6.1：有较低 V_{OL} 的反相器，是有较短的从高到低开关时间吗？

不是.

对于电阻负载反相器，

$$V_{OL} = V_{DD} - V_{T0} + \frac{1}{k_n R} - \sqrt{\left(V_{DD} - V_{T0} + \frac{1}{k_n R}\right)^2 - \frac{2V_{DD}}{k_n R}}$$

电阻增大，V_{OL} 变小，但是从高到低的开关时间变长了.

6.2

a. 对于短沟道情况

$\mu_n C_{ox} = 98.3\ \mu A/V^2$，$\left(\frac{W}{L}\right)_n = 10$　$V_{T,n} = 0.53V$，$E_{Cn} L_n = 0.4V$

对 10 打的电路

[设 $V_{DD} = 1.2V$)
当 $V_{out} = V_{OL}$ 时.

$$I_D = \frac{V_{DD} - V_{OL}}{R} = \frac{k_n}{2} \frac{1}{\left(1 + \frac{V_{OL}}{E_{Cln}}\right)} \left(2(V_{in} - V_{T,n}) V_{OL} - V_{OL}^2\right)$$

$$\frac{1.2 - V_{OL}}{10 \times 10^3} = \frac{98.3}{2} \times 10^{-6} \times \frac{1}{1 + \frac{V_{OL}}{0.4}} \left(2(1.2 - 0.53) V_{OL} - V_{OL}^2\right)$$

$$1.2 - V_{OL} = \frac{98.3}{2} \times 10^{-2} \times \frac{0.4}{0.4 + V_{OL}} \left(1.34 V_{OL} - V_{OL}^2\right)$$

$$(1.2 - V_{OL})(0.4 + V_{OL}) = 49.15 \times 10^{-2} \times 0.4 \left(1.34 V_{OL} - V_{OL}^2\right)$$

$$0.48 + 0.8 V_{OL} - V_{OL}^2 = 10^{-2}\left(26.3444 V_{OL} - 19.66 V_{OL}^2\right)$$

$$48 + 80 V_{OL} - 100 V_{OL}^2 = -19.66 V_{OL}^2 + 26.3444 V_{OL}$$

$$80.34 V_{OL}^2 - 53.6556 V_{OL} - 48 = 0$$

$$V_{OL} = 1.17V$$

$$\frac{(V_{GS} - V_T) E_{Cn} L_n}{V_{GS} - V_T + E_{Cn} L_n} = 0.25V$$

晶体管不工作在线性区

所以工作在饱和区

$$\frac{V_{DD}-V_{OL}}{R} = W \cdot V_{sat} \cdot C_{ox} \cdot \frac{(V_{th}-V_{T0})^2}{(V_{th}-V_{T0})+E_c L_n}$$

$$= \frac{\mu_n C_{ox}}{2} \cdot \left(\frac{W}{L}\right)_n \frac{(V_{th}-V_{T0})^2}{1+\left(\frac{V_{th}-V_{T0}}{E_c L_n}\right)}$$

解得 $V_{OL} = 0.375\,V$

$$V_{50\%} = \frac{V_{OH}+V_{OL}}{2} = \frac{1.2+0.375}{2} = 0.7875\,V$$

$$\tau_{pHL} = \frac{C_{load}(V_{OH}-V_{50\%})}{I_c}$$

$$I_c = \frac{1}{2}\left(I_{c,V_{OH}} + I_{c,V_{50\%}}\right)$$

$$I_{c,V_{OH}} = \frac{k_n}{2} \cdot \frac{(V_{in}-V_{T,n})^2}{(V_{in}-V_{T,n})+E_{cn} L_n} = 8.248 \times 10^{-5}\,A$$

$$I_{c,V_{50\%}} = I_{c,V_{OH}} - \frac{V_{DD}-V_{50\%}}{R} = 4.123 \times 10^{-5}\,A$$

$$\tau_{pHL} = \frac{10\times10^{-15} \times (1.2-0.7875)}{\frac{1}{2}\times(8.248+4.123)\times10^{-5}} = 6.6688\times10^{-11}\,s$$

