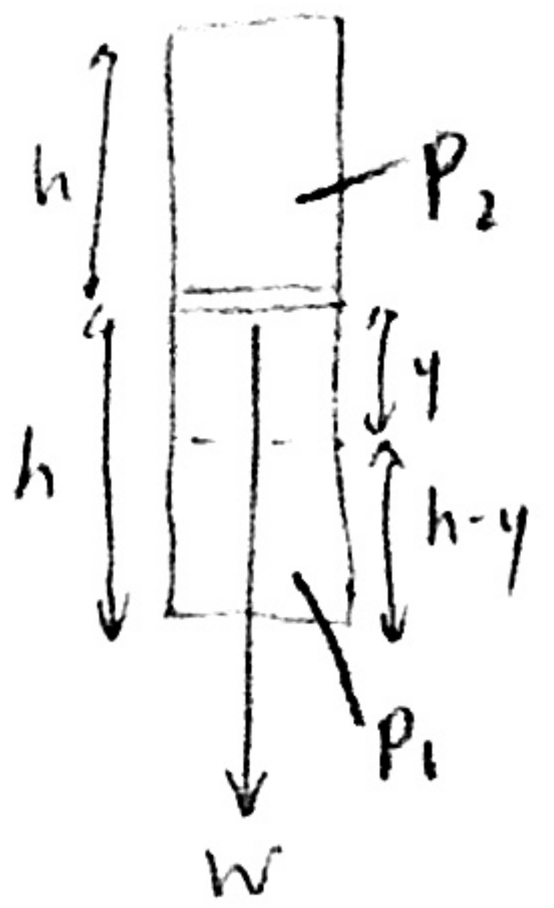
Thermodynamics (solutions)

1.



Assuming the gas is made up of diatomic molecules.

COE) work done in lowering W

$= Wy = \frac{5}{2}\Delta PV$ (increase in internal energy)

$= \frac{5}{2}\left[P_1A(h-y) + P_2A(h+y) - 2P_0Ah\right]$

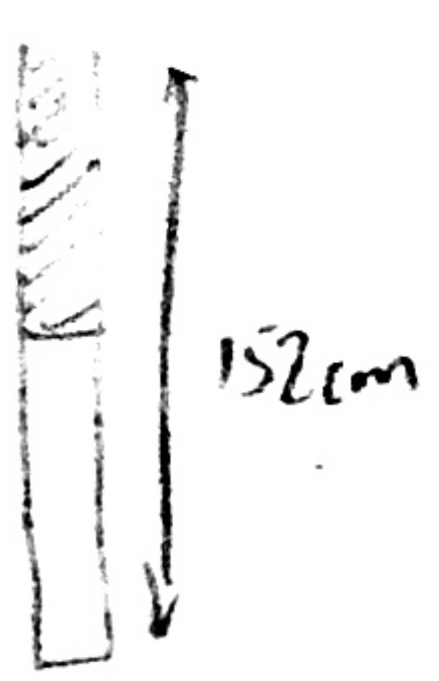
$W \gg 1 \Rightarrow \quad Wy = \frac{5}{2}A\left[P_1(h-y) + P_2(h+y)\right]$

