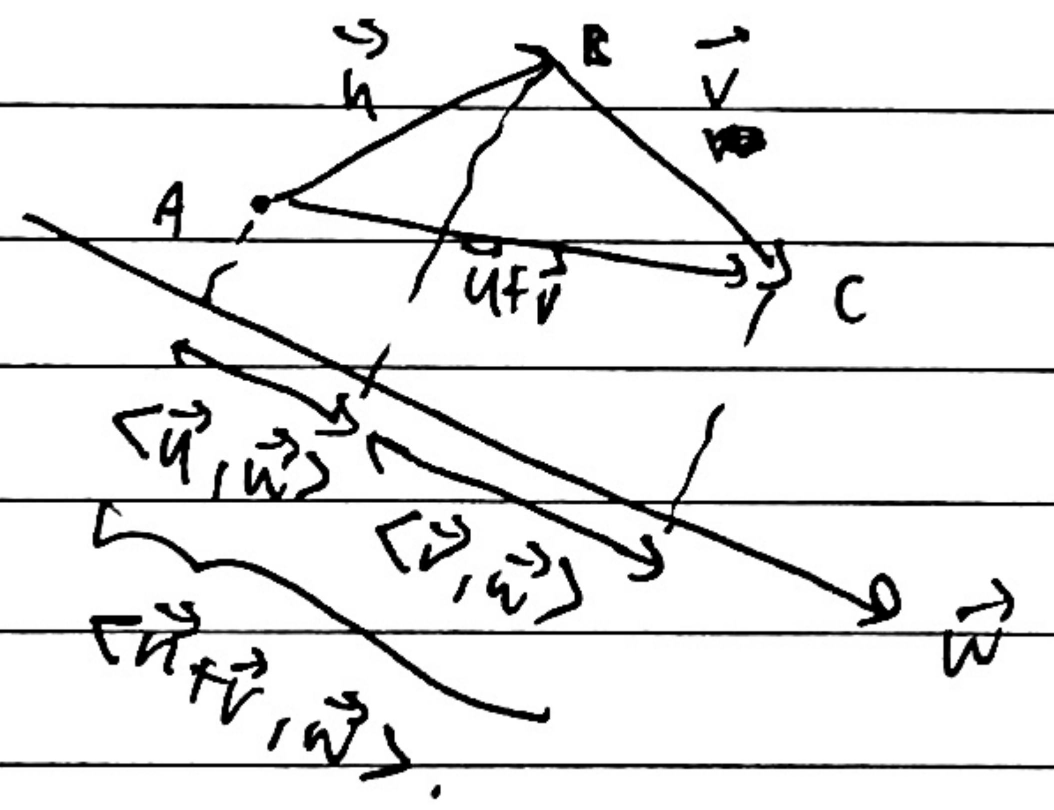
$\langle \vec{u}, \vec{u} \rangle = |\vec{u}||\vec{u}| \cos 0^\circ = |\vec{u}|^2$ $\quad \langle \vec{u}, \vec{v} \rangle = \langle \vec{v}, \vec{u} \rangle$

$\cos^{-1}\left(\frac{\langle \vec{u}, \vec{v} \rangle}{\sqrt{\langle \vec{u}, \vec{u} \rangle \langle \vec{v}, \vec{v} \rangle}} \right) = \theta.$

$\langle \vec{u} + \vec{v}, \vec{w} \rangle = \langle \vec{u}, \vec{w} \rangle + \langle \vec{v}, \vec{w} \rangle$

$\langle \lambda \vec{u}, \vec{w} \rangle = \lambda \langle \vec{u}, \vec{w} \rangle$

Take $\vec{w}$ as the number line. WLOG, let $\vec{w}$ be of unit length.



Tuesday: Prepare for quiz on complex numbers

Thursday: Do all exercises on geometry.