Math 180 Homework 1

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4.1.2 Which of the following statements about graphs G and H are true? Substantiate your answers!

(ii) G and H are isomorphic if and only if there exists a bijection $f : E(G) \to E(H)$.
False, by counter example: let $G = (V, E) = (\{1\}, \emptyset)$ and $H = (V', E') = (\{1, 2\}, \emptyset)$; then there exists a bijection $f : E \to E'$ but G and H are not isomorphic.

(iii) If there exists a bijection $f : V(G) \to V(H)$ such that every vertex $u \in V(G)$ has the same degree as $f(u)$, then G and H are isomorphic.
True,

(iv) If G and H are isomorphic, then there exists a bijection $f : V(G) \to V(H)$ such that every vertex $u \in V(G)$ has the same degree as $f(u)$, then G and H are isomorphic.

(v) If G and H are isomorphic, then there exists a bijection $f : E(G) \to E(H)$.
True; since G and H are isomorphic, by definition, there exists a bijection $f : V(G) \to V(H)$ such that $\{x, y\} \in E(G) \Leftrightarrow \{f(x), f(y)\} \in E(H)$ for distinct $x, y \in V(G)$.

(vi) G and H are isomorphic if and only if there exists a map $f : V(G) \to V(H)$ such that for any two vertices $u, v \in V(G)$, we have $\{u, v\} \in E(G) \Leftrightarrow \{f(u), f(v)\} \in E(H)$.

(vii) Every graph on n vertices is isomorphic to some graph on the vertex set $\{1, 2, \ldots, n\}$.

(viii) Every graph on $n \geq 1$ vertices is isomorphic to infinitely many graphs.

4.1.4 Show that a graph G with n vertices is assymetric if and only if $n!$ distinct graphcs on the set $V(G)$ are isomorphic to G.

4.1.6 How many graphs on the vertex set $\{1, 2, \ldots, 2n\}$ are isomorphic to the graph consisting of n vertex-disjoint edges?

4.2.1

4.2.2

4.3.5

4.3.6

4.3.6

P0

P1 1.

2.

3.

4.