Math 131A Homework 1

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Q1 (a) Continuity: Since $f(t,y) = \dfrac{1+y}{t}$ is only undefined at $t=0$ and the given interval $t \in [1,2]$ does not include 0, $f(t,y)$ is continuous on the given interval.
Lipschitz: $\dfrac{\delta f(t,y)}{\delta y} = \dfrac{1}{t} \leq \dfrac{1}{1} = 1 = L$.
Therefore this IVP is well-posed by theorem 5.6.

(b) Continuity: $f(t,y) = y\,cos(t)$ is continuous on $t \in [0,1]$ because both y and $cos(t)$ are defined everywhere.
Lipschitz: $\dfrac{\delta f(t,y)}{\delta y} = cos(t) \leq cos(0) = 1 = L$.
Therefore this IVP is well-posed by theorem 5.6.

Q2 (a) $y(1.5) \approx 2 + 0.5 \cdot \dfrac{1+1}{1+2} = \dfrac{7}{3} \approx 2.333333$

$y(2) \approx \dfrac{7}{3} + 0.5 \cdot \dfrac{1+1.5}{1+\frac{7}{3}} = \boxed{\dfrac{65}{24}} \approx 2.708333$

(b)

h	y(2)
0.5	2.7083333333333335
0.2	2.729166194327493
0.1	2.7355407599225927
0.01	2.741056919124695

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def q2b():
    h_vals = [0.5, 0.2, 0.1, 0.01]  # step sizes
    t_0, y_0, t_f = 1.0, 2.0, 2.0  # initial/final values

    for h in h_vals:
        print(f"h={h}")
        t, y, steps = t_0, y_0, int((t_f-t_0)/h)
        for _ in range(steps):
            y_prev = y
            y = y_prev + h*(1+t)/(1+y_prev)
            t += h
        print(f"y={y}\n")
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(c) Exact solution: $y(2) = \sqrt{14} - 1 \approx 2.741657387$; we can see that smaller h values resulted in approximations that are closer to the exact $y(2)$ value as expected.

Q3 (a) $\dfrac{df(t,y)}{dt} = \dfrac{\delta f(t,y)}{\delta t} + \dfrac{\delta f(t,y)}{\delta y} \cdot \dfrac{\delta y}{\delta t} = -y^2e^{-t} + 2ye^{-t} \cdot y^2e^{-t} = \boxed{-y^2e^{-t} + 2y^3e^{-2t}}$

(b) $y(0.5) \approx y(0) + 0.5y'(0) + \dfrac{0.5^2}{2}y''(0) = 1 + 0.5 \cdot (1^2 \cdot e^0) + \dfrac{0.5^2}{2} \cdot (1 \cdot 1 \cdot e^0) = 1 + \dfrac{1}{2} + \dfrac{1}{8} = \dfrac{13}{8}$

$y(1) \approx \dfrac{13}{8} + 0.5 \cdot (\dfrac{13^2}{8^2} \cdot e^{-0.5}) + \dfrac{0.5^2}{2} \cdot (\dfrac{3}{8} \cdot \dfrac{13}{8} \cdot e^{-0.5})$

(c)

(d)