Math 131A Homework 2

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8.1 Prove the following:

(a) $\lim \dfrac{(-1)^n}{n} = 0$

(c) $\lim \dfrac{2n-1}{3n+2} = \dfrac{2}{3}$

8.4 Let (t_n) be a bounded sequence, i.e., there exists M such that $|t_n| \leq M$ for all n, and let (s_n) be a sequence such that $\lim s_n = 0$. Prove $\lim (s_n t_n) = 0$.

8.5 (a) Consider three sequences (a_n), (b_n) and (s_n) such that $a_n \leq s_n \leq b_n$ for all n and $\lim a_n = \lim b_n = s$. Prove $\lim s_n = s$. This is called the "squeeze lemma."

(b) Suppose (s_n) and (t_n) are sequences such that $|s_n| \leq t_n$ for all n and $\lim t_n = 0$. Prove $\lim s_n = 0$.

8.6 Let (s_n) be a sequence in $\mathbb{R}$.

(a) Prove $\lim s_n = 0$ if and only if $\lim |s_n| = 0$.

(b) Observe that if $s_n = (-1)^n$, then $\lim |s_n|$ exists, but $\lim s_n$ does not exist.

8.9 Let (s_n) be a sequence that converges.

(a) Show that if $s_n \geq a$ for all but finitely many n, then $\lim s_n \geq a$.

(b) Show that if $s_n \leq b$ for all but finitely many n, then $\lim s_n \leq b$.

(c) Conclude that if all but finitely many s_n belong to $[a, b]$, then $\lim s_n$ belongs to $[a, b]$.

8.10 Let (s_n) be a convergent sequence, and suppose $\lim s_n > a$. Prove there exists a number N such that $n > N$ implies $s_n > a$.

9.1 (a)

(b)

9.3

9.9

9.10 (a)

(b)

9.11

9.12

10.5

10.6

10.7

P1