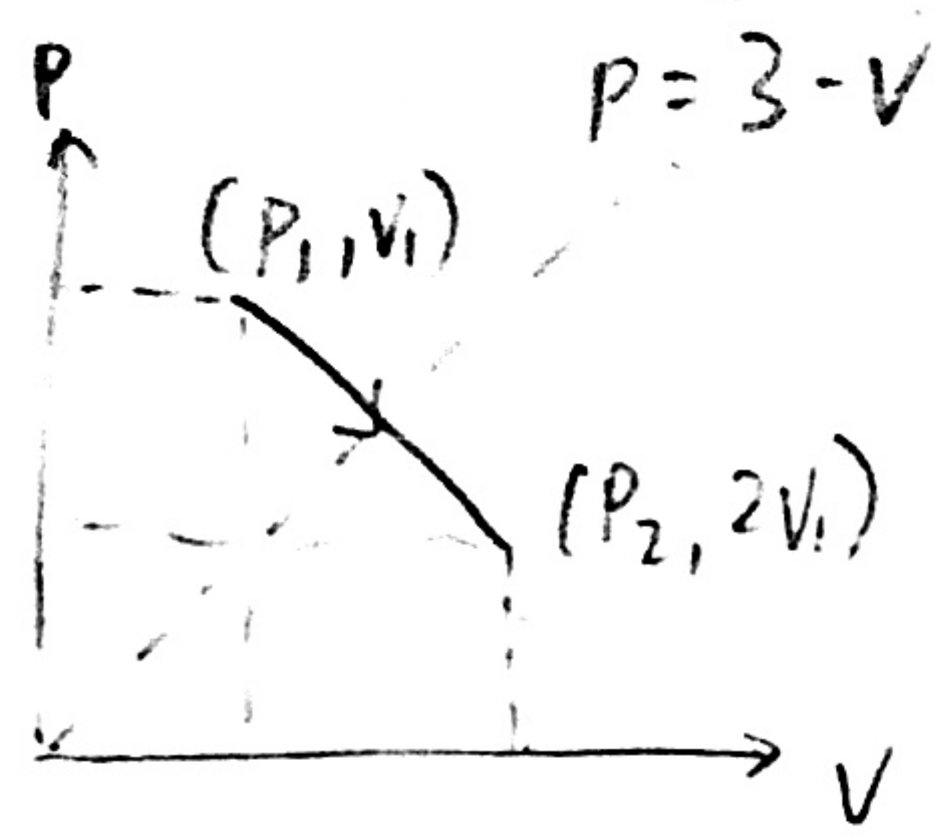
Equilibrium) $(P_1 - P_2)A = W$

$P_1A(h-y) = P_2A(h+y) = nRT$

$\therefore \quad y = \sqrt{\frac{5}{7}}\,h \Rightarrow \boxed{\frac{h-y}{h} = 1 - \sqrt{\frac{5}{7}} \approx 15\%}$

2.





As mercury gets pushed out of the tube, the pressure decreases linearly with increase in volume.

