Synthetic aperture non-synchronous measurements（合成孔径非同步测量）

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合成孔径阵列测量理论与方法

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上海交通大學
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研究背景(1)

Lenna

Cameraman

光学相机的CCD阵列

如果我们有一个传声器阵列，
是否可以对**声源/声场可视化**?

研究问题

近场声全息技术 **声波束形成**

近场+ 低频 **远场+ 高频**

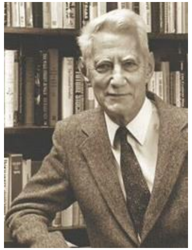
奈奎斯特-香农采样定理

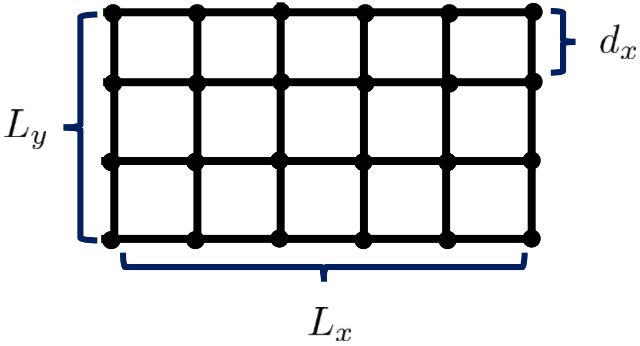
$c = 344m/s$

$L = \min\{L_x, L_y\}$

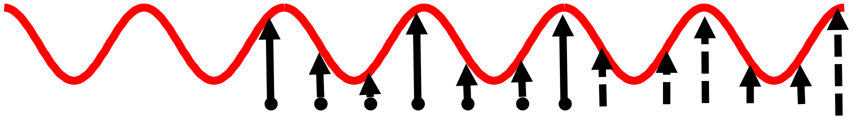
$f_{\min} = \frac{c}{L}$

$f_{\max} = \frac{c}{2d_x}$





对于有限带宽的声源

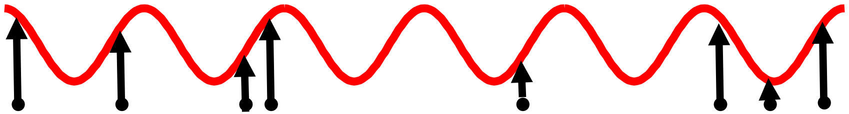


单次测量

阵列测量的外推

- **有外推误差的困扰**
- **没有新信息**

对于声源(可以稀疏表示)

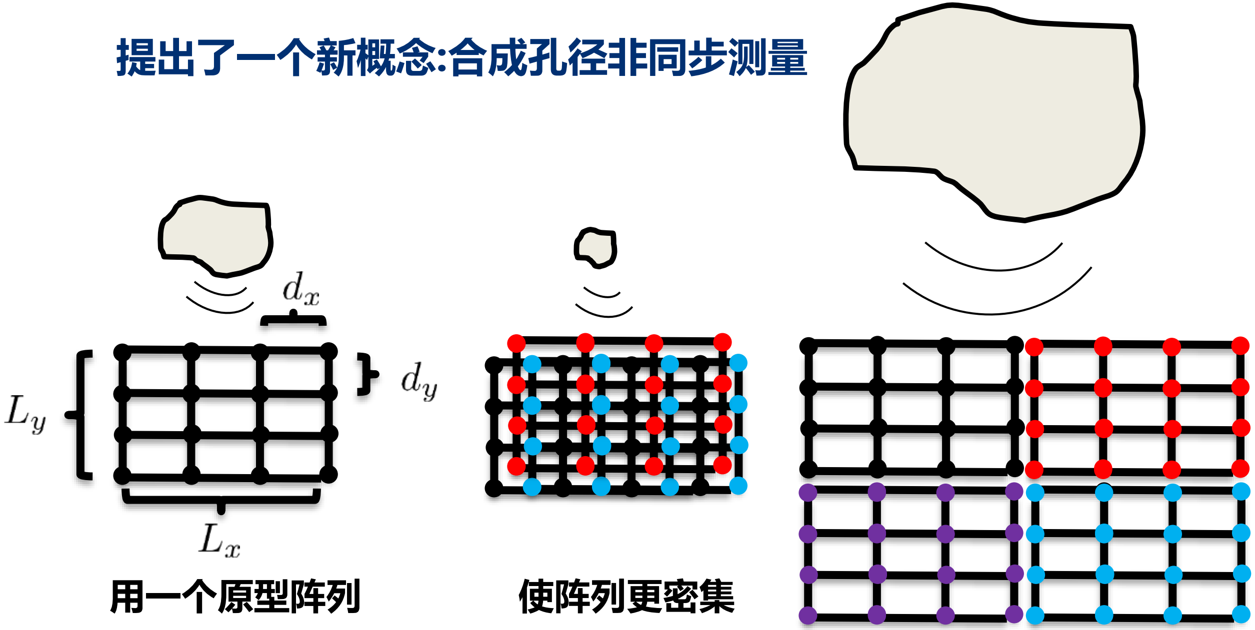


随机测量（压缩感知）

- **通常系统需要重新设计**
- **在声学中很难构造感知矩阵**

合成孔径非同步测量(1)

