# Stats 100A Homework 2

Jiaping Zeng

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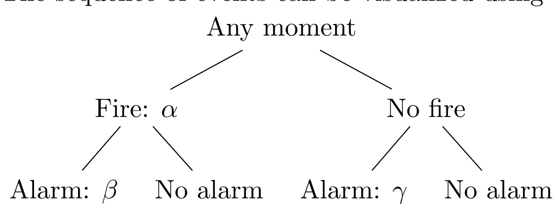
## Problem 1

The sequence of events can be visualized using the chart below:




By adding up the instances where alarm occurs, we can see that $P(\text{alarm}) = \beta + \gamma$.
In addition, $P(\text{fire\&alarm}) = P(\text{fire})P(\text{alarm}|\text{fire}) = \alpha\beta$.
Then, $P(\text{fire}|\text{alarm}) = \dfrac{P(\text{fire\&alarm})}{P(\text{alarm})} = \boxed{\dfrac{\alpha\beta}{\beta + \gamma}}$.
Let $\alpha = 1\%$, $\beta = 98\%$ and $\gamma = 5\%$. Then,
$P(\text{alarm}|\text{fire}) = \beta = 0.98$
$P(\text{fire}|\text{alarm}) = \dfrac{\alpha\beta}{\beta + \gamma} = \dfrac{0.01 * 0.98}{0.98 + 0.05} = 0.0095$
The values are very different because the probablity of a fire is very low.

## Problem 2

1. $P(X > 4) = p(5) + p(6) = 0.5$

2. $P(X = 6|X > 4) = \frac{0.3}{0.5} = 0.6$

3. $\mathbb{E}(X) = \sum_x xp(x) = 1 * 0.1 + 2 * 0.1 + 3 * 0.1 + 4 * 0.2 + 5 * 0.2 + 6 * 0.3 = \boxed{4.2}$
   $\text{Var}(X) = \mathbb{E}[(X - \mathbb{E}(X))^2] = 10.24*0.1+4.84*0.1+1.44*0.1+0.04*0.2+0.64*0.2+3.24*0.3 = \boxed{2.76}$
   $\text{SD}(X) = \sqrt{\text{Var}(X)} = \sqrt{2.22} \approx \boxed{1.66}$

4. $\mathbb{E}(h(X)) = \sum_x h(x)p(x) = -20 * 0.1 - 10 * 0.1 + 0 * 0.1 + 10 * 0.2 + 20 * 0.2 + 100 * 0.3 = \boxed{33}$
   $\text{Var}(h(X)) = \mathbb{E}[(h(X) - \mathbb{E}(h(X)))^2] = 2809*0.1+1849*0.1+1089*0.1+529*0.2+169*0.2+4489*0.3 = \boxed{2061}$
   $\text{SD}(h(X)) = \sqrt{\text{Var}(h(X))} = \sqrt{2061} \approx \boxed{45.40}$
   The units of $\mathbb{E}(h(X))$ and $\text{Var}(h(X))$ are dollars and dollars-squared respectively.

## Problem 3

$\mathbb{E}(Z) = 1 * p + 0 * (1-p) = \boxed{p}$
$\mathbb{E}(Z^2) = 1^2 * p + 0^2 * (1-p) = \boxed{p}$
$\text{Var}(Z) = (1-p)^2 * p + (0-p)^2 * (1-p) = \boxed{-p^2 + p}$

## Problem 4

1. $\mathbb{E}(aX) = \sum_x axp(x) = a \sum_x xp(x) = aE(X)$

2. $\mathbb{E}(X+b) \sum_x (x+b)p(x) = \sum_x xp(x) + \sum_x bp(x) = \sum_x xp(x) + b \sum_x p(x) = \mathbb{E}(X) + b$

3. $\text{Var}(aX) = \mathbb{E}[(aX - \mathbb{E}(aX))^2] = \mathbb{E}[(aX - a\mathbb{E}(X))^2] = \mathbb{E}[a^2(X - \mathbb{E}(X))^2] = a^2\mathbb{E}[(X - \mathbb{E}(X))^2]$
   $= a^2\text{Var}(X)$

4. $\text{Var}(X+b) = \mathbb{E}[(X + b - \mathbb{E}(X+b))^2] = \mathbb{E}[(X + b - \mathbb{E}(X) - b)^2] = \text{Var}(X)$

5. Let $\mu = \mathbb{E}(X)$ which is a constant term. Then, $\text{Var}(X) = \mathbb{E}[(X-\mu)^2] = \mathbb{E}(X^2 - 2\mu X + \mu^2) = \mathbb{E}(X^2) - 2\mu\mathbb{E}(X) + \mu^2 = \mathbb{E}(X^2) - \mu^2 = \mathbb{E}(X^2) - \mathbb{E}(X)^2$

6. $\mathbb{E}(Z) = \mathbb{E}(\frac{X-\mu}{\sigma}) = \frac{\mathbb{E}(X-\mu)}{\sigma} = \frac{\mathbb{E}(X)-\mu}{\sigma} = \boxed{0}$
   $\text{Var}(Z) = \text{Var}(\frac{X-\mu}{\sigma}) = \frac{\text{Var}(X-\mu)}{\sigma^2} = \frac{\text{Var}(X)}{\sigma^2} = \boxed{1}$