Note $P_1 = 2P_2 \Rightarrow$ PV curve is symmetric with respect to $P = V$

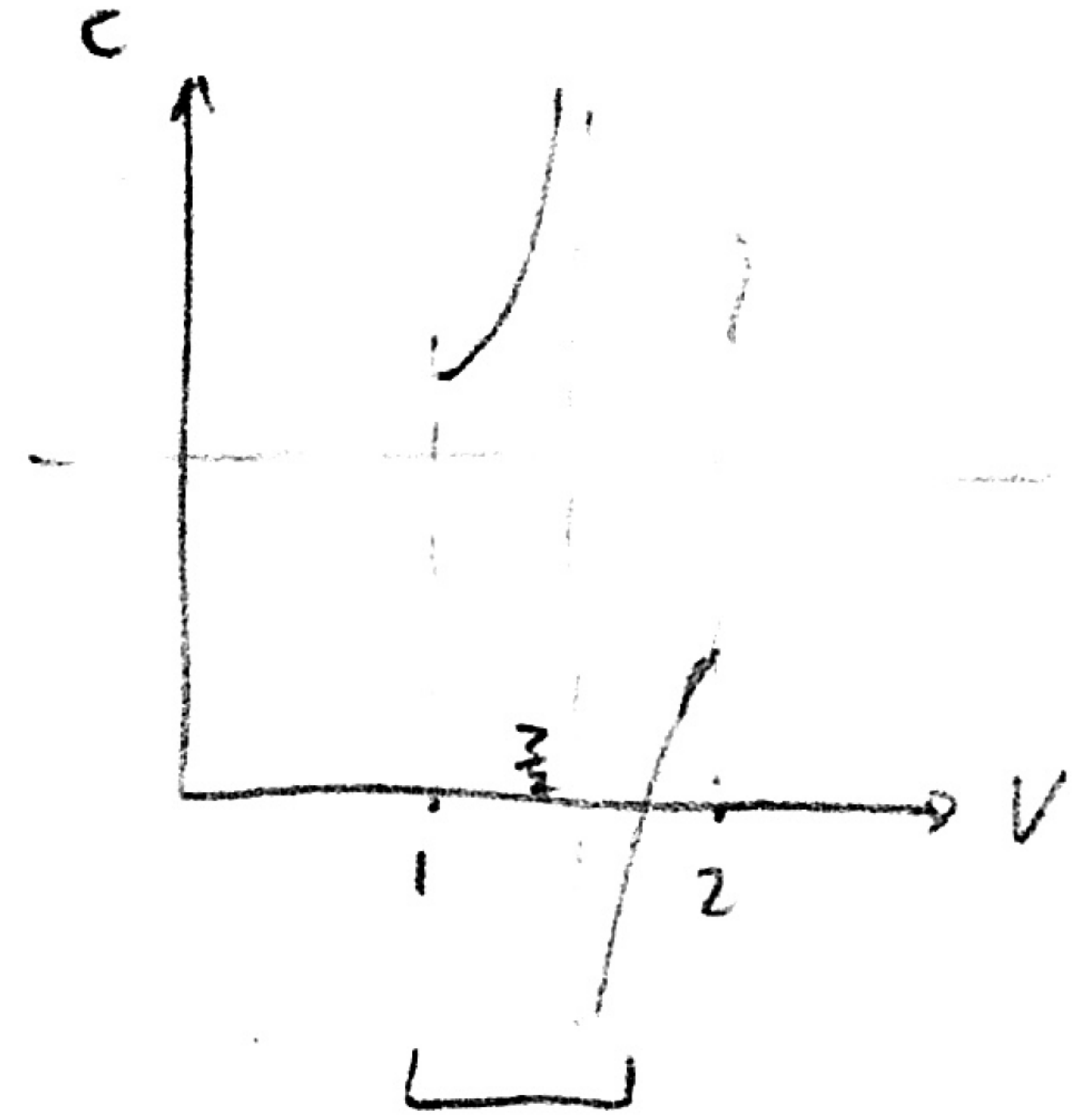
$\Rightarrow$ maximum temperature when $(P, V) = \left(\frac{3}{4}P_1, \frac{3}{2}V_1\right)$

First law) $\frac{5}{2}nR\Delta T = C\Delta T - P\Delta V$

$P_1V_1 = nRT$, but also $P = 3 - V$

$\therefore (3-2V)\Delta V = \Delta T \Rightarrow \frac{5}{2}\Delta T = C\Delta T - \frac{3-V}{3-2V}\Delta T$

$\Rightarrow C = \frac{21-12V}{6-4V}$



We only need to care about this part.

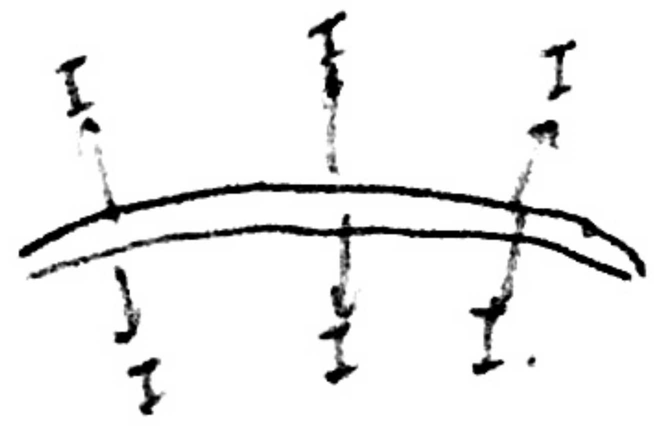
Between V to $\frac{3}{2}V$, $\Delta T > 0 \Rightarrow$ heat injected

Between $\frac{3}{2}V$ to $\frac{7}{4}V$, $\Delta T < 0$, $C < 0 \Rightarrow$ heat injected

At other area, heat not injected

$\boxed{\text{Answer} = \frac{27}{16}nR}$

3. $I = \sigma A T^4$ (Stefan Boltzmann)



"Black on both sides" ⇒ shields also radiate inwards

Surface of probe reradiate a total received intensity of $2I$ at a new temperature T_1.

