Math 177 Homework 3

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Section 3.1

1. (i) $L = 1000a_{\overline{5}|10\%} + 500v^5 a_{\overline{5}|10\%} = \boxed{4967.68}$

   (ii) $B_3 = L(1+10\%)^3 - 1000s_{\overline{3}|10\%} = \boxed{3301.98}$

   (iii) $I_4 = B_3 * 10\% = \boxed{330.20}$

   $P_4 = 1000 - I_4 = \boxed{669.80}$

   (iv) $B_8 = 500_{\overline{2}|10\%} = \boxed{867.77}$

2. $v = \frac{1}{1+\frac{9\%}{12}}$; $B_{40} = \sum_{n=1}^{20} 1000v^n(1-2\%)^{39+n} = \boxed{6889.11}$

4. (i) Let X be the monthly payment amount. Then $20000 = X(12 + a_{\overline{36}|\frac{6\%}{12}}) \implies \boxed{X = 445.72}$

   $B_1 = 20000 - 12X = \boxed{14651.36}$

   (ii) $20000 = X(a_{\overline{12}|\frac{3\%}{12}} + v_{\frac{3\%}{12}}^{12} a_{\overline{36}|\frac{5\%}{12}}) \implies \boxed{X = 452.61}$

   $B_1 = 20000(1+\frac{3\%}{12})^{12} - Xs_{\overline{12}|\frac{3\%}{12}} = \boxed{15101.68}$

5. Bank Y monthly interest: $(1+i)^6 = 1 + \frac{14\%}{2} \implies i = 0.0113403$; $P_t = \frac{19800}{36} = 550 \implies I_t = iB_{t-1} = \frac{12\%}{12}[19800 - (t-1)P_t] \implies Price = P_t a_{\overline{20}|i} + \sum_{n=16}^{36}(v_i^{n-16} I_n) = \boxed{10857.27}$

6. $L = \sum_{i=1}^{n} P_n = \sum_{i=1}^{n}(K_n - I_n) = K_T - I_T$

9. $B_{10} = L = 1000$

   $B_{20} = B_{10}(1-5\%)^{10} = 598.74$

   $Xa_{\overline{10}|10\%} = B_{20} \implies \boxed{X = 97.44}$

11. (a) $1000 = K(2a_{\overline{144}|\frac{12\%}{12}} - a_{\overline{72}|\frac{12\%}{12}}) \implies \boxed{K = 9.89}$

t	K_t	I_t	P_t	B_t
0	$-$	$-$	$-$	1000
1	9.89	10.00	-0.11	1000.11
2	9.89	10.00	-0.11	1000.22
3	9.89	10.00	-0.11	1000.33
4	9.89	10.00	-0.11	1000.44
5	9.89	10.00	-0.11	1000.55
6	9.89	10.01	-0.12	1000.67
7	9.89	10.01	-0.12	1000.79
8	9.89	10.01	-0.12	1000.91
9	9.89	10.01	-0.12	1001.03
10	9.89	10.01	-0.12	1001.15
11	9.89	10.01	-0.12	1001.27
12	9.89	10.01	-0.12	1001.39

(b) Using the table above, we can see that P_{t+1} is indeed equal to $P_t(1+i)+K_{t+1}-K_t$ for $1 \leq t \leq 12d$.

(c) $B_{72} = 1000 + 0.11s_{\overline{72}|1\%} = 1011.52$
$B_{144} = 1011.52 - (2K - iB_{72})s_{\overline{72}|1\%} = \boxed{0.02}$

13. (a) $P_6 = K_6 - I_6 = 500 - (1000a_{\overline{10}|i} - a_{\overline{1}|i})i = 500(1 - 2a_{\overline{10}|i}i + vi) = 500[-2(1-v^{10}) - (1-vi)] = 500(2v^{10} - v)$

(b) $P_6 = K_6 - I_6 = 500 - (1000a_{\overline{10}|i} - a_{\overline{1}|i})i = 500(1 - 2a_{\overline{10}|i}i + vi) = 500[1 - i(2a_{\overline{10}|i} - v)]$

(c) $P_6 = P_1(1+i)^5 = (500 - Li)(1+i)^5$

Section 3.2

1. $B_t = L(1+t)^t - Ks_{\overline{t}|i} = Ka_{\overline{n}|i}(1+i)^t - Ks_{\overline{t}|i} = (Ks_{\overline{t}|i} + Ka_{\overline{n-t}|i}) - Ks_{\overline{t}|i} = Ka_{\overline{n-t}|i}$
$B_t = L(1+t)^t - Ks_{\overline{t}|i} = L + Lis_{\overline{t}|i} - Ks_{\overline{t}|i} = L + s_{\overline{t}|i}(Li - K) = L + P_1 s_{\overline{t}|i}$

4. Let X be the monthly payment amount. Then $L = Xa_{\overline{60}|0.01}$ and $B_t = Xa_{\overline{60-t}|0.01}$. $B_t = \frac{L}{2} \implies \boxed{t = 34.41}$.

