Develpment of NSCBC for compressible Navier-Stokes equations in OpenFOAM®: Subsonic Non-Reflecting Outflow

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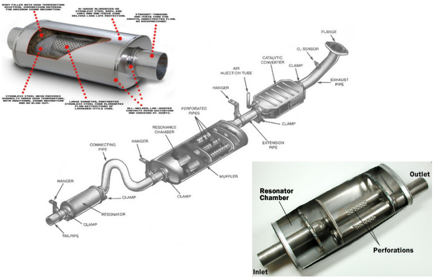
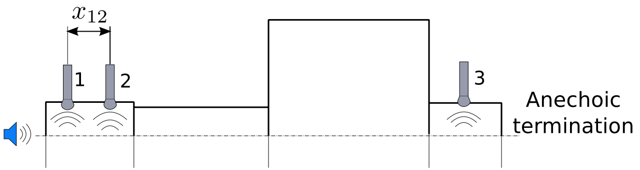
Introduction

An ideal muffler for a ICE should work as a **low pass filter**: attenuation should be applied to the fluctuating flow which is associated with the acoustic pressure fluctuation, while the steady or mean flow should be allowed to pass unimpeded through the muffler.



- The performance of a silencer can be described by two key parameters:
  - the **attenuation of the pressure fluctuations** crossing the muffler over a pre-defined wide frequency range, usually 20-2000 Hz
  - the **pressure drop** associated with convective and dissipative effects of the mean flow: it affects both the acoustic attenuation performance and the “back pressure” seen by the engine

Acoustic performance of silencers



The acoustic performance of acoustic silencers is determined by the **Transmission Loss**, defined as the ratio of Sound Pressure Level spectrums (L_p) of **incident waves**:

$$TL(f_n) = 20 log_{10} \frac{{p_{ups}}^{+}(f_n)}{{p_{dws}}^{+}(f_n)}$$

- all acoustic frequencies of the field of interest must be excited: **large-band acoustic sources**
- **anechoic termination** is needed

Since the gas pressure in the domain is the result of superimposition of incident waves coming from the source and reflections from the boundaries, *it is impossible to directly measure the incident components of the pressure pulsation*. Hence, **algorithms for data postprocessing** based on the general hypotheses of the linear acoustics are used (**two-sensor method**).

Motivation

The **objective of this study** is:

- to develop a three-dimensional time-domain approach based on the CFD simulation to evaluate the transmission loss of silencers and resonators without and with mean flow
- to examine the influence of mean flow on the acoustic attenuation performance of complex silencers

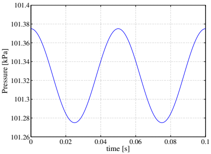
... but:

- an **inlet b.c. to model different acoustic sources** is needed
- **acoustic simulations of compressible flows** require an accurate control of wave reflections from the computational domain boundaries. Acoustic waves are often modified by numerical dissipation
- the `waveTransmisive` b.c. in OpenFOAM® **is not perfectly non reflecting**; small acoustic waves are reflected to the inner domain
- there is the **need for sophisticated boundary conditions (NSCBC)**, which can handle correctly the transmission and the reflection of acoustic waves on boundaries

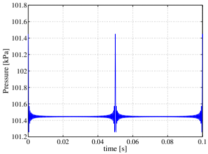
acousticSourceFvPatchField

A **boundary condition** `acousticSourceFvPatchField` **to model different types of acoustic sources** has been developed in the OpenFOAM® technology.

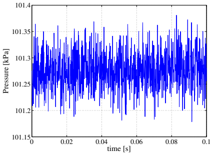
SINGLE SINUSOID



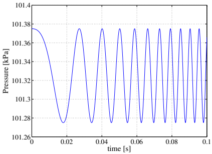
PULSE



WHITE NOISE



FREQUENCY SWEEP




```
inlet
{
type            acousticSourceTotalPressure;
sourceType      "whiteNoise";
U               U;
phi             phi;
rho             rho;
psi             none;
gamma           1.4;
refPressure     100000;
f0              10;
fn              2000;
step            10;
amplitude       50;
value           uniform 100000;
}
```

- **Different kind of time-varying perturbations** are applied at the inlet boundary patch
- Ad-hoc developed **run time controls** ensure correct case setup and avoid **aliasing** due to poor frequency resolution or to **non physical frequency signals** distorting the spectrum in the chosen range (Oppenheim and Schaffer)

Non-reflecting (anechoic) outlet b.c.

