Math 177 Homework 6

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Section 7.1

1. $D_{mac} = \dfrac{nFv_j^n + \sum_{t=1}^n tFrv_j^t}{Fv_j^n + \sum Frv_j^t} = \dfrac{10(1.118^{-1} + 2 * 1.118^{-2}) + 3 * 110 * 1.118^{-3}}{10(1.118^{-1} + 1.118^{-}2) + 110 * 1.118^{-3}} = \boxed{2.729364}$

2. $D_{mac} = \dfrac{nFjv_j^n + \sum_{t=1}^n tFjv_j^t}{Fv_j^n + \sum Fjv_j^t} = \dfrac{nFjv_j^n + \sum_{t=1}^n tFjv_j^t}{Fv_j^n + Fja_{\overline{n}|j}} = \dfrac{nFjv_j^n + \sum_{t=1}^n tFjv_j^t}{Fv_j^n + F(1 - v_j^n)} = njv_j^n + \sum_{t=1}^n tjv_j^n = njv_j^n + j(Ia)_{\overline{n}|j} = \ddot{a}_{\overline{n}|j} \implies \boxed{D = \ddot{a}_{\overline{6}|10\%} = 4.790787}$

5. $D_{mac} = \dfrac{(Ia)_{\overline{n}|i}}{a_{\overline{n}|}i} = \dfrac{\frac{\ddot{a}_{\overline{n}|i} - nv^n}{i}}{a_{\overline{n}|i}} = \dfrac{\ddot{a}_{\overline{n}|i}}{1 - v^n} - \dfrac{nv^n}{1 - v^n} = \dfrac{1}{d} - \dfrac{n}{is_{\overline{n}|i}}$

6. $D_{mac} = \dfrac{D_1PV_1 + D_2PV_2}{PV_1 + PV_2} = \dfrac{12.7 * 0.8835F_1 + 14.6 * 1.3049F_2}{0.8835F_1 + 1.3049F_2} = 13.5 \implies F_1 = 67.005728, F_2 = 32.994272 \implies PV_{total} = 0.8835F_1 + 1.3049F_2 = \boxed{102.25}$

8. $\frac{d}{di}[L(1+i)^D] = LD(1+i)^{D-1} + (1+i)^D \frac{dL}{di} = LD(1+i)^{D-1} - \frac{LD(1+i)^D}{1+i} = 0$

14. $\frac{d}{dj}[D_{mac}(j)] = \frac{d}{dj}(\sum_t tw_t) = \sum_t t\frac{d}{dj}(w_t) = \sum_t t\frac{d}{dj}(\frac{K_tv^t}{L}) = \sum_t \frac{tDLK_tv^{t+1} - t^2LK_tv^{t+1}}{L^2} < 0$

Section 7.2

5. (a) $v^{10} = A_5v^5 + A_{15}v^{15}$
   $10v^{10} = 5A_5v^5 + 15A_{15}v^{15}$
   $\implies \boxed{A_5 = 0.310461, A_{15} = 0.805255}$

   (b) (i) $v^{10} = 0.40v^5 + A_{t_2}v^{t_2}$
   $10v^{10} = 5 * 0.40v^5 + t_2A_{t_2}v^{t_2}$
   $\implies \boxed{t_2 = 0.843197, A_{t_2} = 19.052997}$

   (ii) $v^{10} = 0.70v^5 + A_{t_2}v^{t_2}$
   $10v^{10} = 5 * 0.70v^5 + t_2A_{t_2}v^{t_2}$
   $\implies t_2 = -0.001875, A_{t_2} = -34.259719 \implies \boxed{\text{no solution}}$

   (c) (i) $v^{10} = A_5v^5 + 0.90v^{t_2}$
   $10v^{10} = 5A_5v^5 + 0.90t_2v^{t_2}$
   $\implies \boxed{A_5 = 0.125850, t_2 = 11.271029 \text{ or } A_5 = 0.430235, t_2 = 21.281183}$

(ii) $v^{10} = A_5 v^5 + 1.5 v^{t_2}$

$10v^{10} = 5A_5 v^5 + 1.5 t_2 v^{t_2}$

$\implies \boxed{A_5 = 0.505572, t_2 = 31.914800}$

(iii) $v^{10} = A_5 v^5 + 0.75 v^{t_2}$

$10v^{10} = 5A_5 v^5 + 0.75 t_2 v^{t_2}$

$\implies \boxed{\text{no real solution}}$

(d) (i) $v^{10} = 0.80 v^{t_1} + A_{15} v^{15}$

$10v^{10} = 0.80 t_1 v^{t_1} + 15 A_{15} v^{15}$

$\implies \boxed{A_{15} = 0.220775, t_1 = 9.205993}$

(ii) $v^{10} = 1.1 v^{t_1} + A_{15} v^{15}$

$10v^{10} = 1.1 t_1 v^{t_1} + 15 A_{15} v^{15}$

$\implies A_{15} = -0.108540, t_1 = 10.315978 \implies \boxed{\text{no solution}}$

(iii) $v^{10} = 0.01 v^{t_1} + A_{15} v^{15}$

$10v^{10} = 0.01 t_1 v^{t_1} + 15 A_{15} v^{15}$

$\implies A_{15} = 1.369372, t_1 = -18.393957 \implies \boxed{\text{no solution}}$

