CMU Computer Vision HW5

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1 Theory: 3D Reconstruction

Q1.1

Assume we have point correspondence: $\{x, x'\}$
We know that $x'Fx = 0$
where $x = \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}, x' = \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}, F = \begin{bmatrix} f_1 & f_2 & f_3 \\ f_4 & f_5 & f_6 \\ f_7 & f_8 & f_9 \end{bmatrix}$

and therefore $x_1x_2f_1 + x_1y_2f_4 + x_1f_7 + y_1x_2f_2 + y_1y_2f_5 + y_1f_8 + x_2f_3 + y_2f_6 + f_9 = 0$

According to the question, $(x_1, y_1) = (0, 0), (x_2, y_2) = (0, 0)$,
so we can see that $f_9(F_{33})$ must be 0 so that the equation above hold.

Q1.2

We have learned in lecture that $E = [t_\times]R$
In this problem, we have pure translation that is parallel to the x-axis,
so $E = t_\times$, where $t_\times = \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix}$
We know that $l' = Ex = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -t_1 \\ t_1 y_1 \end{bmatrix}$
We can see that the coefficient of the x component is 0, so l' is parallel to the x-axis

$E^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & t_1 \\ 0 & -t_1 & 0 \end{bmatrix}$
$l = Ex' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & t_1 \\ 0 & -t_1 & 0 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ t_1 \\ -t_1 y_2 \end{bmatrix}$
Therefore, l is also parallel to the x-axis

Q1.3

Let the original 3d location of the robot to be X
then at time a, the robot's location is : $X_a = R_a X + t_a$
at time b, the robot's location is: $X_b = R_b X + t_b$
Then, $X = R_a^{-1}(X_a - t_a)$
so $X_b = R_b R_a^{-1}(X_a - t_a) + t_b = R_b R_a^{-1} X_a - R_b R_a^{-1} t_a + t_b$
Therefore, $R_{rel} = R_b R_a^{-1}, t_{rel} = -R_b R_a^{-1} t_a + t_b$

Using the formula given in the lectures,
$E = [t_{rel\times}] R_{rel}$
$F = K^{-T} E K^{-1} = K^{-T} [t_{rel\times}] R_{rel} K^{-1}$

Q1.4

I did not do this problem.

2 Photometric Stereo

Q2.1

The dot product is the value of $\cos(\theta)$, where θ is the angle between the normal and the light direction.
When the light is perpendicular to the surface, then the cross section of the light is the same as dA.
When the light is not perpendicular to the surface, then the cross section of the light is larger than dA, which means the only portion of the light reach dA.
The viewing direction does not matter because the object is Lambertian,
and the reflected light is the same for all directions.

Q2.2

The implicit function of the surface is $F = z - f(x, y)$
We know that the gradient of this implicit function is also the normal to this surface
Therefore, $\nabla F = (f_x, f_y, 1)$
Thus, the normal can be $(f_x, f_y, 1)$ multiplied by any scalars.

Q2.3

I did not do this part

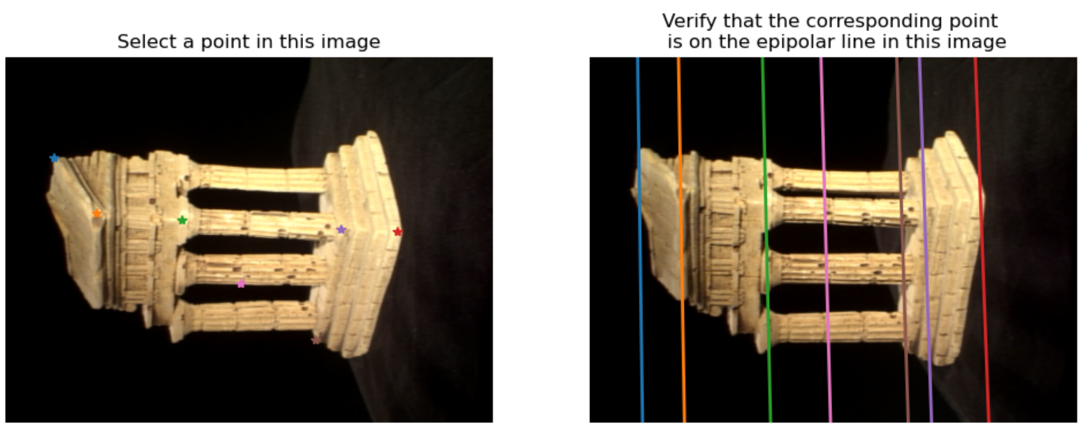
Q2.4

I did not do this part

3 3D Reconstruction

Q3.2.1

$$F = \begin{bmatrix} -9.82915851e-10 & 1.32237382e-07 & -1.12586594e-03 \\ 5.72771833e-08 & -2.96995020e-09 & 1.17849254e-05 \\ 1.08270210e-03 & -3.05057289e-05 & -3.05057289e-05 \end{bmatrix}$$



