pefaithon $x \in \mathbb{R}$ is called maximum of S if $x \in \mathbb{R}$ S $X \in \mathbb{R}$		No.:	Date:		
Completuress Arism Completuress Arism R has the properties f1x, \(\frac{1}{2}\), \(\infty\), \(\infty\) and R? R has the properties f1x, \(\frac{1}{2}\), \(\infty\), \(\infty\) and R? Internals. For a c b, \((n_1 \L) = \) x \(\infty\) x \(\infty\), \(\infty\) a \(\infty\), \(\infty\) \(\infty\), \(\infty\)		MATH 104 LECTURE 2 NOTES			
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R has the previous f1x, +1 \in \text{ A 110 has from } \end{array} what's the difference between (b. and R? Internals. For a < b, (a, L) = \(\frac{1}{2} \times \times \times \) \(\frac{1}{2} \times \times \times \) \(\frac{1}{2} \times \times \times \times \) \(\frac{1}{2} \times \times \times \times \) \(\frac{1}{2} \times \times \times \times \times \) \(\frac{1}{2} \times \times \times \times \times \) \(\frac{1}{2} \times \times \times \times \times \times \) \(\frac{1}{2} \times \time		amplethness Axiom	mnx, min,		
Internals. For a < b, [a, L] = { x < R : a < x < b }. (open internal) [a, b] = { x < R : a < x < b }. (closed internal) [a, b] = { x < R : a < x < b }. (semi-upon internal) (a, m) = { x < R : a < x > b }. (semi-upon internal) (a, m) = { x < R : a < x }. Man, mon. Let S \(\sigma \). We a nonempty set . Definition X \(\sigma \) R is called maximum of S if x \(\sigma \) R is \(\sigma \) Y \(\sigma \) S, \(\sigma \) X \(\sigma \) \(\sigma \) R is and \(\sigma \) Y \(\sigma \) S. The consider the number of S if \(\sigma \) R is \(\sigma \) A is \(\sigma \) R is \(\sigma \) A is \(\sigma \) R is \(\sigma \) A is \(\sigma \) R is \(\sigma \) A is \(\sigma \) R is \(\sigma \) A is \(\sigma \) R is \(\sigma \) A is \(umputures	s axion.	
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[915] = { x ∈ R; a < x ≤ b}. [dexed heaver] (a,b) = { x ∈ R, a < x ≤ b}. [semi-upon heaver]) (a, m) = { x ∈ R, a < x ≤ b}. [semi-upon heaver]) (a, m) = { x ∈ R, a < x ≤ b}. Max, mn. Let S ⊆ R. be a nonempty set. Defaition x ∈ R is called maxhirm of S if x ∈ R S & Y ∈ S, Y ≤ x. I ∈ R n cylich minimum of S if 0 = ∈ S and by G S, Z ≤ Y. Lx: Prove S = {e.i.} has no max. Suppose an the instrum, x p the max. Oxx < 1 ⇒ x to ∈ S. Then consider the number x to x to x ∈ S > x which is fiver. Hunce we found a number within (0,1) that is higger than x ⇒ underdiction. Assume the unitary, 3 a = max S. ⇒ b < b < 1. Put, consider x = Bot1 ∈ S. ab < x (->fe-). ∴ S has no max. Definition Say M ∈ R is an upperbound of S if the ES, x ≤ M If such M exists, say S is bounded above, Say m ∈ R is a lower bound of S if 0 the ES. M ≤ x. If m exists, S is bounded below. If s is bounded both above and below, then S is bounded.					
[a,b]= \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \	Internis.	FOC acb, (MIL) = { X ER: 9< X < b }.	(open interul)		
Max, mn. Let S.S.R. be a nanemyty set. Definition A ER 15 could maximum of S if X ERS & YYES, Y & X. IER 17 could maximum of S if X ERS & YYES, Y & X. IER 17 could maximum of S if X ERS & YYES, Y & X. IER 17 could maximum of S if X ERS & YYES, Y & X. It Suppose an tre confram, X 18 the max. O < 1 < X to 1 = 1 Suppose an tre confram, X 18 the max. Then consider the number X to 1 = X to 1 > X which is far. Hence we found a number within (0,1) that is bigger than X = 1 confradition. Assume the confram, 3 do = max S. = 1 O < do < 1. Put, consider X = Rott ES. do < X (->to 1		[4,5]= \(\text{X} \in \text{R}; \q \lext{X} \leq \text{6}\).	(dond Interval)		
Mak, mn. Let S.S.R. be a nonempty set. Definition XER is called maximum of S if XERS & YYES, YEX. IER 1 cylled Minimum of S if XERS & YYES, YEX. IER 1 cylled Minimum of S if D = S and By 6S, Z < Y. Ex: prove S = (0.1) has he max. O< 1 < XEI < 11 = 1 Suppose on the contramy, X is the max. O <xci< td=""><td></td><td>(a,b)= ? x ER, a < x < b}</td><td>(semi-upen inter</td><td>41)</td></xci<>		(a,b)= ? x ER, a < x < b}	(semi-upen inter	41)	
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Ex: prove S=[01] has no max. O<\frac{1}{2} < \frac{1}{2} = \frac{1}{2}	pefairion	1 XER is called maximum of S if XERS & YYES, YEX.			
Ex: prove S=[01] has no max. O<\frac{1}{2} < \frac{1}{2} = \frac{1}{2}		ZERA rylled Minjourn of 5 if ZES	and ty 65	₹ < Y.	
Suppose on the confrag. X is the max. Oxx<1 => \frac{x+1}{2} \in S. Then consider the number \frac{x+1}{2} \frac{x+1}{2} \times \times \frac{x+1}{2} \times			<i>,</i>	•	
Supple an the language, x 1s the max. Oxx<1 => \frac{\text{xt1}}{2} \in \text{Xt1} \text		Ex: Prove S= (011) 495 ho max.			
Then consider the number xf! $\frac{x+1}{2} \times x \in 1 \times x$ which is true. Hence we found a number within (0,1) that is bigger than x 2) contradiction Assume the instany, $\frac{1}{2} \times x = \frac{x}{2} = x = x = x = x = x = x = x = x = x = $		Suppose an the contrary, x is the max.	0 <x<1< td=""><td>×+1 € S.</td></x<1<>	×+1 € S.	
Assume the matring, 300 = max S. 2) 0 < 66 < 1. But, consider $x = \frac{Bofl}{2} \in S$. $6 < x = f < f < f < f < f < f < f < f < f < f$					
Assume the matring, 300 = max S. 2) 0 < 66 < 1. But, consider $x = \frac{Bofl}{2} \in S$. $6 < x = f < f < f < f < f < f < f < f < f < f$		three we found a number within (0,1) that is higger than x			
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Definition Say MER is an upperbound of S if \(\forall \times \) \(\t		= 70 < 6 < 1.			
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Say m t R is a loner bound of S if O HXES, m < X. If m exists, S is bounded below. If S is bounded both above and below, then S is bounded,		If such M exists, say 5 to bounded above.	,	extracting	
If m exists, S is bounded below. If S is bounded both above and below, then S is bounded,				The 2nd condition	
If S is bonded both above and below, then S is bounded,			∀x ES. m≤x.		
		If m exists, S is bounded below.			
If set S admits a maximum/ninimum. Then it is bounded about/below respectively. Pophazic		If S is bonded both above and below, then S is bounded,			
If set S admits a maximum/nimium, then it is bounded about/below respectively. Thatic					
Prophazic		If set 5 admits a maximum/nominum, then if B	bounded about / belo.	~ respectively	
				Pop bazic`	

