Section 4
The Cost of Production I

Reference:

- N. Gregory Mankiw and Mark P. Taylor (2023), "*Microeconomics*", Cengage Learning, Chapter 5
- Pindyck, R.S. und D.L. Rubinfeld (2012), *"Microeconomics"*, 9th Edition, Prentice Hall, Chapter 7

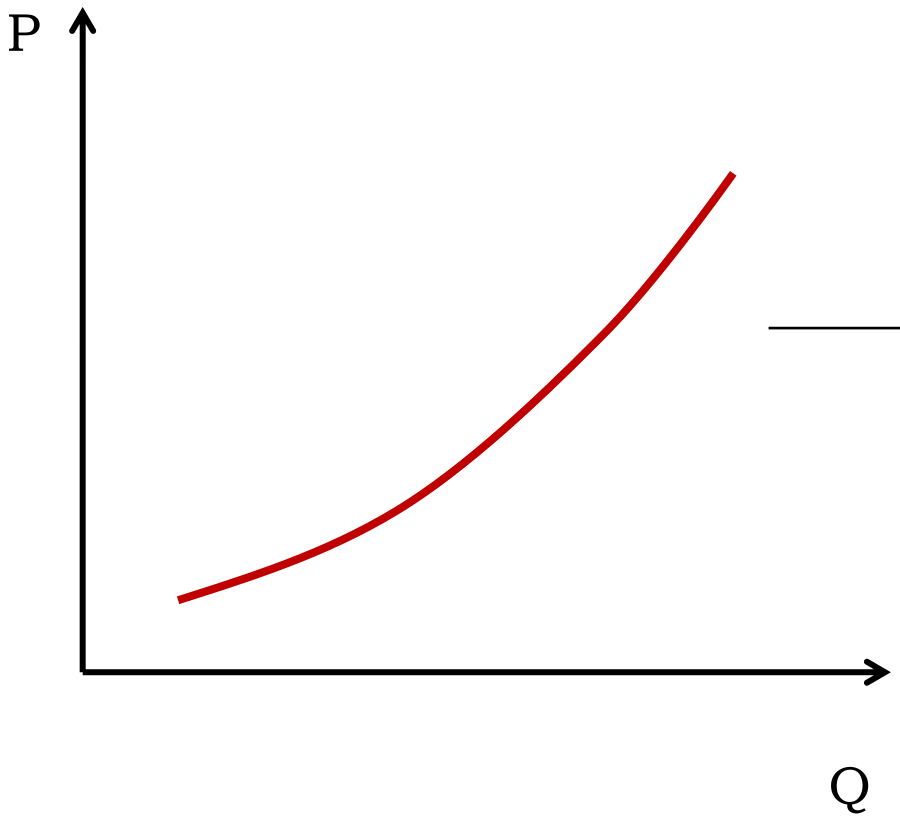
The slides of this section are mainly based on the 6th edition of the book by Mankiw and Taylor (2023). In some slides we reproduce figures, sentences and definitions given in the book.

Introductory Video: How Intelligent Machines Are Revolutionizing Cow Farming

https://www.youtube.com/watch?v=tLjI_eixBQk

Production Function

Supply Function, Production and Cost



Behind the supply function:

- **Production function**
- **Cost function**

Simplified production process

- **Production**:
  - **Inputs:** resources, such as labor, capital, raw materials and energy
  - **Output:** the amount of a good or service produced by a company
  - **Production technology**: combination of inputs in order to produce goods and services

Production process: bus company

Inputs:

- ✓ Labor (nr. Workers)
- ✓ Capital (nr. Buses)
- ✓ Energy

→

Output

Bus kilometres
or
Passenger kilometres

Scale of Production, Cost and Productivity

Two types of technologies → Different combination of inputs (capital, labor, energy) → Different production cost

Production Function

- **Production function**: a simplified mathematical representation of the production process (in this case just two inputs)

$$\mathbf{Q = f\,(L,C)} \quad \text{where L =Labor and C= capital}$$

- Indicates the **highest output** that a firm can produce **for every** specified **combination of inputs** given the state of technology.

The Short Run Versus the Long Run

Normally we distinguish between two periods when we analyze production processes:

- **The short run** describes the period in which firms may adapt their production by changing some variable factors (labor or raw material)
  ➔ **some variable (L) and some fixed (C) inputs** ➔ $\boxed{Q = f(L; \bar{C})}$