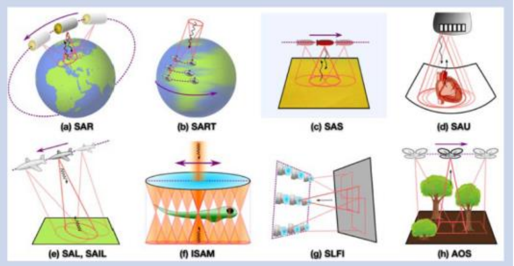
提出了一个新概念:合成孔径非同步测量

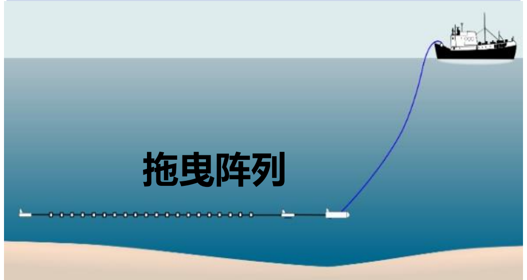


用一个原型阵列

使阵列更密集

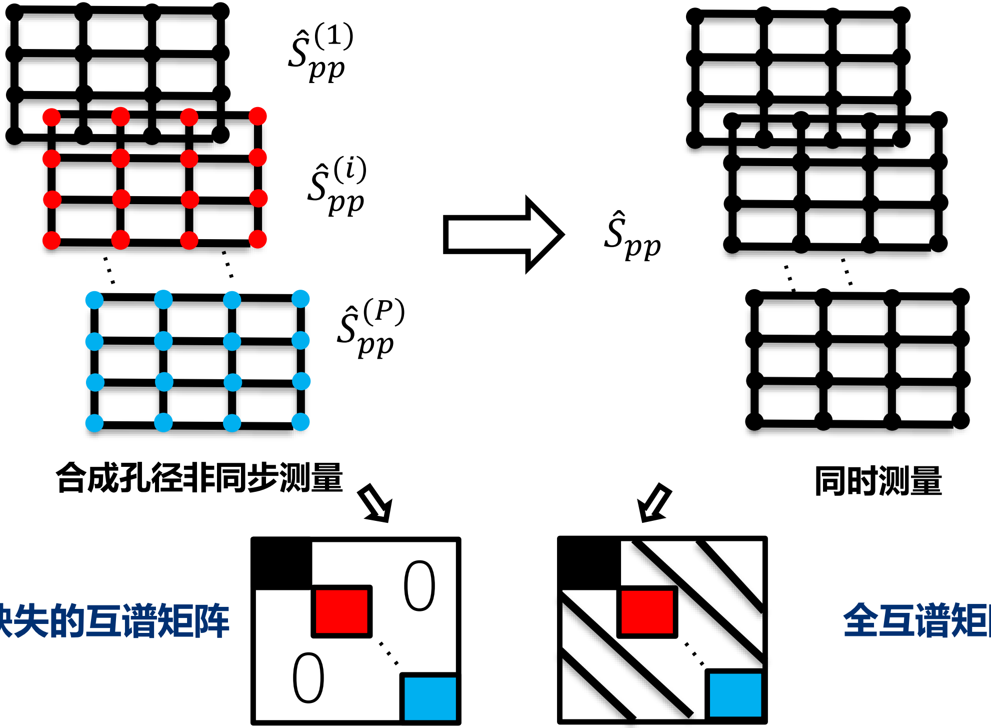
使阵列更大





- 合成孔径雷达(SAR)
- 合成孔径射电望远镜（SART）
- 合成孔径声纳（SAS）
- 合成孔径超声（SAU）
- 合成孔径激光雷达（SAL）
- 干涉式合成孔径显微镜（ISAM）
- 结构化的光场成像（SLFI）机载光学切片

合成孔径非同步测量(2)



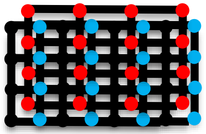
数据缺失的互谱矩阵

全互谱矩阵

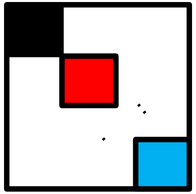
传播公式 $p(r_{m,i},\omega;\zeta)=\int_{\Gamma}G(r_{m,i},\omega|\mathbf{r})s(\mathbf{r},\omega;\zeta)\,\mathrm{d}\Gamma(\mathbf{r})+n(r_{m,i},\omega;\zeta).$

合成孔径非同步测量(3)

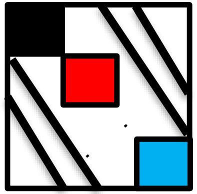
非侵入式方法



合成孔径非同步测量

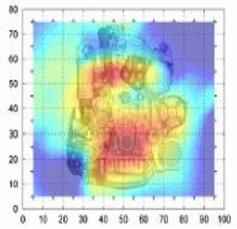


数据缺失的互谱矩阵

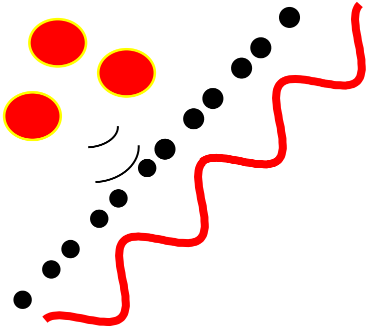


互谱矩阵补全

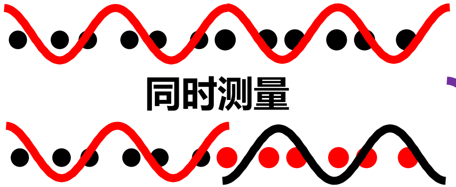
反馈给经典的逆问题算法



声源重构

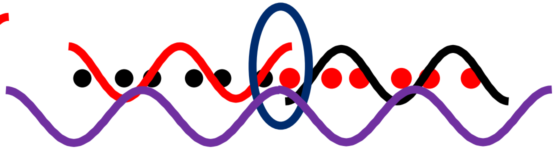


低秩:信号由有限数量的不相关源产生(具有某种潜在的结构)



同时测量

非同步测量

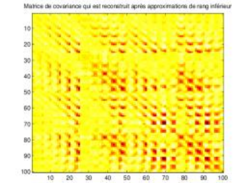


空间基函数

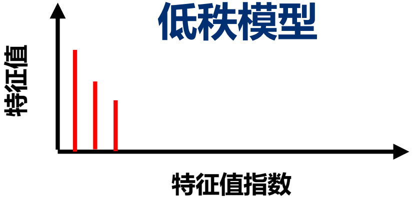
增强声场的空间连续性

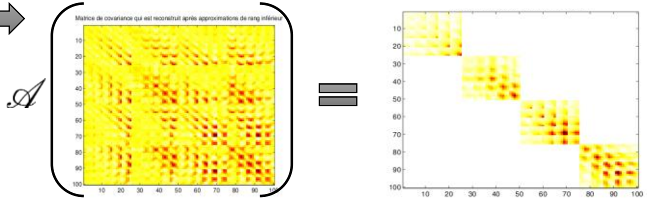
合成孔径非同步测量(4)

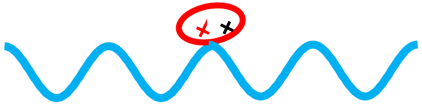
$$\text{find} \quad \mathbf{S} \in \mathbf{C}^{MP\times MP}$$



$$\text{such that} \quad \begin{cases} \operatorname{Rank}(\mathbf{S}) = r \\ \|\mathscr{A}(\mathbf{S}) - \hat{\mathbf{S}}_{pp}^{m}\|_F \le \epsilon \\ \mathbf{S}^* = \mathbf{S} \succeq 0 \\ \|\mathbf{\Psi}\mathbf{S}\mathbf{\Psi}^* - \mathbf{S}\|_F \le \varepsilon \end{cases}$$







Liang Yu etc. Spectral matrix completion by Cyclic Projection and application to sound source reconstruction from non-synchronous measurements, Journal of Sound and Vibration, 2016

合成孔径非同步测量(5)

核范数最小化

$$
\begin{aligned}
\underset{\mathbf{S}}{\text{minimize}} \quad & \|\mathbf{S}\|_* \\
\text{subject to} \quad & \|\mathscr{A}(\mathbf{S}) - \hat{\mathbf{S}}_{pp}^m\|_F \le \epsilon \\
& \|\mathbf{\Psi}\mathbf{S}\mathbf{\Psi}^* - \mathbf{S}\|_F \le \varepsilon \\
& \mathbf{S}^* = \mathbf{S} \succeq 0,
\end{aligned}
$$

