

# Math 115A Homework 2

Jiaping Zeng

5/5/2020

2. Let  $V$  be a finite dimensional vector space and  $W$  a subspace. Show that  $V$  and  $W \times V/W$  are isomorphic by finding an explicit isomorphism.
5. A differential operator on  $\mathbb{R}_n[x]$  is a linear combination of expressions of the form  $x^a \frac{d^b}{dx^b}$  where  $a - b \leq 0$  and  $b \leq n$ . We can consider a differential operator as a linear map  $\mathbb{R}_n[x] \rightarrow \mathbb{R}_n[x]$ .
- (a) Let  $D : \mathbb{R}_2[x] \rightarrow \mathbb{R}_2[x]$  be the differential operator given by  $2 - 4\frac{d}{dx} + 2x\frac{d^2}{dx^2}$ . Find the matrix of  $D$  relative to the basis  $\{x^2, (x-1)^2, (x+1)^2\}$ .
- (b) Does the differential equation  $2f - 4\frac{df}{dx} + 2x\frac{d^2f}{dx^2} = 0$  have any solutions  $f \in \mathbb{R}_2[x]$ ?
- (c) Suppose  $E : \mathbb{R}_2[x] \rightarrow \mathbb{R}_2[x]$  is a differential operator and that the matrix of  $E$ , relative to the basis  $\{1, x, x^2\}$  is

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}.$$

Find  $E$ .