



Outline for Today

1st slot (45 min) - Lecture:

- 1. Recap
- 2. Goals
- 3. Over-Smoothing Problem
- 4. Training Pipeline Construction
- 5. Summary & Take-Home Messages

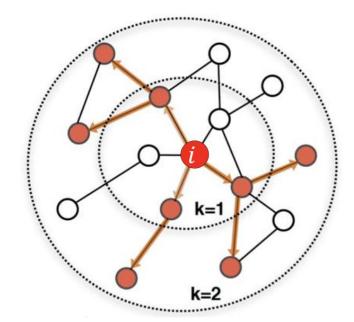
2nd slot (45 min) - Project 1 distribution.

Recap: GNN Intro

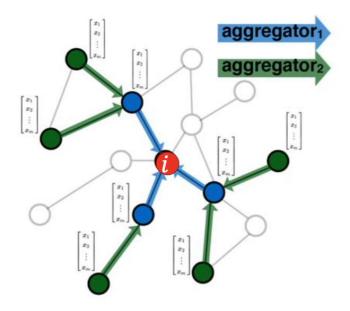


Recap: Graph Neural Networks

Idea: node's neighbourhood defines a computation graph.



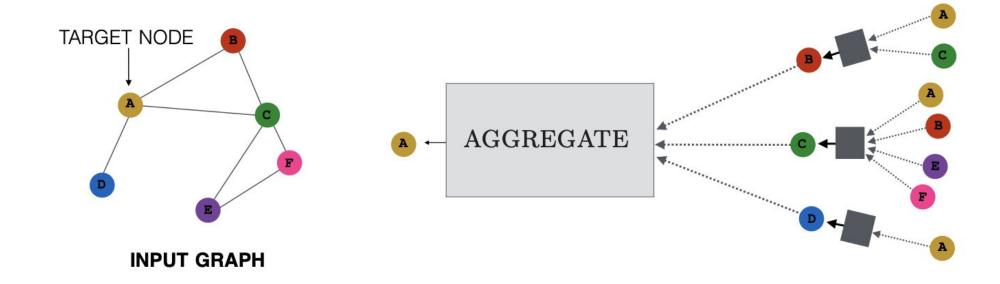
Determine node computation graph



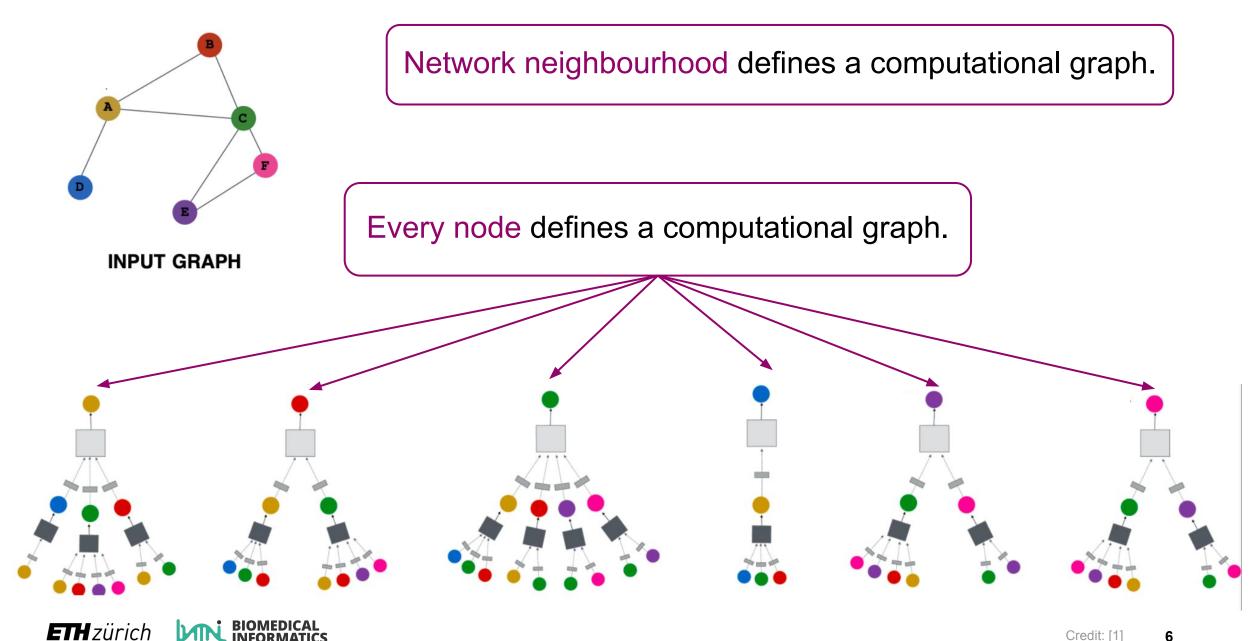
Propagate & transform the information

Recap: Neural Message Passing

Key idea: generate node embeddings based on local network neighbourhood.



Recap: Neural Message Passing



Recap: Neural Message Passing

1. **AGGREGATE** function takes as input the set of embeddings of the nodes in v's graph neighborhood $\mathcal{N}(v)$ and generates the message based on this aggregated information.