Q3.2.2

I did not do this part

Q3.3.1

$$E = \begin{bmatrix} -2.27212417e-03 & 3.06787864e-01 & -1.66258424e+00 \\ 1.32881826e-01 & -6.91514535e-03 & 4.32863830e-02 \\ 1.66718696e+00 & 1.33349695e-02 & 6.72134034e-04 \end{bmatrix}$$

Q3.3.2

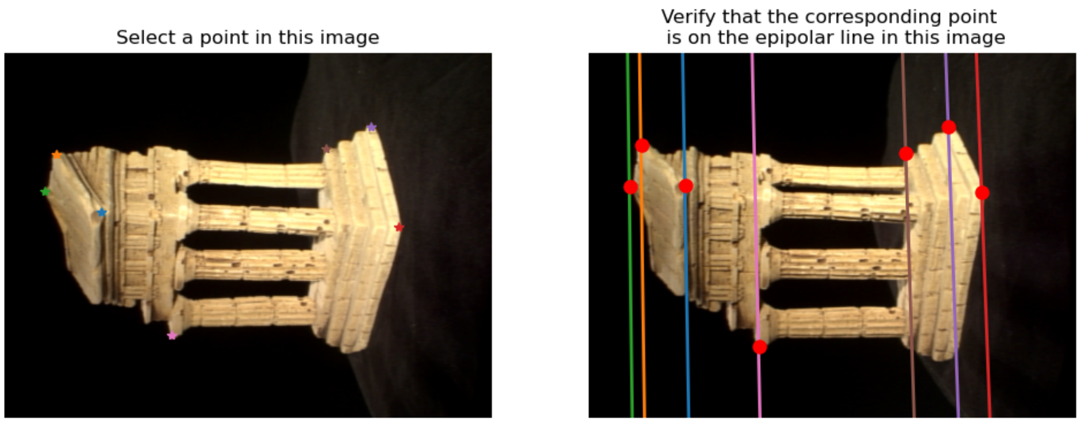
$$A = \begin{bmatrix} yp_3^T - p_2^T \\ p_1^T - xp_3^T \\ y'p_3'^T - p_2'^T \\ p_1'^T - x'p_3'^T \end{bmatrix}$$

where p_i^T is ith row of camera matrix C_1, $p_i'^T$ is ith row of camera matrix C_2

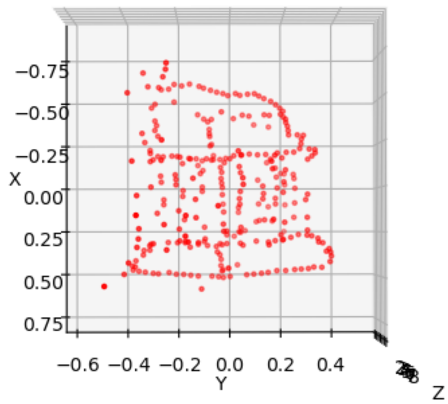
Q3.3.3

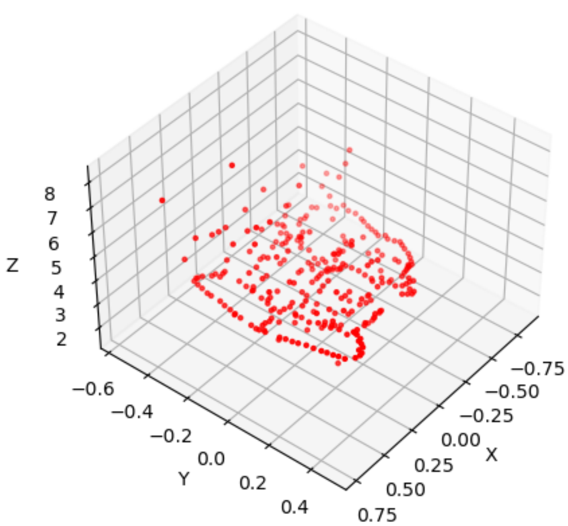
See the implementation in my code.