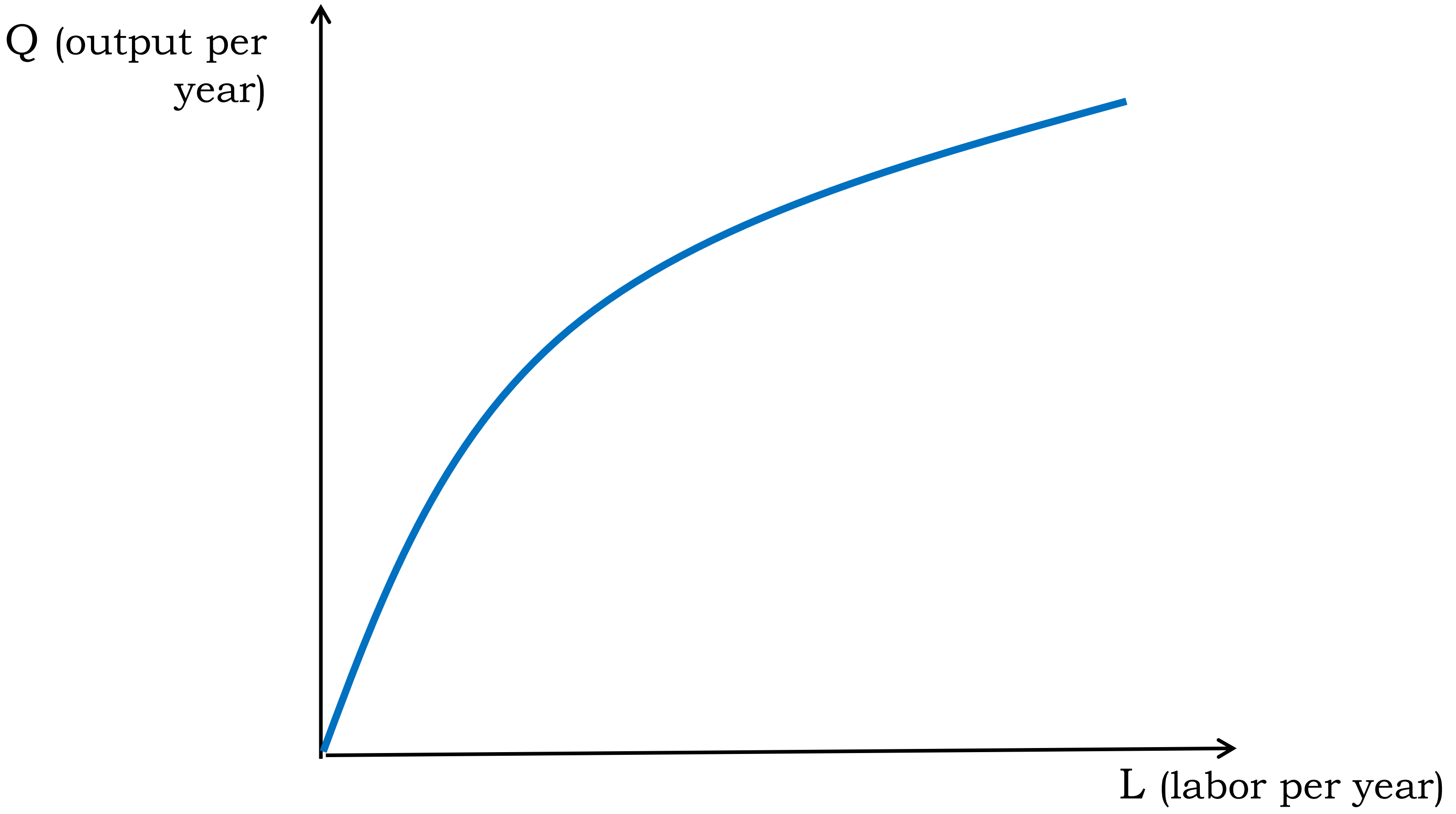
- **The long run** describes the period which is long enough for firms to adapt their production by changing all factors **including** capital.
  ➔ **all inputs variable** ➔ $\boxed{Q = f(L, C)}$

↳ Difference between variable and fixed costs

↳ Difference between costs in the short run and in the long run

Short Run Case: Production with One Variable Input, Labor (L), and One Fixed Input, Capital ($\bar{C}$)

$\boxed{Q = f(L; \bar{C})}$



Total and Marginal Product

- The **total product** measures the total amount of output produced in physical units of measurement (tons, kWh, ...)

- The **marginal product of a production factor (MP)** is defined by the additional output produced as one input is increased by 1 unit (other production factors held constant). Marginal product for labor (MP):

$$MP = \frac{\Delta \text{Output}}{\Delta \text{Labour Input}} = \frac{\Delta Q}{\Delta L}$$

- Using calculus (partial derivative) Q = f (L,C) :

$$MP = \frac{\partial Q}{\partial L}$$

Diminishing Marginal Product (one input)