7. $X(1+6\%)^{10} - X = \frac{10X}{a_{\overline{10}|6\%}} - X + 356.54 \implies \boxed{X = 825.00}$

8. Under option (i), let X be the annual payment amount. Then, $Xa_{\overline{10}|8.07\%} = 2000 \implies X = 299.00$. Then, under option (ii), $\sum_{n=0}^{9} 200 + 200(10-n)i) = 10X \implies 11000i + 2000 = 2990 \implies \boxed{i = 0.09}$

11. Let $t-1, t, t+1$ be the dates of the given consecutive payments. Then, $P_t = B_t - B_{t-1} = 5190.72 - 5084.68 = 106.04$ and $P_{t+1} = B_{t+1} - B_t = 5084.68 - 4973.66 = 111.02$. Then, $i = \frac{P_{t+1}}{P_t} - 1 = 0.0469634 \implies I_t = iB_{t-1} = 243.77 \implies K = P_t + I_t = \boxed{349.81}$

13. Scheme (i): Let each payment be X. Then, $L = Xa_{\overline{n}|i} \implies X = \frac{L}{a_{\overline{n}|i}}$; $I = nX - L = \boxed{L(na_{\overline{n}|i} - 1)}$

Scheme (ii): $I = Li\sum_{t=0}^{n-1} \frac{n-t}{n} = \boxed{\frac{1}{2}Li(n+1)}$

14. $A = 125000 a_{\overline{5}|5\%} = \boxed{541184.58}$

$B = 75000 a_{\overline{5}|5\%} = \boxed{324710.75}$

$C = 10000 (Da)_{\overline{5}|5\%} = \boxed{134104.67}$

$A + B + C = 541184.58 + 324710.75 + 134104.67 = 1000000$

15. As shown below, $n = 10$ and $Payment = 58.40$

t	K_t	I_t	P_t	B_t
0	–	–	–	1000.00
1	100	10.00	90.00	910.00
2	100	9.10	90.90	819.10
3	100	8.19	91.81	727.29
4	100	7.27	92.73	634.56
5	100	6.35	93.65	540.91
6	100	5.41	94.59	446.32
7	100	4.46	95.54	350.78
8	100	3.51	96.49	254.29
9	100	2.54	97.46	156.83
10	100	1.57	98.43	58.40

16. (a) Scheme (i): $B_t = \dfrac{L a_{\overline{n-t}|i}}{a_{\overline{n}|i}} = \dfrac{L(1 - v_i^{n-t})}{1 - v_i^n}$

Scheme (ii): $B_t = \dfrac{L a_{\overline{12(n-t)}|i}}{a_{\overline{12n}|i}} = \dfrac{L(1 - v_j^{12(n-t)})}{1 - v_j^{12n}}$

Since $v_i^n = v_j^{12n}$, the above are equivalent.

(b) Scheme (i): $I_{(i)} = \dfrac{nLi}{1 - v_i^n}$

Scheme (ii): $I_{(ii)} = \dfrac{nLj}{1 - v_j^{12n}}$

Since $j = \frac{i^{(12)}}{12}$, $j < \frac{i}{12} \implies I_{(i)} > I_{(ii)}$

17. $B_n = \frac{3}{4} L \implies a_{\overline{n}|i} = \frac{3}{4} a_{\overline{2n}|i} \implies v^n - 1 = \frac{3}{4} v^{2n} - \frac{3}{4} \implies v^n = \frac{1}{3} \implies \boxed{I_{n+1} = \dfrac{2}{3} K}$

26. $I_n = 153.86 \implies i B_{n-1} = 153.86 \implies i(X - P_{n-1}) = 153.86 \implies X = 7240$

$I_n = 153.86 \implies K(1 - v) = 153.86 \implies K = 1384.74$

$P_1 = K - I_1 = K - iX = \boxed{479.74}$

29. (a) $K_1 v + K_2 v^2 + ... K_n v^n = \frac{1+i}{1+i} + \frac{(1+i)^2}{(1+i)^2} + ... + \frac{(1+i)^n}{(1+i)^n} = n$

(b) $B_t = K_{t+1} v + K_{t+2} v^2 + ... + K_n v^{n-t} = \frac{(1+i)^{t+1}}{(1+i)} + \frac{(1+i)^{t+2}}{(1+i)^2} + ... + \frac{(1+i)^n}{(1+i)^{n-t}} = (n-t)(1+i)^t$