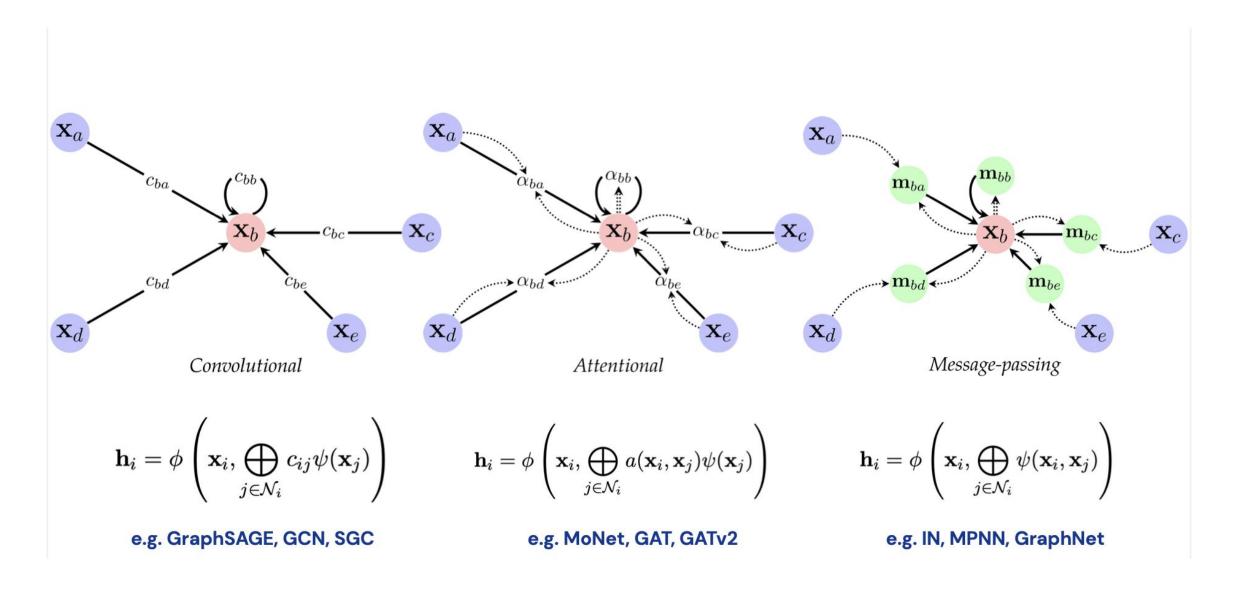
$$h_v^0 = x_v,\,orall\,v\,\in\,V$$

For each step (layer)
$$k$$
 = 1, ..., K : $m^{k-1}_{\mathcal{N}(v)} = \mathrm{AGGREGATE}\left(\{h^{k-1}_u, \, orall \, u \, \in \, \mathcal{N}(v) \, \}\right)$

2. **UPDATE** function combines this message with previous node's v embedding to create the new embedding (k = 1, ..., K):

$$h_v^k = \operatorname{UPDATE}(\, h_v^{k-1}, \, m_{\mathcal{N}(v)}^{k-1})$$

Recap: Zoo of GNN Architectures



Notes on Normalization

Recap:
$$h_v^k = f^k \left(W^k rac{\sum_{u \in \mathcal{N}(v)} h_u^{k-1}}{|\mathcal{N}(v)|} + B^k \, h_v^{k-1}
ight), \, orall \, v \in V$$

Normalization Option 1 - simply average

$$h^k_v = f^k \left(W^k \sum_{u \in \mathcal{N}(v)} \underbrace{\frac{h^{k-1}_u}{\sqrt{|\mathcal{N}(v)||\mathcal{N}(u)|}}}_{} + B^k \, h^{k-1}_v
ight), \, orall \, v \in V$$

Normalization Option 2 – symmetric normalization

Option 2 – in some tasks information from very high-degree nodes may not be very useful for inferring membership.



Goals for Today's Lecture

- 1. How to stacking more GNN layers successfully
 - a. What is an over-smoothing problem.
 - b. How to overcome it designing layer connectivity.
- 2. Training Pipeline Construction
 - a. Different Prediction Heads.
 - b. Losses and Metrics.
 - c. How to split the data.

After this Lecture - we are finally ready to train!