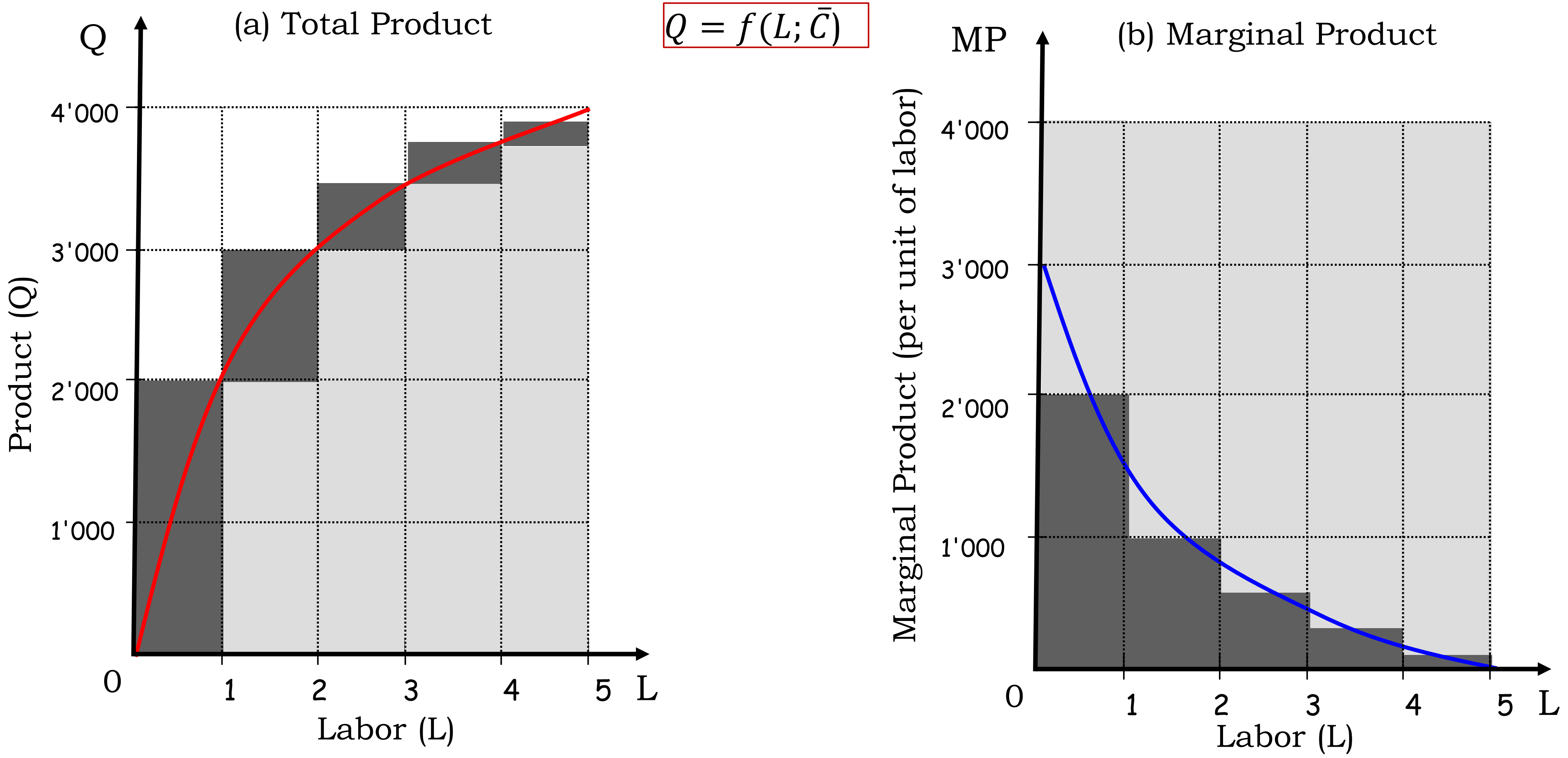
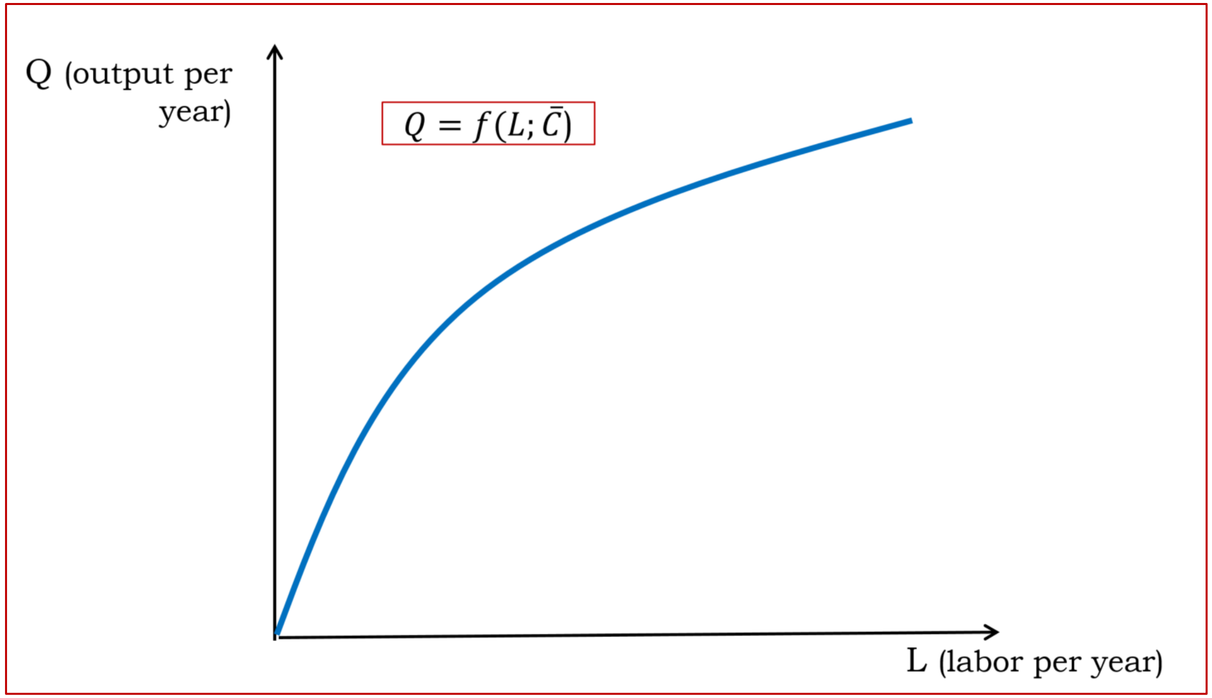


Figure: The marginal product shown as the derivative of the total product.

