# Math 177 Week 1 Notes

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- Accumulated amount function: $A(t) =$ [accumulated amount at time t]
  - Accumulation function: $a(t) =$ when the initial fund is 1 at time 0
  - $a(0) = 1$ and $A(t) = A(0)a(t)$
- Effective rate of interest in the $t^{th}$ period: $i_t = \dfrac{a(t) - a(t-1)}{a(t-1)} = \dfrac{A(t) - A(t-1)}{A(t-1)}$
  - $a(t) = (1 + i_t)a(t-1) = \prod_{j=1}^{t}(1 + i_j)$
  - Compound interest: $a(t) = (1+i)^t \implies i_t = i$
  - Simple interest: $a(t) = 1 + it \implies i_t = \dfrac{i}{1 + i(t-1)}$
- Equivalent rates of interest: if two rates result in the same accumulated values at each point in time
- Present value (PV): "*how much should I invest today to have a given amount at the end of t years?*"
  - Discount function: $PV = \dfrac{1}{a(t)}$
  - For compound interest: $PV = \dfrac{1}{(1+i)^t} = \left(\dfrac{1}{i+i}\right)^t$
    * Present value factor/discount factor: $v = \dfrac{1}{1+i}$
- Equation of value: the equation balacing the current values of cash inflows and outflows
  - Note: a reference point must be chosen
  - Current value at a given time: [accumulated value prior to given time] + [present value occuring on or after given time]
- Nominal interest ($i^{(m)}$): interest expressed as an annual amount payable in equal installments during the year
  - Compounding period: $\dfrac{1}{m}$ years
  - Interest rate per period: $\dfrac{i^{(m)}}{m}$
  - Equivalent effective annual interest: $1 + i = \left(1 + \dfrac{i^{(m)}}{m}\right)^m$
- Effective rate of discount in the $t^{th}$ period: $d_t = \dfrac{a(t) - a(t-1)}{a(t)} = \dfrac{A(t) - A(t-1)}{A(t)}$

- $d_t = \dfrac{i_t}{1+i_t}$ and $i_t = \dfrac{d_t}{1-d_t}$
- Simple discount: $\dfrac{1}{a(t)} = 1 - d_t \implies a(t) = \dfrac{1}{1-d_t}$

- Interest payable
  - In arrears: payable at the end of an interest period (standard way)
  - In advance: payable at the start of an interest period

- Nominal discount ($d^{(m)}$):
  - Compounding period: $\dfrac{1}{m}$ years
  - Interest rate per period: $\dfrac{d^{(m)}}{m}$
  - Equivalent effective annual interest: $1 - d = \left(1 - \dfrac{d^{(m)}}{m}\right)^m$

- Force of interest: $\delta_t = \dfrac{a'(t)}{a(t)} = \dfrac{A'(t)}{A(t)}$
  - $A(t) = A(0)e^{\int_0^t \delta_s ds}$
  - When $\delta_t$ is constant, we may drop the subsubscript and denote it as $\delta$
    * $a(t) = e^{\int_0^t \delta ds} = e^{\delta t}$
  - $i^{(\infty)} := \lim_{m\to\infty} i^{(m)} = ln(1+i) = \delta$