

# Supplementary details on experiments of effective reproduction number estimation with trend filtering

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## 1 Supplementary details on experimental settings of RtEstim for Section 3.1

We run 10-fold cross validation (CV) to choose the best tuning parameter from the candidate set of size 50, i.e.,  $\lambda = \{\lambda_1, \dots, \lambda_{50}\}$ . Specifically, we divide the all samples (except the first and last entries) into ten folds evenly and randomly, and build models on each sample set by leaving a fold out across all hyperparameters. We select the tuning parameter that gives the lowest averaged *deviance* between the estimated reproduction numbers and the observed samples averaged over all folds.

## 2 Supplementary results of experiments for Fig 3 in Section 3.2

The full KL divergence values for Poisson and negative Binomial incidence without excluding the outliers are shown in Fig 9 and Fig 10 respectively. Both y-axes are displayed on a logarithmic scale for a better visualization, since a few values are much larger than others.

Comparing among Rt estimations by EpiLPS, RtEstim and EpiEstim with *weekly* sliding windows, KL values computation excludes the first week of Rt estimates for all approaches. The outliers are mainly EpiLPS and EpiEstim Rt estimations in Scenario 2, which is generally an easy problem except for the changepoint at where the two segments are discontinuous. Since EpiEstim produces a continuous Rt curve, the changepoint can be difficult to recover and make estimation less accurate.

## 3 Experimental results of methods using monthly sliding windows for Section 3.1

Comparing across Rt estimations by EpiLPS, RtEstim and EpiEstim with *monthly* sliding windows, KL divergece values computation excludes the first month of Rt estimates for all approaches.

The relative performance of EpiEstim with monthly sliding windows, in general, is not as good as its weekly sliding window based on the relative positions of its boxes and the counterparts of the other methods, except for the Scenario 2 with negative Binomial incidence. A reason is that EpiEstim with longer sliding windows