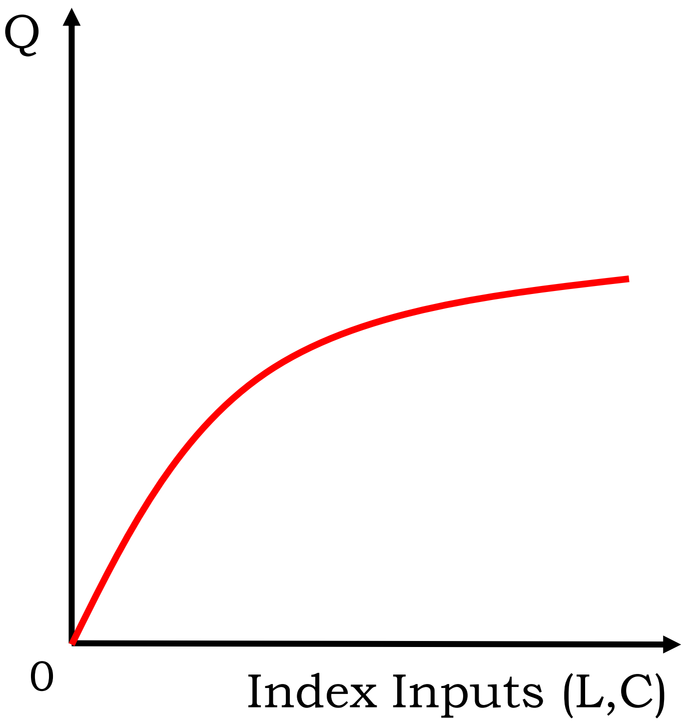
Diminishing Marginal Product

- **Diminishing** (Decreasing) **Marginal Product:** property that affirms that the marginal product of an input declines as the quantity of the input increases (other inputs fixed).
- This property is no law of nature like the law of gravitation but an empirical principle.
- Diminishing marginal product is shown graphically as a concave production curve.

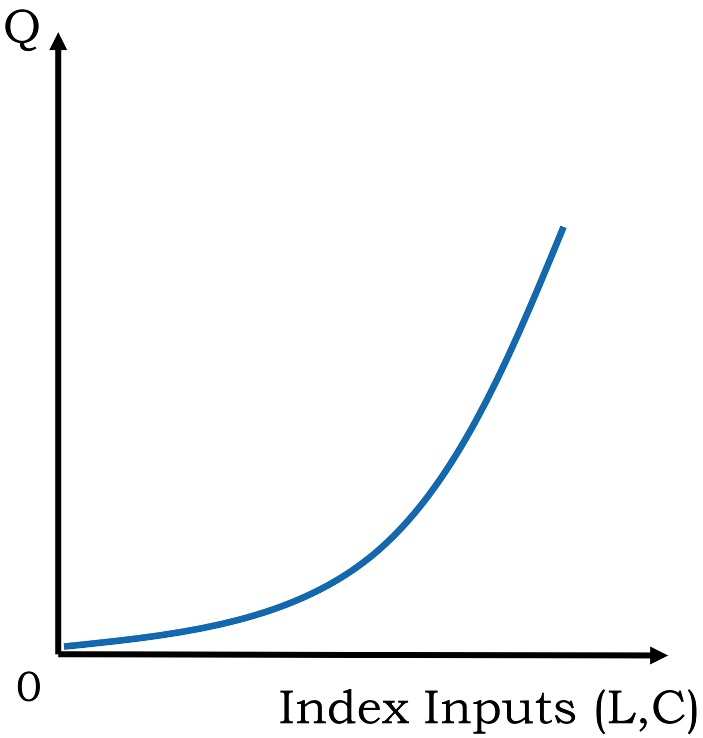


Long Run Case: Production with two variable Inputs, Labor (L), and Capital (C)

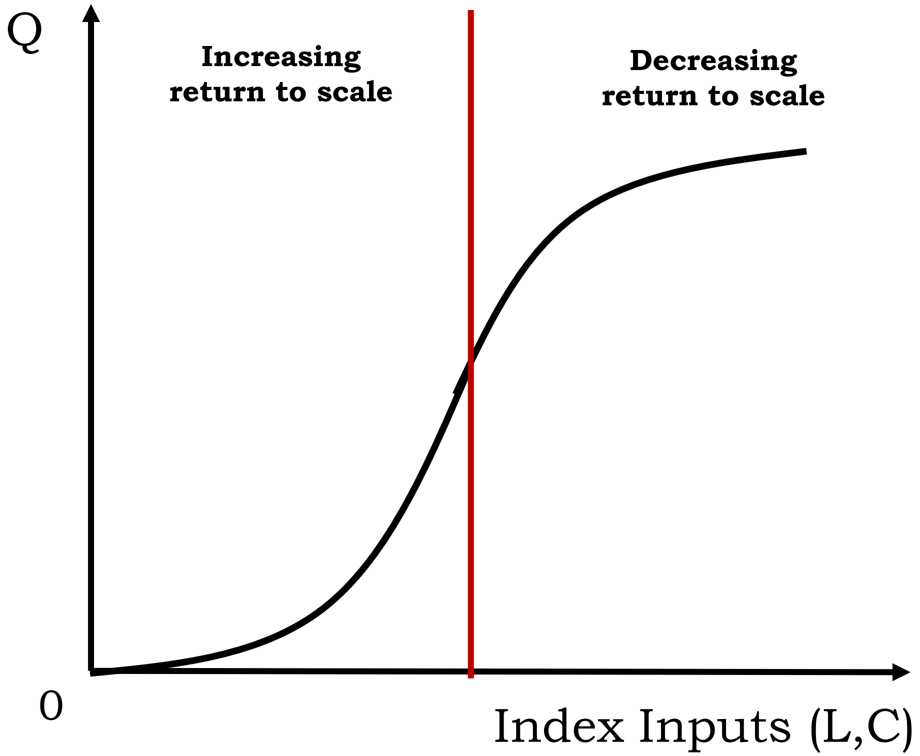
$$\boxed{Q = f(L, C)}$$



Decreasing
Production function



Increasing
Production function



Mathematical Representation: Cobb Douglas Production Function

- Cobb Douglas Production Function: $Q = z\, C^{\alpha} L^{\beta}$
  - where z, α and β are constants and C and L are the usual variable amounts of Capital and Labor.