Q3.4.1


Select a point in this image
Verify that the corresponding point
is on the epipolar line in this image

Q3.4.2

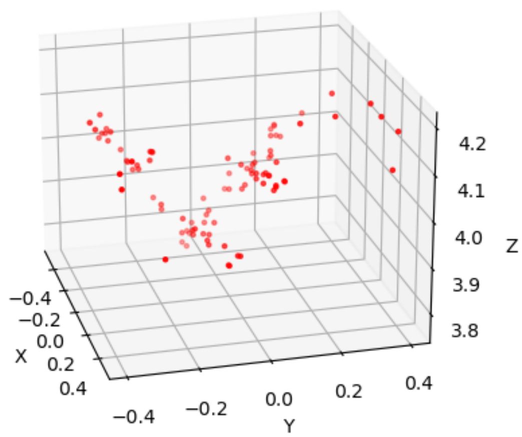




The temple seems flat because the range of z axis is larger than the range of x and y axes.

Q3.5.1

Without using Ransac, I cannot create a 3d reconstruction from the noisy data.
With Ransac, the 3d reconstruction result is below:



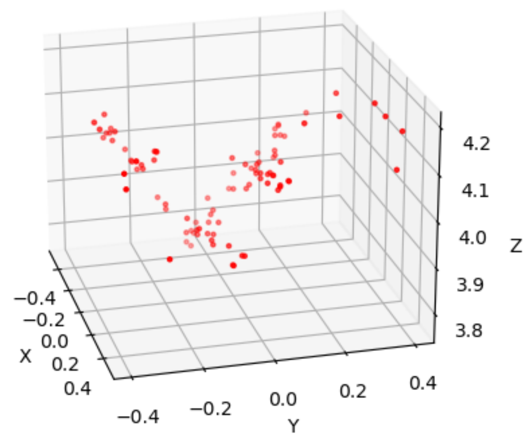
I used the distance between the matched point and the corresponding epipolar line as my error metrics.
If the distance is less than 1, I consider the pair of points as inlier.
Otherwise, I consider the pair as outlier.

Q3.5.2

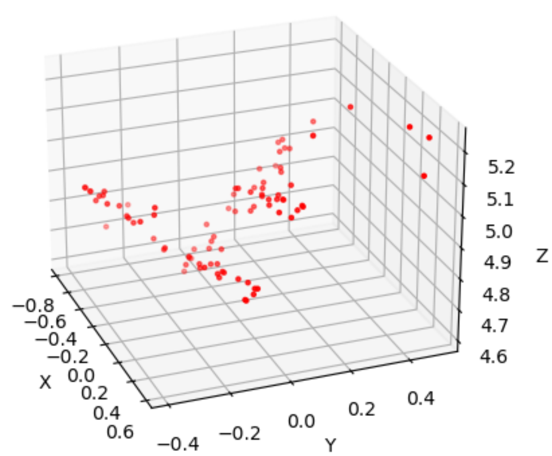
See the implementation in my code.

Q3.5.3

Without optimization:



With optimization:

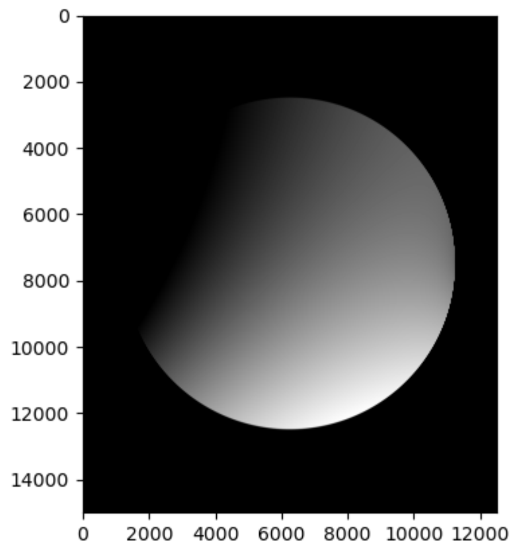


Reprojection error without optimization: 9012.918600582721
Reprojection error without optimization: 6.234553329638631