(e) $v^{10} = 0.40 v^{t_1} + 0.90 v^{t_2}$

$10v^{10} = 0.40 t_1 v^{t_1} + 0.90 t_2 v^{t_2}$

$\implies t_1 = -635.103151, t_2 = -622.957535 \implies \boxed{\text{no solution}}$

6. $1000000 v^{12} = A_{t_0} v^{t_0} + 15000 \ddot{a}_{\overline{12}|10\%}$

$12 * 1000000 v^{12} = t_0 A_{t_0} v^{t_0} + 15000 (Ia)_{\overline{11}|10\%}$

$\implies \boxed{A_{t_0} = 961145.0766, t_0 = 16.149954}$

7. (a) $100(v^2 + v^4 + v^6) = A_1 v + A_5 v^5$

$100(2v^2 + 4v^4 + 6v^6) = A_1 v + 5A_5 v^5$

$\implies \boxed{A_1 = 71.441655, A_5 = 229.411364}$

(b) Yes because we can see that $P_A^{''}(i_0) > P_L^{''}(i_0)$ by plugging in the values above.

Note 20

1. $P_A(i_0) = P_L(i_0) \implies \sum_t A_t v^t = \sum_t L_t v^t$, then:

2.4 $P_A^{'}(i_0) = P_L^{'}(i_0) \implies \frac{d}{di} \sum_t A_t v^t = \frac{d}{di} \sum_t L_t v^t$ by differentiating both sides of the above equation. Then, $D_{mod,A}(i_0) = -\dfrac{\frac{d}{di}\sum_t A_t v^t}{\sum_t A_t v^t} = -\dfrac{\frac{d}{di}\sum_t L_t v^t}{\sum_t L_t v^t} = D_{mod,L}(i_0)$

2.5 $P_A^{'}(i_0) = P_L^{'}(i_0) \implies \sum_t t A_t v^t = \sum_t t L_t v^t$ using 2.6 shown below. Then, $D_{mac,A}(i_0) = \dfrac{\sum_t t A_t v^t}{\sum_t A_t v^t} = \dfrac{\sum_t t L_t v^t}{\sum_t L_t v^t} = D_{mac,L}(i_0)$.

2.6 By differentiating $P_A(i_0) = P_L(i_0)$, $P_A^{'}(i_0) = P_L^{'}(i_0) \implies \frac{d}{di} \sum_t A_t v^t = \frac{d}{di} \sum_t L_t v^t \implies \sum_t A_t(-t v^{t+1}) = \sum_t L_t(-t v^{t+1}) \implies \sum_t t A_t v^{t+1} = \sum_t t L_t v^{t+1} \implies \sum_t t A_t v^t = \sum_t t L_t v^t$

2. As shown above, $P_A(i_0) = P_L(i_0) \implies \sum_t A_t v^t = \sum_t L_t v^t$ and $P_A^{'}(i_0) = P_L^{'}(i_0) \implies \sum_t tA_t v^t = \sum_t tL_t v^t$. Then:

   2.7 $P_A^{''}(i_0) > P_L^{''}(i_0) \implies \sum_t t^2 A_t v^t > \sum_t t^2 L_t v^t$ as shown below in 2.9, which implies that $\frac{d^2}{di^2}P_A(i) > \frac{d^2}{di^2}P_L(i)$. Then, dividing the previous inequality by $P_A(i_0) = P_L(i_0)$ results in $\dfrac{\frac{d^2}{di^2}P_A(i)}{P_A(i)} = \dfrac{\frac{d^2}{di^2}P_L(i)}{P_L(i)} \implies C_{mod,A}(i_0) > C_{mod,L}(i_0)$.

   2.8 $P_A^{''}(i_0) > P_L^{''}(i_0) \implies \sum_t t^2 A_t v^t > \sum_t t^2 L_t v^t$ as shown below in 2.9. Then, dividing the previous inequality by $P_A(i_0) = P_L(i_0)$ results in $\dfrac{\sum_t t^2 A_t v^t}{\sum_t A_t v^t} = \dfrac{\sum_t t^2 L_t v^t}{\sum_t L_t v^t} \implies C_{mac,A}(i_0) > C_{mac,L}(i_0)$.

   2.9 By differentiating $P_A^{'}(i_0) = P_L^{'}(i_0)$, $P_A^{''}(i_0) > P_L^{''}(i_0) \implies \frac{d}{di}\sum_t tA_t v^t > \frac{d}{di}\sum_t tL_t v^t \implies \sum_t tA_t(-tv^{t+1}) > \sum_t tL_t(-tv^{t+1}) \implies \sum_t t^2 A_t v^{t+1} > \sum_t t^2 L_t v^{t+1} \implies \sum_t t^2 A_t v^t > \sum_t t^2 L_t v^t$.

   2.10 Since $D_{mac}(i_0)$ is constant on both sides and given 2.9 shown above, $\sum_t [t - D_{mac}(i_0)]^2 A_t v^t > \sum_t [t - D_{mac}(i_0)]^2 L_t v^t$.