THEORY

- Implementation of a true non-reflecting outlet based on the NSCBC theory: variables which are not imposed by physical boundary conditions are computed on the boundaries by solving the conservation equations as in the domain
- Wave propagation is assumed to be associated only with the hyperbolic part of the Navier-Stokes equations, **waves associated with the diffusion process are neglected**
- In characteristics analysis, **absence of reflection is enforced by correcting the amplitude of the ingoing characteristic** (wave reflected by the boundary) to zero. Partial reflection is needed for a well-posed problem
- **Local viscous terms and tranverse terms have been neglected in the formulation of the governing equations (LODI)**
- Also:
  - the method allows a control of the different waves crossing the boundaries
  - no extrapolation procedure is used

VALIDATION

- Shock Tube
- Engine Silencers

Non-reflecting (anechoic) outlet b.c.

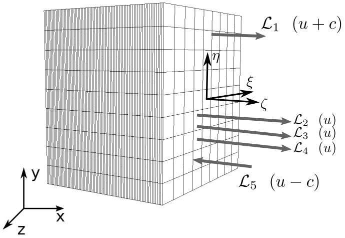
For each cell face at the boundary end, the governing equations written in a **local reference frame** (**ξ, η, ζ**) are:

Continuity:

$$\frac{\partial \rho}{\partial t} + d_1 + \frac{\partial \rho u_2}{\partial \eta} + \frac{\partial \rho u_3}{\partial \zeta} = 0$$

Momentum:

$$\begin{aligned}
\frac{\partial \rho u_1}{\partial t} + u_1 d_1 + \rho d_3 + \frac{\partial \rho u_1 u_2}{\partial \eta} + \frac{\partial \rho u_1 u_3}{\partial \zeta} &= 0 \\
\frac{\partial \rho u_2}{\partial t} + u_2 d_1 + \rho d_4 + \frac{\partial \rho u_2 u_2}{\partial \eta} + \frac{\partial \rho u_2 u_3}{\partial \zeta} &= -\frac{\partial p}{\partial \eta} \\
\frac{\partial \rho u_3}{\partial t} + u_3 d_1 + \rho d_5 + \frac{\partial \rho u_3 u_2}{\partial \eta} + \frac{\partial \rho u_3 u_3}{\partial \zeta} &= -\frac{\partial p}{\partial \zeta}
\end{aligned}$$



Each reference frame has its origin in the cell face center and the vector ζ is set as perpendicular to the cell face.

Energy:

$$\frac{\partial \rho E}{\partial t} + \frac{1}{2}(u_k \cdot u_k) d_1 + \frac{d_2}{\gamma - 1} + \rho u_1 d_3 + \rho u_2 d_4 + \rho u_3 d_5 + \frac{\partial [(\rho E + p) u_2]}{\partial \eta} + \frac{\partial [(\rho E + p) u_3]}{\partial \zeta} = -\nabla \cdot q$$

Extension to local Cartesian coordinates

- For each cell face at the boundary end, a local reference frame (ξ, η, ζ) has been defined:

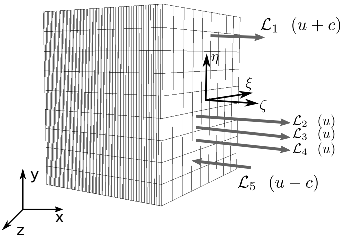
$$
\begin{aligned}
x &= x(\xi, \eta, \zeta) \\
y &= y(\xi, \eta, \zeta) \\
z &= z(\xi, \eta, \zeta)
\end{aligned}
$$

- Each reference frame has its origin in the cell face center and the vector ζ is set as perpendicular to the cell face.
- The governing equations for the **global reference frame** take the form:

$$\frac{\partial \boldsymbol{U}}{\partial t} + \frac{\partial \boldsymbol{F}_1}{\partial x} + \frac{\partial \boldsymbol{F}_2}{\partial y} + \frac{\partial \boldsymbol{F}_3}{\partial z} = -\boldsymbol{\nabla} p$$

where

$$
\begin{aligned}
\boldsymbol{U} &= \frac{\hat{\boldsymbol{U}}}{J} \\
\frac{\partial \boldsymbol{F}_1}{\partial x} &= \frac{\hat{\boldsymbol{F}}_1 x_\xi + \hat{\boldsymbol{F}}_2 x_\eta + \hat{\boldsymbol{F}}_3 x_\zeta}{\boldsymbol{J}} \\
\frac{\partial \boldsymbol{F}_2}{\partial y} &= \frac{\hat{\boldsymbol{F}}_1 y_\xi + \hat{\boldsymbol{F}}_2 y_\eta + \hat{\boldsymbol{F}}_3 y_\zeta}{\boldsymbol{J}} \\
\frac{\partial \boldsymbol{F}_3}{\partial z} &= \frac{\hat{\boldsymbol{F}}_1 z_\xi + \hat{\boldsymbol{F}}_2 z_\eta + \hat{\boldsymbol{F}}_3 z_\zeta}{\boldsymbol{J}}
\end{aligned}
$$



NSCBC approach for b.c.