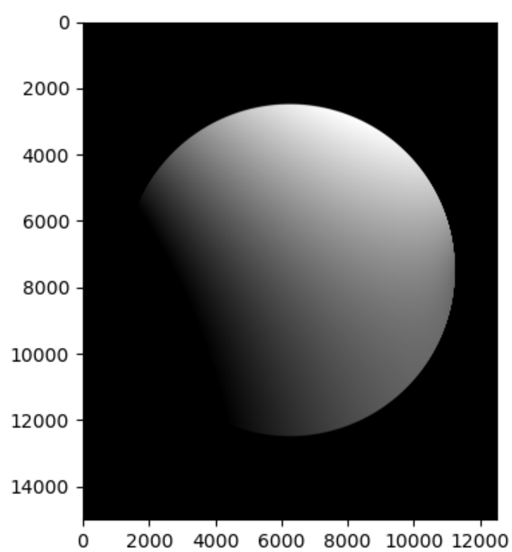
4 Calibrated Photometric Stereo

Q4.1

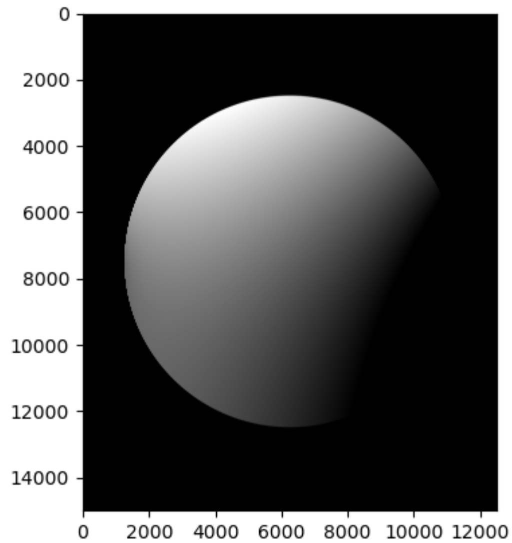
light direction: $(1, 1, 1)/\sqrt{3}$



light direction: $(1, -1, 1)/\sqrt{3}$



light direction: $(-1,-1,1)/\sqrt{3}$



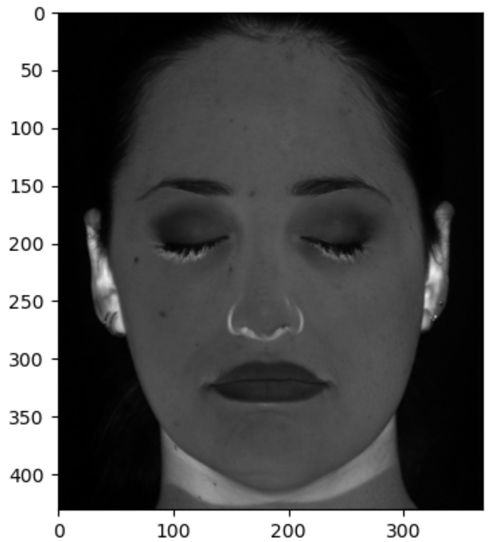
Q4.2.1

See the implementation in my code.

Q4.2.2

I construct $A = L^T, y = I$

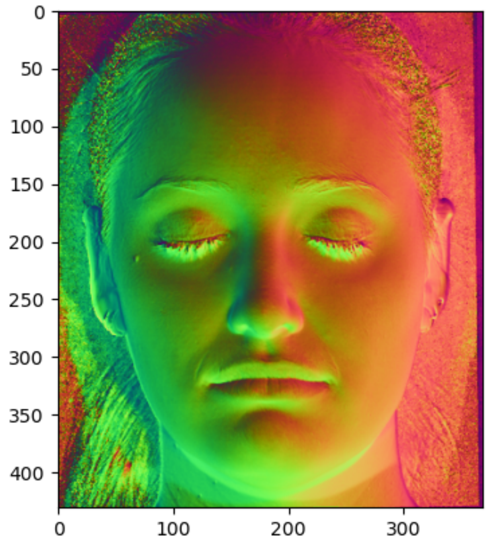
Q4.2.3



It looks like more light got reflected from her ears,
the area around her neck, and the areas around the edges of her nose.
I think this is because these areas are not as smooth as the rest of the face.

Q4.2.4

The normals match my expectation of the curvature of the face



Q4.3.1

I did not do this part.

Q4.3.2

I did not do this part.

5 Extra Credit

I (Jiaqi Geng) have hosted two online study sessions with Qichen Fu.