- This production function is commonly used by economists, because it is capable of "illustrating" some attractive results of production theory but also represent some production processes.

Marginal Products:

$$\boldsymbol{Q = zC^{\alpha}L^{\beta}}$$

$$\boldsymbol{MP_L = z\beta C^{\alpha}L^{\beta-1}}$$

$$\boldsymbol{MP_C = z\alpha C^{\alpha-1}L^{\beta}}$$

Relation: Production and Total Cost

Example: Bus Kilometres ; very simple production function → $\boxed{Q = f(L;\bar{C})}$

Number of workers	Output - Bus Kilometres	MP of Labor	Number of Busses	Cost of capital	Labor Cost	Total Cost of inputs
0	0	0	10	30	0	30
1	50	50	10	30	10	40
2	90	40	10	30	20	50
3	120	30	10	30	30	60
4	140	20	10	30	40	70
5	150	10	10	30	50	80

The intuition behind the estimation of a production function with Econometric Methods (Bus Companies Operating in Switzerland, N=57)

$$Q = z\, L^{\beta} C^{\alpha} \quad \rightarrow \quad \ln Q = \ln z + \beta \ln L + \alpha \ln C + \varepsilon$$

Data collection process

Econometric analysis

	Q	L	lnQ	lnL	lnC
1	34	2	3.526361	.6931472	4.477337
2	53	2	3.970292	.6931472	4.584968
3	583	4	6.368187	1.386294	7.385231
4	177	6	5.17615	1.791759	5.590987
5	92	2	4.521789	.6931472	5.030438
6	63	2	4.143135	.6931472	4.584968
7	149	6	5.003946	1.791759	5.976351
8	208	9	5.337538	2.197225	6.280396
9	153	5	5.030438	1.609438	5.874931
10	124	6	4.820282	1.791759	6.021023
11	219	6	5.389072	1.791759	6.775366
12	596	21	6.390241	3.044523	7.003974
13	4357	213	8.379539	5.361292	9.168685
14	259	17	5.556828	2.833213	6.461468
15	98	3	4.584968	1.098612	6.39693
16	8	1	2.079442	0	2.70805
17	197	6	5.283204	1.791759	5.717028
18	405	18	6.003887	2.890372	6.878326

