Math 115A Homework 2

Jiaping Zeng

5/5/2020

2. Let V be a finite dimensional vector space and W a subspace. Show that V and $W \times V/W$ are isomorphic by finding an explicit isomorphism.

5. A differential operator on $\mathbb{R}_n[x]$ is a linear combination of expressions of the form $x^a \frac{d^b}{dx^b}$ where $a-b \leq 0$ and $b \leq n$. We can consider a differential operator as a linear map $\mathbb{R}_n[x] \to \mathbb{R}_n[x]$.

   (a) Let $D : \mathbb{R}_2[x] \to \mathbb{R}_2[x]$ be the differential operator given by $2 - 4\frac{d}{dx} + 2x\frac{d^2}{dx^2}$. Find the matrix of D relative to the basis $\{x^2, (x-1)^2, (x+1)^2\}$.

   (b) Does the differential equation $2f - 4\frac{df}{dx} + 2x\frac{d^2 f}{dx^2} = 0$ have any solutions $f \in \mathbb{R}_2[x]$?

   (c) Suppose $E : \mathbb{R}_2[x] \to \mathbb{R}_2[x]$ is a differential operator and that the matrix of E, relative to the basis $\{1, x, x^2\}$ is
   $$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}.$$
   Find E.

6. Consider the linear map $X : \mathbb{R}_n[x] \to \mathbb{R}_n[x]$ given by $X(p) = \frac{dp}{dx} + \frac{x^n}{n!}p(0)$. Calculate the dimension of
   $$C(X) = \{T \in Hom(\mathbb{R}_n[x], \mathbb{R}_n[x]) \mid T \circ X = X \circ T\}.$$