Math 177 Homework 1

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4/8/2020

Section 1.1

1. After 1 year: $A(1) = 10000 * 1.04 = \boxed{10400}$
   After 2 years: $A(2) = 10000 * (1.04)^2 = \boxed{10816}$
   After 3 years: $A(3) = 10000 * (1.04)^3 = \boxed{11248.64}$
   Year 1 interest: $A(1) - A(0) = 10400 - 10000 = \boxed{400}$
   Year 2 interest: $A(2) - A(1) = 10816 - 10400 = \boxed{416}$
   Year 3 interest: $A(3) - A(2) = 11248.64 - 10816 = \boxed{432.64}$

3. Balance after 12 months: $A(12) = 10000 * (1.01)^3 * (1.0075)^{12-3} = \boxed{11019.70}$
   Average compound monthly interest: $11019.70 = 10000 * (1+i)^{12} \implies \boxed{i = 0.812\%}$

4. Let the 10th year be the reference point; then $10000 = 10000*(1+4\%)^{10} - (1+5\%)*K*(1+4\%)^{10-4} - (1+5\%)*K*(1+4\%)^{10-5} - K*(1+4\%)^{10-6} - K*(1+4\%)^{10-7}$. Solving for K results in $\boxed{K = 979.93}$.

6. Joe's accumulated amount (simple interest): $10 * (1 + 0.11 * 10) + 30 * (1 + 0.11 * 5) = 67.5$
   Tina's accumulated amount (compound interest): $10 * (1 + 0.0915)^n + 30 * (1 + 0.0915)^{2n}$
   We can solve for n by setting the two amounts equal: $67.5 = 10 * (1 + 0.0915)^n + 30 * (1 + 0.0915)^{2n}$ which results in $\boxed{n = 3.364610}$.

11. To simplify the calculations, we can look at the accumulation after $17 * 67 = 1139$ days.
    (a) 17-day rate of $\frac{3}{4}\%$: $a(67) = (1 + 0.0075)^{67} = 1.649752$
    67-day rate of 3%: $a(17) = (1 + 0.03)^{17} = 1.652848$
    Therefore $\boxed{\text{67-day rate of 6\%}}$ results in more rapid growth.
    (b) 17-day rate of $\frac{3}{2}\%$: $a(67) = (1 + 0.015)^{67} = 2.711595$
    67-day rate of 6%: $a(17) = (1 + 0.06)^{17} = 2.692773$
    Therefore $\boxed{\text{17-day rate of } \frac{3}{2}\%}$ results in more rapid growth.

12. (a) Amount after 6 months: $1000 * (1 - 9\%) * (1 + 25\%) * (1 - 1.5\%) = 1120.4375$
    6-month return: $(1120.4375 - 1000)/1000 = \boxed{12.04\%}$
    (b) Amount after 6 months: $1000 * (1 - 9\%) * (1 - \frac{3.50}{4.00}) * (1 - 1.5\%) = 784.30625$
    6-month return: $(784.30625 - 1000)/1000 = \boxed{-21.57\%}$

Section 1.2

2. $v = \frac{1}{1+10\%} = 0.909091$

   Child 1 (age 1): $PV_1 = 25000 * v^{18-1} + 100000 * v^{21-1} = 19810.47952$

   Child 2 (age 3): $PV_2 = 25000 * v^{18-3} + 100000 * v^{21-3} = 23970.68023$

   Child 3 (age 6): $PV_3 = 25000 * v^{18-6} + 100000 * v^{21-6} = 31904.97538$

   Total: $PV = PV_1 + PV_2 + PV_3 = \boxed{75686.14}$

5. Pick July 1, 2021 as time of reference and let payment needed then be K. Then, $200*(1+4\%)^{2021-2020} + 300 * v^{2022-2021} = 100 * (1+4\%)^{2021-2017} + K$. Solving for K results in $\boxed{K = 379.48}$.

7. Let time 0 be the time of reference. Then, $100 + 200 * v^n + 300 * v^{2n} = 600 * v^{10} \implies 100 + 200 * 0.75941 + 300 * (0.75941)^2 = 600 * (\frac{1}{1+i})^{10}$. Solving for i results in $\boxed{i = 0.03511}$.

8. Let the cost of the machine in scenario i be M_i.

   (a) $M_1 = 20 * (24000) * (1 + \frac{0.75\%}{12})^{4*12} = \boxed{494613.54}$

   (b) $M_2 - 200000 * (\frac{1}{1+0.75\%})^{4*12} = M_1 \implies M_2 = \boxed{634336.37}$

   (c) $M_3 = 0.15 * M_3 * (\frac{1}{1+0.75\%})^{4*12} + M_1 \implies M_3 = \boxed{552512.50}$

12. Let i be the implied annual effective interest rate and the time of reference be 2 years from now. Then we get $1000 * (1+i)^2 + 1092 = 2000 * (1+i)$ which has $\boxed{\text{no real solution}}$.

16. (a) We can find B_1 by adding the initial balance (B_0) with interest and all transactions with their respective interests: $\boxed{B_1 = B_0 * (1+i) + \sum_{k=1}^{n} a_k * [1 + i * (1 - t_k)]}$

    (b)

    (c)

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Section 1.4

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Section 1.5

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