快速迭代收缩阈值算法(FISTA)

1: 初始$\mathbf{G}_0 = \mathbf{S}_0 = 0 \in \mathbb{C}^{MP \times MP}$，$t_1 = 1$；$\mu$ 是步长，
λ_0 是初始正则化参数且 λ_d 是初始正则化参数
2: 当 $\lambda_k \ge \lambda_d$ 时
3: For $1:N_m$ （N_m 是每个正则化的最大迭代步数）
4: $\mathbf{G}_k = \mathbf{G}_{k-1} - \mu(\mathcal{A}(\mathbf{G}_{k-1}) - \hat{\mathbf{S}}_{pp}^m)$
5: $\tilde{\mathbf{S}}_k = \text{shrink}(\mathbf{G}_k, \lambda_k \mu)$
6: $\mathbf{S}_k = \mathbf{\Psi}\tilde{\mathbf{S}}_k\mathbf{\Psi}^*$
7: $t_{k+1} = \frac{1}{2}(1 + \sqrt{1 + 4t_k^2})$
8: $\mathbf{G}_{k+1} = \mathbf{S}_k + \frac{t_k - 1}{t_{k+1}}(\mathbf{S}_k - \mathbf{S}_{k-1})$
9: End
10: If Stopping criteria $\le SC$
11: break
12: End for if
13: $\lambda_k = \max(\eta\lambda_{k-1}, \lambda_d)$
14: 回到第 2 步

增广拉格朗日乘子法（ALM）

1: 初始$\mathbf{Y}_1 = \mathbf{E}_1 = \mathbf{0} \in \mathbb{C}^{MP \times MP}$ (零矩阵)；数据缺失的互谱矩阵是$\hat{\mathbf{S}}_{pp}^m$ 且 μ 是给定的惩罚参数。
2: For $k = 1:N_m$ （N_m 是最大迭代步数）
3: $\mathbf{G}_k = \hat{\mathbf{S}}_{pp}^m - \mathbf{E}_k + \mu^{-1}\mathbf{Y}_k$
4: $\tilde{\mathbf{S}}_k = \text{shrink}(\mathbf{G}_k, \mu)$
5: $\mathbf{S}_{k+1} = \mathbf{\Psi}\tilde{\mathbf{S}}_k\mathbf{\Psi}^H$
6: $\mathbf{E}_{k+1} = \overline{\mathcal{A}}(\hat{\mathbf{S}}_{pp}^m - \mathbf{S}_{k+1} + \mu^{-1}\mathbf{Y}_k)$
7: $\mathbf{Y}_{k+1} = \mathbf{Y}_k + \mu(\hat{\mathbf{S}}_{pp}^m - \mathbf{S}_{k+1} - \mathbf{E}_{k+1})$
8: If Stopping criterion $\le \varepsilon_1$
9: break
10: End if
11: End
12: Output $\mathbf{S}_{k+1}$ (全互谱矩阵)

交替方向乘子法（ADMM）

1: 初始$\mathbf{Y}_1 = 0 \in \mathbb{C}^{MP \times MP}$,随机分配矩阵$\mathbf{M}_1 \in \mathbb{C}^{MP \times MP}$
且输入矩阵$\hat{\mathbf{S}}_{pp}^m$. μ, γ 和 λ是给是定参数.
2: For $k = 1:N_m$ （N_m 是最大迭代步数）
3: $\mathbf{G}_k = \mathbf{M}_k - (1/\mu)\mathbf{Y}_k$
4: $\tilde{\mathbf{S}}_k = \text{shrink}(\mathbf{G}_k, \lambda/\mu)$
5: $\mathbf{S}_{k+1} = \mathbf{\Psi}\tilde{\mathbf{S}}_k\mathbf{\Psi}^H$
6: $\mathbf{E}_k = \mathbf{S}_{k+1} + (1/\mu)\mathbf{Y}_k$
7: $\mathcal{A}(\mathbf{M}_{k+1}) = 1/(\mu+1)\mathcal{A}(\hat{\mathbf{S}}_{pp}^m + \mu\mathbf{E}_k)$
8: $\overline{\mathcal{A}}(\mathbf{M}_{k+1}) = \overline{\mathcal{A}}(\mathbf{E}_k)$
9: $\mathbf{Y}_{k+1} = \mathbf{Y}_k + \gamma\mu(\mathbf{S}_{k+1} - \mathbf{M}_{k+1})$
10: If Stopping criterion $\le \varepsilon_2$
11: break
12: End for if
13: End
14: 输出$\mathbf{S}_{k+1}$ (全互谱矩阵).

[1] Liang Yu etc. Acoustical source reconstruction from non-synchronous sequential measurements by Fast Iterative Shrinkage Thresholding Algorithm, Journal of Sound and Vibration, 2017

[2] Liang Yu etc. Fast iteration algorithms for implementing the acoustic beamforming of non-synchronous measurements, Mechanical Systems and Signal Processing, 2019

合成孔径非同步测量(6)

最大范数最小化

$$\underset{\boldsymbol{S}}{\text{minimize}} \quad \left\|\mathcal{A}(\boldsymbol{S}) - \hat{\boldsymbol{S}}_{pp}^{m}\right\|_F + \beta\|\boldsymbol{S}\|_{\max}$$