Q= bus km
L= number of workers
C= number of seats

```
reg lnQ lnL lnC if ANNO==1994

      Source |       SS           df       MS      Number of obs   =        57
-------------+----------------------------------   F(2, 54)        =    346.43
       Model |  102.934979         2  51.4674895   Prob > F        =    0.0000
    Residual |  8.02253141        54  .148565397   R-squared       =    0.9277
-------------+----------------------------------   Adj R-squared   =    0.9250
       Total |   110.95751        56  1.98138411   Root MSE        =    .38544

------------------------------------------------------------------------------
         lnQ |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         lnL |   .1939337    .087965     2.20   0.032     .0175745    .3702929
         lnC |   .8347022   .0804377    10.38   0.000     .6734344    .9959701
       _cons |  -.1541987   .3347774    -0.46   0.647    -.8253869    .5169895
------------------------------------------------------------------------------
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$$\ln Q = -0.154 + 0.194\, \ln L + 0.835\, \ln C$$

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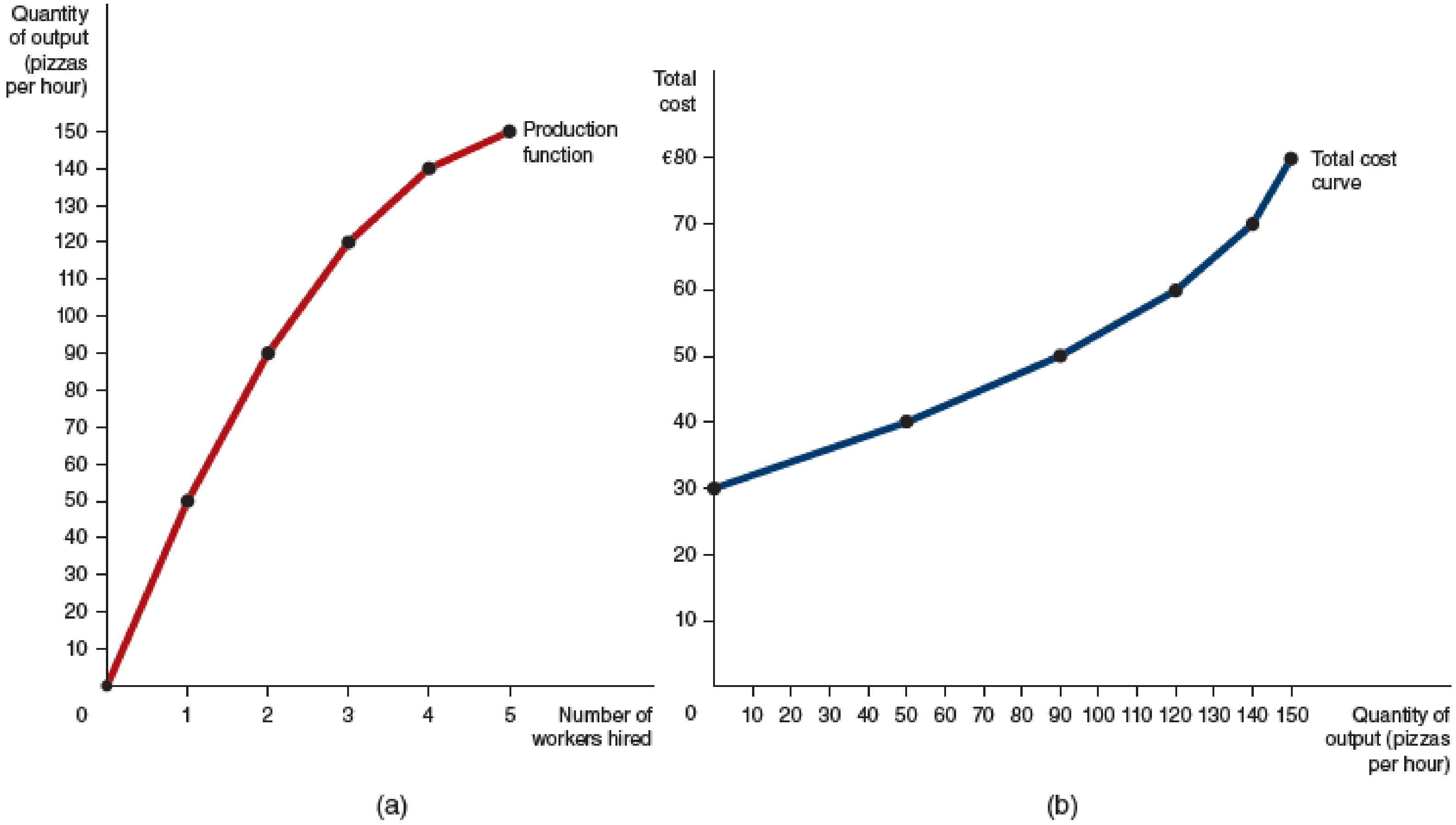
Swiss Federal Institutes of Technology

Production Function and Cost Function



Source: Mankiw and Taylor (2023), "Microeconomics"

Productivity and Technological change

Partial Productivity Indicators

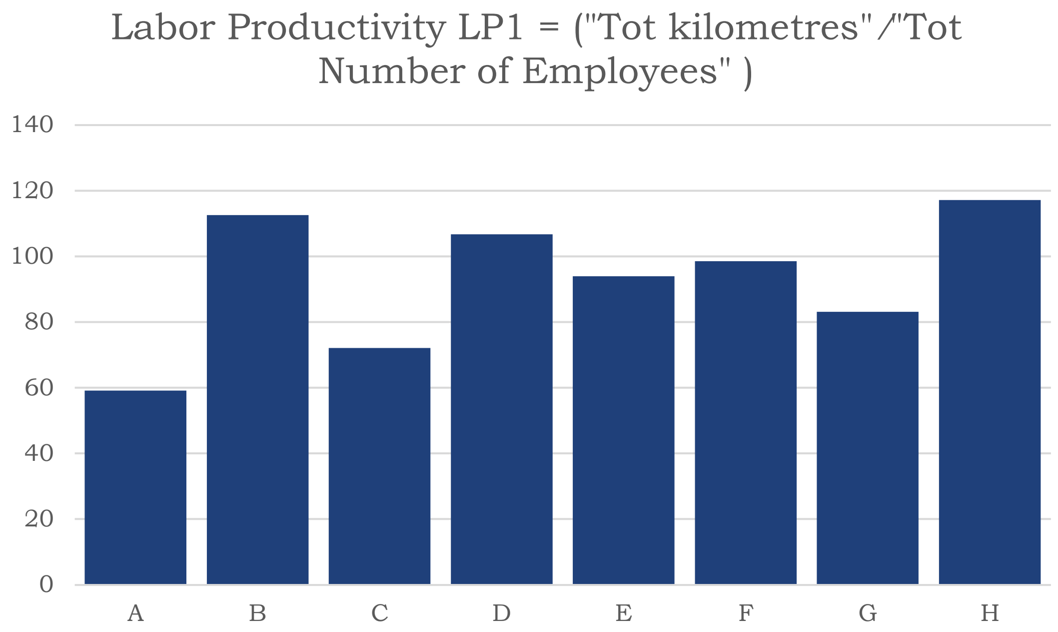
- A **partial productivity** measure relates a firm’s output to a single input factor.

