Final

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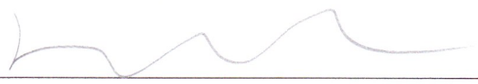
Instructions:

- The exam will begin at 12 am June 9 PDT. You will be given **24 hours** to complete and submit your works. The submission window will be closed at 12 am June 10 PDT.
- **No late submission** will be considered. Make sure to allow enough time to complete and submit your works. Make-ups for the exam are permitted only under exceptional circumstances, as outlined in the UCLA student handbook.
- The exam will be **open book/open notes**. You may also use a physical and/or online calculator, provided it supports the same or less functionality than the officially accepted models on SOA Exam[1].
- **You must show your works to receive credit.**
- **You must sign the code of conduct:**

> I assert, on my honor, that I have not received assistance of any kind from any other person while working on the exam and that I have not used any non-permitted materials or technologies during the period of this evaluation.
>
> Signature: 

Any deviation from the rules may render your exam void. Also, if needed, you may be contacted after the exam and asked for additional explanations of solutions for problems on the exam.

- **A Gradescope link for submitting your works will be provided.** Your submission should meet a set of criteria:
  (a) Your submission must be a single PDF file.
  (b) The code of conduct, your name, UID, and either physical or electronic signature must appear on the first page. (See above for an example.)

  There will be several ways to achieve this. The following is a set of common examples:
  - The exam template file will be designed to satisfy the above criteria. So you may simply print it out, fill in the necessary forms, and write down your solution of each of the problems. And then you may either scan it or take a (high-resolution and high-contrast) picture of it.
  - You may use letter size blank papers as soon as all the above criteria are met.
  - You may directly write on the exam PDF file, such as using a tablet.
  - You may use a word processor or LaTeX to prepare your submission electronically.

[1] https://www.soa.org/education/exam-req/exam-day-info/edu-calculators/

Question 1. Suppose that the instantaneous forward rate takes the form:

$$\delta_t = Xt + Y, \qquad 0 \leq t \leq 3.$$

You are also given the following spot interest rates:

Period of Time	Spot Interest Rate
1 Year	5.65%
2 Years	6.82%

Compute the three-year spot rate.

$f_{[0,1]} = 5.65\%$ $\quad f_{[0,2]} = 6.82\%$ $\quad f_{[0,3]} = ?$

$$\begin{cases} 5.65\% = X + Y \\ 6.82\% = 2X + Y \end{cases} \Rightarrow X = 1.17\%, \quad Y = 5.65\% - 1.17\% = 4.48\%$$

$$\Rightarrow f_{[0,3]} = 3X + Y = \boxed{7.99\%}$$

Question 2. Demetrius is comparing two investment options:

Option 1: A 15-year annuity-immediate with annual payments of $10X$.

Option 2: A 15-year increasing annuity-immediate which also makes annual payments. The payment at the end of year 1 is equal to 50. At the end of year 2, and at the end of each year through year 15, each subsequent payment is increased by X.

Demetrius finds that both annuities have the same present value at an annual effective interest rate of 8.874%. Calculate X.

Option 1: $PV_1 = 10X \cdot a_{\overline{15}|}$

Option 2: $PV_2 = X \cdot (Ia)_{\overline{15}|} + (50 - X)v$

$$a_{\overline{15}|8.874\%} = \frac{1 - v^{15}}{8.874\%} = 8.120997$$

$$(Ia)_{\overline{15}|8.874\%} = \frac{\ddot{a}_{\overline{15}|} - 15v^{15}}{8.874\%} = 52.417335$$

$$PV_1 = PV_2 \Rightarrow 10X \cdot a_{\overline{15}|} = X \cdot (Ia)_{\overline{15}|} + 50v - Xv$$

$$X\left[10a_{\overline{15}|} - (Ia)_{\overline{15}|} + v\right] = 50v$$

$$\boxed{X = 1.545705}$$

Question 3. Aaleyah takes out a 8-year loan of L, which is to be repaid by making 8 annual repayments at the end of each year using the amortization method. Interest on the loan is charged at an annual effective rate of i. Aaleyah repays the loan with a increasing series of payments, i.e., she repays 100 in year one, 200 in year two, 300 in year three, ..., and 800 in year eight. The amount of principal repaid in year seven is equal to 636.