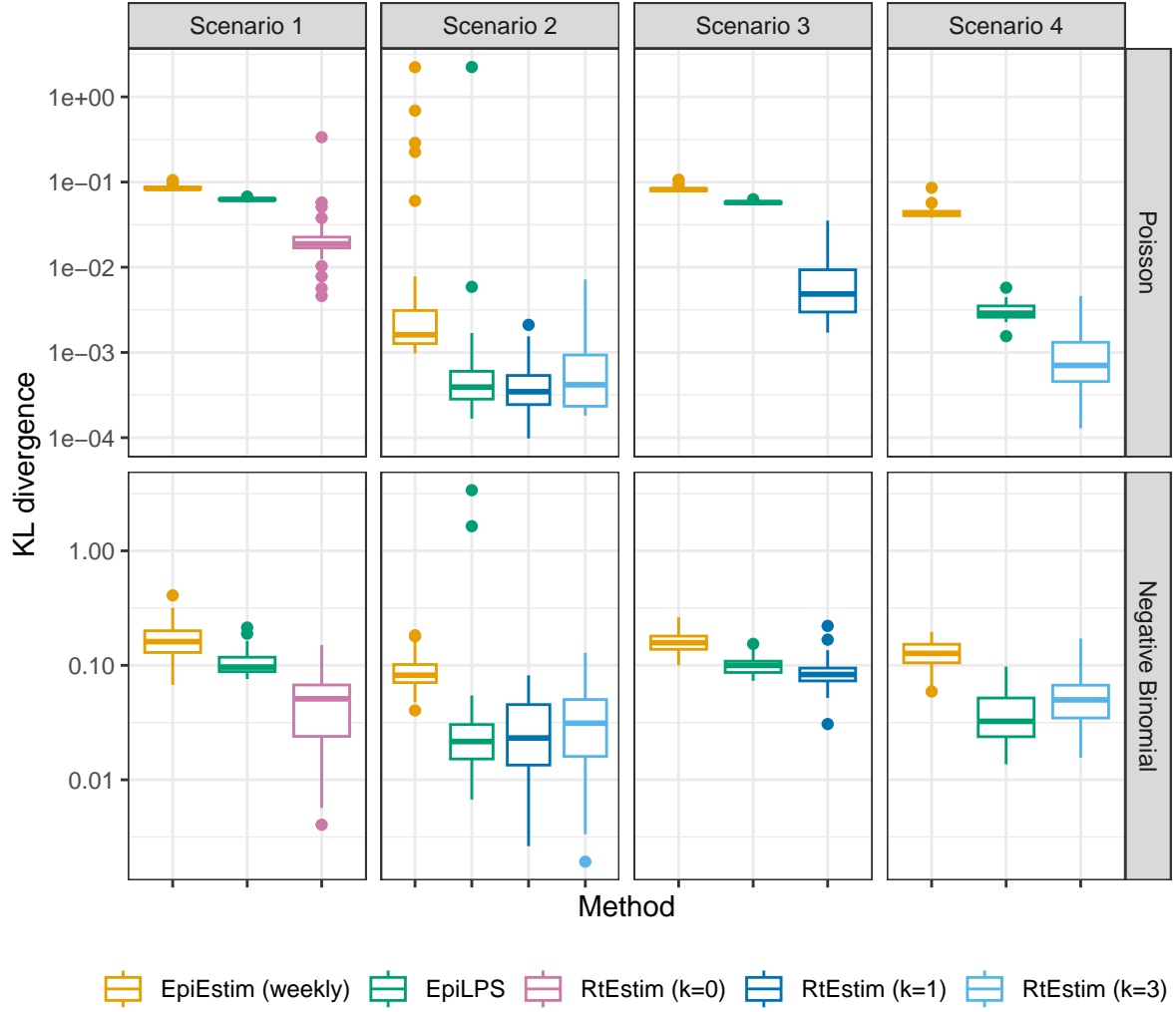


Fig 10. KL divergence for methods with weekly sliding windows. Y-axis is on a logarithmic scale.

assume similarity of neighboring  $R_t$  across longer periods, and thus, is smoother and less accurate compared to the one with shorter sliding windows.

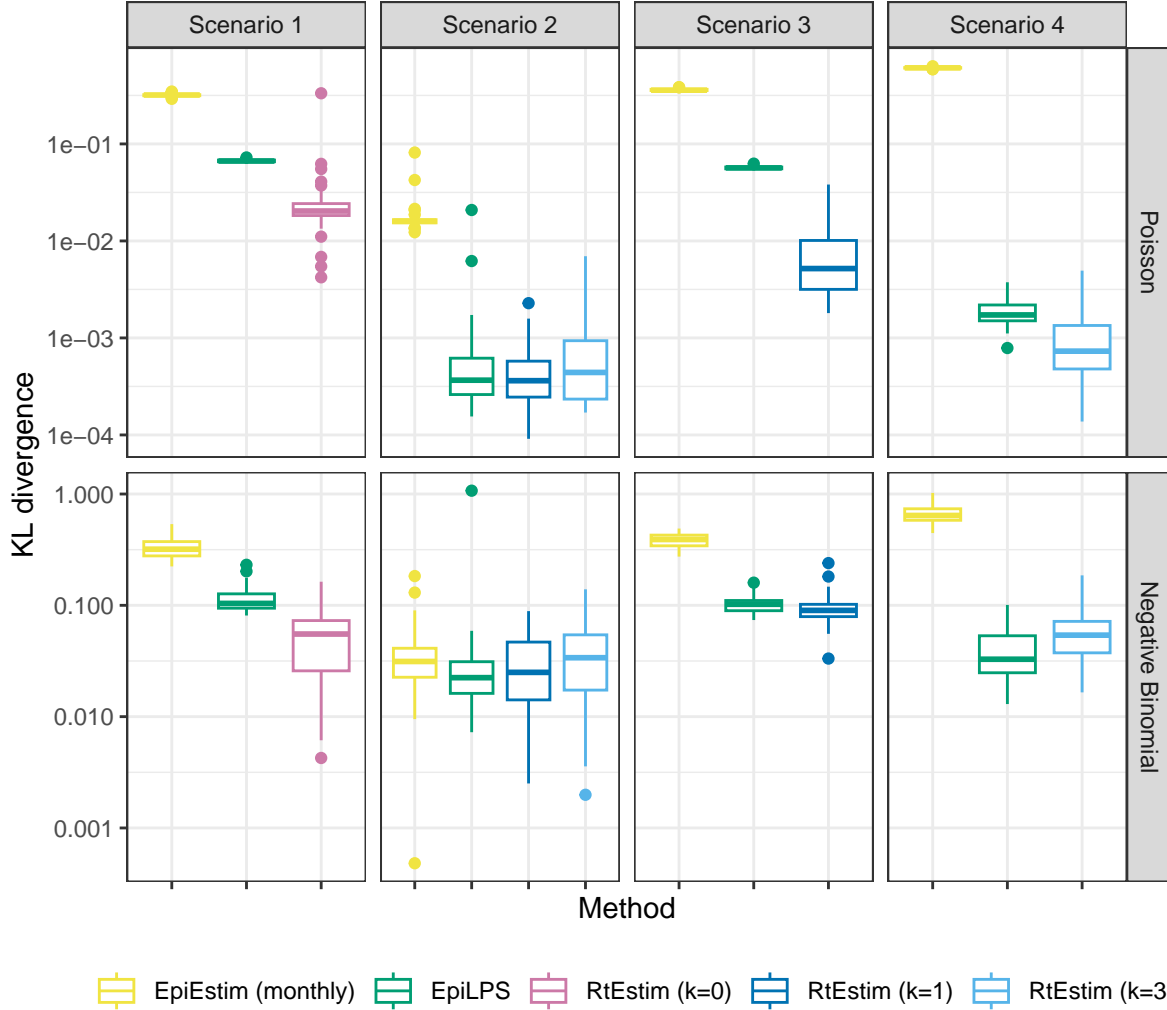


Fig 11. KL divergence for methods with monthly sliding windows. Y-axis is on a logarithmic scale.