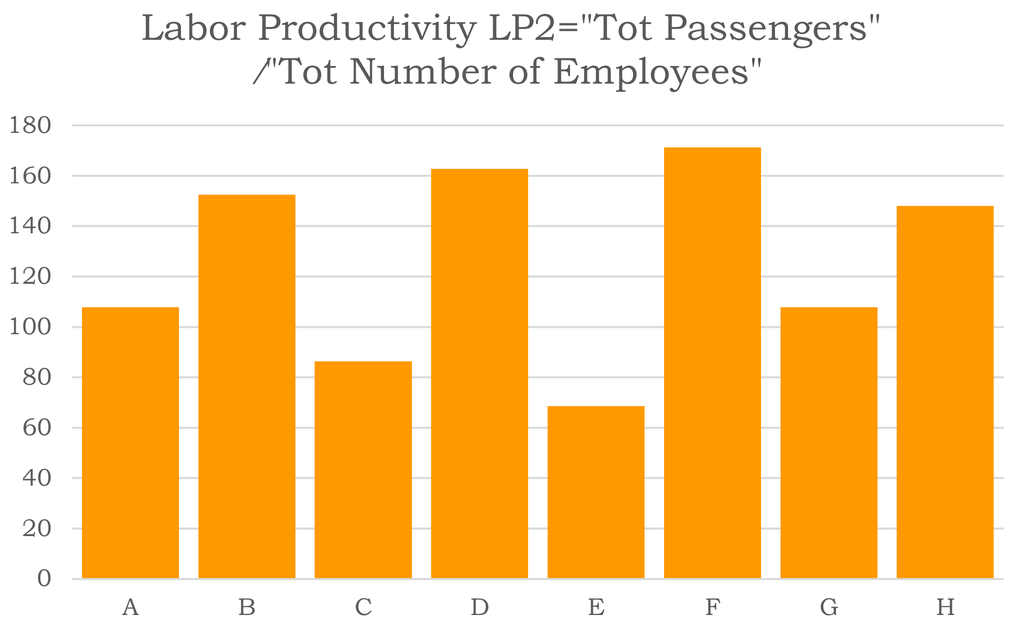
- A **partial productivity** indicator can be defined as the ratio of final output produced to a single input factor used in the production process.

- For instance labor productivity index can be defined as:
  Labor Productivity = Q/L

- LP1: Tot kilometres/Tot Number of Employees
- LP2: Tot Passengers/Tot Number of Employees

Labor Productivity in Transportation Sector





Total Productivity Indicators

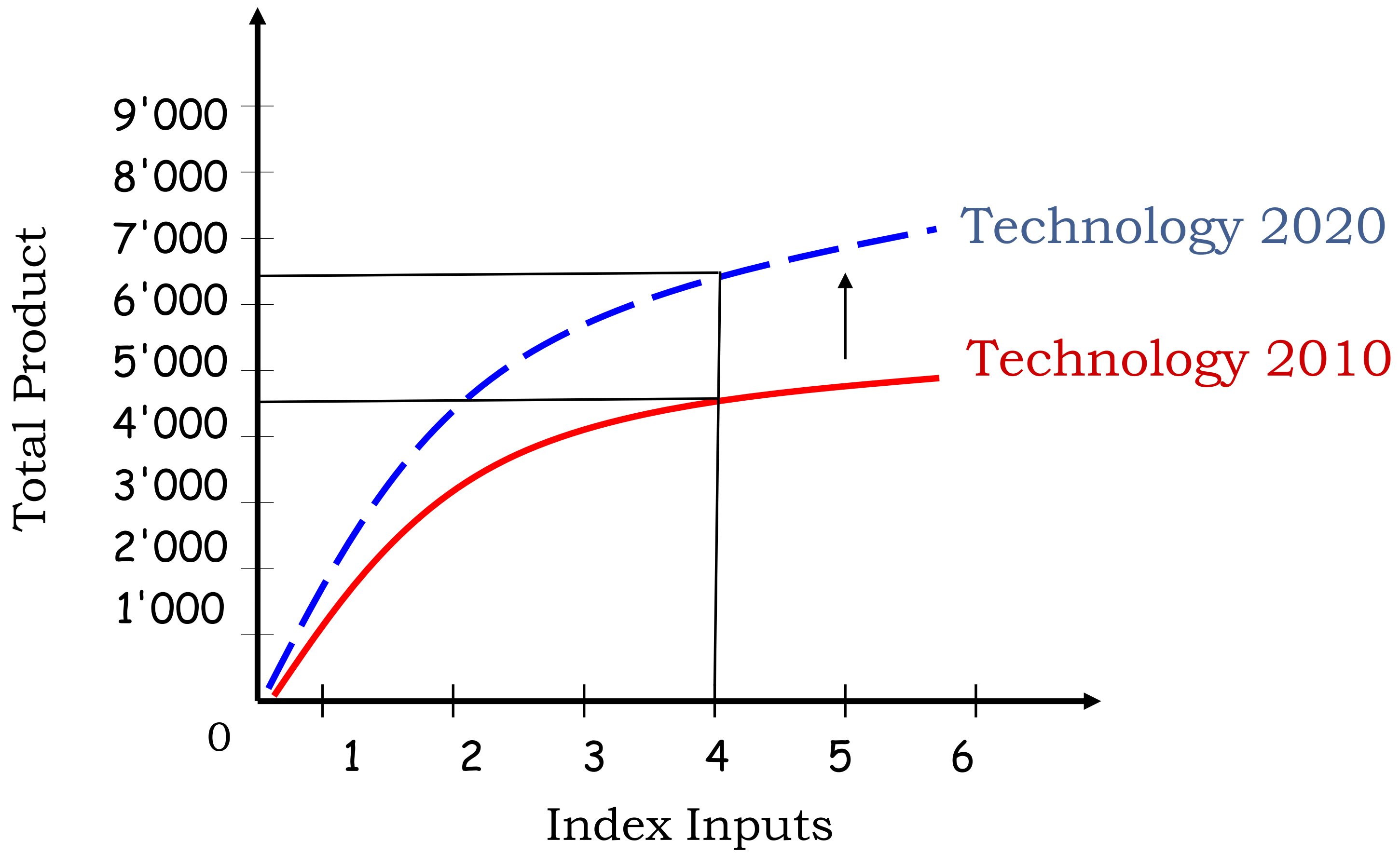
- **Total factor productivity (TFP)** can be defined as the ratio of a total output to an index of total input, that is:

$$\mathbf{TFP} = \frac{\mathbf{Output}}{\mathbf{Input\ index}}$$

- TFP measures are generally used as relative measures between at least two firms (country) or one firm (country) in two different years.

- The weights used in these indicators are usually cost shares in the input index

Technological Change and the Production Function



Appendix

TFP important issues

- TFP can differ between firms at one point in time for the following 3 reasons:
  - Cost efficiency,
  - Scale efficiency differences,
- Moreover TFP can differ between firms over time (ceteris paribus) for an additional reason:
  - Technical change (frontier shift).

Total factor productivity change

- The approaches to the TFP growth can be divided into two groups:
  - Price-based index numbers
  - Indicators based on cost frontier or production frontier analysis (SFA, DEA)

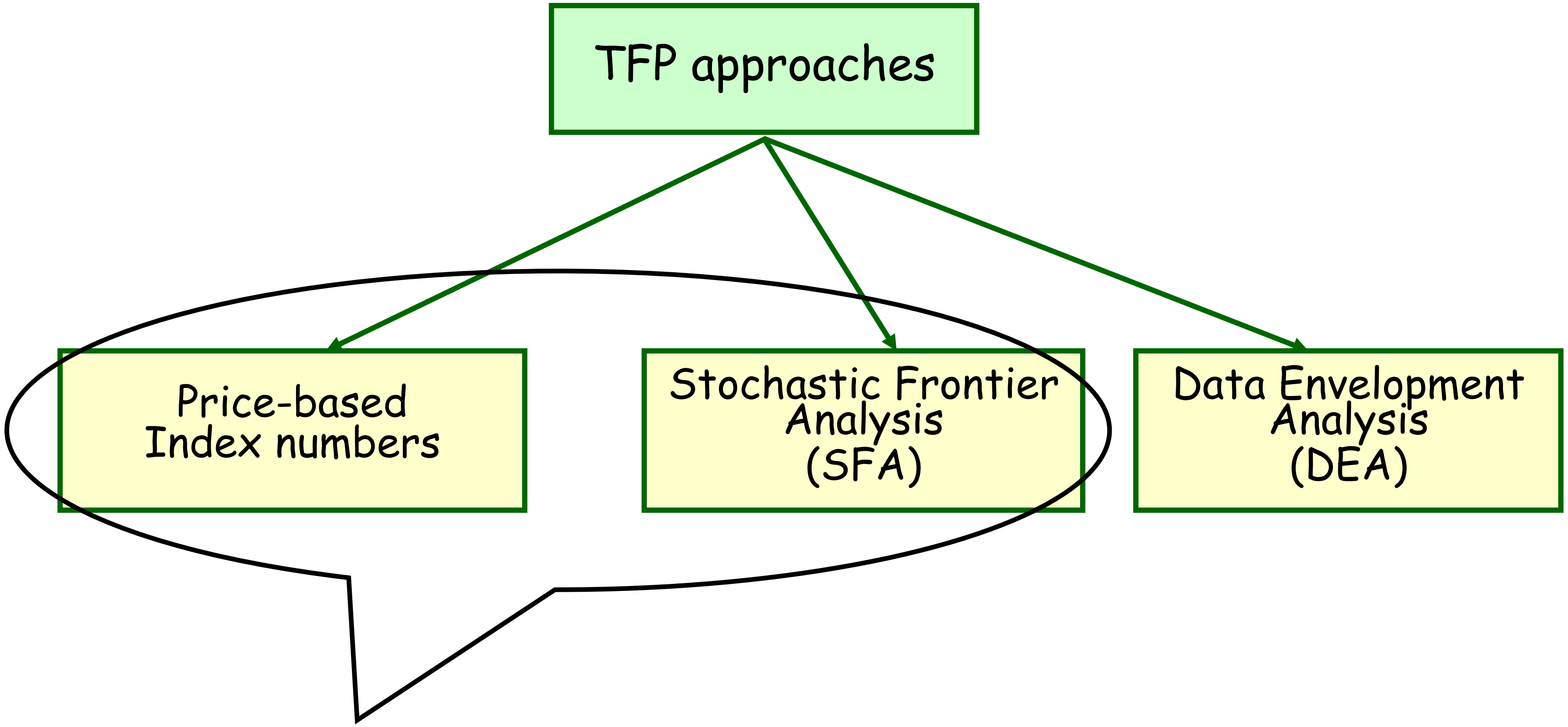
- **Price-based index numbers** are simple TFP indicators: Tornqvist index and Fisher Ideal index are two of the most commonly used measures.