Calculate L. (*Hint:* You might find the prospective method helpful for determining the value of i.)

$B_7 = B_6 - P_7 \Rightarrow K_8 v = K_7 v + K_8 v^2 - P_7$

$800v = 700v + 800v^2 - 636$

$v = 0.956315557$

$L = \sum_{s=1}^{n} K_s v^s$

$= 100v + 200v^2 + \ldots + 800v^8$

$= 100(Ia)_{\overline{8}|}$

$= \boxed{2806.01}$

Question 4. Sherry wishes to invest in a bond. There are two bonds available:

(i) Bond X: A $\$100$ par value, n-year bond with 4% annual coupons and maturing at par.

(ii) Bond Y: A $\$200$ par value, n-year bond with annual coupons of $\$3$ and maturing at par.

Sherry realizes that both bonds have the same purchase price at an annual effective yield rate of 6.5%. Calculate n.

Bond X: $P_X = 100 v^n_{6.5\%} + 100 \cdot 4\% \cdot a_{\overline{n}|6.5\%}$

Bond Y: $P_Y = 200 v^n_{6.5\%} + 3 \cdot a_{\overline{n}|6.5\%}$

$P_X = P_Y \Rightarrow 100 v^n + 4 a_{\overline{n}|} = 200 v^n + 3 a_{\overline{n}|}$

$a_{\overline{n}|} = 100 v^n$

$n = 31.995386 \approx \boxed{32}$

Question 5. Junho invests 2500, at the beginning of the year, in a fund which credits interest at the end of each six-month period at a nominal annual rate of i convertible semiannually. The interest payments are immediately reinvested in a separate fund earning interest at an annual effective rate of 5%.

At the end of ten years, immediately after the interest is credited, Junho withdraws all the balance from both funds. From this 10-year investment, he earns an annual effective yield of 6.484%.

Calculate i.

Fund 1: Since each interest payment is immediately reinvested, account balance stays constant at 2500 and each interest payment is $2500 \cdot \frac{i}{2} = 1250i$

Fund 2: Account balance at year 10: $1250i \cdot s_{\overline{20}|\frac{i}{2}}$

Then,

$$2500 \cdot (1 + 6.484\%)^{10} = 2500 + 1250i \cdot 2 \cdot s_{\overline{10}|5\%}$$

$$i = \frac{2500 \cdot (1.06484^{10} - 1)}{2500 \, s_{\overline{10}|5\%}}$$

$$\boxed{i \approx 6.95\%}$$

Question 6. A bond has the present value of $P(5\%)$ and Macaulay duration of $D_{\text{mac}}(5\%)$ at an annual effective yield rate of 5%. Using the first order Macaulay approximation, the present values of the bond at different yield rates are estimated as follows:

Yield Rate	Estimated Present Value
4.8%	$\$160,000$
5.1%	$\$150,000$

Compute $P(5\%)$ and $D_{\text{mac}}(5\%)$.

$$P(i) \approx P(i_0)\left(\frac{1+i_0}{1+i}\right)^{D_{\text{mac}}(i_0)} \Rightarrow \begin{cases} 160000 = P(5\%)\left(\frac{1+5\%}{1+4.8\%}\right)^{D_{\text{mac}}(5\%)} \\ 150000 = P(5\%)\left(\frac{1+5\%}{1+5.1\%}\right)^{D_{\text{mac}}(5\%)} \end{cases}$$

Let $X = P(5\%)$ and $Y = D_{\text{mac}}(5\%)$, then

$$\begin{cases} 160000 = X \cdot \left(\frac{1.05}{1.048}\right)^{Y} \\ 150000 = X \cdot \left(\frac{1.05}{1.051}\right)^{Y} \end{cases} \Rightarrow X = 153258.75,\ Y = 22.58$$

So $\boxed{P(5\%) = 153258.75 \text{ and } D_{\text{mac}}(5\%) = 22.58}$

Question 7. A company must pay a benefit of $\$2,500$ to a customer in two years. To provide for this benefit, the company will buy one-year and four-year zero-coupon bonds. The company wants to immunize itself from small changes in interest rates on either side of 9%. What face amount of the four-year bonds should be purchased?

