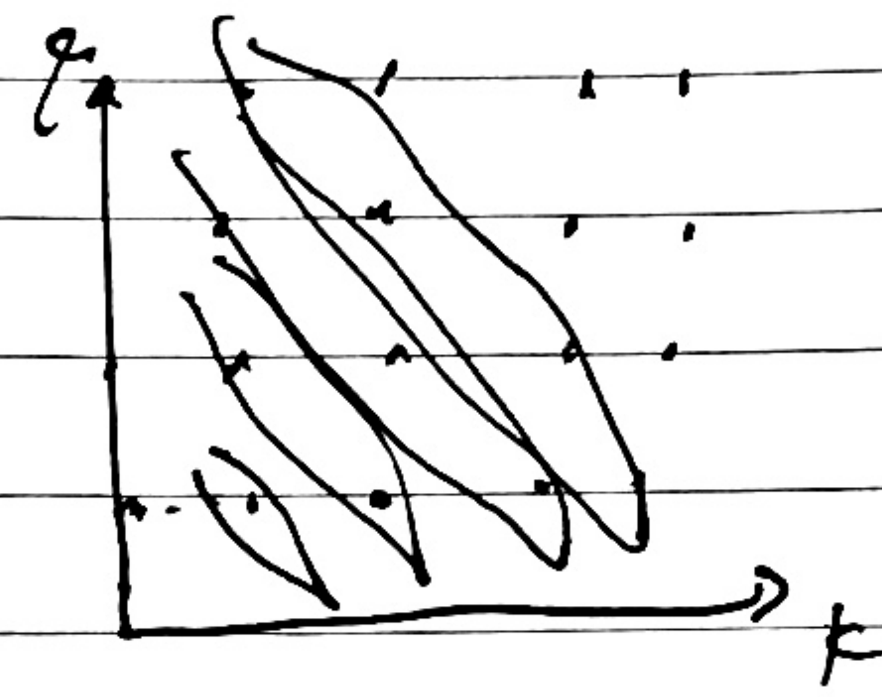
No.: Date: 02/09/2021

MATH H110 LECTURE 3 NOTES.

$e^{z+w} = e^z e^w$. $\qquad e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots$

$\text{LHS} = e^{z+w} = \sum_{n\geq 0} \frac{(z+w)^n}{n!}$



$\text{RHS} = \left(\sum_{k\geq 0} \frac{z^k}{k!}\right)\left(\sum \frac{w^k}{k!}\right)$

group by equal sum

$\text{LHS} = \sum_{n\geq 0} \frac{(z+w)^n}{n!} = \sum_{n\geq 0} \frac{1}{n!} \sum_{k+l=n} \binom{n}{k} z^k w^l$.

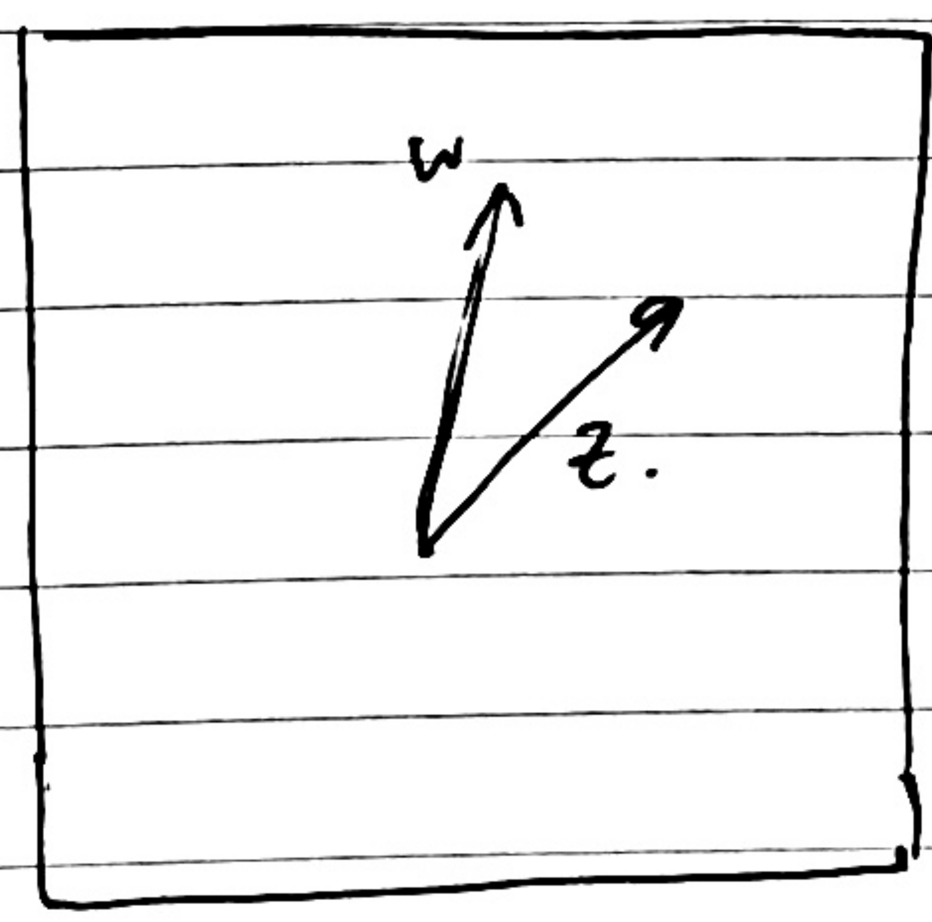
$= \sum_{n\geq 0} \frac{1}{n!} \sum_{k+l=n} \frac{n!}{k!\,l!} z^k w^l = \sum_{n\geq 0} \sum_{k+l=n} \frac{z^k}{k!} \cdot \frac{w^l}{l!}$

$= \sum_{k\geq 0} \frac{z^k}{k!} \sum_{l\geq 0} \frac{w^l}{l!} = \text{RHS}$.

$f(z+w) = f(z)f(w)$

Take w to be very small $\Rightarrow f'(z) = f(z)f'(0) = cf(z)$

$\therefore f(z) = e^{az}$



$z \mapsto wz$.

$wz = |w|\,u$

unit vector of complex w.

Let $u = \cos\theta + i\sin\theta$.

$uz = u(a+bi)$

$= (\cos\theta + i\sin\theta)(a+bi)$

$= (a\cos\theta - b\sin\theta) + (b\cos\theta + a\sin\theta)i$

rotation of θ anticlockwise $\begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}a \\ b\end{bmatrix}$.

Chebyshev polynomial

$\cos n\theta = T_n(\cos\theta)$

i.e. $T_3(x) = 4x^3 - 3x$

expresses $\cos n\theta$ in the form of $\cos\theta$.

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Bezout's Theorem

$P(z_0)=0 \Leftrightarrow P(z)=Q(z)(z-z_0)$.

By polynomial division, $P(z)=(z-z_0)Q(z)+R(z)$.
$\deg R<1 \Rightarrow R$ is constant. Substitute $z=z_0$,

$$P(z_0)=R(z_0)=c \Rightarrow c=0. \quad \therefore R=0.$$