Math 151B Homework 4

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Q1 After expanding the expression we have

$y(t_{i+1}) = y(t_i) + ahf(t_i, y(t_i)) + bhf(t_{i-1}, y(t_{i-1})) + chf(t_{i-2}, y(t_{i-2})) + dhf(t_{i-3}, y(t_{i-3}))$

Then we can expand both sides using Taylor polynomials as follows:

$LHS = y(t_{i+1}) = y(t_i) + hy'(t_i) + \frac{h^2}{2}y''(t_i) + \frac{h^3}{6}y'''(t) + \frac{h^4}{24}y^{(4)}(t) + O(h^5)$

$RHS = y(t_i) + ahy'(t_i) + bhy'(t_i - h) + chy'(t_i - 2h) + dhy'(t_i - 3h)$

$= y(t_i)+ahy'(t_i)+bh[y'(t_i)-hy''(t_i)+\frac{h^2}{2}y'''(t_i)-\frac{h^3}{6}y^{(4)}(t)+O(h^4)]+ch[y'(t_i)-2hy''(t_i)+2h^2y'''(t_i)-\frac{4h^3}{3}y^{(4)}(t)+O(h^4)]+dh[y'(t_i)-3hy''(t_i)+\frac{9h^2}{2}y'''(t_i)-\frac{9h^3}{2}y^{(4)}(t)+O(h^4)]$

$= y(t_i)+(a+b+c+d)hy'(t)+(-b-2c-3d)h^2y''(t_i)+(\frac{b}{2}+2c+\frac{9d}{2})h^3y'''(t_i)+(-\frac{b}{6}-\frac{4c}{3}-\frac{9d}{2})h^4y^{(4)}(t_i)+O(h^5)$

Then by matching coefficients we have

$a + b + c + d = 1, b - 2c - 3d = \frac{1}{2}, \frac{b}{2} + 2c + \frac{9d}{2} = \frac{1}{6}, -\frac{b}{6} - \frac{4c}{3} - \frac{9d}{2} = \frac{1}{24}$

$\implies \boxed{a = \frac{55}{24}, b = -\frac{59}{24}, c = \frac{37}{24}, d = \frac{-3}{8}}$

Q2

Q3