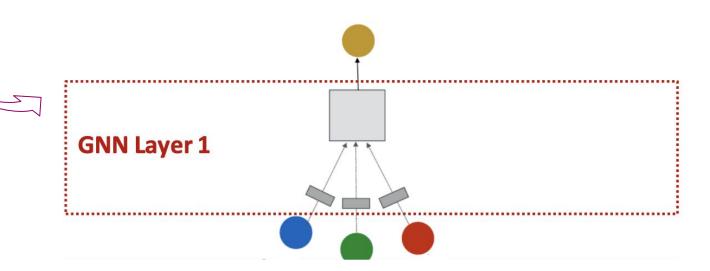
Deep Learning with GNNs



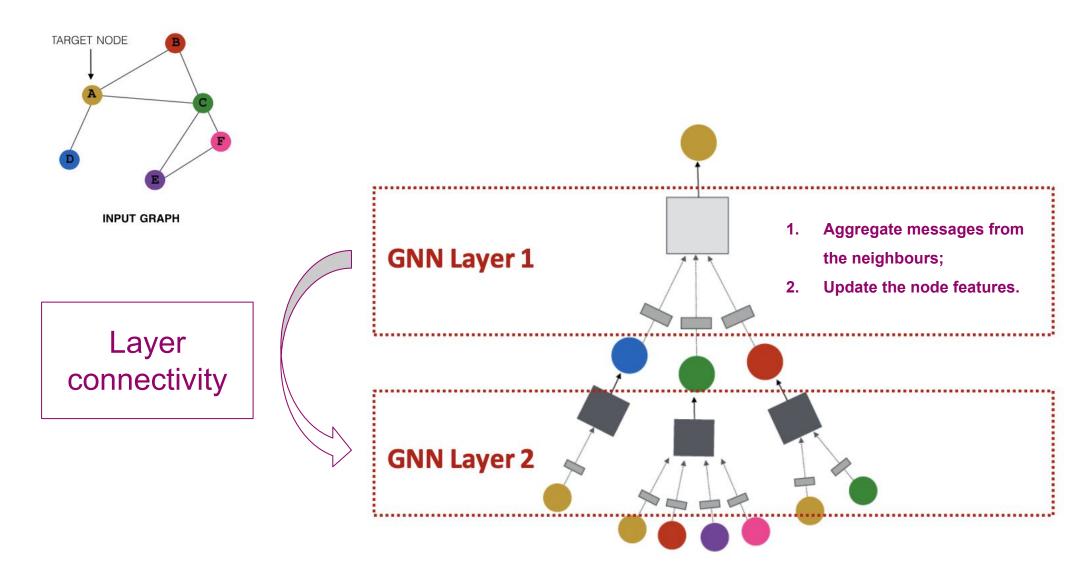
Neural Message Passing: Summary

Two-step process:

- 1. **Aggregate** messages from the neighbours
- 2. **Update** the node features.



Stacking GNN Layers



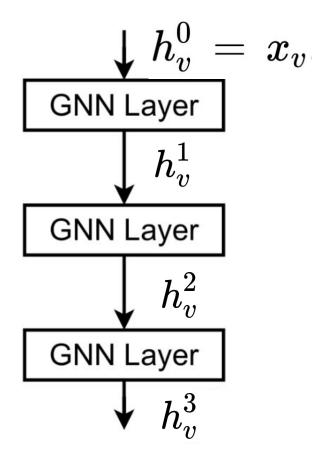


Stacking GNN Layers

The standard way: Stack GNN layers sequentially

Input: Initial raw node feature $oldsymbol{x}_v$

Output: Node embeddings after K GNN layers \boldsymbol{h}_v^K



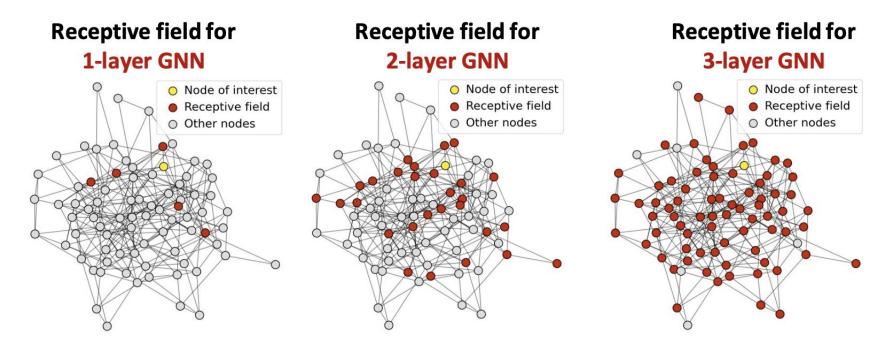
Over-Smoothing Problem



Over-Smoothing Problem

Over-smoothing: the issue of stacking many GNN layers → all the node embeddings converge to the same value.

Receptive field of GNN: the set of nodes that determine the embedding of a node of interest.

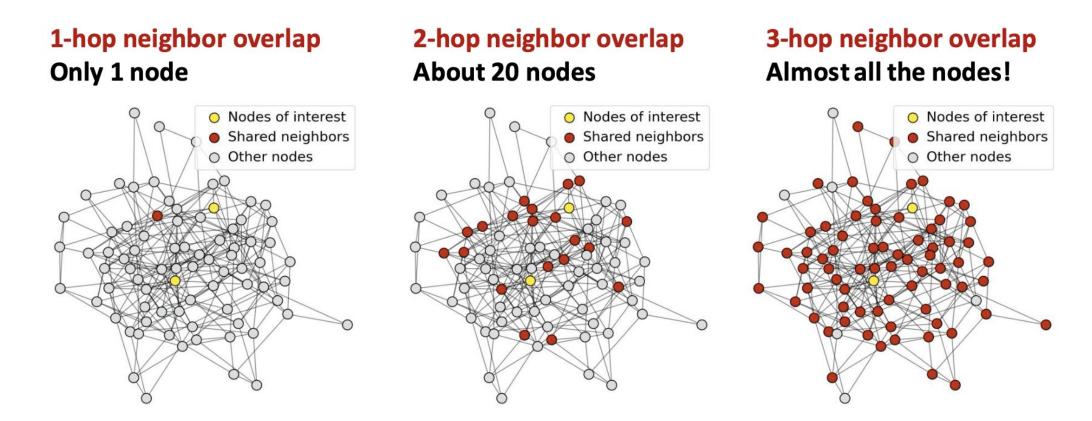


K-layer GNN: each node has a receptive field of *K*-hop neighborhood.



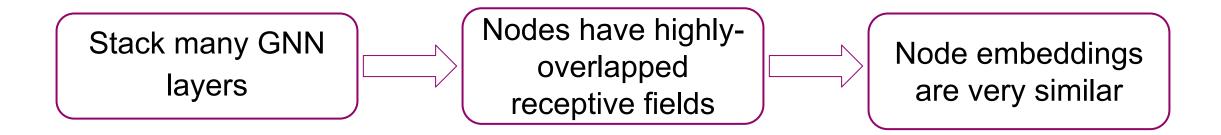
Over-Smoothing Problem

Receptive field overlap for two nodes: the shared neighbors quickly grows when we increase the number of hops (number of GNN layers).



