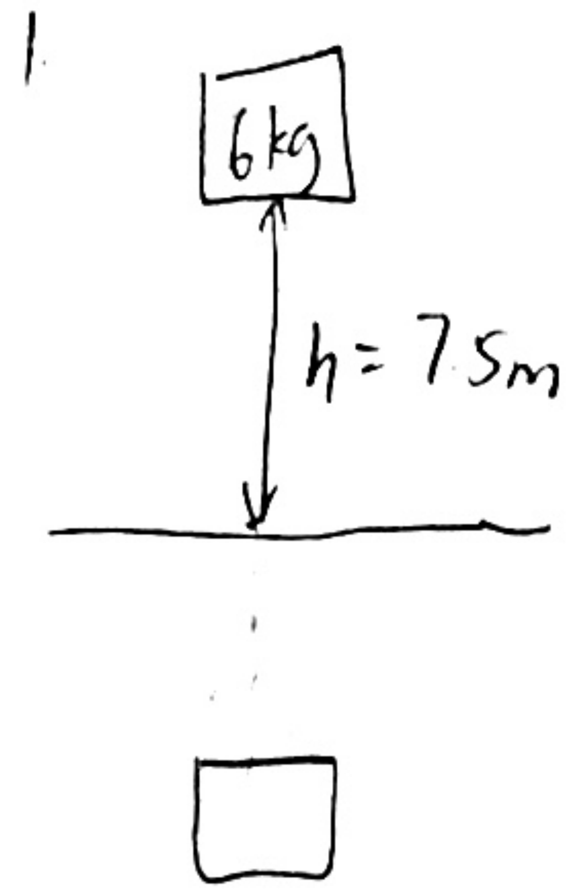
Fluid Dynamics (Solutions)

1.



$$V_0 = \sqrt{2gh}$$

$$F = ma = mg - \rho_w V g$$

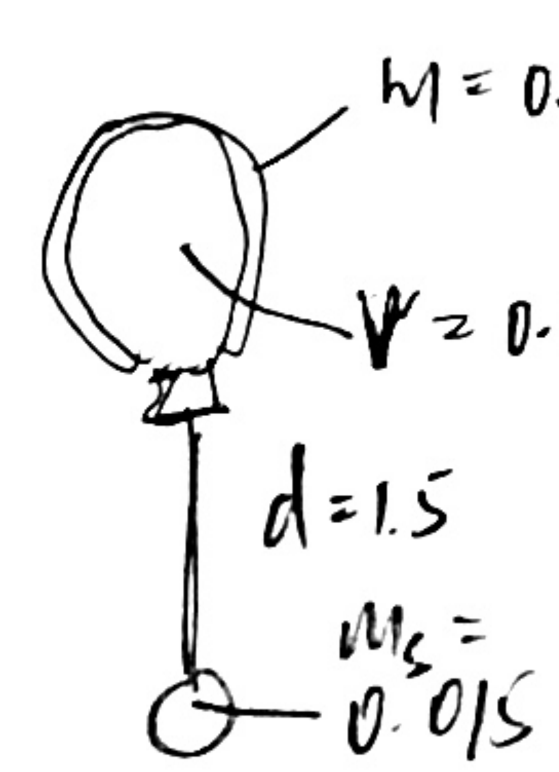
$$\Rightarrow a = g - \frac{\rho_w V g}{m} = g - \frac{\rho_w \frac{m}{\rho} g}{m} = \left(1 - \frac{\rho_w}{\rho}\right) g < 0.$$

Constant acceleration

$$v = 0 \Rightarrow V_0 + at = 0 \Rightarrow t = \frac{V_0}{\left(\frac{\rho_w}{\rho} - 1\right) g}$$

$$S = V_0 t + \frac{1}{2} a t^2 = \frac{V_0^2}{2\left(\frac{\rho_w}{\rho} - 1\right) g} = \boxed{\frac{h}{\frac{\rho_w}{\rho} - 1}}$$

2.