The vector d given by characteristic analysis (Thompson) can be written as:

$$\boldsymbol{d} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{bmatrix} = \begin{bmatrix} \frac{\partial m_1}{\partial \xi} \\ \frac{\partial c^2 m_1}{\partial \xi} + u_1 \frac{\partial p}{\partial \xi} \\ u_1 \frac{\partial u_1}{\partial \xi} + \frac{1}{\rho} \frac{\partial p}{\partial \xi} \\ u_1 \frac{\partial u_2}{\partial \xi} \\ u_1 \frac{\partial u_3}{\partial \xi} \end{bmatrix} = \begin{bmatrix} \frac{1}{c^2} \left[L_2 + \frac{1}{2} (L_5 + L_1) \right] \\ \frac{1}{2} (L_5 + L_1) \\ \frac{1}{\rho c} (L_5 - L_1) \\ L_3 \\ L_4 \end{bmatrix}$$

where:

$$\boldsymbol{L} = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \\ L_5 \end{bmatrix} = \begin{bmatrix} \lambda_1 \left(\frac{\partial p}{\partial \xi} - \rho c \frac{\partial u_1}{\partial \xi} \right) \\ \lambda_2 \left(c^2 \frac{\partial \rho}{\partial \xi} - \frac{\partial p}{\partial \xi} \right) \\ \lambda_3 \frac{\partial u_2}{\partial \xi} \\ \lambda_4 \frac{\partial u_3}{\partial \xi} \\ \lambda_5 \left(\frac{\partial p}{\partial \xi} + \rho c \frac{\partial u_1}{\partial \xi} \right) \end{bmatrix}$$

$$\lambda_1 = u_1 - c$$
$$\lambda_2 = \lambda_3 = \lambda_4 = u_1$$
$$\lambda_5 = u_1 + c$$

λ_i is the characteristic velocity associated to L_i

- L_i is the amplitude variation of the i_{th} characteristic wave crossing the boundary
- L_1 is the **incoming characteristic** reflected by the boundary

Subsonic non-reflecting outflow

- A perfectly subsonic non-reflecting outflow (L_1=0) might lead to an ill-posed problem (**mean pressure at the outlet would result to be undetermined**)
- **Corrections must be added** to the treatment of the b.c. to make the problem well posed. The amplitude of the incoming wave is then set as:

$$L_1 = K\left(p - p_\infty\right)$$

that in global coordinates becomes:

$$L_1 = \sigma \cdot \frac{|1 - \mathrm{M}^2|}{\sqrt{2}J\rho l}$$

  - M is the max. Mach number defined over the patch
  - σ is a constant leading the pressure drift. $0.1 < \sigma < \pi$ (Strickwerda)
  - l is a characteristic size of the domain
  - J is the Jacobian marix

The resulting formulation makes the b.c. **partially non reflecting** and the **problem well-posed**.

Numerical solution

- Governing equations have been solved by a **multistage time stepping scheme** in $t^{n,k}$:

$$t^{n,k} \equiv t^n + k \cdot \delta t = t^n + \frac{k}{K}\Delta t \qquad k \in [1;K] \tag{1}$$

  where $t^{n,k}$ is a variable local fractional time-step.

- The method consists of the iteration of **two main steps**:

  1) Evaluation of backward spatial derivatives at t^n and of the fluxes at time $t^n + \frac{k}{K}\Delta t$; conservation equations are solved sequentially. The solution is first order in time.

  2) Fluxes and source terms calculated at the previous step are used to find the solution at time $t^n + \frac{k+1}{K}\Delta t$. The time accuracy of this method is of the second order at this stage.

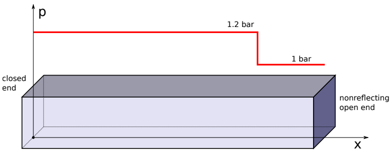
- The process is iterated until the solution at the new time $t^{n+1} \equiv t + \Delta t$ is calculated.

- The time stepping algorithm:
  - requires a **relatively small amount of memory storage**
  - it is **more stable and accurate**
  - it allows for **larger global time steps** in the simulation than a traditional explicit method.

