Math 177 Homework 4

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Section 4.1

1. Since yield rate $7.7\% > 7.2\%$, $(b) > (a)$ and $(d) > (c)$. In addition, since the yield rate is greater than the coupon rate for all options, $(a) > (c)$ and $(b) > (d)$. Therefore, $(b) > (a) > (d) > (c)$.

2. $115.84 = Cv_{3.5\%}^{24} + 100 * 3.5\% a_{\overline{24}|3\%} \implies \boxed{C = 114.99}$

3. $5083.49(1+j)^{20} = 10000 \implies j = 0.0344081$; $X = 10000v_j^{20} + 10000 * 10\% a_{\overline{20}|j} = \boxed{12227.90}$

4. Let n be the number of half-years. $Fv_{2.5\%}^{16} + F * 3\% a_{\overline{16}|2.5\%} = Fv_{2.5\%}^{n} + F * 2.75\% a_{\overline{n}|2.5\%} \implies n = 42.84 \implies \boxed{21.42 \text{ years}}$

6. Let j be the quarterly yield rate and i the nominal annual yield rate. Then, $800 = 1000v_j^{100} + 1000 * 2.5\% a_{\overline{100}|j} \implies j = 0.0316179 \implies \boxed{i = 0.126417}$

7. $L = P = 1000v_{4\%}^{20} + 1000 * 5\% a_{\overline{20}|4\%} = 1135.903263$
   Net gain $= 50s_{\overline{20}|3\%} + 1000 - L(1+7\%)^{10} = \boxed{109.03}$

11. I. False since $i_2 > i_1 \implies K_2 < K_1 \implies P_2 < P_1$.
    II. True since $i_2 > i_1 \implies v_{i_2} < v_{i_1} \implies r_2 a_{\overline{n}|i_2} > r_1 a_{\overline{n}|i_1}$.
    III. False. $i_2 > i_1 \implies v_{i_2} < v_{i_1} \implies PV_A > PV_B$

12. Let prices of the bonds be X and Y with coupon rates $2r$ and r respectively.
    $X = 100 + 100(2r - 1.5\%)a_{\overline{n}|3\%}$; $Y = 100 + 100(r - 1.5\%)a_{\overline{3\%}|n}$
    $X + Y = 240$ and $X - Y = 24 \implies X = 132, Y = 108$
    $\implies n = 13.05$ and $r = 0.0225$
    Therefore the coupon rates are 2.25% and 4.50% respectively.

15. $P = 1000v_{5\%}^{40} + 1000 * 4\% a_{\overline{40}|5\%} = 828.409137$
    $P = Cv_{5\%}^{20} + 1000 * 4\% a_{\overline{20}|5\%} \implies \boxed{C = 875.38}$

18. $g = \frac{Fr}{C} \implies F = \frac{Cg}{r}$, $r = \frac{Cg}{F}$, $Fr = Cg$, then
    (4.2E) $P = Cv_j^n + Fra_{\overline{n}|j} = Cv_j^n + Cga_{\overline{n}|j}$
    (4.3E) $P = C + (Fr - Cj)a_{\overline{n}|j} = P = C + (Cg - Cj)a_{\overline{n}|j} = C + C(g-j)a_{\overline{n}|j}$
    (4.4E) $P = K + \frac{r}{j}(F - K) = K + \frac{Fr}{j} - \frac{r}{j}K = K + \frac{Cg}{j} - \frac{Cg}{Fj}K = K + \frac{g}{j}(C - K)$

Section 4.2

1. Total amount paid: $F + Frn$

   Total interest repaid: $Frn - F(r-j)a_{\overline{n}|j}$

   Total priciple repaid: $F + F(r-j)a_{\overline{n}|j}$

4. $B_{t+1} = B_t(1+j) - K_{t+1} = 90(1+3.3\%) - 100 * 2.5\% = \boxed{90.47}$

5. $100(r - 3.5\%) = 1.00 \implies r = 0.045$; $136 = 100 + 100(r - 3.5\%)a_{\overline{n}|3.5\%} \implies \boxed{n = 26}$

Section 5.1

4. $\sum_{n=0}^{3} C_n^A v^n = 0 \implies -5 + 3.72v + 4v^3 = 0 \implies \boxed{j_A = 0.253304}$

   $\sum_{n=0}^{3} C_n^B v^n = 0 \implies -5 + 3v + 1.7v^2 + 3v^3 = 0 \implies \boxed{j_B = 0.253280}$

   We can set the two transactions equal: $-5+3.72v+4v^3 = -5+3v+1.7v^2+3v^3 \implies i = 0.111111$ or $i = 0.25$. Then by substituting values of i we can see that $B > A$ for $0.111111 < i < 0.25$ and $A > B$ otherwise.