There are two principles in play: Conservation of momentum and conservation of string (string remains taut)

Let v be the initial velocity of balloon just before the string becomes taut and v' be the speed of both balloon and rock immediately afterwards

COM) $mV = (m + m_s) V'$

For balloon

$$F = \rho V g - mg = ma$$

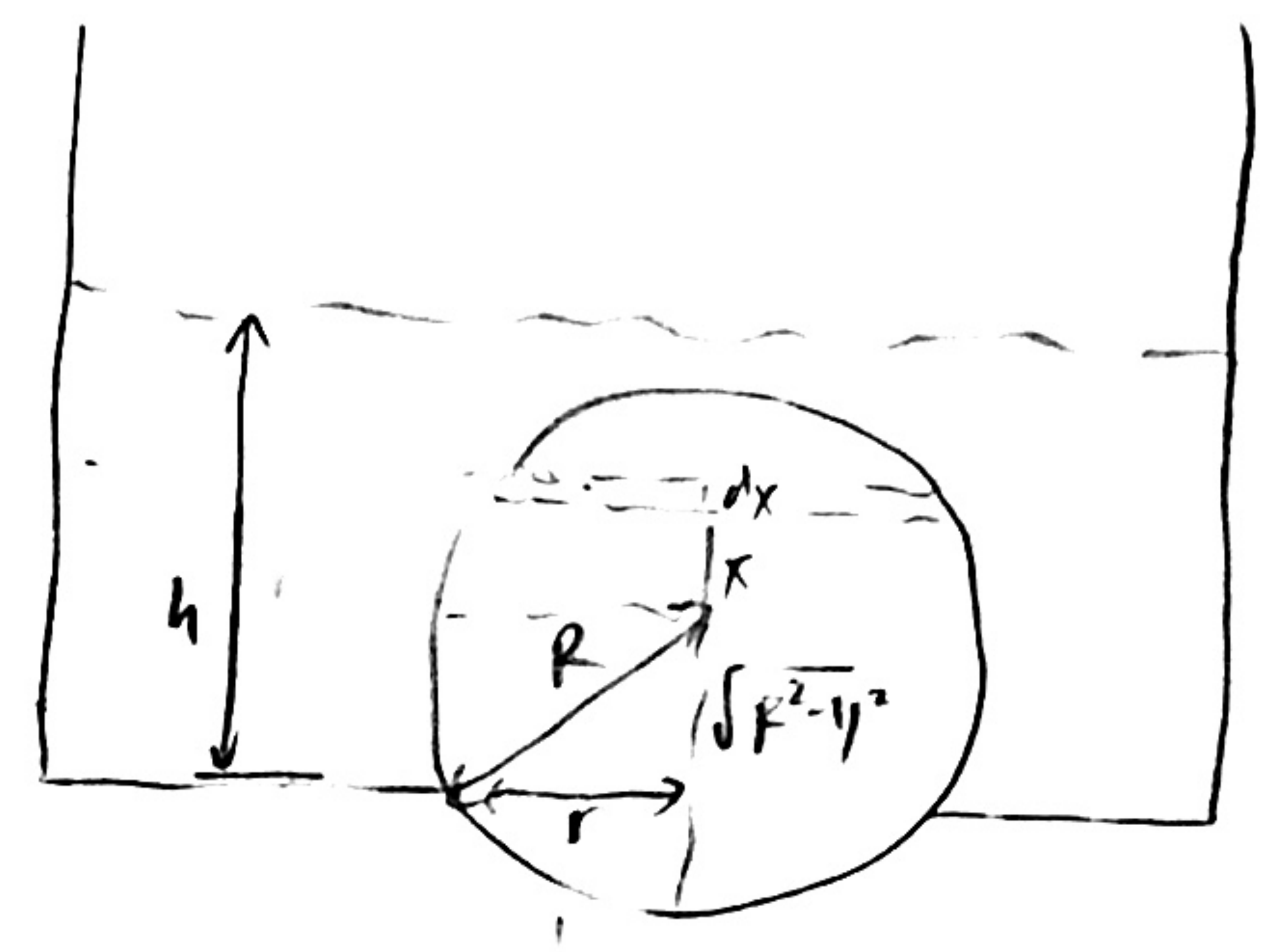
$$\Rightarrow a = \frac{\rho V g}{m} - g > 0.$$