Over-Smoothing Problem & Receptive Field

Node embedding is determined by its receptive field \rightarrow if two nodes have highly-overlapped receptive fields, then their embeddings are highly similar.



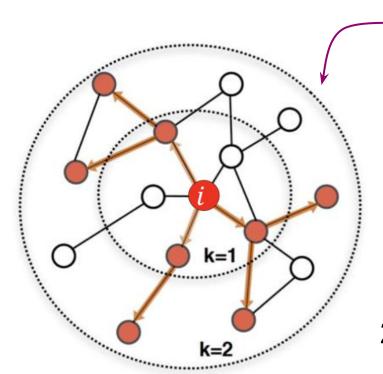
Why it is bad?

- Using node embeddings we are unable to differentiate the nodes.
- Node-specific information becomes "washed out" after several iterations of GNN message passing.



Over-Smoothing Problem: General Advice

Be cautious when adding GNN layers!



Adding more GNN layers does not always help

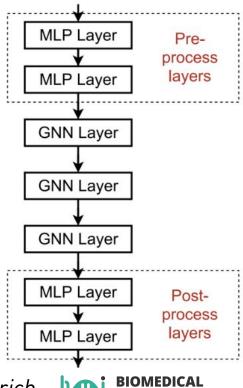
- 1. Analyze the necessary receptive field to solve your problem.
- 2. In Random Search set max number of GNN layers to be a bit more than the receptive field. Do not set it to be unnecessarily large!

How to Improve: Expressive Power

Increase the expressive power within each GNN layer.



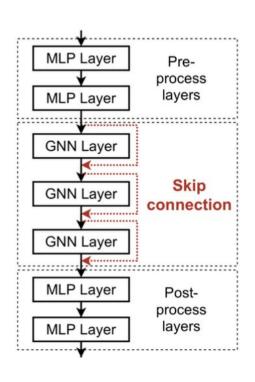
Add MLP layers (applied to each node) before and after GNN layers, as pre-process layers and post-process layers.



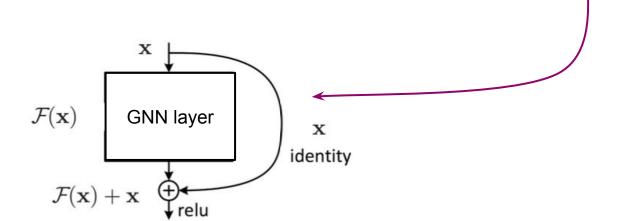
- Pre-processing layers: important when encoding node features is necessary.
- Post-processing layers: important when reasoning / transformation over node embeddings are needed.

Design GNN Layer Connectivity: Adding Residual Connections

Add residual connections in GNN to prevent node attributes from becoming "washed out".



We can increase the impact of earlier layers on the final node embeddings.



Design GNN Layer Connectivity: Adding Residual Connections

Recap:
$$h_v^k = f^k \left(W^k rac{\sum_{u \in \mathcal{N}(v)} h_u^{k-1}}{|\mathcal{N}(v)|} + B^k h_v^{k-1}
ight), \, orall \, v \in V$$

AGGREGATE

UPDATE

FINAL residual UPDATE = UPDATE +
$$h_v^{k-1}$$

Sometimes these are also called skip connections.

Design GNN Layer Connectivity: Interpolation Method

Recap:
$$h_v^k = f^k \left(W^k rac{\sum_{u \in \mathcal{N}(v)} h_u^{k-1}}{|\mathcal{N}(v)|} + B^k h_v^{k-1}
ight), \, orall \, v \in V$$

AGGREGATE

UPDATE

FINAL UPDATE =
$$lpha_1$$
UPDATE + $lpha_2 \, h_v^{k-1}$

$$oldsymbol{lpha}_1, oldsymbol{lpha}_2 \in [0,1]^d$$
 and $oldsymbol{lpha}_2 = oldsymbol{1} - oldsymbol{lpha}_1$

Design GNN Layer Connectivity: Alternative ways for **Skip** Connections

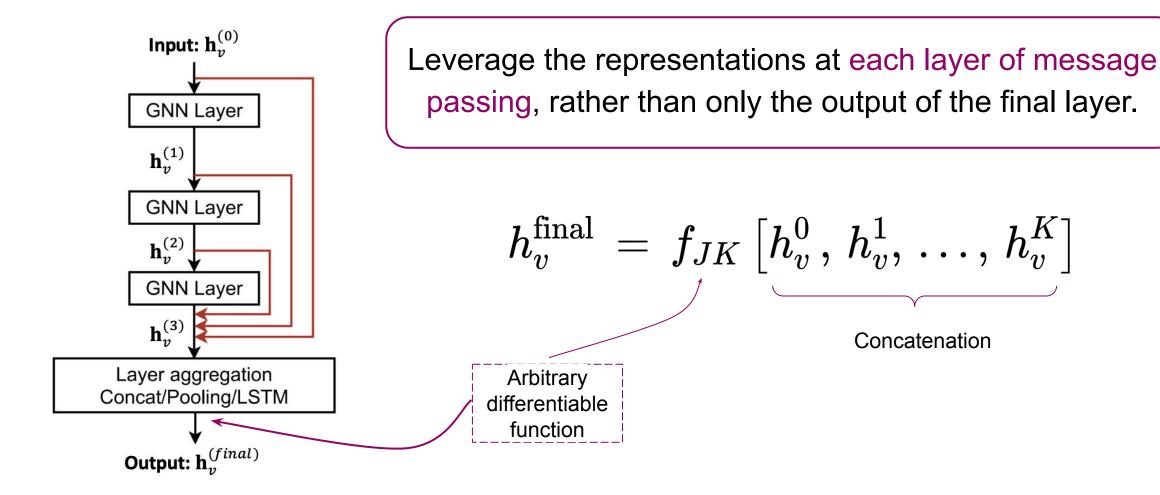
Recap:
$$h_v^k = f^k \left(W^k rac{\sum_{u \in \mathcal{N}(v)} h_u^{k-1}}{|\mathcal{N}(v)|} + B^k \, h_v^{k-1}
ight), \, orall \, v \in V$$
 AGGREGATE

UPDATE

FINAL UPDATE = [UPDATE,
$$h_v^{k-1}$$
|

Key intuition: to encourage the model to disentangle information during message passing – separate the information coming from the neighbors from the current representation of each node.