$2I = \sigma A T_1^4 \Rightarrow \boxed{T_1 = \sqrt[4]{2}\, T}$

For N protecting shields, net radiation must be I ⇒ $\boxed{T_N = \sqrt[4]{N+1}\; T}$

6.

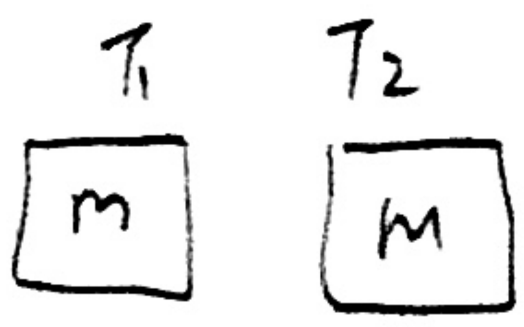
N molecules are compressed into a ten-times smaller space than they had originally occupied

$$\Delta S = k \ln\left(\frac{W_2}{W_1}\right) = k \ln\left(\frac{1}{10^N}\right) = -Nk\ln 10.$$

$$\Delta Q = T\Delta S = -NkT\ln 10$$
$$= -nRT\ln 10 = 10\, p_0 V_0 \ln 2$$

work done $= (10 \times 10^5) \times (1 \times 10^{-2}) \times \ln 10$
$\approx \boxed{23 \text{ kJ}}$

4.



$\Delta S_1 + \Delta S_2 \geq 0$ (second law of thermo)

$$\Rightarrow \frac{\Delta Q_1}{T_1} + \frac{\Delta Q_2}{T_2} \geq 0$$

$$\Rightarrow \Delta Q_1 T_2 + \Delta Q_2 T_1 \geq 0$$

Since the bodies have same mass, $\Delta Q_i \propto \Delta T_i$

$$\Rightarrow (\Delta T_1) T_2 + (\Delta T_2) T_1 \geq 0$$
$$\Rightarrow \Delta(T_1 T_2) \geq 0$$

⇒ $T_1 T_2$ is monoinvariant (can only increase)