对于PMOS管

当 $V_{out} = V_{OL}$,

$$\frac{V_{DD}-V_{OL}}{R} = \frac{k_n}{2} \cdot \left(2(V_{in}-V_{T,n})V_{OL} - V_{OL}^2\right)$$

解得 $V_{OL} = 0.1789\,V$

$$V_{OL} < (V_{GS}-V_T) = 0.67\,V$$

饱和区公式

$$V_{50\%} = \frac{V_{OH}+V_{OL}}{2} = \frac{1.2+0.1789}{2} = 0.68945\,V$$

$$I_{c,V_{OH}} = \frac{k_n}{2}(V_{in}-V_{T,n})^2 = 2.206\times10^{-4}\,A$$

$$I_{c,V_{50\%}} = 2.206\times10^{-4} - \frac{1.2-0.68945}{10^4} = 1.69345\times10^{-4}\,A$$

$$\tau_{pHL} = \frac{10\times10^{-15}(1.2-0.68945)}{\frac{1}{2}\times(2.206+1.69345)\times10^{-4}} = 2.61723\times10^{-11}\,s$$

b. 充电时

$$I = \frac{V_{DD} - V_{out}}{R} = C_{Load} \frac{dV_{out}}{dt}$$

$$\int_{V_{OL}}^{V_{50\%}} \frac{dV_{out}}{1.2 - V_{out}} = \frac{1}{10^4 \times 10 \times 10^{-15}} \tau_{PLH}$$

$$\tau_{PLH} = -10^{-10} \ln(1.2 - V_{out}) \Big|_{V_{OL}}^{V_{50\%}}$$

短沟道: $\tau_{PLH} = 10^{-10} \ln\left(\frac{1.2 - 0.375}{1.2 - 0.7875}\right) = 6.93 \times 10^{-11} s.$

长沟道: $\tau_{PLH} = 10^{-10} \ln\left(\frac{1.2 - 0.1789}{1.2 - 0.68945}\right) = 6.93 \times 10^{-11} s.$

6.8 CMOS 反相器:

nMOS: $V_{T0,n} = 0.5V$, $\mu_n C_{ox} = 9.8 \mu A/V^2$ $E_{c,n} L_n = 0.4V$

pMOS: $V_{T0,p} = -0.48V$ $\mu_p C_{ox} = 46 \mu A/V^2$ $E_{c,p} L_p = 1.8V$

$V_{DD} = 1.2V$. $L_n = L_p = 40nm$. $C_{out} = 200fF$

40nm 可认为是短沟道

a. 求 W_n, W_p, 设 $V_{th} = 0.59V$. $\tau_{rise} = 100ps$

$$V_{th} = \frac{V_{T0,n} + \sqrt{k}\,(V_{DD} - |V_{T0,p}|)}{1 + \sqrt{k}} = 0.59$$

$$\Rightarrow k = 0.479 = \frac{W_p}{W_n} \cdot \frac{E_{c,n} L_n}{E_{c,p} L_p}$$

$$\frac{W_p}{W_n} = 0.479 \times \frac{1.8}{0.4} = 2.1555$$

$$\tau_{rise} = \frac{C_{out} \cdot \Delta V}{I_{average}} = 100 \times 10^{-12} s$$

$$I_{average} = \frac{200 \times 10^{-15} \times 0.96}{10^{-10}} = 1.92 \times 10^{-3} A$$

$V_{out} = 1.2V$ 時

$$I_1 = \frac{1}{2}\mu_p C_{ox}\frac{W_p}{L_p}\cdot\frac{1}{1+\frac{V_{SG}+V_{tn,p}}{E_{c,p}L_p}}\cdot(V_{SG}+V_{tn,p})^2$$

$$= 8.5166\times10^{-6}\cdot\frac{W_p}{L_p}$$

$V_{out} = 1.08V$ 時

$$I_2 = \frac{1}{2}\mu_p C_{ox}\frac{W_p}{L_p}\cdot\frac{1}{1+\frac{V_{SD}}{E_{c,p}L_p}}\cdot(2(V_{SG}+V_{tn,p})V_{SD}-V_{SD}^2)$$

$$= 3.4155\times10^{-6}\frac{W_p}{L_p}$$

$$I_{average} = 1.92\times10^{-3} = \frac{1}{2}(8.5166\times10^{-6}+3.4155\times10^{-6})\frac{W_p}{L_p}$$