Design GNN Layer Connectivity: Jumping Knowledge Connections



Training Pipeline Construction



GNN Layers in Practice

Many DL modules can be incorporated into/with a GNN layer:

- Normalization for speed training, preventing overtraining
 - Node level batch, layer, etc → GraphNorm [12] (extension for the GNN).
- Dropout for regularization, robustness
 - Node level increasing the expressive power of GNN [10]
 - Edge level increasing the expressive power of GNN, helps
 with over-fitting and over-smoothing [11]
 - Linear transformation

Depending on the task, combinations of different GNN layer variants

E.g., network with GCN layers (convolution) followed by GAT layers (attention)

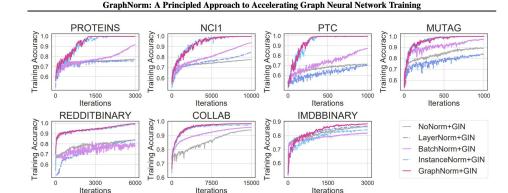
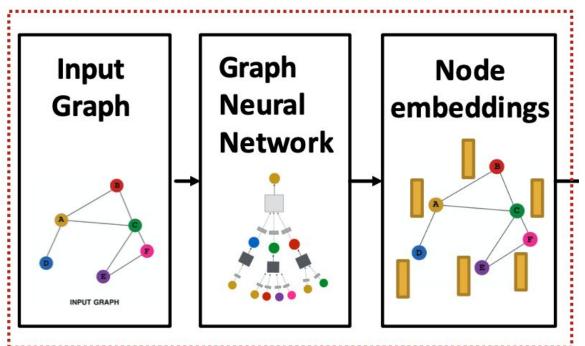


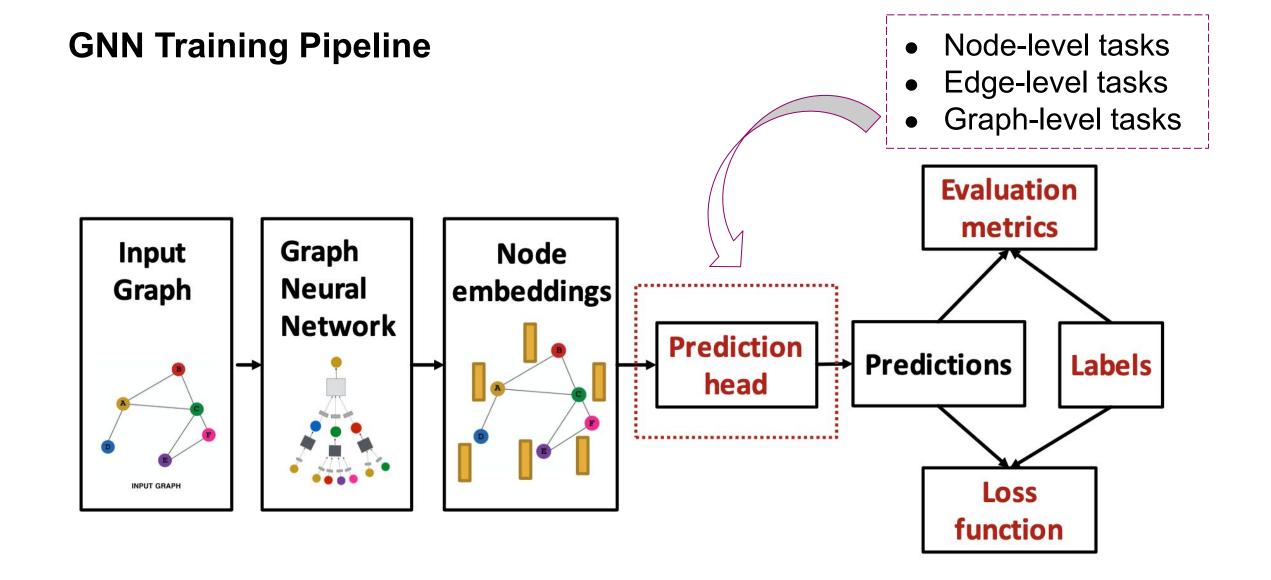
Figure 2. Training performance of GIN with different normalization methods and GIN without normalization in graph classification tasks. The convergence speed of our adaptation of InstanceNorm dominates BatchNorm and LayerNorm in most tasks. GraphNorm further improves the training over InstanceNorm especially on tasks with highly regular graphs, e.g., IMDB-BINARY (See Figure 5 for detailed illustration). Overall, GraphNorm converges faster than all other methods.