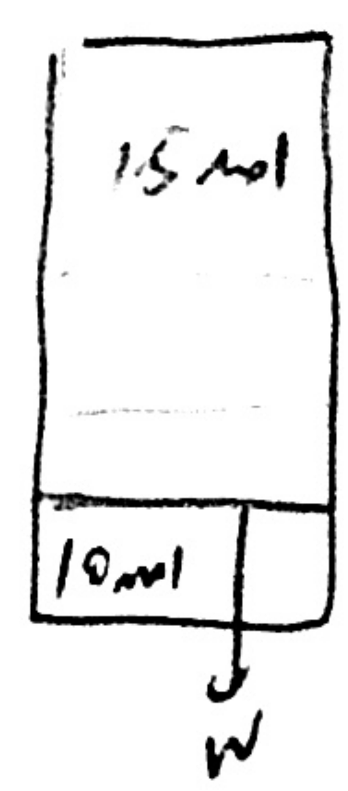
∴ Maximum energy

$$= \boxed{\left(\frac{T_1 + T_2}{2}\right) mc - \sqrt{T_1 T_2}\, mc.}$$

5. Number of microstates available to system of N objects within volume $V \propto V^N$.

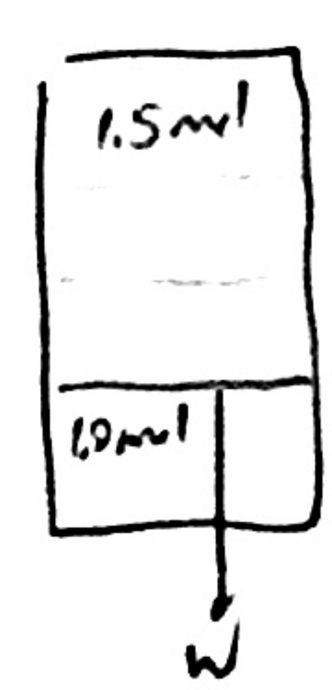
$$\Delta S = k \ln \frac{W_2}{W_1} = k \ln\left(\frac{(5V)^{5N_A}}{(2V)^{2N_A}(3V)^{3N_A}}\right)$$
$$= \boxed{kN_A (5\ln 5 - 2\ln 2 - 3\ln 3)}$$

7.



$$P_1 V_1 = n_H RT$$
$$P_2\left(\frac{V_1}{4}\right) = n_O RT$$
$$RT = \frac{P_1 V_1}{n_H} = \frac{P_2 V_1}{4 n_O}$$
$$\Rightarrow P_2 = \frac{8}{3} P_1$$

$$(P_2 - P_1) A = W \Rightarrow \frac{5}{3} P_1 A = W$$



$$P_1' V_1' = n_H R T_2$$
$$P_2'\left(\frac{V_1'}{3}\right) = n_O R T_2$$
$$RT_2 = \frac{P_1' V_1'}{n_H} = \frac{P_2'\left(\frac{V_1'}{3}\right)}{n_O} \Rightarrow P_2' = 2P_1'$$

$$(P_2' - P_1') A = W \Rightarrow P_1' A = W$$

$$\Rightarrow P_1' = \frac{5}{3} P_1$$

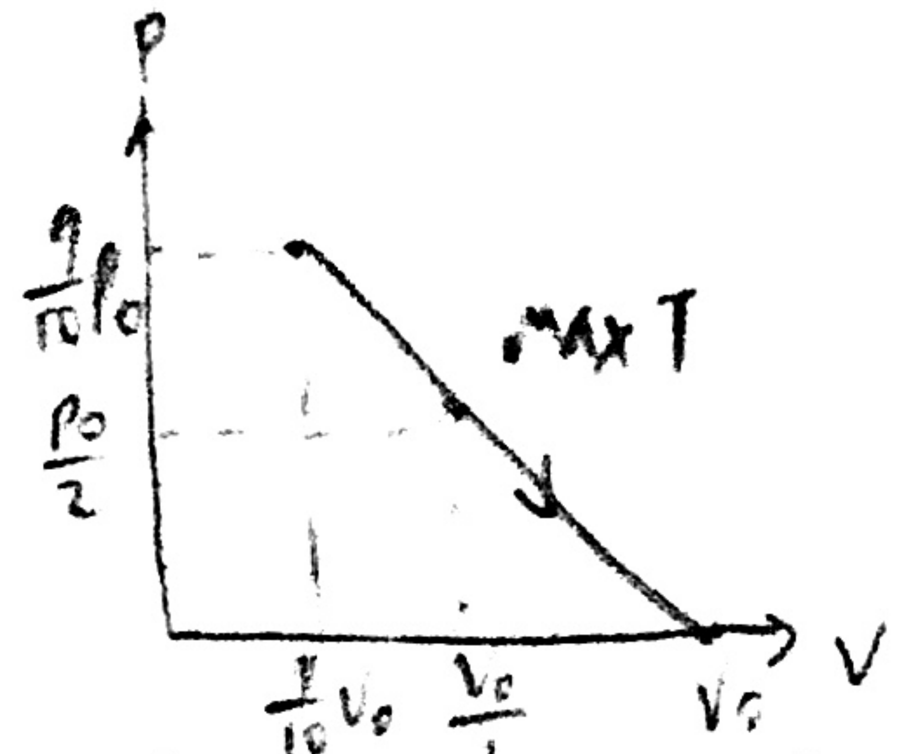
$$P_1' \frac{3V_0}{4} = n_H R T_2, \quad P_1 \frac{4V_0}{5} = n_H RT$$

$$\Rightarrow T_2 = \frac{P_1'\left(\frac{3}{4} V_0\right)}{P_1 \frac{4V_0}{5}} T = \boxed{\frac{25}{16} T}$$