$W_p = 12.872\mu m$.

$$W_n = \frac{W_p}{2.155} = 5.9634\mu m$$

b.

$$t_p = \frac{t_{pLH}+t_{pHL}}{2}$$

$$t_{pLH} = \frac{C_{load}}{k_p}\frac{2}{E_{c,p}\cdot L_p}\cdot\frac{V_{50\%}(V_{DD}-|V_{T,p}|+E_{c,p}L_p)}{(V_{DD}-|V_{T,p}|)^2}$$

$$= 43.78\ ps$$

$$t_{pHL} = \frac{C_{load}}{k_n}\frac{2}{E_{c,n}L_n}\frac{V_{50\%}((V_{DD}-V_{T,n})+E_{c,n}L_n)}{(V_{DD}-V_{T,n})^2}$$

$$= \frac{200\times10^{-15}}{98\times10^{-6}\times\frac{5.9634}{40\times10^{-3}}}\cdot\frac{2}{1.8}\cdot\frac{0.6((1.2-0.5)+0.4)}{(1.2-0.5)^2}$$

$$= 92.19\ ps$$

$$t_p = \frac{t_{pHL}+t_{pLH}}{2} = 67.985\ ps$$

c.

$$t_{pHL} \approx \frac{C_{load}\frac{1}{2}V_{DD}}{\xi W_n(V_{DD}-V_T)}$$

短沟道条件下 电源电压对延迟影响很小.

$$V_{th} = \frac{V_{T0,n} + \sqrt{k}\,(V_{DD} - |V_{T0,p}|)}{1+\sqrt{k}}$$

V_{DD} 减小. V_{th} 减小.

6.9 $V_{th} = 0.58V$, $C_{out} = 5fF$

$C_{db,n} = 0.16fF + 1.7W_n$

$C_{db,p} = 0.13fF + 1.4W_p$

a. 短沟道:

$$V_{th} = 0.58V = \frac{V_{T0,n} + \sqrt{k}\,(V_{DD} - |V_{T0,p}|)}{1+\sqrt{k}}$$

$$\longrightarrow \quad k = 0.3265 = \frac{W_p}{W_n} \cdot \frac{E_{cn} L_n}{E_{cp} L_p}$$

$$\frac{W_p}{W_n} = 1.47$$

$$t_{pHL} = \frac{C_{load}}{k_n} \cdot \frac{2}{E_{cn} L_n} \cdot \frac{V_{sat}\,((V_{DD} - V_{T,n}) + E_{cn} L_n)}{(V_{DD} - V_{T,n})^2}$$

$t_{pHL} \le 35ps$

$$\frac{200 \times 10^{-15}}{98 \times 10^{-6} \frac{cm}{40nm} 2.13} \cdot \frac{2}{0.4} \cdot \frac{0.6\,((1.2-0.5)+0.4)}{(1.2-0.5)^2} \le 35 \times 10^{-12} s$$

$W_n \ge 0.57 \mu m$

$W_p \ge 0.867 \mu m$

长沟道:

$$V_{th} = \frac{V_{T0,n} + \sqrt{k_R} \cdot (V_{DD} - |V_{T0,p}|)}{1+\sqrt{k_R}} = 0.58$$

$$\frac{1}{k_R} = \frac{k_p}{k_n} = \frac{W_p}{2.13 W_n} = 0.3265$$

$$\frac{W_p}{W_n} = 0.696$$

$$t_{pHL} = \frac{C_{load}}{\mu_n C_{ox} \frac{W_n}{L_n} (V_{DD} - V_{T0,n})} \left(\frac{2V_{T0,n}}{V_{DD} - V_{T0,n}} + \ln\left(\frac{4(V_{DD} - V_{T0,n})}{V_{DD}} - 1 \right) \right) \le 35ps$$