核范数和最大范数

能量边界

主要区别在于它们的特征向量的边界

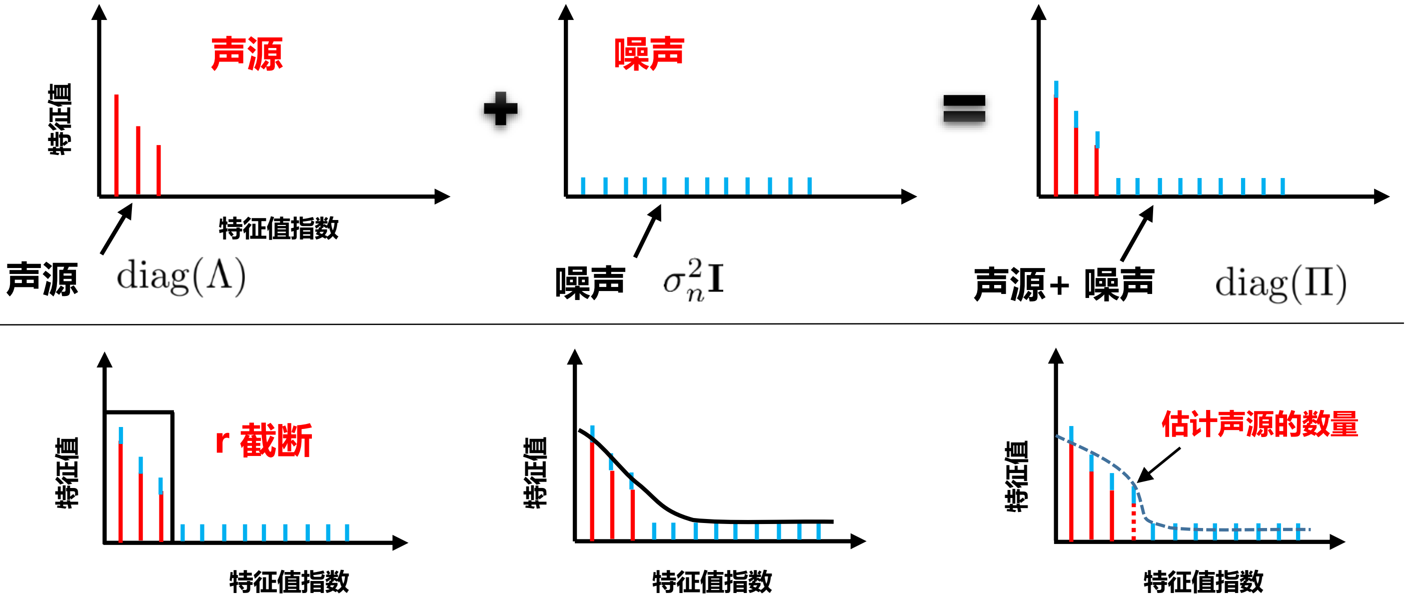
范数	定义
核范数	$\|\|\boldsymbol{S}\|\|_{*}=\inf\{\sum_j \sigma_j^2(\boldsymbol{S}): \boldsymbol{S} = \sum_j \sigma_j^2(\boldsymbol{S}) u_j v_j^T, u_j \in \mathbb{R}^m, v_j \in \mathbb{R}^n, \|u_j\|_2 = \|v_j\|_2 = 1\}$
最大范数	$\|\|\boldsymbol{S}\|\|_{\max}=\min\{\|\boldsymbol{L}\|_{2,\infty}\|\boldsymbol{R}\|_{2,\infty}: \boldsymbol{S} = \boldsymbol{L}\boldsymbol{R}^*\}$ 两个L2行范数的乘积
ℓ_2 行范数	$\|\|\boldsymbol{S}\|\|_{2,\infty}=\max_j\left(\sum_k \boldsymbol{S}_{jk}^2\right)^2$
最大范数(选择性的)	$\|\|\boldsymbol{S}\|\|_{\max}=K_G \inf\{\sum_j \sigma_j^2(\boldsymbol{S}): \boldsymbol{S} = \sum_j \sigma_j^2(\boldsymbol{S}) u_j v_j^T, u_j \in \mathbb{R}^m, v_j \in \mathbb{R}^n, \|u_j\|_\infty = \|v_j\|_\infty = 1\}$

Grothendieck 常数

一般认为，最大范数更适用于单元分布不均匀的矩阵补全问题 峰值边界

[1] Liang Yu etc. The max-norm minimization in non-synchronous measurements, Proceedings of the 23rd International Congress Acoustics, 2019
[2] Liang Yu etc. Low Complexity Modeling of Cross-Spectral Matrix and Its Application in the Non-Synchronous Measurements of Microphones Array, IEEE Access, 2021

合成孔径非同步测量(7)



秩最小化
声源个数提前估计

核范数最小化
声源个数不需要知道

最大范数最小化
高频/低信噪比

余亮等.传声器阵列特征值滤波去噪方法, 声学学报, 2021

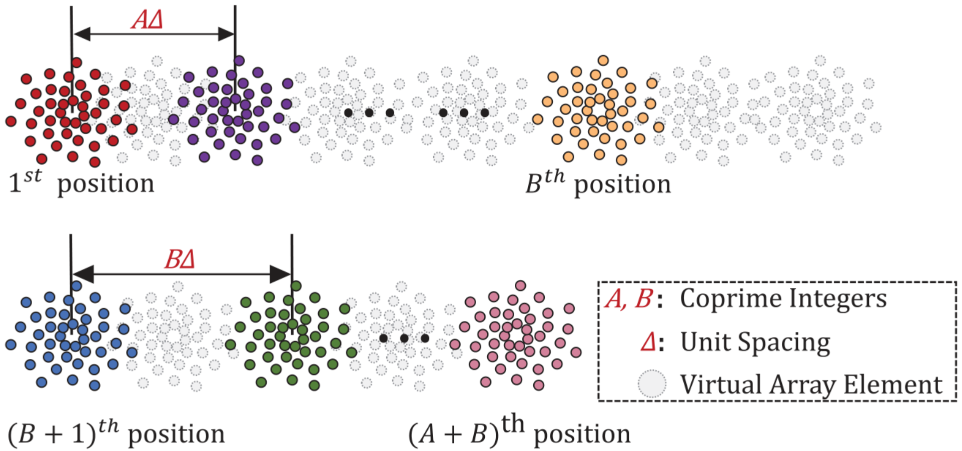
余亮等.基于最优收缩的传声器阵列部分相干噪声去噪方法, 声学技术, 2021

合成孔径非同步测量(8)

测量位置的选择

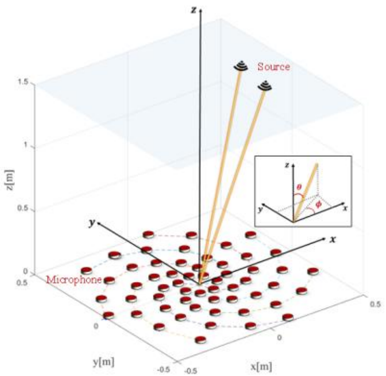
实现互质位置的合成孔径非同步测量(CP-NSM)来改进传统的合成孔径非同步测量(NSM)

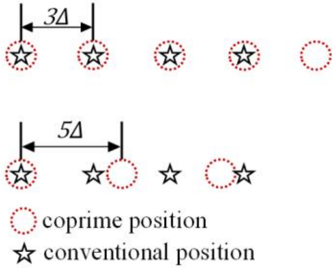
- **coprime准则**

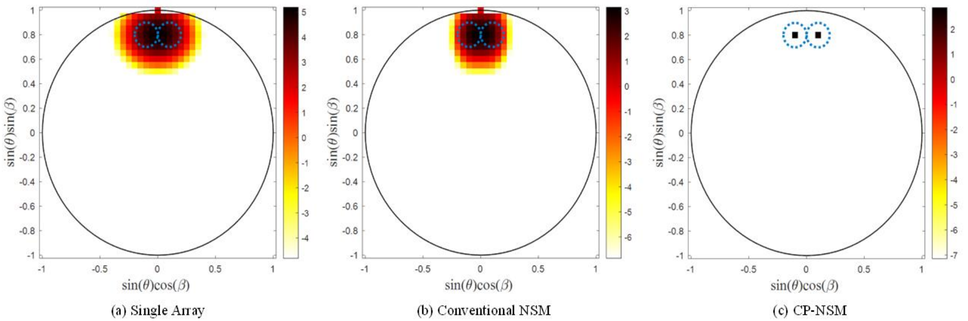


- **互质位置的阵列布置稀疏, 与传统NSM方法对比，减小了麦克风之间的互耦效应, 二次扩大了合成阵列孔径。**

合成孔径非同步测量(9)







