Math 131A Homework 3

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10.9

11.2

11.5

12.2

12.4

12.9

12.10

12.12

14.5

P1 Let (s_n) be the sequence

$$s_n = \frac{n^2+1}{n^2+2n} \sin n.$$

Prove that (s_n) has a convergent subsequence.

P2 Let (s_n) be a sequence that contains eveyr integer. Prove that there is a subsequence of (s_n) which diverges to $-\infty$.

P3 Suppose (s_n) is a sequence and (t_k) is a subsequence of (s_n) such that (t_k) converges. Prove that $\lim t_k \leq \limsup s_n$.

P4 For each series, determine whether the series (1) converges to a real number, (2) diverges to $+\infty$, (3) diverges to $-\infty$, or (4) none of these. Prove your answers.

(a) $\sum_{n=1}^{\infty} \frac{\cos^2(n)}{n^2}$

(b) $\sum_{n=1}^{\infty} \frac{n-1}{n^2}$

(c) $\sum_{n=1}^{\infty} (-1)^n$

(d) $\sum_{n=1}^{\infty} \frac{n+1}{n^3-1}$