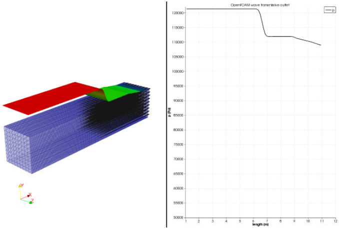
NonReflecting NSCBC vs `waveTransmissive`

SHOCK TUBE simulation:

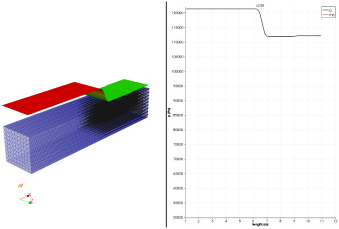
- 40500 hexahedral cells
- $p_{max} = 1.2$ bar
- $p_0 = 1.0$ bar, $T_0 = 293$ K



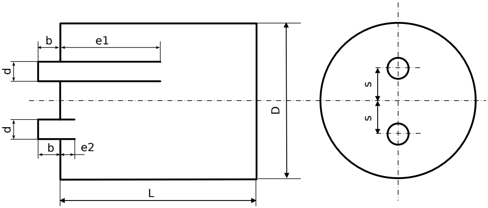
OpenFOAM wave transmissive BC



LODI nonreflecting BC



Case study: reverse-flow silencers

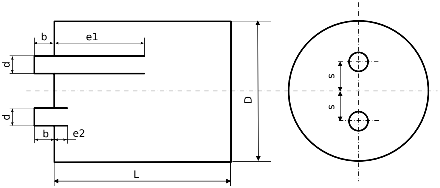


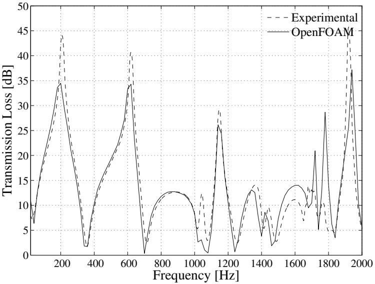
Silencer	l [mm]	w [mm]	d [mm]	b [mm]	e1 [mm]	e2 [mm]	s [mm]
RC-l1	494	197	50	17	17	17	50
RC-l2	494	197	50	17	257	17	50
RC-m	377	197	50	17	167	17	50
RC-s	127	197	50	17	17	17	50

Case setup

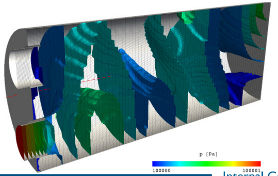
- **solver**: `pisoFoam`
- **temporal discretisation**: Crank-Nicholson scheme
- **differential operators**: standard finite volume discretisation of Gaussian integration
- **working fluid**: air
- **boundary conditions**:
  - inlet : pressure pulse with frequency content f$\in$[20;2000] Hz (step 20 Hz)
  - outlet: non-reflective NSCBC anechoic boundary condition
  - walls : adiabatic, no-slip condition
- **time step** limited by the CFL criterion (max. Courant=0.4). Max time-step: 10^{-6} s
- **perturbation period** $T = 1/\min(f_{min}, f_{step})$. Two periods were needed to reach full convergence in the simulation. Max time step used guarantees a sampling frequency that satisfies the Nyquist sampling law

Reverse-flow silencers: long chamber 1 (AVL)

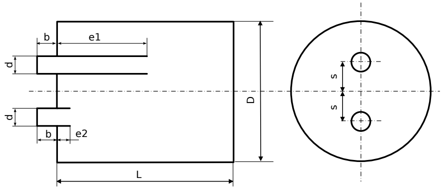


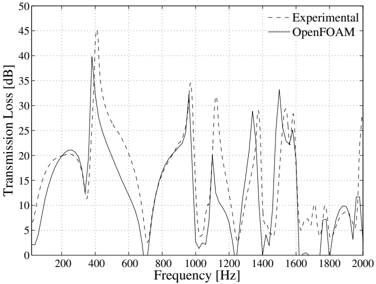


Silencer	l [mm]	w [mm]	d [mm]	b [mm]	e1 [mm]	e2 [mm]	s [mm]
RC-l1	494	197	50	17	17	17	50

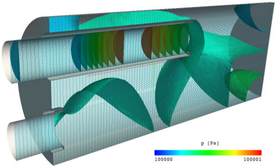


Reverse-flow silencers: long chamber 2 (AVL)