MER is the supremum of S (that is bounded above) if M is the smallest 24 fin 14101. upper bound. (sup 5) m fill is the infimum of a bounded below set S if m is the largest loner bound (inf s) Exercite: 12 Sup S for S= [0,1] ¥ x ∈ [0,1), |> x (since o< x < 1) => 1 is an upporbound. To show I is the smallest upper bound, suffices to show any XXI cannot be an upperboard In fact, if x < 0, \frac{1}{2} = 5 and \frac{1}{2} > X. 环 O < X < 1, congider ! X < 1 任 E S. => if x<1, x (nnof be "up lerbound. M= srps => {M B an apperbound of S M is the smallest among such (A) & M 13 41 upper 50 und. ampleteress Axion of R Rudamental property of cent numbers that you can use in the course. morand) admits supremum (as 9 real). ns does not admit sprenion. Completeness axiom fails for nathonal numbers. $g^2\{x \in Q: x^2 < 2\}$.

Mathonal numbers have a gap of 12. the tedeknd outs is a nyponous wary of defining R form 42. insequences of Any nonempty subset of SCIR bounded below admits intimum. (as a real) 2) Architectean property: For any a,b>0, 7h EN st. na>b.

(3) Penseness of OR: Per every a < b, 3r EOR st. a < r < b.

OR has gaps but not too many.

FOP bazic

Pre 0 f 0 f (2) Assume on the confrage, AntN, assume na < b. let S= 5 na: n E MJ. S is not enpty, since a=1.9 ES. Also, b is on upper bound.

By completeness axion, sup S exists. Let M= Sup S. nou, M-a (<M) is not an upper bound, so 7 No a ES st. M-a < no a. -2) M < (nof1) a ES. (Confredicting the fact shat M = SupS)

-. Vsing the completeness axiom

800F of (3)

Suffices to show Jm, n s.t. an < bn
(n#>0)

Since b-a>0, we can fick n(b-a)>1 by Archimedean principle => nb-na>1.

By Archaredean principle, exists k > max { [an], 16n] so that -k<an<6n4

unsider for ses K= {j \ Z; -k \ j \ \ } KEK= [jek; ga < j].

Both sets use honempty since k 6 k, ke.

let m=min{jEk: an <j} Then -k < an <m.

Since m >-k, m-1 EK. >> Then Since m is the minimum element of kz,

 $m-1 \notin k_2$ -) $m-1 \leqslant an$ $m-1 \leqslant an \leqslant a_n + 1 \leqslant bn$