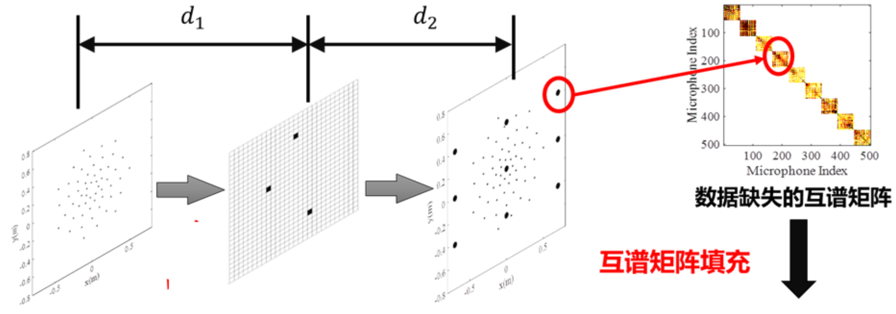
单次测量　　传统 NSM　　CP-NSM

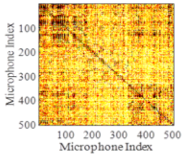
仿真结果表明：在较低的频率和较强的噪声干扰下，CP-NSM能够实现较高的空间分辨率。

Ning Chu, Qin Liu, Liang Yu etc. Non-Synchronous Measurements of a Microphone Array at Coprime Positions, IEEE Signal Processing Letters, 2021

合成孔径非同步测量(10)

功率传播前向模型



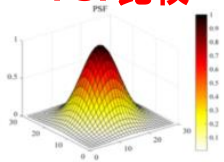


原型阵列（单个阵列） 声源平面 非同步测量阵列（单个阵列在不同位置测量）

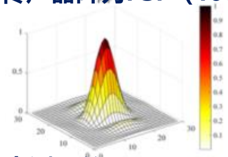
全互谱矩阵

$$
\begin{cases}
y = Hx + e \\
h_{n_1,n_2} = \dfrac{\left|g_{n_1}^H g_{n_2}\right|^2}{\left\|g_{n_1}\right\|_2^4}
\end{cases}
$$

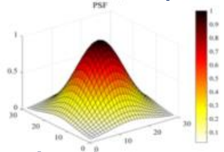
PSF比较



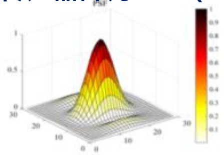
原型传声器阵列PSF（1000Hz）



非同步测量的PSF（1000Hz）



原型传声器阵列PSF（800Hz）



非同步测量的PSF（800Hz）

合成孔径非同步测量(11)

联合最大后验（JMAP）

$$J_{JMAP}(\boldsymbol{x},\boldsymbol{V_x},\boldsymbol{V_e}) = -\ln[\mathrm{p}(\boldsymbol{x},\boldsymbol{V_x},\boldsymbol{V_e},\boldsymbol{y})]$$

$$\begin{cases} \widehat{\boldsymbol{x}} = \operatorname{argmin}_{(\boldsymbol{x})} J_{JMAP}(\boldsymbol{x},\widehat{\boldsymbol{V_x}},\widehat{\boldsymbol{V_e}}) \\ \widehat{\boldsymbol{V_x}} = \operatorname{argmin}_{(\boldsymbol{V_x})} J_{JMAP}(\widehat{\boldsymbol{x}},\boldsymbol{V_x},\widehat{\boldsymbol{V_e}}) \\ \widehat{\boldsymbol{V_e}} = \operatorname{argmin}_{(\boldsymbol{V_e})} J_{JMAP}(\widehat{\boldsymbol{x}},\widehat{\boldsymbol{V_x}},\boldsymbol{V_e}) \end{cases}$$

1. 初始化参数 $\boldsymbol{V_x}$, $\boldsymbol{V_e}$, $\boldsymbol{x}$

2. 迭代 i = 1, 2, 3,… do

$$\begin{cases} \left[V_{x_{j,j}}\right]_{JMAP}^{\{c\}} = \dfrac{\beta_{x_j}^{\{c\}}}{\alpha_{x_j}^{\{c\}}} = \dfrac{\beta_{x_j}^{\{c-1\}} + \dfrac{{x_j}^2}{2}}{\alpha_{x_j}^{\{c-1\}} + \dfrac{3}{2}} \\ \left[V_{e_{i,i}}\right]_{JMAP}^{\{c\}} = \dfrac{\beta_{e_i}^{\{c\}}}{\alpha_{e_i}^{\{c\}}} = \dfrac{\beta_{e_i}^{\{c-1\}} + \dfrac{1}{2}\left(y_i - \boldsymbol{H}_i\boldsymbol{x}^{\{c-1\}}\right)^2}{\alpha_{e_i}^{\{c-1\}} + \dfrac{3}{2}} \\ \boldsymbol{x}^{\{c\}} = \left(\boldsymbol{H}^T[\boldsymbol{V_e^{\{c\}}}]^{-1}\boldsymbol{H} + [\boldsymbol{V_x^{\{c\}}}]^{-1}\right)^{-1}\boldsymbol{H}^T[\boldsymbol{V_e^{\{c\}}}]^{-1}\boldsymbol{y} \end{cases}$$

用上一次元素的迭代结果作为下一次元素的初始化参数

变分贝叶斯(VB)

目标:最小化*KL*散度【最大化变分下限】

$$\begin{aligned}\hat{q}(\theta) =& \arg\min_{q(\theta)}\{KL[q(\theta):\mathcal{P}(\boldsymbol{x},\boldsymbol{V_x},\boldsymbol{V_e},\boldsymbol{y})]\} \\ =& \arg\min_{q(\theta)}\{-\int_\theta q(\theta)\ln\frac{\mathcal{P}(\theta,\boldsymbol{y})}{q(\theta)}d\theta\},\end{aligned}$$

$$p(\boldsymbol{x},\boldsymbol{V_x},\boldsymbol{V_e},\boldsymbol{y}) \propto q_1(\boldsymbol{x})q_2(\boldsymbol{V_x})q_3(\boldsymbol{V_e})$$