Output of a GNN: set of node embeddings $\ h_v^K, \, orall \, v \in V$



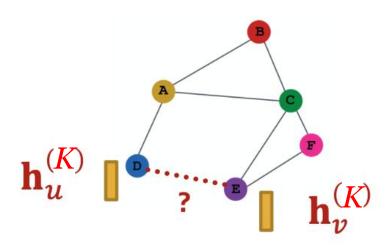


Prediction: Node & Edge Level

Node-level prediction: directly make prediction using node embeddings $\ h_v^K, \, orall \, v \, \in \, V$

- Classification: classify among c categories.
- Regression: regress on c targets.

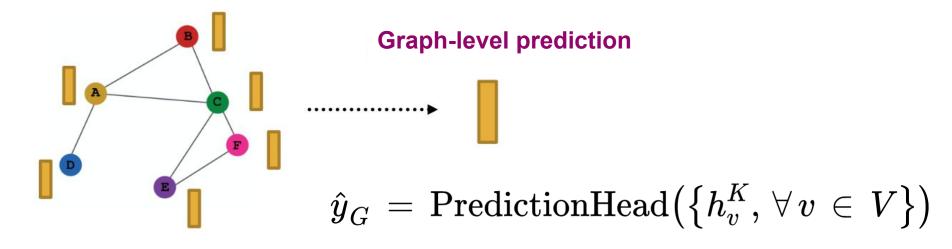
Link-level prediction: predict the link between two nodes, make prediction using pairs of node embeddings.



- 1. Linear(Concatenate $[h_u^K, h_v^K]$,
- 2. Average,
- 3. Hadamard $h_u^K * h_v^K$,
- 4. Dot Product,
- 5. Weighted L-1, L-2.

Prediction: Graph Level

Graph-level prediction: make prediction using all the node embeddings in our graph.

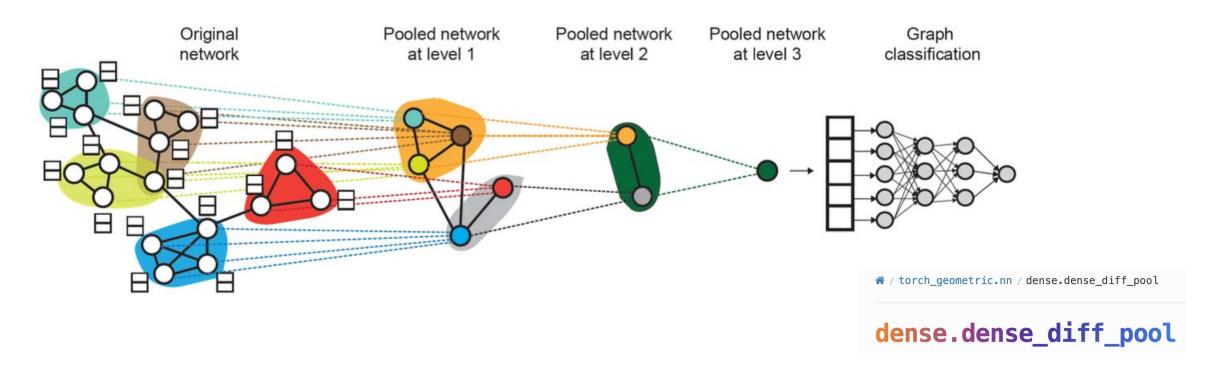


- 1. Global mean pooling $\hat{y}_G = \operatorname{Mean}ig(ig\{h_v^K,\,orall\,v\in\,Vig\}ig)$
- 2. Global max pooling $\ \hat{y}_G = \operatorname{Max}ig(ig\{h_v^K,\,orall\,v\in\,Vig\}ig)$
- 3. Global sum pooling $\ \hat{y}_G = \mathrm{Sum}ig(ig\{h_v^K, \, orall \, v \in Vig\}ig)$

Issue: Global pooling over a (large) graph will lose information.

Graph Pooling

- 1. Attention-based Pooling [8].
- 2. Hierarchical Pooling graph clustering to perform the pooling [9].