8. $U = \frac{3}{2}RT$

$p = -\frac{P_0}{V_0}V + P_0$

$PV = nRT$



Highest temperature ⇒ highest value of PV

$PV = \left(-\frac{P_0}{V_0}V + P_0\right)V = -\frac{P_0}{V_0}V^2 + P_0V$

$\frac{d}{dV}(PV) = \frac{d}{dV}\left(-\frac{P_0}{V_0}V^2 + P_0V\right) = -\frac{2P_0}{V_0}V + P_0$

$= 0$ (at max PV)

$\Rightarrow P_0 = \frac{2P_0}{V_0}V \Rightarrow \boxed{V = \frac{V_0}{2}}$

$\therefore$ max $PV = \left(-\frac{P_0}{V_0}\frac{V_0}{2} + P_0\right)\frac{V_0}{2} = \frac{P_0V_0}{4}$

$\therefore \boxed{\text{Max } T = \frac{1}{nR}\left(\frac{P_0V_0}{4}\right)}$

First law) $\frac{3}{2}RT = C\Delta T - p\Delta V$

$p = 1 - V$

$(1-V)V = nRT \Rightarrow \quad \Delta T = \Delta((1-V)V)$

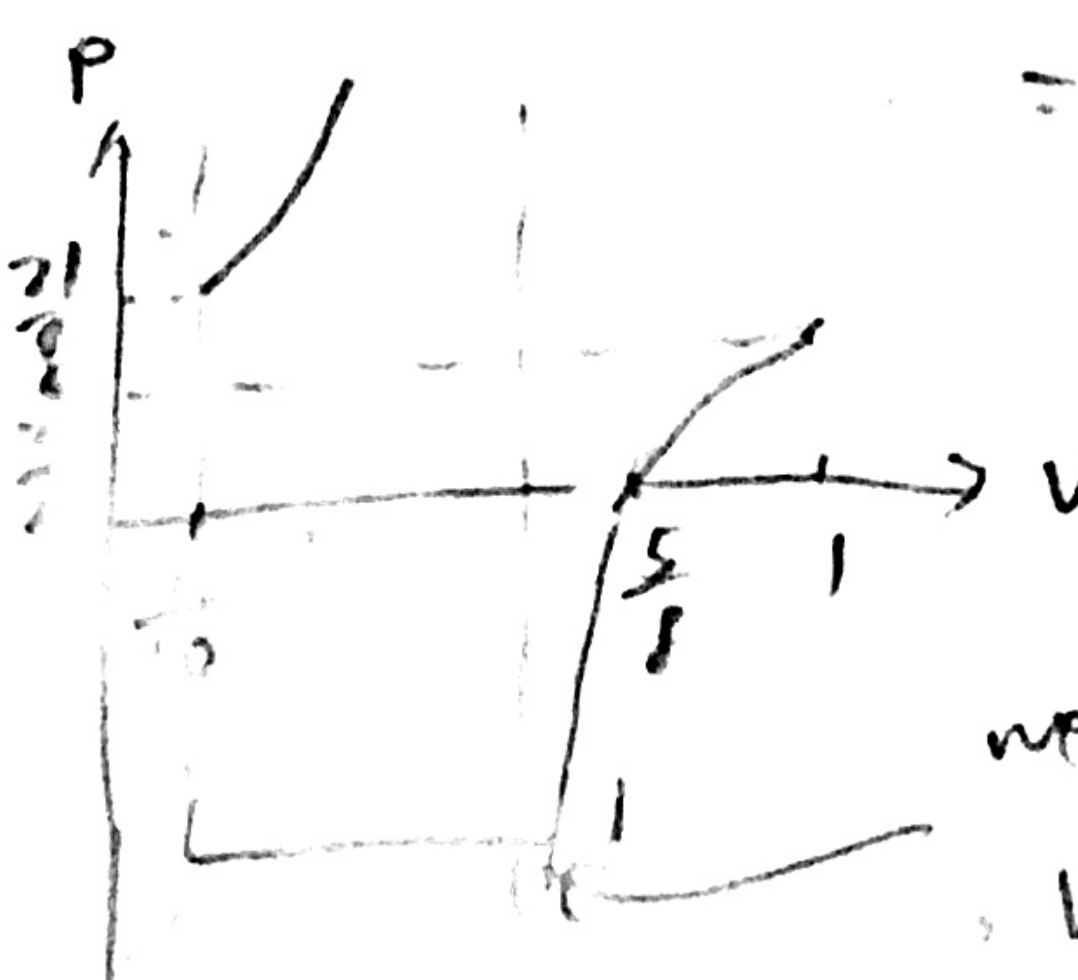
$= -2V\Delta V + \Delta V$

$= (1-2V)\Delta V$

$\Rightarrow \quad \frac{3}{2}\Delta T = C\Delta T - \frac{1-V}{1-2V}\Delta T$

$\Rightarrow \quad C = \frac{3}{2} + \frac{1-V}{1-2V} = \frac{2-2V+3-6V}{2-4V}$

$= \frac{5-8V}{2-4V}$