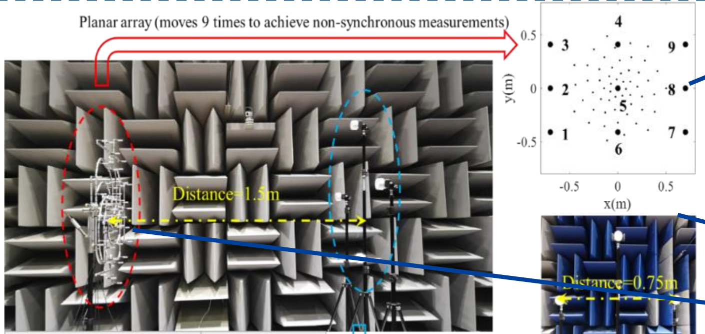
$$\begin{cases} q_1(\boldsymbol{x}) = \mathcal{N}(\boldsymbol{x}|\boldsymbol{\mu},\boldsymbol{\Sigma}) \\ q_2(\boldsymbol{V_x}) = \prod_j \mathcal{IG}\left(v_{x_j}\middle|\alpha_{x_j},\beta_{x_j}\right) \\ q_3(\boldsymbol{V_e}) = \prod_i \mathcal{IG}\left(v_{e_i}\middle|\alpha_{e_i},\beta_{e_i}\right) \end{cases}$$

1. 初始化参数$\boldsymbol{V_x}$, $\boldsymbol{V_e}$, $\boldsymbol{\Sigma}$, $\boldsymbol{\mu}$

2. 迭代 i = 1, 2, 3,… do

$$\begin{cases} \left[V_{x_{j,j}}^{-1}\right]_{VBA}^{\{c\}} = \dfrac{\alpha_{x_j}^{\{c\}}}{\beta_{x_j}^{\{c\}}} = \dfrac{\alpha_{x_j}^{\{c-1\}} + \frac{1}{2}}{\beta_{x_j}^{\{c-1\}} + \frac{1}{2}\left(\left[\mu_j^{\{c-1\}}\right]^2 + \boldsymbol{\Sigma}_{j,j}^{\{c-1\}}\right)} \\ \left[V_{e_{i,i}}^{-1}\right]_{VBA}^{\{c\}} = \dfrac{\alpha_{e_i}^{\{c\}}}{\beta_{e_i}^{\{c\}}} = \dfrac{\alpha_{e_i}^{\{c-1\}} + \frac{1}{2}}{\beta_{e_i}^{\{c-1\}} + \frac{1}{2}[\boldsymbol{H}_i\boldsymbol{\Sigma}^{\{c-1\}}\boldsymbol{H}_i^T + (y_i - \boldsymbol{H}_i\boldsymbol{\mu}^{\{c\}})^2]} \\ \boldsymbol{\Sigma}^{\{c\}} = \left(\boldsymbol{H}^T\left[\boldsymbol{V_e^{\{-1\}}}\right]^{\{c\}}\boldsymbol{H} + \left[\boldsymbol{V_x^{\{-1\}}}\right]^{\{c\}}\right)^{-1} \\ \boldsymbol{\mu}^{\{c\}} = \boldsymbol{\Sigma}^{\{c\}}\boldsymbol{H}^T\left[\boldsymbol{V_e^{\{-1\}}}\right]^{\{c\}}\boldsymbol{y} \end{cases}$$

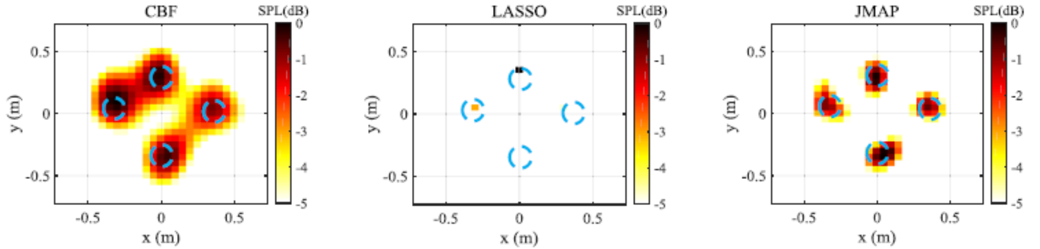
合成孔径非同步测量(12)



Sampling frequency	50000Hz
Sampling time	10s
Experimental environment	Anechoic chamber

阵列在9个位置
的非同步测量

56通道平面螺旋阵
列及4个单极子声源

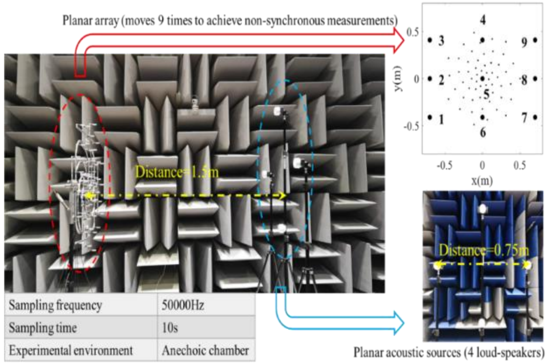


CBF LASSO JMAP

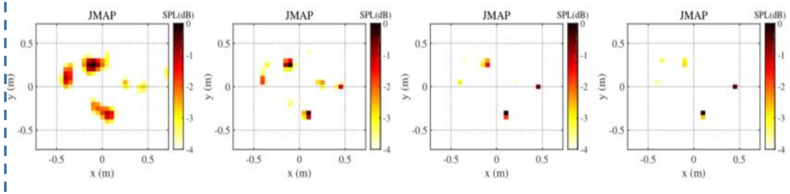
3种方法下4个单极子声源的成像结果（窄带400Hz，中心频率1000Hz）

Ning Chu, Yue Ning, Liang Yu etc. A High-Resolution and Low-Frequency Acoustic Beamforming Based on Bayesian Inference and Non-Synchronous Measurements, IEEE Access, 2020

合成孔径非同步测量(13)