Other Pooling Methods

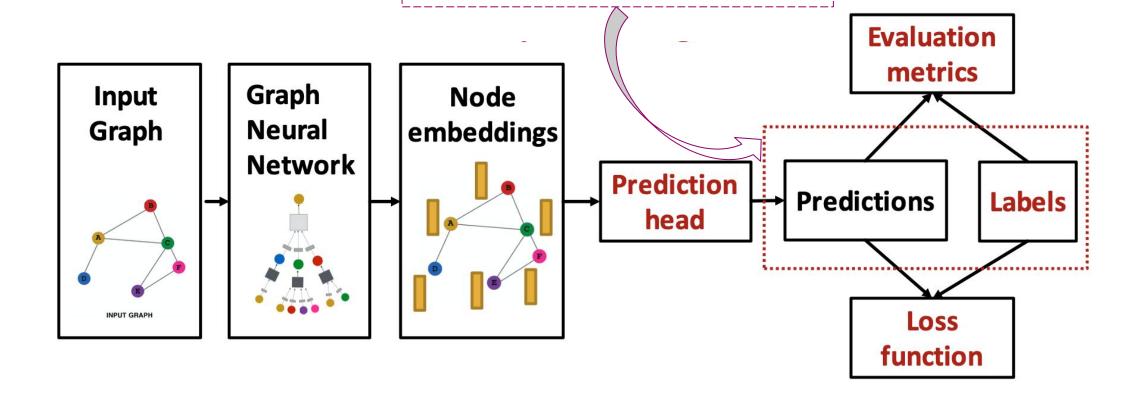
Pooling Layers	
global_add_pool	Returns batch-wise graph-level-outputs by adding node features across the node dimension, so that for a single graph \mathcal{G}_i its output is computed by
global_mean_pool	Returns batch-wise graph-level-outputs by averaging node features across the node dimension, so that for a single graph \mathcal{G}_i its output is computed by
global_max_pool	Returns batch-wise graph-level-outputs by taking the channel-wise maximum across the node dimension, so that for a single graph \mathcal{G}_i its output is computed by
KNNIndex	A base class to perform fast k -nearest neighbor search (k -NN) via the faiss library.
L2KNNIndex	Performs fast k -nearest neighbor search (k -NN) based on the L_2 metric via the faiss library.
MIPSKNNIndex	Performs fast k -nearest neighbor search (k -NN) based on the maximum inner product via the faiss library.
TopKPooling	top_k pooling operator from the "Graph U-Nets", "Towards Sparse Hierarchical Graph Classifiers" and "Understanding Attention and Generalization in Graph Neural Networks" papers.
SAGPooling	The self-attention pooling operator from the "Self-Attention Graph Pooling" and "Understanding Attention and Generalization in Graph Neural Networks" papers.
EdgePooling	The edge pooling operator from the "Towards Graph Pooling by Edge Contraction" and "Edge Contraction Pooling for Graph Neural Networks" papers.
ASAPooling	The Adaptive Structure Aware Pooling operator from the "ASAP: Adaptive Structure Aware Pooling for Learning Hierarchical Graph Representations" paper.
PANPooling	The path integral based pooling operator from the "Path Integral Based Convolution and Pooling for Graph Neural Networks" paper.
MemPooling	Memory based pooling layer from "Memory-Based Graph Networks" paper, which learns a coarsened graph representation based on soft cluster assignments
max_pool	Pools and coarsens a graph given by the <code>torch_geometric.data.Data</code> object according to the clustering defined in <code>cluster</code> .
avg_pool	Pools and coarsens a graph given by the <pre>torch_geometric.data.Data</pre> object according to the clustering defined in <pre>cluster</pre> .



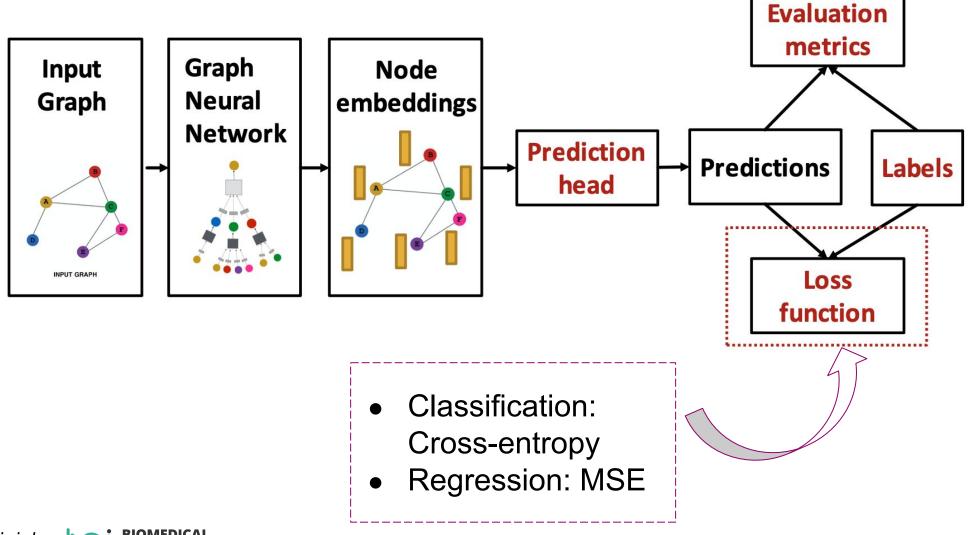


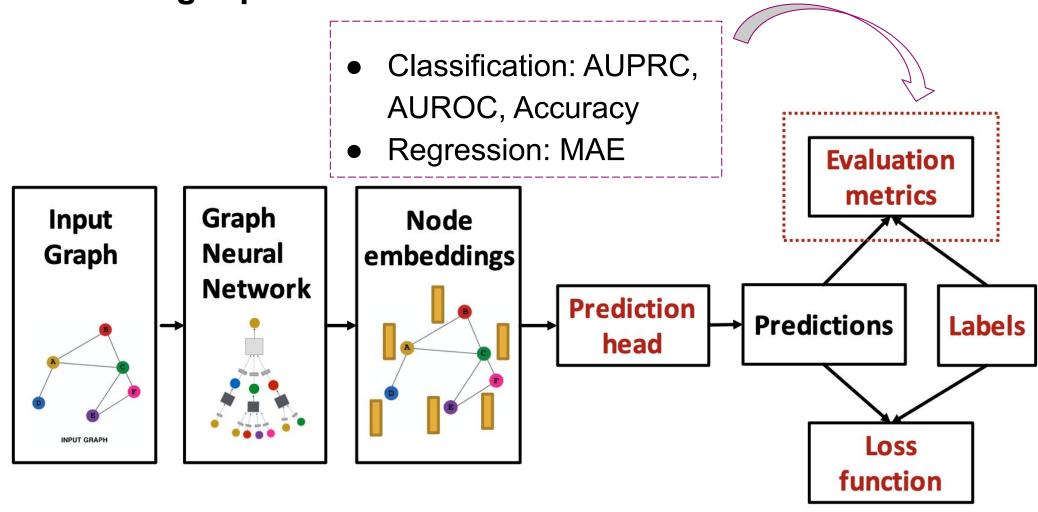
Depends on the task/settings:

- Supervised labels,
- Unsupervised signals.

