Math 132 Homework 1

Jiaping Zeng

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1.1.5 $(\overline{2-i})^2$

Answer: $(\overline{2-i})^2 = (2+i)^2 = 4+4i-1 = 3+4i$

1.1.7 $(x+iy)^2$

Answer: $(x+iy)^2 = x^2+2xyi-y^2 = (x^2-y^2)+i(2xy)$

1.1.8 $i\overline{(2+i)^2}$

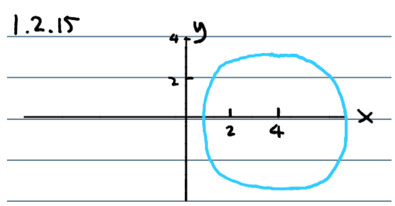
Answer: $i\overline{(2+i)^2} = i\overline{3+4i} = i(3-4i) = 4+3i$

1.2.10 $\left|\overline{(2+3i)^8}\right|$

Answer: $\left|\overline{(2+3i)^8}\right| = \left|(2+3i)^8\right| = |2+3i|^8 = \sqrt{13}^8 = 13^4 = 28561$

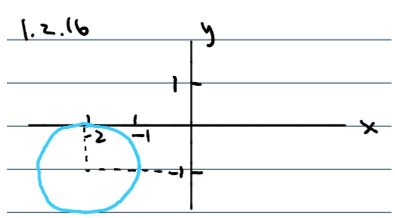
1.2.15 $|z-4|=3$

Answer: Since $|z|=3$ is a circle of radius 3 at the origin, $|z-4|=3$ is a circle of radius 3 centered at $(4,0)$.



1.2.16 $|z+2+i|=1$

Answer: Similar to the above, $|z+2+i|=1$ is a circle of radius 1 centered at $(-2,-1)$.



1.2.34 (a) Use the triangle inequality to show that $|z-1| \leq 2$ for $|z| \leq 1$.

(b) Explain your result in (a) geometrically.

(c) Is the upper bound in (a) best possible?

1.3.5 $-3-3i$

Answer: $r=\sqrt{(-3)^2+(-3)^2}=3\sqrt{2}$; $\cos\theta=\frac{x}{r}=\frac{-3}{3\sqrt{2}}=-\frac{\sqrt{2}}{2}, \sin\theta=\frac{y}{r}=\frac{-3}{3\sqrt{2}}=-\frac{\sqrt{2}}{2} \implies \theta=\frac{5\pi}{4} \implies \operatorname{Arg} z=-\frac{3\pi}{4}$. Then $-3-3i=3\sqrt{2}(\cos(-\frac{3\pi}{4})+i\sin(-\frac{3\pi}{4}))$.

1.3.6 $-\frac{\sqrt{3}}{2}+\frac{i}{2}$

Answer: $r=\sqrt{\frac{3}{4}+\frac{1}{4}}=1$; $\cos\theta=\frac{x}{r}=-\frac{\sqrt{3}}{2}, \sin\theta=\frac{y}{r}=\frac{1}{2} \implies \theta=\frac{5\pi}{6}=\operatorname{Arg} z$. Then $-\frac{\sqrt{3}}{2}+\frac{i}{2}=\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}$.

1.3.21 $(1+i)^{30}$

Answer: $(1+i)^{30}=\sqrt{2}^{30}(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})^{30}=2^{15}(\cos\frac{30\pi}{4}+i\sin\frac{30\pi}{4})=2^{15}(0+i)=2^{15}i$, so $\operatorname{Re} z=0$ and $\operatorname{Im} z=2^{15}$.

1.5.7 $e^{-1-i\frac{\pi}{6}}$

Answer: $e^{-1-i\frac{\pi}{6}}=e^{-1}\cdot e^{-i\frac{\pi}{6}}=e^{-1}(\cos(\frac{-\pi}{6})+i\sin(\frac{\pi}{6}))=e^{-1}\cos(\frac{-\pi}{6})+ie^{-1}\sin(\frac{\pi}{6})$, so $a=e^{-1}\cos(\frac{-\pi}{6})$ and $b=e^{-1}\sin(\frac{\pi}{6})$.

P1 Let $z=x+iy$ be a complex number. Show that:

(a) $\bar{\bar{z}}=z$

(b) $\operatorname{Re}(z)=\frac{z+\bar{z}}{2}$

(c) $\operatorname{Im}(z)=\frac{z-\bar{z}}{2i}$

P2 Let $z_1=x_1+iy_1$ and $z_2=x_2+iy_2$ be complex numbers. Show that:

(a) $\overline{z_1+z_2}=\overline{z_1}+\overline{z_2}$

(b) $\overline{z_1 z_2}=\overline{z_1}\cdot\overline{z_2}$

P3 (a) Show that $|z-4|\leq 6$ if $|z-3i|\leq 1$

(b) Show that $|z-4|\geq 4$ if $|z-3i|\leq 1$

(c) Show that $\left|\frac{1}{z-4}\right|\leq\frac{1}{2}$ if $|z-1|\leq 1$