Math 177 Homework 2

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Section 2.1

2. Let the refernce time point be the end of the 10th year. We can start by find the interest rate using the first payment option: $900s_{\overline{10}|i} = 1000a_{\overline{\infty}|i} \implies 900 * \dfrac{(1+i)^{10}-1}{i} = 1000 * \dfrac{1-v^{\infty}}{i} \implies i = 0.0775839375$. We can then use i to calculate K as follows: $Ks_{\overline{5}|i} * (1+i)^{10-5} = 1000a_{\overline{\infty}|i} \implies K * \dfrac{(1+i)^5-1}{i} * (1+i)^5 = 1000 * \dfrac{1-v^{\infty}}{i} \implies \boxed{K = 1519.42}$.

4. Smith makes his deposits over $(2034-2010+1)*12 = 300$ months and the accumulated account pays over $(2059-2035+1)*12 = 300$ months. Then we can setup the following equation with the refernce time being December 31, 2034: $1000s_{\overline{300}|\frac{0.12}{12}} = Ya_{\overline{300}|\frac{0.12}{12}} \implies \boxed{Y = 19788.47}$.

5. (i) January 1, 2015: $100s_{\overline{7}|\frac{0.09}{12}} = \boxed{715.95}$

   (ii) January 1, 2016: $100s_{\overline{19}|\frac{0.09}{12}} = \boxed{2033.97}$

   (iii) February 1, 2017: $100[s_{\overline{19}|\frac{0.09}{12}} * \left(1+\dfrac{0.105}{12}\right)^9 * \left(1+\dfrac{0.12}{12}\right)^4 + s_{\overline{9}|\frac{0.105}{12}} * \left(1+\dfrac{0.12}{12}\right)^4 + s_{\overline{4}|\frac{0.12}{12}}] = \boxed{3665.12}$

   (iv) Interest on February 28, 2017: $3665.12 * \frac{0.12}{12} = \boxed{36.65}$

6. Using year 3n as the reference point, we can set up the following equation: $8000 = 98s_{\overline{n}|i}(1+i)^{2n} + 196s_{\overline{2n}|i}$. We can then combine the previous equation with the given $(1+i)^n = 2.0$ into a system of equations with two variables and two unknowns. Solving numerically results in $n = 6.00$ and $\boxed{i = 0.1225}$.

8. Since it is implied that $i = 0.1$, $\displaystyle\sum_{t=1}^{10} s_{\overline{t}|0.1} = \sum_{t=1}^{10} \frac{1.10^t-1}{1.10} = 10(\ddot{s}_{\overline{10}|0.1} - 10) = \boxed{11S - 100}$.

9. $I_t = is_{\overline{t-1}|i}$. $\displaystyle\sum_{t=1}^{n} I_t = \sum_{t=1}^{n}(1+t)^{t-1} - n = s_{\overline{n}|i} - n$. This relationship represents that interest is the difference between accumulated value and deposit.

11. (a) $(1+i)^n = \dfrac{s_{\overline{2n}|i}}{s_{\overline{n}|i}} - 1 = \dfrac{210}{70} - 1 = \boxed{2}$

    $s_{\overline{n}|i} = \dfrac{(1+i)^n-1}{i} = \dfrac{1}{i} \implies i = \dfrac{1}{s_{\overline{n}|i}} = \boxed{\dfrac{1}{70}}$

    $s_{\overline{3n}|i} = s_{\overline{n}|i} + s_{\overline{2n}|i}(1+i)^n = 70 + 210 * 2 = \boxed{490}$

(b) Let $u = (1+i)^n$. Then, $\dfrac{X}{Y} = \dfrac{s_{\overline{3n}|i}}{s_{\overline{n}|i}} = u^2 + u + 1 \implies u = \dfrac{-1+\sqrt{-3+\frac{4X}{Y}}}{2} \implies v^n = \dfrac{1}{u} = \boxed{\dfrac{2}{-1+\sqrt{-3+\frac{4X}{Y}}}}$

(c) Let $w = 1+i$. Then, $s_{\overline{n}|i} = w^2 s_{\overline{n-2}|i} + w + 1 \implies 48.99 = 36.34w^2 + w + 1 \implies w = 1.135490 \implies \boxed{i = 0.135490}$

12. $s_{\overline{n}|0.11} = \dfrac{1.11^n - 1}{0.11} \implies 1.11^n = 0.11 s_{\overline{n}|0.11} + 1 = 15.08$; $AV = s_{\overline{n}|0.11} + 1.11^n s_{\overline{m}|0.07} = \boxed{640.72}$

17. Annuity A: $X = 55a_{\overline{20}|i}$

Annuity B: $X = 30a_{\overline{10}|i} + 60v^{10}a_{\overline{10}|i} + 90v^{20}a_{\overline{10}|i}$

Solving numerically results in $i = 0.0717734$ and $\boxed{X = 574.72}$.