7. (a) Since $C_n < 0$ for $0 \leq n \leq 23$ and $C_{24} > 0$, there exists a unique $i > -1$.

   (b) Since i is unique, $F_{24} > 0 \implies Y - 150000 - 24(10000) - 2(10000) - 740000 > 0 \implies \boxed{Y > 938800}$

9. (a) Net profit: $1000000(1+i)^{15} + (950000 - 5 * 10000\ddot{s}_{\overline{1}|4\%})(1+i)^{14} + (910000 - 4 * 10000\ddot{s}_{\overline{2}|4\%})(1+i)^{13} + (870000 - 4 * 10000\ddot{s}_{\overline{3}|4\%})(1+i)^{12} + (840000 - 3 * 10000\ddot{s}_{\overline{4}|4\%})(1+i)^{11} + (910000 - 3 * 10000\ddot{s}_{\overline{5}|4\%})(1+i)^{10} + (910000 - 2 * 10000\ddot{s}_{\overline{6}|4\%})(1+i)^{9} + (910000 - 2 * 10000\ddot{s}_{\overline{7}|4\%})(1+i)^{8} + (910000 - 4 * 10000\ddot{s}_{\overline{8}|2\%})(1+i)^{7} + (910000 - 10000\ddot{s}_{\overline{9}|4\%})(1+i)^{6} + (910000 - 10000\ddot{s}_{\overline{10}|4\%})(1+i)^{5} + (910000 - 10000\ddot{s}_{\overline{11}|4\%})(1+i)^{4} + (910000 - 10000\ddot{s}_{\overline{12}|4\%})(1+i)^{3} + (910000 - 10000\ddot{s}_{\overline{13}|4\%})(1+i)^{2} + (910000 - 10000\ddot{s}_{\overline{14}|4\%})(1+i)^{1} - 69 * 300000$

   (b) We can find the value of i by setting the above expression to equal to 0 and solve for i.

11. $-1000000 - \int_0^5 200000e^{-\delta t}dt + \int_1^3 250000(1+t)e^{-\delta t}dt + \int_3^5 400000(5.5-t)e^{-\delta t}dt = 0 \implies \boxed{\delta = 0.371795}$

Section 5.2

1. $(1+X)^2 = \frac{1310000+250000}{1000000} * \frac{1265000+150000}{1310000} * \frac{1540000+250000}{1265000} * \frac{1420000+150000}{1540000} \implies \boxed{X = 0.0913523}$

2. $1 + 0\% = \frac{12}{10} * \frac{X}{12+X} \implies \boxed{X = 60}$

   $10(1+Y) + 60(1 + \frac{6}{12}Y) = 60 \implies \boxed{Y = -0.25}$

3. Fund after 1 year (dollar-weighted): $100000(1+x) - 8000(1 + \frac{9}{12}x)$

   Time-weighted: $1 + x = \frac{103992}{100000(1+x) - 8000(1+\frac{9}{12}x)}$

   $\implies \boxed{x = 0.0624991}$

4. 6 months: $1 + Y = \frac{40}{50} * \frac{80}{40+20} * \frac{157.50}{80+80} \implies Y = 0.05 \implies i = (1+X)^2 - 1 = 0.1025$

   12 months: $1.1025 = \frac{40}{50} * \frac{80}{40+20} * \frac{175}{80+80} * \frac{X}{175+75} \implies \boxed{X = 236.25}$

6. Account K: $100(1+i) - X(1+\frac{6}{12}i) + 2X(1+\frac{3}{12}i) = 125$
   Account L: $1+i = \frac{125}{100} * \frac{105.8}{125-X}$
   $\implies X = 10, \boxed{i = 0.15}$

Section 5.3

2. $i = \dfrac{2I}{2F(t_1)+N} = \dfrac{2I}{2F(t_1)+(F(t_2)-F(t_1)+I)} = \dfrac{2I}{F(t_1)+F(t_2)-I}$

3. $16147 = -10000 * (1+15\%)^t * (1+9\%)^{10-t} + Xs_{\overline{t}|15\%} * (1+9\%)^{10-t} + Xs_{\overline{10-t}|9\%} \implies \boxed{t=6}$