we just need to calculate heat injected in this region

9.

(a) $P = A\epsilon\sigma T^4$

$= \left[4\pi(6.96\times10^8)^2\right]\cdot 1\cdot(5.67\times10^{-8})(5800)^4$

$\approx \boxed{3.91\times10^{26}\text{ W}}$

(b) $I \propto \frac{1}{r^2}$

$\Rightarrow I = \sigma T^4\left(\frac{R_S}{R_{ES}}\right)^2$

$= (5.67\times10^{-8})(5800)^4\left(\frac{6.38\times10^8}{150\times10^{11}}\right)^2$

$= \boxed{0.116}\text{ Wm}^{-2}$

(c) Suppose equilibrium temperature is T_E.

$(4\pi R_E^2)\sigma T_E^4 = (\pi R_E^2)I$

$\Rightarrow \boxed{T_E = \left[\frac{I}{4\sigma}\right]^{\frac{1}{4}}} \approx \boxed{279\text{K}}$

(d) $15^\circ C \approx 288\text{K} > 279\text{K}$.

Greenhouse effect, cloud cover

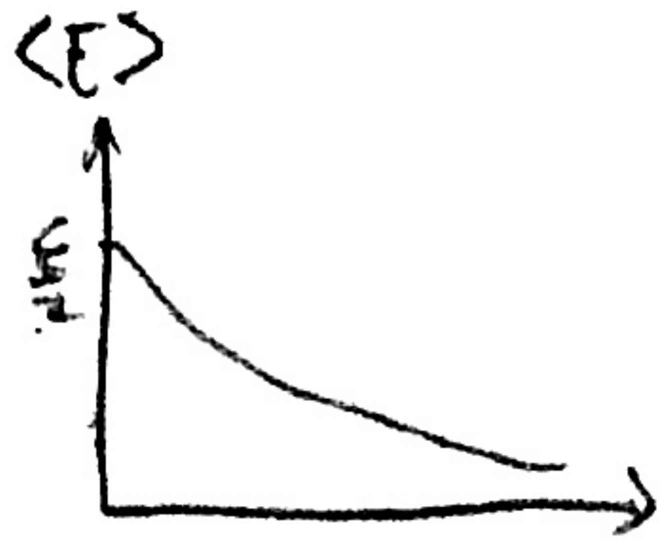
(e) $4\pi R_M^2\sigma T_M^4 = (\pi R_M^2)I'$

$\Rightarrow T_M = \left[\frac{I'}{4\sigma}\right]^{\frac{1}{4}} = \left[\frac{T^4\left(\frac{R_S}{R_{MS}}\right)^2}{4}\right]^{\frac{1}{4}}$

$\approx \boxed{227\text{ K}}$

11.

