I certify on my honor that I have neither given nor received any help, or used any non-permitted resources, while completing this evaluation.

Signature:

Date: 12/14/2020

Q2 (a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

Q3 (a) Prove that $r(k, 2) = k$ for all $k \geq 2$.

Answer: Since we have $r(k, \ell) = 1 + \max\{|V(G)| : \omega(G) < k \text{ and } \alpha(G) < \ell\}$, we can start by finding $|V(G)|$ that satisfies both $\omega(G) < k$ and $\alpha(G) < 2$. Since $\alpha(G) < 2$, G has to be a complete graph, or we else can take two vertices that are not connected to have $\alpha(G) \geq 2$. Then to have $\omega(G) < k$ with G begin a complete graph, we can only have up to $k - 1$ vertices. Therefore $\max\{|V(G)| : \omega(G) < k \text{ and } \alpha(G) < \ell\} = k - 1$ and $r(k, 2) = 1 + \max\{|V(G)| : \omega(G) < k \text{ and } \alpha(G) < 2\} = k$.

(b) Prove that $r(k, \ell) \leq r(k - 1, \ell) + r(k, \ell - 1)$ for all $k, \ell \geq 2$.

Answer:

(c) Use parts (a) and (b) above to obtain an upper bound for $r(5, 5)$.

Answer: By part (a) we have $r(k, 2) = k$ and by symmetry we also have $r(2, \ell) = \ell$. Then using part (b), We have

$$
\begin{aligned}
&r(5,5) \\
&\leq r(4,5) + r(5,4) \\
&\leq (r(3,5) + r(4,4)) + (r(4,4) + r(5,3)) \\
&= r(3,5) + 2r(4,4) + r(5,3) \\
&\leq r(2,5) + r(3,4) + 2(r(3,4) + r(4,3)) + r(4,3) + r(5,2) \\
&= r(2,5) + 3r(3,4) + 3r(4,3) + r(5,2) \\
&\leq 5 + 3(r(2,4) + r(3,3)) + 3(r(3,3) + r(4,2)) + 5 \\
&= 10 + 3r(2,4) + 6r(3,3) + 3r(4,2) \\
&\leq 10 + 3 \cdot 4 + 6(r(2,3) + r(3,2)) + 3 \cdot 4 \\
&= 34 + 6 \cdot 6 \\
&= 70.
\end{aligned}
$$

Q4 (a)

(b)

(c)

(d)

Q5 (a)

(b)

(c)