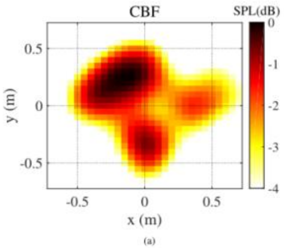


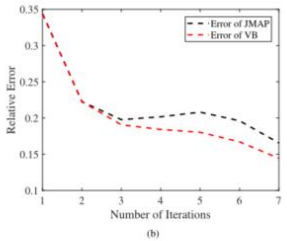
JMAP



第2次迭代 第4次迭代 第6次迭代 第8次迭代

JMAP迭代的成像结果

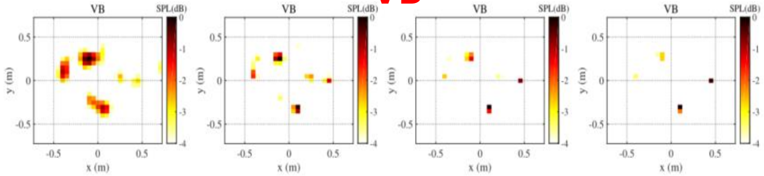




基于非同步测量的CBF成像结果（1000Hz）

JMAP和VB迭代中声源定位的相对误差（1000Hz）

VB



第2次迭代 第4次迭代 第6次迭代 第8次迭代

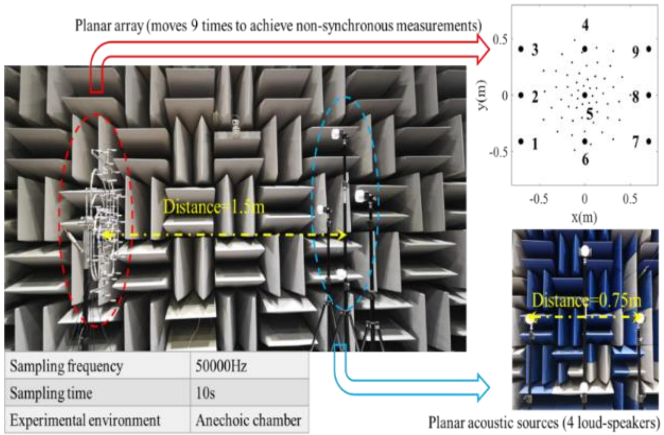
VB迭代的成像结果

VB的优点来自矩阵求逆和正则化参数的自适应估计

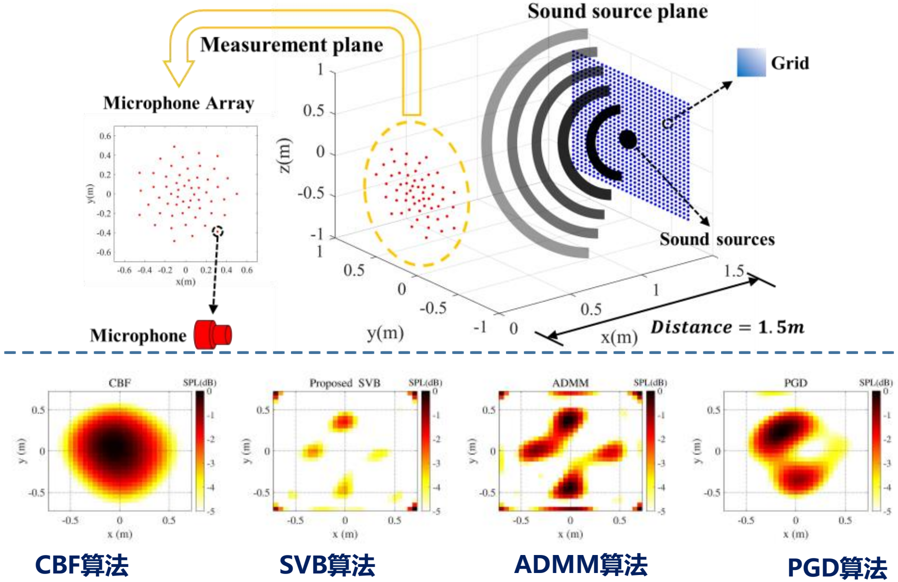
Ning Chu, Yue Ning, Liang Yu etc. A Fast and Robust Localization Method for Low-Frequency Acoustic Source: Variational Bayesian Inference based on Non-Synchronous Array Measurements, IEEE Transactions on Instrumentation and Measurement, 2020

合成孔径非同步测量(14)

- **SVB是一种子空间迭代集成变分贝叶斯方法，将概率测度空间中的子空间优化方法融入到所提出的支持向量机方法中，解决所涉及的病态逆问题。**



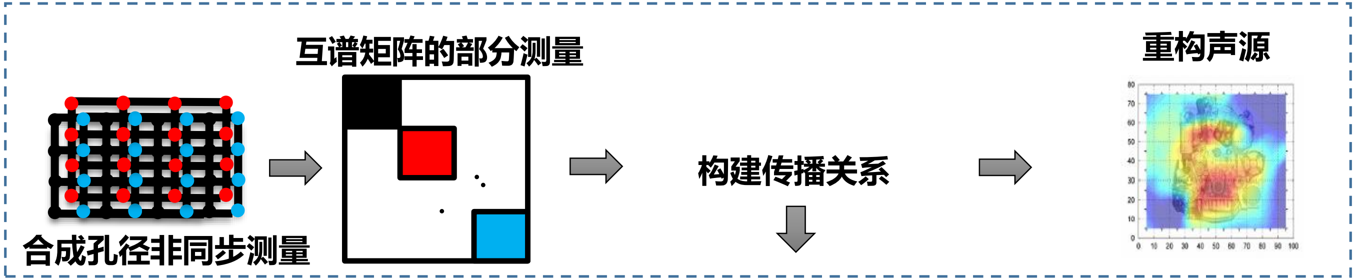
Sampling frequency	50000Hz
Sampling time	10s
Experimental environment	Anechoic chamber



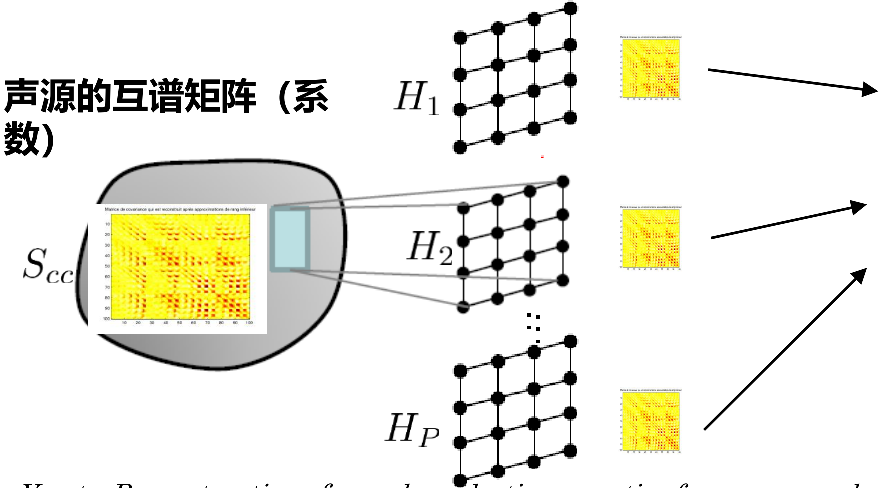
该工作基于合成孔径非同步测量框架，未来可以在测度空间基于贝叶斯算法进一步研究高分辨率高精度的定位成像算法