$\boxed{E}$ E is either, 0 or ε



Probability of system being in energy state of 0

$$\Rightarrow P(0) = \frac{1}{1+e^{-\frac{\varepsilon}{k_BT}}}.$$

$$P(\varepsilon) = \frac{e^{-\frac{\varepsilon}{k_BT}}}{1+e^{-\frac{\varepsilon}{k_BT}}}$$

$$\langle E\rangle = 0\cdot P(0) + \varepsilon P(\varepsilon) = \frac{\varepsilon e^{-\frac{\varepsilon}{k_BT}}}{1+e^{-\frac{\varepsilon}{k_BT}}}$$

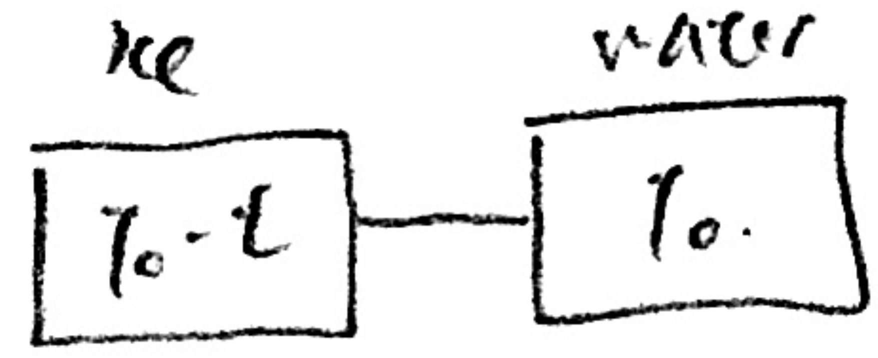
$$= \boxed{\frac{\varepsilon}{1+e^{\frac{\varepsilon}{k_BT}}}}$$

12. Probability of molecules being at height z (and therefore having energy of mgz)

$$P(z) \propto e^{-\frac{mgz}{k_BT}}$$

$$\therefore \boxed{n(z) = n(0)\, e^{-\frac{mgz}{k_BT}}.}$$

10.



In the final state, both temperature must be the same

Initial config

	n-part ice	remainder ice	water frozen to ice	water
Temp	$T_0 - t$	$T_0 - t$	T_0	T_0
Mass	$\frac{1}{n} m$	$\frac{n-1}{n} m$	0	m

Final config

Temp	$T_0 - \Delta T$	T_0	T_0	T_0
mass	$\frac{1}{n} m$	$\frac{n-1}{n} m$	Δm	$m - \Delta m$

Two invariants: Conservation of energy & entropy

$$\text{(CoE)} \quad \underbrace{\int_{T_0-t}^{T_0} \frac{n-1}{n} m a T\, dT}_{\text{energy gained by remainder ice}} - \underbrace{(\Delta m)\lambda}_{\text{energy lost by water frozen to ice}} - \underbrace{\int_{T_0-\Delta T}^{T_0-t} \frac{1}{n} m a T\, dT}_{\text{energy lost by n-part ice}} = 0.$$

$$\text{(CoS)} \quad \underbrace{\frac{n-1}{n} m a t}_{\text{gain in entropy of remainder ice}} - \underbrace{\frac{(\Delta m)\lambda}{T_0}}_{\text{loss in entropy due to } \Delta m \text{ of water frozen to ice}} - \underbrace{\frac{1}{n} m a (\Delta T - t)}_{\text{loss in entropy due to lowered temperature of n-part ice}} = 0$$

Solve for $\Delta T \Rightarrow \Delta T = \sqrt{n}\, t$

$\therefore$ lowest temperature $= T_0 - \Delta T = \boxed{T_0 - \sqrt{n}\, t}$