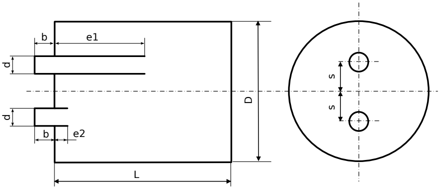


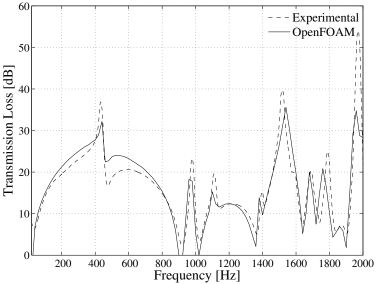


Silencer	l [mm]	w [mm]	d [mm]	b [mm]	e1 [mm]	e2 [mm]	s [mm]
RC-l2	494	197	50	17	257	17	50

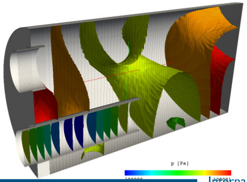


Reverse-flow silencers: mid chamber (AVL)

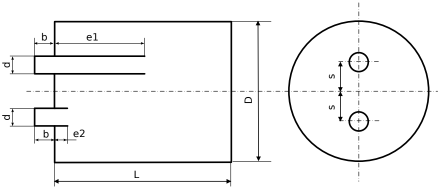


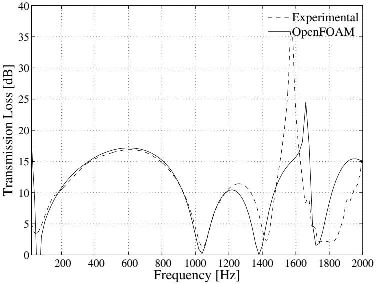


Silencer	l [mm]	w [mm]	d [mm]	b [mm]	e1 [mm]	e2 [mm]	s [mm]
RC-m	377	197	50	17	167	17	50

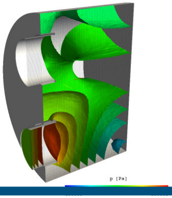


Reverse-flow silencers: short chamber (AVL)

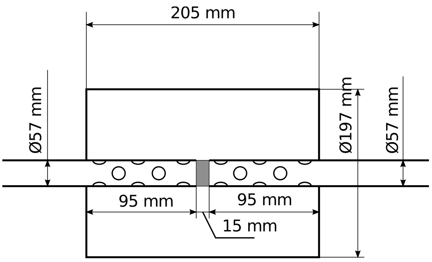


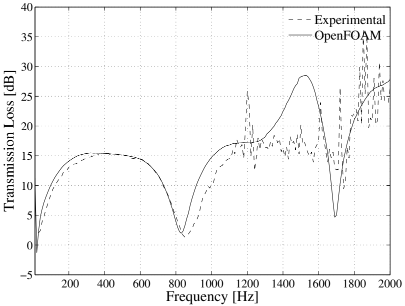


Silencer	l [mm]	w [mm]	d [mm]	b [mm]	e1 [mm]	e2 [mm]	s [mm]
RC-s	127	197	50	17	17	17	50



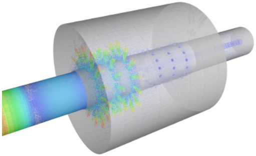
Single-plug perforated muffler (AVL)





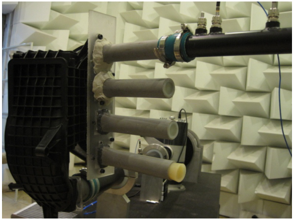
Silencer	d [mm]	D [mm]	l_a [mm]	l_p [mm]	t_w [mm]	d_h [mm]	porosity [%]
P1	57	197	95	15	2	5	5

- porosity = 5%
- plug length = 95 mm
- chamber length= 205 mm
- zero mean flow



Conclusions

- NSCBC written in local coordinates for compressible subsonic Navier-Stokes equations
- **Non-reflecting condition for subsonic outflows** based on the NSCBC approach
- **Multistage time stepping scheme** for the semi-implicit solution of the NSCBC
  - faster convergency
  - allows for higher timesteps when coupled with a transient solver
  - improved robustness
- Validation on non-linear acoustics



CURRENT WORK

- Prediction of the acoustic performance of complex devices with **non-zero mean flow**
- **Application of the NSCBC to compressible LES simulation**: in-cylinder cold flow
- **LES simulation**: implementation of a synthetic turbulent inlet b.c.

Thanks for your attention!

Internal Combustion Engine
Group

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