Math 132 Homework 7

Jiaping Zeng

11/16/2020

4.4.2 $\dfrac{\sin z}{e^z}, z_0 = 1 + 7i.$

Answer: $\dfrac{\sin z}{e^z}$ is analytic everywhere, so $R = \infty$.

4.4.3 $\dfrac{z}{z-3i}, z_0 = 0.$

Answer: $\dfrac{z}{z-3i}$ is analytic for $z \neq 3i$, so $R = |z_0 - 3i| = |0 - 3i| = 3$.

4.4.5 $\dfrac{z+1}{z-i}, z_0 = 2 + i.$

Answer: $\dfrac{z+1}{z-i}$ is analytic for $z \neq i$, so $R = |z_0 - i| = |2 + i - i| = 2$.

4.4.6 $\dfrac{\sin z}{z^2+4}, z_0 = 3.$

Answer: $\dfrac{\sin z}{z^2+4}$ is analytic for $z \neq \pm 2i$, so $R = \min\{|z_0 - 2i|, |z_0 + 2i|\} = \min\{|3 - 2i|, |3 + 2i|\} = \min\{\sqrt{13}, \sqrt{13}\} = \sqrt{13}$.

4.5.1 $\dfrac{1}{1+z^2}, 1 < |z|.$

Answer: Let $w = \frac{1}{z^2}$; then $1 < |z| \implies \left|\frac{1}{z^2}\right| = |w| < 1$. By substitution we have $\dfrac{1}{1+z^2} = \dfrac{\frac{1}{z^2}}{1+\frac{1}{z^2}} = w \cdot \dfrac{1}{1-(-w)}$. Since $|w| < 1$, we can use the geometric series as follows: $w \cdot \dfrac{1}{1-(-w)} = w\displaystyle\sum_{n=0}^{\infty}(-w)^n = \sum_{n=0}^{\infty}(-1)^n w^{n+1} = \sum_{n=0}^{\infty}\frac{(-1)^n}{z^{2n+2}}$.

4.5.2 $\dfrac{3+z}{2-z}, 2 < |z|.$

Answer:

4.5.4 $z + \dfrac{1}{z}, 1 < |z - 1|.$

Answer:

4.5.13 $\dfrac{z}{(z+2)(z+3)}, 2 < |z| < 3.$

Answer: By partial fraction we have $\dfrac{z}{(z+2)(z+3)} = \dfrac{A}{z+2} + \dfrac{B}{z+3} \implies z = A(z+3) + B(z+2) \implies A = -2, B = 3$.

P1 Use the geometric series formula to derive the power series expansion for the given function f centered at the given point z_0:

(a) $f(z) = \frac{z}{1-z}$, centered at $z_0 = 0$.

(b) $f(z) = \frac{z^2+1}{z-1}$, centered at $z_0 = 0$.

(c) $f(z) = \frac{1}{(1-z)^3}$, centered at $z_0 = 0$.

(d) $f(z) = \frac{1}{1-2z^3}$, centered at $z_0 = 0$.

P2 Find the Laurent series for $\frac{1}{(z-i)(z+2i)}$ in the given domain:

(a) $A_{1,2}(0)$

(b) $A_{0,3}(i)$

(c) $A_{3,\infty}(i)$