$$V^2 - V_i^2 = 2as \Rightarrow V = \sqrt{2ad}$$

$$\Rightarrow V = \sqrt{2\left(\frac{\rho V g}{m} - g\right) d}$$

$$\Rightarrow \boxed{V' = \frac{m}{m + m_s} \sqrt{2dg\left(\frac{\rho V}{m} - 1\right)}}$$

3.



Buoyant force $= \rho V g - \rho g h \pi r$

As height decreases, Buoyant force increases which results in the ball coming out.

$$V = \int dV = \int_{-\sqrt{R^2 - r^2}}^{R} \pi (R^2 - x^2)\, dx$$

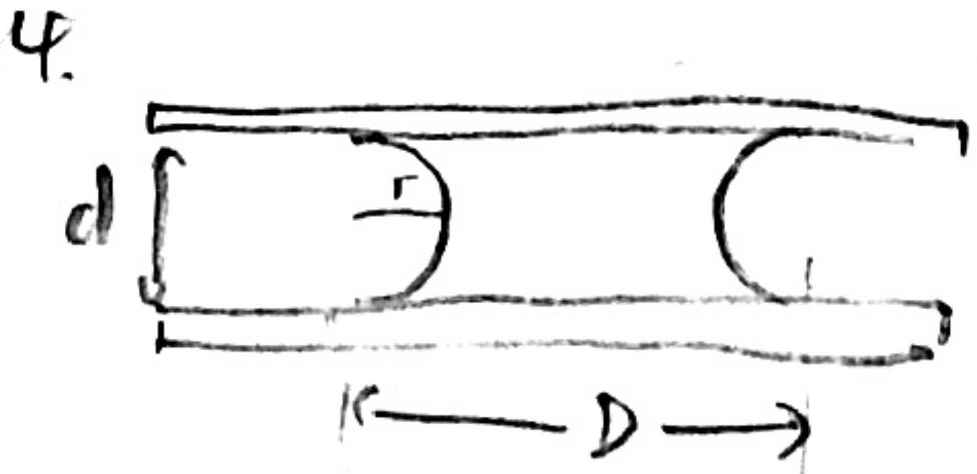
$$= \pi \left[R^2 x - \frac{x^3}{3} \right]_{-\sqrt{R^2 - r^2}}^{R} = \frac{2}{3} \pi R^3 + \pi \sqrt{R^2 - r^2} \left(\frac{2R^2 + r^2}{3} \right)$$

For $B = mg$,

$$B = \frac{\rho \pi}{3} \left(2R^3 + \sqrt{R^2 - r^2} (2R^2 + r^2) \right) g - \rho g h \pi r^2 = mg$$

$$\Rightarrow \boxed{h = \frac{\frac{\rho \pi}{3} \left(2R^3 + \sqrt{R^2 - r^2} (2R^2 + r^2) \right) - m}{\rho \pi r^2}}$$

4.



$r = \frac{d}{2}$ (radius of curvature)

$$\boxed{\text{Young-Laplace Eqn} \quad \Delta p = \gamma \left(\frac{1}{R_x} + \frac{1}{R_y} \right)}$$

$$\Delta p = \frac{\gamma}{r} = \gamma \left(\frac{2}{d} \right) = \frac{2\gamma}{d}$$

$\Rightarrow$ pressure in the disc $= P_0 - \frac{2\gamma}{d}$

$$F = 2 \cdot \pi \left(\frac{D}{2} \right)^2 \cdot \frac{2\gamma}{d} = \boxed{\frac{\pi D^2 \gamma}{d}}$$