$\rightarrow \quad W_n \geq 0.1638\ \mu m$

$W_p \geq 0.114\ \mu m$

b. $(\frac{W}{L})_n = 10 \quad (\frac{W}{L})_p = 15 \quad C_{load} = 5fF$

短沟道:

$$t_{rise} = \frac{C_{load}\,\Delta V}{I_{average}}$$

上升:

$V_{out} = 0.12V$时, $I_{vout} = \frac{1}{2}\mu_p C_{ox}\frac{W_p}{L_p}\cdot\frac{1}{1+\frac{V_{SG}+V_{To.p}}{E_{c.p}L_p}}(V_{SG}+V_{To.p})^2 = 127.75\ \mu A$

$V_{out} = 1.08V$时. $I_{vout} = \frac{1}{2}\mu_p C_{ox}\frac{W_p}{L_p}\cdot\frac{1}{1+\frac{V_{SD}}{E_{cp}L_p}}(2(V_{SG}+V_{To.p})V_{SD}-V_{SD}^2) = 51.232\ \mu A$

$I_{average} = \frac{1}{2}(127.75+51.232)\ \mu A = 89.491\ \mu A$

$$t_{rise} = \frac{5\times10^{-15}\times(1.08-0.12)}{89.491\times10^{-6}} = 53.636\ ps$$

下降:

$V_{out} = 0.12V$时. $I_{Vout} = \frac{1}{2}\mu_n C_{ox}\frac{W_n}{L_n}\cdot\frac{1}{1+\frac{V_{DS}}{E_{cn}L_n}}\left(2(V_{GS}-V_{to.n})V_{DS}-V_{DS}^2\right) = 57.9\ \mu A$

$V_{out} = 1.08V$时. $I_{Vout} = \frac{1}{2}\mu_n C_{ox}\frac{W_n}{L_n}\cdot\frac{1}{1+\frac{V_{GS}+V_{to.n}}{E_{cn}L_n}}(V_{GS}-V_{to.n})^2 = 87.31\ \mu A$

$I_{average} = \frac{1}{2}(87.31+57.9)\ \mu A = 72.6\ \mu A$

$$t_{fall} = \frac{5\times10^{-15}\times(1.08-0.12)}{72.6\times10^{-6}} = 66.11\ ps.$$

长沟道:

上升:

$V_{out} = 0.12V$时. $I_{Vout} = \frac{1}{2}\mu_p C_{ox}\frac{W_p}{L_p}(V_{SG}+V_{to.p})^2 = 178.85\ \mu A$

$V_{out} = 1.08V$时 $I_{Vout} = \frac{1}{2}\mu_p C_{ox}\frac{W_p}{L_p}(2(V_{SG}+V_{to.p})V_{SD}-V_{SD}^2) = 54.65\ \mu A$

$I_{average} = \frac{1}{2}(178.85+54.65)\ \mu A = 116.75\ \mu A$

$$t_{rise} = \frac{5\times10^{-15}\times(1.08-0.12)}{116.75\times10^{-6}} = 41.113\ ps$$

下降：

$V_{out} = 0.12V$时，$I_{out} = \frac{1}{2}\mu_n C_{ox}\frac{W_n}{L_n}\frac{1}{1+\frac{V_{DS}}{E_{cn}L_n}}\left(2(V_{GS}-V_{tan})V_{DS}-V_{DS}^2\right) = 75.26\mu A$

$V_{out} = 1.08V$时，$I_{out} = \frac{1}{2}\mu_n C_{ox}\frac{W_n}{L_n}\frac{1}{1+\frac{V_{GS}+V_{tan}}{E_{cn}L_n}}(V_{GS}-V_{tan})^2 = 240.1\mu A$

$$I_{average} = \frac{1}{2}(75.26+240.1)\mu A = 157.68\mu A$$

$$t_{fall} = \frac{5\times10^{-15}\times(1.08-0.12)}{157.68\times10^{-6}} = 30.441\ ps$$