Redington immunization: ① $P_A(i_0) = P_L(i_0)$

② $P'_A(i_0) = P'_L(i_0) \Leftrightarrow \sum_t t A_t v_0^t = \sum_t t L_t v_0^t$

③ $P''_A(i_0) > P''_L(i_0) \Leftrightarrow \sum_t t^2 A_t v_0^t = \sum_t t^2 L_t v_0^t$

Let the face values of the 1-year and 4-year bonds be X and Y respectively. Then,

① : $Xv + Yv^4 = 2500v^2$

② : $Xv + 4Yv^4 = 2 \cdot 2500v^2$

$\Rightarrow$ $X = 1529.05$, $Y = 990.08$

Check ③ :

$$Xv + 16Yv^4 \overset{?}{>} 4 \cdot 2500v^2$$

$$\Rightarrow \quad 12625.16 > 8416.80 \quad \checkmark$$

Therefore $\boxed{X = 1529.05,\ Y = 990.08}$

Question 8. Otonashi Corporation expects to borrow money from Idomizu Bank over the next three years. Under this arrangement, Otonashi will borrow $\$1,000,000$ at the beginning of year one, two, and three. (So the loan amount will be $\$1,000,000$ in year one, $\$2,000,000$ in year two, and $\$3,000,000$ in year three.) Otonashi will repay the entire loan at once at the end of year three.

Otonashi has agreed to pay Idomizu the one-year spot interest rate each year. Otonashi wants to hedge the risk of changing interest rates and lock in a fixed interest rate for the next three years. To do this, Otonashi enters into an accreting interest rate swap.

You are given the following prices for $\$100$ par value zero-coupon bonds:

Time to Maturity	Price per 100
1 Year	97
2 Year	X
3 Year	89
4 Year	83

The swap rate on Otonashi's swap is 4.09%. Determine X.

$P_1 = 0.97 \quad P_2 = \frac{X}{100} \quad P_3 = 0.89$

$Q_1 = 10^6 \quad Q_2 = 2 \cdot 10^6 \quad Q_3 = 3 \cdot 10^6$

$$R = \frac{\sum_{i=1}^{n} Q_{t_i}(P_{t_{i-1}} - P_{t_i})}{\sum_{i=1}^{n} Q_{t_i} P_{t_i}}$$

$$\Rightarrow \quad 4.09\% = \frac{10^6(1-0.97) + 2\cdot 10^6(0.97 - \frac{X}{100}) + 3\cdot 10^6(\frac{X}{100} - 0.89)}{10^6 \cdot 0.97 + 2\cdot 10^6 \cdot \frac{X}{100} + 3\cdot 10^6 \cdot 0.89}$$

$$\Rightarrow \quad \boxed{X = 92.45}$$

Question 9. Honningbrew Meadery enters into a forward contract with a beekeeper to purchase 1000 pounds of honey in one year. The spot price of honey (per pound) is $\$1.50$ when the contract is made.

(a) Suppose that the delivery price (per pound) is determined as $\$1.57$, based on the continuously compounded risk-free rate of interest r for a zero coupon bond with one year to maturity. Find the value of r.

$$F_{0,T} = S_0 e^{rT} \Rightarrow 1570 = 1500 e^{r}$$

$$r = 0.0486105 \approx \boxed{4.56\%}$$

(b) Assume that six months has passed since Monningbrew entered into the contract. The spot price of honey (per pound) has risen to $\$1.55$, and the continuous compounded risk-free rate of interest is 4.77% per annum for 6 month maturities. Determine the value of Honningbrew's forward contract.

$$P = S_t - F_{0,T} e^{-r(T-t)}$$

$$= 1550 - 1570 e^{-0.5(1-0.5)}$$

$$= 1500 - 1570 e^{-0.25}$$

$$\approx \boxed{277.28}$$