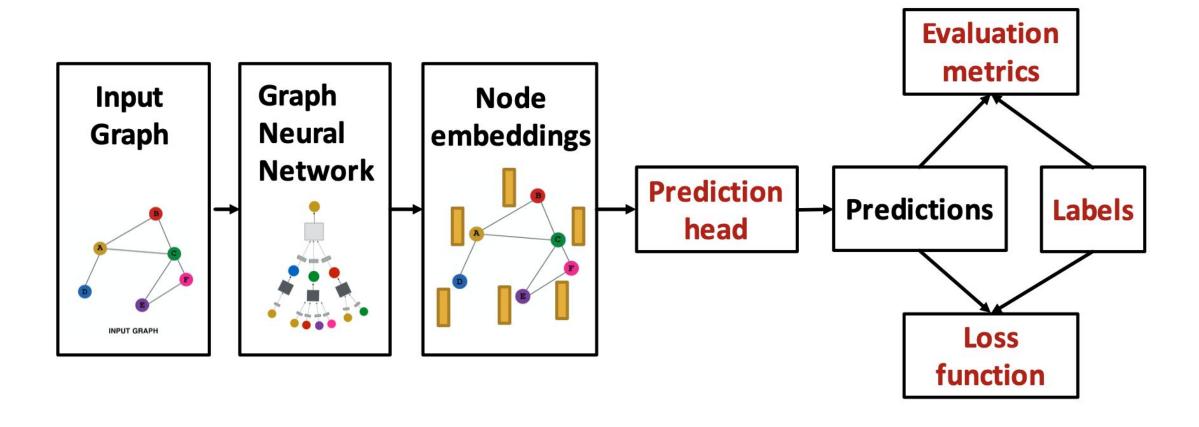












How do we split our dataset into train / validation / test set?



Why Splitting Graphs is special

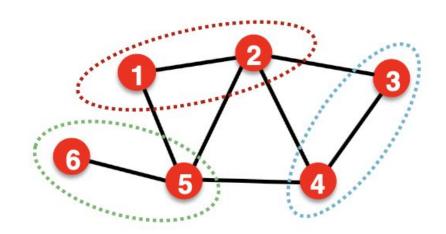
Often, the data points are independent (i.e image classification).

Splitting a graph dataset is different!

Example:

Node classification: data points are NOT independent.

Training
Validation
Test



Node 5 will affect our prediction on node 1, because it will participate in message passing→affect node 1's embedding.

Transductive setting

The input graph can be observed in all the dataset splits (training /validation /test set). We only split the labels.

Training
Validation
Test

- 1. At training time: we compute embeddings using the entire graph, and train using node 1&2's labels \rightarrow i.e these labels are the part of the supervised loss computation.
- 2. At validation time: we compute embeddings using the entire graph, and evaluate on node 3&4's labels.

Transductive setting

The input graph can be observed in all the dataset splits (training /validation /test set). We only split the labels.

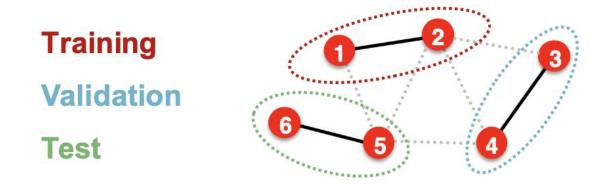


- The dataset consists of one graph.
- The entire graph can be observed in all dataset splits, we only split the labels.
- Only applicable to node / edge prediction tasks.

Use case: graph is static and won't change in the future.

Inductive setting

We break the edges to create the multiple graphs. Training / validation / test sets are on different graphs.



- 1. At training time: we compute embeddings using the graph over node 1&2, and train using node 1&2's labels.
- 2. At validation time: we compute embeddings using the graph over node 3&4, and evaluate on node 3&4's labels.

Inductive setting

We break the edges to create the multiple graphs. Training / validation / test sets are on different graphs.



- The dataset consists of multiple graphs.
- Each split can only observe the graph(s) within the split. A successful model should generalize to unseen graphs.
- Applicable to node / edge / graph tasks.

Use case: graph is dynamic.

Take-Home Messages

- 1. Stacking GNN layers: what to do with over-smoothing
 - a. Deciding number of layers carefully;
 - b. Residual and skip connections,
- 2. The full training pipeline of a GNN:
 - a. The output of a GNN: node embeddings,
 - b. Different prediction heads: node/edge/graph level,
 - c. Losses & Metrics,
 - d. How to split the data: Transductive and Inductive settings.

BIOMEDICAL INFORMATICS

Slides & Image Credits

- 1. CS224W: Machine Learning with Graphs
- 2. Graph Representation Learning Book
- 3. Geometric Deep Learning Grids, Groups, Graphs, Geodesics, and Gauges
- 4. <u>Deep Residual Learning for Image Recognition</u>
- 5. Column Networks for Collective Classification
- 6. Representation Learning on Graphs with Jumping Knowledge Networks
- 7. <u>node2vec: Scalable Feature Learning for Networks</u>
- 8. ORDER MATTERS: SEQUENCE TO SEQUENCE FOR SETS
- 9. <u>Hierarchical Graph Representation Learning with Differentiable Pooling</u>
- 10. <u>DropGNN: Random Dropouts Increase the Expressiveness of Graph Neural Networks</u>
- 11. <u>DROPEDGE: TOWARDS DEEP GRAPH CONVOLU- TIONAL NETWORKS ON NODE</u>
 <u>CLASSIFICATION</u>
- 12. GraphNorm: A Principled Approach to Accelerating Graph Neural Network Training