## 4 Time comparisons of methods for Section 3.2

EpiEstim with both weekly and monthly sliding windows are very fast and converge in less than 0.1 seconds. Piecewise constant  $R_t$ Estim (with  $k=0$ ) estimates can be generated within 0.1 seconds as well. EpiLPS is slightly slower, but still very fast and within 1 second for all experiments. Piecewise linear and cubic  $R_t$ Estim (with  $k=1$  and 3 respectively) are slower, but mostly within 10 seconds.

It is remarkable that our  $R_t$ Estim computes 50 lambda values with 10-fold CV for each experiment, which results in 500 times of modelling per experiment. The running times are no more than 10 seconds for most of the experiments, which means the running time for each time of modelling is very fast, and on average can be less than 0.02 seconds. The other two methods only run once for a fixed set of hyperparameters for each experiment.

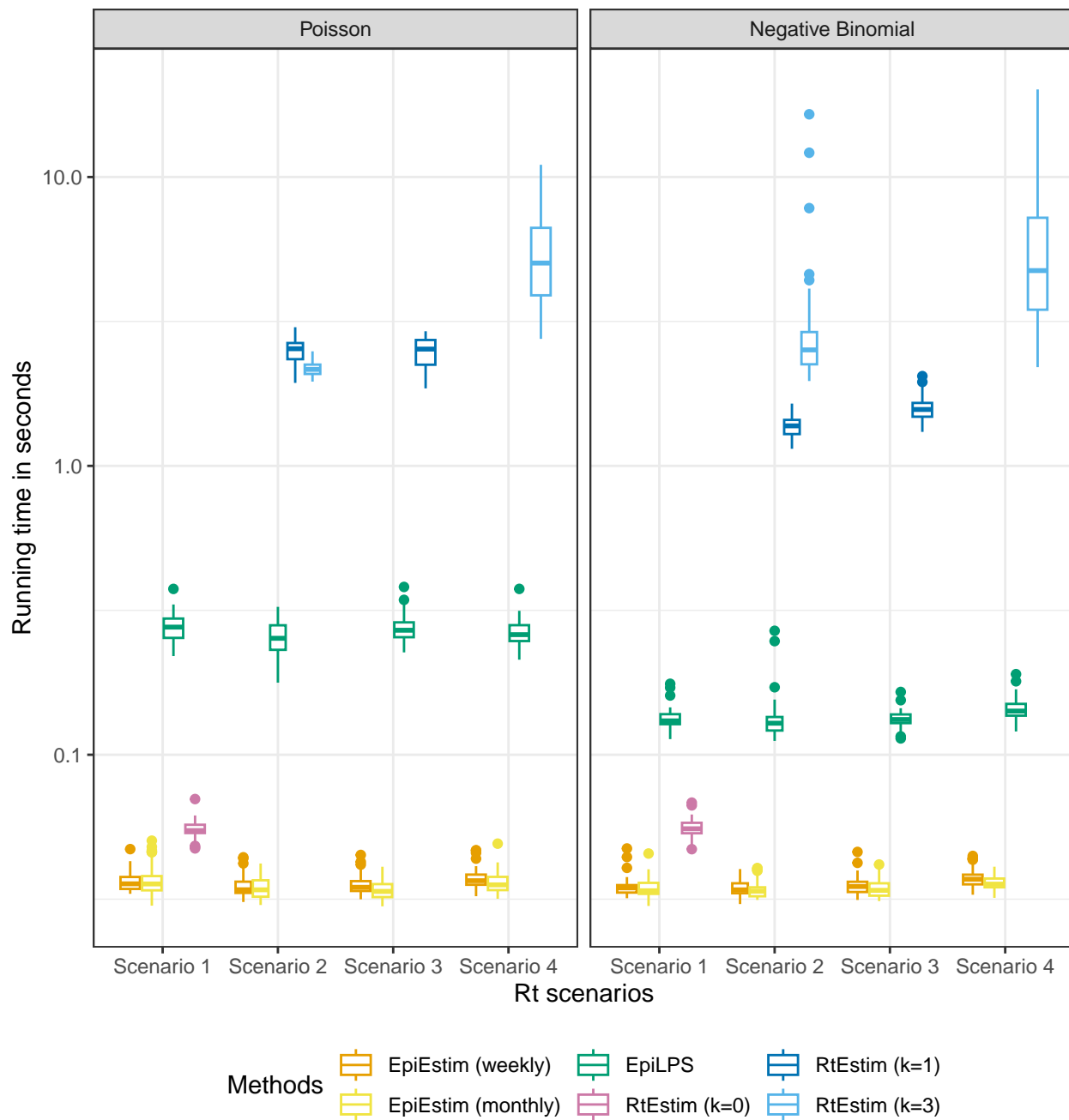


Fig 12. Time comparisons of methods (excluding one outlier of `RtEstim (k=1)` in Scenario 2 with negative Binomial incidence). Y-axis is on a logarithmic scale.