Math 177 Summary & Formulas

Jiaping Zeng

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Chapter 1

- Key topics: interest rates, accumulation, force of interest
- Important formulas:
  - Nominal interest ($i^{(m)}$): $1+i=\left(1+\frac{1^{(m)}}{m}\right)^m$
  - Present value factor: $v=\frac{1}{1+i}$
  - Rate of discount: $d=\frac{i}{1+i}$
  - Force of interest (δ_t): $a(t)=e^{\int_0^\delta \delta_s ds}$

Chapter 2

- Key topic: annuities
- Important formulas:
  - Annuity-immediate: $s_{\overline{n}|i}=\frac{(1+i)^n-1}{i}$, $a_{\overline{n}|i}=\frac{1-v^n}{i}$
  - Annuity-due: $\ddot{s}_{\overline{n}|i}=\frac{(1+i)^n-1}{d}$, $\ddot{a}_{\overline{n}|i}=\frac{1-v^n}{d}$
  - Arithmetically increasing annuity: $(Is)_{\overline{n}|i}=\frac{\ddot{s}_{\overline{n}|i}-n}{i}$, $(Ia)_{\overline{n}|i}=\frac{\ddot{a}_{\overline{n}|i}-nv^n}{i}$

Chapter 3

- Key topic: loans
- Important formulas:
  - Basic/recursive definition: $B_0=L$, $I_t=iB_{t-1}$, $P_t=K_t-I_t$, $B_t=B_{t-1}-P_t$
  - Loan with level payments: $B_t=a_{\overline{n-t}|}$, $I_t=1-v^{n+1-t}$, $P_t=v^{n+1-t}$

Chapter 4

- Key topic: bonds
- Important formulas:
  - Price of bond: $P=Cv_j^n+Fra_{\overline{n}|j}$

- Premium/Discount (when $C = F$): $P = F + F(r - j)a_{\overline{n}|j}$
- Makeham's formula ($K = Cv_j^n$): $P = K + \frac{r}{j}(F - k)$

Chapter 5

- Key topic: rate of return
- Important formulas:
  - Net present value: $P(i) = \sum_{k=0}^{n} C_k v_i^{t^k}$ (C_k: net flow at time t)
  - Dollar-weighted rate of return (i_D): $A(1 + i_D) + \sum_{k=1}^{n} C_k(1 + i_D(1 - t_k)) = B$, ($A, B$: amount in fund at start, end of the year)
  - Time-weighted rate of return (i_T): $1 + i_T = \prod_{k=1}^{n+1} \frac{[\text{amount just before } t_k]}{[\text{amount just after } t_{k-1}]}$

Chapter 6

- Key topics: spot rates, forward rates
- Important formulas:
  - Forward interest rate ($f_{[t_1,t_2]}$): $(1 + r_{t_1})^{t_1}(1 + f_{[t_1,t_2]})^{t_2 - t_1} = (1 + r_{t_2})^{t_2}$
  - Spot rate (r_t) from forward rates: $(1 + r_n)^n = \prod_{t=1}^{n}(1 + f_{[t-1,t]})$

Chapter 7

- Key topic: cash flows
- Important formulas:
  - Macaulay duration: $D_{mac}(i) = \frac{\sum_{t=1}^{n} tK_t(1 + i)^{-t}}{\sum_{t=1}^{n} K_t(1 + i)^{-t}}$
  - Modified duration: $D_{mod}(i) = -\frac{\frac{d}{di}P(i)}{P(i)}$, ($P(i) = \sum_{t=1}^{n} K_t(1 + i)^{-t}$: present value of cash flow)
  - For flat term structure: $D_{mod}(i) = vD_{mac}(i)$
  - Macaulay convexity: $C_{mac}(i) = \frac{\sum_{t=1}^{n} t^2 K_t v^t}{\sum_{t=1}^{n} K_t v^t}$
  - Modified convexity: $C_{mod}(i) = \frac{\frac{d^2}{di^2}P(i)}{P(i)}$