Math 164 Homework 2

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1. Let $f : \mathbb{R}^n \to \mathbb{R}$ be a smooth function. Prove that $\nabla^2 f(x) = D(\nabla f(x)) = \nabla(\nabla f(x))$.
   Answer:

2. Find the gradient and hessian of the following functions.

   (a)

3. Let $\sigma(\cdot) : \mathbb{R} \to \mathbb{R}$ be a scalar function and apply elementwise to its input vectors.

4. In each of the following problems justify your answer using optimality conditions.

   (a) Compute the gradient $\nabla f(x)$ and Hessian $\nabla^2 f(x)$ of the Rosenbrock function

   $$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

   Show that $x^\star = [1, 1]^T$ is the only local minimizer of this function, and that the Hessian matrix at that point is positive definite.
   Answer:

   $$\nabla f(x) = \begin{bmatrix} \frac{\delta f}{\delta x_1} \\ \frac{\delta f}{\delta x_2} \end{bmatrix} = \begin{bmatrix} 400x_1^3 - 400x_1x_2 + 2x_1 - 2 \\ 200x_2 - 200x_1^2 \end{bmatrix}$$