- An important shortcoming of these measures is that they can only give an overall estimate of total growth that could be driven by changes in efficiency, scale economies as well as technical progress.

- The decomposition of growth into different components is only possible using a cost or production frontier model.

- The results are, however, sensitive to the adopted weights and the measurement units of input factors, thus difficult to interpret.

Empirical methods to measure TFP



- We will discuss only these approaches

Tornqvist TFP change index

- The Tornqvist index in this form provides an indication of the overall growth rate:

$$\ln(TFP_t / TFP_s) = \ln \frac{Output\ Index_{st}}{Input\ Index_{st}} = \ln Output\ Index_{st} - \ln Input\ Index_{st} =$$

$$= \frac{1}{2}\sum_{i=1}^{N}(v_{is}+v_{it})(\ln y_{it} - \ln y_{is}) - \frac{1}{2}\sum_{j=1}^{K}(\omega_{is}+\omega_{it})(\ln x_{it} - \ln x_{is})$$

Revenue share

Cost share

Quantity of the ith output in the tth period

Quantity of the ith input in the tth period

Example: Tornqvist TFP change index (one output)

0.5*((1+1)*(LN(G3/G2)))

FIRM	TIME	L	C	SL	SC	Q	Output index	Input Index	TFP
A	1	5	30000	0.72	0.28	24000000			
A	2	5	32000	0.65	0.35	26000000	0.080	0.020	0.060
A	3	5	32000	0.66	0.34	30000000	0.143	0.000	0.143
A	4	5	32100	0.72	0.34	31000000	0.033	0.001	0.032
A	5	4.8	32100	0.65	0.35	31500000	0.016	-0.028	0.044
A	6	4.8	32100	0.66	0.34	32000000	0.016	0.000	0.016

=(0.5*((E3+E2)*(LN(C3/C2)))) +(0.5*((F3+F2)*(LN(D3/D2))))