Math 177 Homework 3

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Section 3.1

1. (i) $L = 1000a_{\overline{5}|10\%} + 500v^5a_{\overline{5}|10\%} = \boxed{4967.68}$

   (ii) $B_3 = L(1+10\%)^3 - 1000s_{\overline{3}|10\%} = \boxed{3301.98}$

   (iii) $I_4 = B_3 * 10\% = \boxed{330.20}$

   $P_4 = 1000 - I_4 = \boxed{669.80}$

   (iv) $B_8 = 500_{\overline{2}|10\%} = \boxed{867.77}$

2. $v = \frac{1}{1+\frac{9\%}{12}}$; $B_{40} = \sum_{n=1}^{20} 1000v^n(1-2\%)^{39+n} = \boxed{6889.11}$

4. (i) Let X be the monthly payment amount. Then $20000 = X(12 + a_{\overline{36}|\frac{6\%}{12}}) \implies \boxed{X = 445.72}$

   $B_1 = 20000 - 12X = \boxed{14651.36}$

   (ii) $20000 = X(a_{\overline{12}|\frac{3\%}{12}} + v^{12}_{\frac{3\%}{12}}a_{\overline{36}|\frac{5\%}{12}}) \implies \boxed{X = 452.61}$

   $B_1 = 20000(1+\frac{3\%}{12})^{12} - Xs_{\overline{12}|\frac{3\%}{12}} = \boxed{15101.68}$

5. Bank Y monthly interest: $(1+i)^6 = 1 + \frac{14\%}{2} \implies i = 0.0113403$; $P_t = \frac{19800}{36} = 550 \implies I_t = iB_{t-1} = \frac{12\%}{12}[19800-(t-1)P_t] \implies Price = P_ta_{\overline{20}|i} + \sum_{n=16}^{36}(v_i^{n-16}I_n) = \boxed{10857.27}$

6. $L = \sum_{i=1}^n P_n = \sum_{i=1}^n (K_n - I_n) = K_T - I_T$

9. $B_{10} = L = 1000$

   $B_{20} = B_{10}(1-5\%)^{10} = 598.74$

   $Xa_{\overline{10}|10\%} = B_{20} \implies \boxed{X = 97.44}$

11. (a)

    (b)

    (c)

13. (a) $P_6 = K_6 - I_6 = 500 - (1000a_{\overline{10}|i} - a_{\overline{1}|i})i = 500(1 - 2a_{\overline{10}|i}i + vi) = 500[-2(1-v^{10}) - (1-vi)] = 500(2v^{10} - v)$

    (b) $P_6 = K_6 - I_6 = 500 - (1000a_{\overline{10}|i} - a_{\overline{1}|i})i = 500(1 - 2a_{\overline{10}|i}i + vi) = 500[1 - i(2a_{\overline{10}|i} - v)]$

    (c) $P_6 = P_1(1+i)^5 = (500 - Li)(1+i)^5$

Section 3.2

1. $B_t = L(1+t)^t - Ks_{\overline{t}|i} = Ka_{\overline{n}|i}(1+i)^t - Ks_{\overline{t}|i} = (Ks_{\overline{t}|i} + Ka_{\overline{n-t}|i}) - Ks_{\overline{t}|i} = Ka_{\overline{n-t}|i}$
   $B_t = L(1+t)^t - Ks_{\overline{t}|i} = L + Lis_{\overline{t}|i} - Ks_{\overline{t}|i} = L + s_{\overline{t}|i}(Li - K) = L + P_1 s_{\overline{t}|i}$

4. Let X be the monthly payment amount. Then $L = Xa_{\overline{60}|0.01}$ and $B_t = Xa_{\overline{60-t}|0.01}$. $B_t = \frac{L}{2} \implies \boxed{t = 34.41}$.

7. $X(1+6\%)^{10} - X = \frac{10X}{a_{\overline{10}|6\%}} - X + 356.54 \implies \boxed{X = 825.00}$

8. Under option (i), let X be the annual payment amount. Then, $Xa_{\overline{10}|8.07\%} = 2000 \implies X = 299.00$. Then, under option (ii), $\sum_{n=0}^{9} 200 + 200(10-n)i) = 10X \implies 11000i + 2000 = 2990 \implies \boxed{i = 0.09}$

11. Let $t-1, t, t+1$ be the dates of the given consecutive payments. Then, $P_t = B_t - B_{t-1} = 5190.72 - 5084.68 = 106.04$ and $P_{t+1} = B_{t+1} - B_t = 5084.68 - 4973.66 = 111.02$. Then, $i = \frac{P_{t+1}}{P_t} - 1 = 0.0469634 \implies I_t = iB_{t-1} = 243.77 \implies K = P_t + I_t = \boxed{349.81}$

13.

14. $A = 125000a_{\overline{5}|5\%} = \boxed{541184.58}$
    $B = 75000a_{\overline{5}|5\%} = \boxed{324710.75}$
    $C = 10000(Da)_{\overline{5}|5\%} = \boxed{134104.67}$
    $A + B + C = 541184.58 + 324710.75 + 134104.67 = 1000000$

15.

16. (a)

    (b)

17. $B_n = \frac{3}{4}L \implies a_{\overline{n}|i} = \frac{3}{4}a_{\overline{2n}|i} \implies v^n - 1 = \frac{3}{4}v^{2n} - \frac{3}{4} \implies v^n = \frac{1}{3} \implies \boxed{I_{n+1} = \dfrac{2}{3}K}$

26.

29. (a) $K_1 v + K_2 v^2 + ... K_n v^n = \frac{1+i}{1+i} + \frac{(1+i)^2}{(1+i)^2} + ... + \frac{(1+i)^n}{(1+i)^n} = n$

    (b) $B_t = K_{t+1}v + K_{t+2}v^2 + ... + K_n v^{n-t} = \frac{(1+i)^{t+1}}{(1+i)} + \frac{(1+i)^{t+2}}{(1+i)^2} + ... + \frac{(1+i)^n}{(1+i)^{n-t}} = (n-t)(1+i)^t$