Section 2.2

1. Monthly effective rate of interest: $j = (1 + \frac{10\%}{2})^{\frac{1}{6}} - 1 = 0.00816485$

   – (a) $50000 = Xa_{\overline{25*12}|i} \implies \boxed{X = 454.35}$

   – (b) $50000 = (454.35 + 100)a_{\overline{n}|i} \implies n = 167.84$. Then, $(X+100)a_{\overline{n}|j} + Yv_j^{169} = 50000 \implies \boxed{Y = 290.30}$

4. Quarterly effective rate of interest: $j = (1+7\%)^{\frac{1}{4}} - 1 = 0.170585$; $450s_{\overline{40}|i}(1+7\%)^5 = Y\ddot{a}_{\overline{4}|7\%} \implies \boxed{Y = 9873.20}$

5. Let j be the 4-year rate of interest. Then, $100\ddot{s}_{\overline{10}|j} = 500\ddot{s}_{\overline{5}|j} \implies j = 0.319508$. Then, $X = 100\ddot{s}_{\overline{10}|j} = \boxed{6194.72}$

6. $\ddot{a}_{\overline{\infty}|i} = 20 \implies i = \frac{1}{19}$. Then, $X = 20 * \dfrac{(1+\frac{1}{19})^4 - 1}{(1+\frac{1}{19})^4} = \boxed{3.709875}$

7. $Xs_{\overline{60}|0.005} = 10000 * (1 + \frac{7.45\%}{2})^{5*2} \implies X = 206.616748$; $10000 = Xa_{\overline{60}|i^{(12)}} \implies i^{(12)} = 0.00733377 \implies \boxed{i = 0.0880052}$

9. Monthly effective rate of interest: $j = \frac{0.09}{12} = 0.0075$

   Annual effective rate of interest: $i = (1 + \frac{0.09}{12})^{12} - 1 = 0.0938069$

   $100\ddot{s}_{\overline{12n}|j} + 1000s_{\overline{n}|i} \geq 100000 \implies \boxed{n \geq 19}$

12. $1000 = 100a_{\overline{4}|0.035} + v^4 a_{\overline{8}|i} \implies \boxed{i = 0.0220788}$

19. $L = Pa_{\overline{n/2}|i} \implies P = \dfrac{L}{a_{\overline{n}|i}} + \dfrac{v^{\frac{n}{2}}L}{a_{\overline{n}|i}} = K + v^{\frac{n}{2}}K \leq 2K$

23. $B - A = s_{\overline{n+1}|i} - s_{\overline{n}|i} = \dfrac{(1+i)^{1+n} - (1+i)^n}{i} = \dfrac{(1+i)^n(1+i-1)}{i} = (1-i)^n$

    $\implies i = \dfrac{(1+i)^n - 1}{A} = \boxed{\dfrac{B-1}{A} - 1}$

    $\implies B - A = (2 - \dfrac{B-1}{A})^n \implies \boxed{n = \dfrac{B-A}{ln(2 - \frac{B-1}{A})}}$

Section 2.3

1. Annual effective rate of interest: $i = (1 + \frac{6\%}{12})^{12} - 1 = 0.0616778$; $PV = 2000 s_{\overline{12}|\frac{6\%}{12}} \dfrac{1 - \left(\frac{1.05}{1.0616778}\right)^{20}}{0.0616778 - 0.05} = \boxed{419253.25}$

2.
   - (i) $1000 * 1.01^{29} * \dfrac{1 - (\frac{0.99}{1.01})^{30}}{1 - \frac{0.99}{1.01}} = \boxed{30407.43}$
   - (ii) $1000 * 1.05^{29} * \dfrac{1 - (\frac{0.99}{1.05})^{30}}{1 - \frac{0.99}{1.05}} = \boxed{59704.03}$
   - (iii) $1000 * 1.10^{29} * \dfrac{1 - (\frac{0.99}{1.10})^{30}}{1 - \frac{0.99}{1.10}} = \boxed{151906.38}$

4. $167.50 = 10 a_{\overline{4}|9.2\%} + 10 v^4 \frac{1}{9.2\% - 0.01K} \implies \boxed{K = 4}$

5. Monthly effective rate of interest: $j = (1 + 6\%)^{\frac{1}{12}} - 1 = 0.00486755$; $100000 = R s_{\overline{12}|j} \dfrac{1 - \left(\frac{1.032}{1.06}\right)^{20}}{0.06 - 0.032} \implies \boxed{R = 547.93}$

11. Sandy: $PV_{\text{Sandy}} = 90 a_{\overline{\infty}|i} + 10 (Ia)_{\overline{\infty}|i} = \frac{100}{i} + \frac{10}{i^2}$
    Danny: $PV_{\text{Danny}} = 180 \ddot{a}_{\overline{\infty}|i} = \frac{180(i+1)}{i}$
    $PV_{\text{Sandy}} = PV_{\text{Danny}} \implies \frac{100}{i} + \frac{10}{i^2} = \frac{180(i+1)}{i} \implies \boxed{i = 0.101720}$

12. Monthly effective rate of interest: $j = (1 + \frac{9\%}{4})^{\frac{1}{3}} - 1 = 0.00744444$; $X = 2(Ia)_{\overline{60}|j} = \boxed{2729.21}$

18. $PV_2 = 2PV_1 \implies 11 a_{\overline{\infty}|i} - (Da)_{\overline{10}|i} = 2(Da)_{\overline{10}|i} \implies i = 0.0930160$. Then, $\boxed{PV_1 = 39.40}$

31a. $PV = 1 + 2v^k + 3v^{2k} + \ldots \implies v^k PV = v^k + 2v^{2k} + 3v^{3k} + \ldots$. Then, $PV - v^k PV = 1 + v^k + v^{2k} + v^{3k} + \ldots = \frac{1}{1 - v^k} \implies PV = \frac{1}{(1-v^k)^2} = \frac{1}{(i a_{\overline{k}|i})^2}$.