

# Stats 100A Midterm

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## Problem 1

Single coin flip:

	head	tail
x	1	0
p(x)	0.5	0.5

$$E(x) = \sum_x xp(x) = 1 * 0.5 + 0 * 0.5 = 0.5$$

$$Var(x) = E[(X - \mu)^2] = (1 - 0.5)^2 * 0.5 + (0 - 0.5)^2 * 0.5 = 0.25$$

100 flips:

$$E(X) = 100E(x) = \boxed{50}$$

$$Var(X) = 100Var(x) = \boxed{25}$$

$$SD(X) = \sqrt{Var(X)} = \sqrt{25} = \boxed{5}$$

$$E(X/100) = \frac{E(X)}{100} = \frac{50}{100} = \boxed{0.5}$$

$$Var(X/100) = \frac{Var(X)}{100^2} = \frac{25}{100^2} = \boxed{0.0025}$$

$$SD(X/100) = \sqrt{Var(X/100)} = \sqrt{0.0025} = \boxed{0.05}$$

$$P(X \in [40, 60]) = \frac{\sum_{x=40}^{60} \binom{100}{x}}{2^{100}}$$

## Problem 2

Single voter:

	A	Not A
x	1	0
p(x)	0.2	0.8

$$E(x) = \sum_x xp(x) = 1 * 0.2 + 0 * 0.8 = 0.2$$

$$Var(x) = E[(x - \mu)^2] = (1 - 0.2)^2 * 0.2 + (0 - 0.2)^2 * 0.8 = 0.16$$

100 Voters:

Distribution:  $\boxed{binom(n = 100, p = 0.2)}$

$$E(X) = 100 * E(x) = \boxed{20}$$

$$Var(X) = 100 * Var(x) = \boxed{16}$$

$$SD(X) = \sqrt{Var(X)} = \sqrt{16} = \boxed{4}$$

$$E(X/100) = \frac{E(X)}{100} = \frac{20}{100} = \boxed{0.2}$$

$$Var(X/100) = \frac{Var(X)}{100^2} = \frac{16}{100^2} = \boxed{0.0016}$$

$$SD(X/100) = \sqrt{Var(X/100)} = \sqrt{0.0016} = \boxed{0.04}$$

### Problem 3

Single throw:

	In A	Not in A
x	1	0
p(x)	$\frac{4m}{10000}$	$1 - \frac{4m}{10000}$

$$E(x) = \sum_x xp(x) = 1 * \frac{4m}{10000} + 0 * (1 - \frac{4m}{10000}) = \frac{4m}{10000}$$

$$Var(x) = E[(\hat{\pi} - \mu)^2] = (1 - \frac{4m}{10000})^2 * \frac{4m}{10000} + (0 - \frac{4m}{10000})^2 * (1 - \frac{4m}{10000}) = \frac{-m^2 + 2500m}{6250000}$$

Distribution:  $\boxed{binom(n = 10000, p = \frac{\pi}{4})}$

$$E(\hat{\pi}) = 10000 * E(x) = \boxed{4m}$$

$$Var(\hat{\pi}) = 10000 * Var(x) = \boxed{\frac{-m^2 + 2500m}{625}}$$

$$SD(\hat{\pi}) = \sqrt{Var(\hat{\pi})} = \boxed{\frac{\sqrt{-m^2 + 2500m}}{25}}$$