Liang Yu etc. Adaptive Imaging of Sound Source Based on Total Variation Prior and a Subspace Iteration Integrated Variational Bayesian Method, IEEE Transactions on Instrumentation and Measurement, 2021

合成孔径非同步测量(15)



声源的互谱矩阵（系数）



$$S_{pp}^{\{1\}} = H_1 S_{cc} H_1^* + S_{nn}^{\{1\}}$$
$$\vdots$$
$$S_{pp}^{\{2\}} = H_2 S_{cc} H_2^* + S_{nn}^{\{2\}}$$
$$S_{pp}^{\{P\}} = H_P S_{cc} H_P^* + S_{nn}^{\{P\}}$$

一般来说很难解决

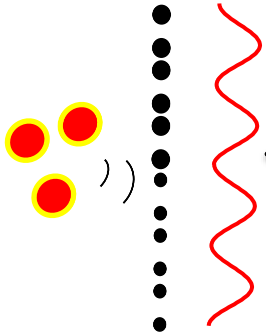
用贝叶斯体系解决

Jerome Antoni, Liang Yu etc. Reconstruction of sound quadratic properties from non-synchronous measurements with insufficient or without references: Proof of concept, Journal of Sound and Vibration, 2015

合成孔径非同步测量(16)

期望最大化（EM）算法

$$S_{cc} = \Lambda E\{\epsilon\epsilon\}^* \Lambda^*$$



$$\left\{\begin{array}{l} S_{pp}^{\{1\}} = H_1 \Lambda \epsilon_1 \epsilon_1^* \Lambda^* H_1^* + S_{nn}^{\{1\}} \\ S_{pp}^{\{2\}} = H_2 \Lambda \epsilon_2 \epsilon_2^* \Lambda^* H_2^* + S_{nn}^{\{2\}} \\ \vdots \\ S_{pp}^{\{P\}} = H_P \Lambda \epsilon_P \epsilon_P^* \Lambda^* H_P^* + S_{nn}^{\{P\}} \end{array}\right.$$

1. 假设 ϵ_i 是已知的

$$\Lambda \leftarrow \{\epsilon_i, S_{pp}^{\{i\}}\}$$

$$H_i \Lambda \epsilon_i = p_i$$

确定性结构

随机变量

2.假设 Λ 是已知的

$$\epsilon_i \leftarrow \{\Lambda, S_{pp}^{\{i\}}\}$$

吉布斯采样(1)

由于H_j的病态矩阵特性，引入了潜在变量ϵ

$$\left\{\begin{array}{l} S_{pp}^{\{1\}} = \mathrm{H}_1 S_{cc} \mathrm{H}_1^* + S_{nn}^{\{1\}} \\ S_{pp}^{\{2\}} = \mathrm{H}_2 S_{cc} \mathrm{H}_2^* + S_{nn}^{\{2\}} \\ \vdots \\ S_{pp}^{\{P\}} = \mathrm{H}_P S_{cc} \mathrm{H}_P^* + S_{nn}^{\{P\}} \end{array}\right. \xrightarrow{c = \Lambda\varepsilon} \left\{\begin{array}{l} S_{pp}^{\{1\}} = \mathrm{H}_1 S_{\varepsilon\varepsilon} \Lambda^* \mathrm{H}_1^* + S_{nn}^{\{1\}} \\ S_{pp}^{\{2\}} = \mathrm{H}_2 \Lambda S_{\varepsilon\varepsilon} \Lambda^* \mathrm{H}_2^* + S_{nn}^{\{2\}} \\ \vdots \\ S_{pp}^{\{P\}} = \mathrm{H}_P \Lambda S_{\varepsilon\varepsilon} \Lambda^* \mathrm{H}_P^* + S_{nn}^{\{P\}} \end{array}\right.$$

核心思想:找到声源的低维子空间(系数)

$$S_{pp}^{j} = H_j \Lambda Diag(\alpha^2) \Lambda^H H_j^H + Diag(\beta_j^2), j = 1,2,\ldots,P$$

α^2：声源的功率(声源是不相关的)

β_j^2：第j个位置的噪声功率

Λ：低维空间的基

$\alpha^2, \beta^2, \Lambda$ 应该被估计？

Liang Yu etc. Acoustical Source Reconstruction from Non-synchronous Measurements by Gibbs Sampling, ICSV 2019

合成孔径非同步测量(17)

吉布斯采样(2)

合成孔径非同步测量的生成模型(分层模型)

$\alpha^2 \sim IG(a_\alpha, b_\alpha)$ $\beta_j^2 \sim IG(a_\beta, b_\beta)$

$\boldsymbol{\Lambda} \sim N_c(0, K^{-1})$ $\boldsymbol{\epsilon}_{ij} \sim N_c(0, \alpha^2)$ $\boldsymbol{n}_{ij} \sim N_c(0, \beta_j^2)$

$\boldsymbol{p}_{ij} = \boldsymbol{G}_j \boldsymbol{\Lambda} \boldsymbol{\epsilon}_{ij} + \boldsymbol{n}_{ij}$, j = 1,2,…, J, i = 1,2,…,I_j

所有随机变量都被赋值为概率密度函数

$$\boldsymbol{S}_{pp}^j = \frac{1}{I_j}\sum \boldsymbol{p}_{ij}\boldsymbol{p}_{ij}^H, j = 1,2,\dots,J$$

J: **非同步测量的数量**

I_j: **快照数**

i: **传声器的编号**

如何估计$\widehat{\Lambda}$, $\widehat{\alpha^2}$, $\widehat{\beta_j^2}$ **从后验分布** $[\boldsymbol{\Lambda}, \alpha^2, \beta_j^2 | \boldsymbol{p}_{ij}]$

吉布斯采样的采样值渐进收敛到后验分布（通过迭代条件分布）

算法 (**从后验概率密度函数中迭代采样值**):

1. **初始化参数** Λ, β^2, α^2, ϵ_{ij}
2. **迭代** i = 1, 2, 3,… do

- $\boldsymbol{\epsilon}_{ij}^{[n]} \leftarrow \left[\boldsymbol{\epsilon}_{ij} \middle| \{\boldsymbol{p}_{ij}\}, \boldsymbol{\Lambda}^{[n-1]}, \beta_j^{2,[n-1]}, \alpha^{2[n-1]}\right]$
- $\boldsymbol{\Lambda}^{[n]} \leftarrow [\boldsymbol{\Lambda} | \{\boldsymbol{p}_{ij}\}, \boldsymbol{\epsilon}_{ij}^{[n]}, \beta_j^{2,[n-1]}, \alpha^{2[n-1]}]$
- $\beta_j^{2,[n]} \leftarrow [\beta^2 | \{\boldsymbol{p}_{ij}\}, \boldsymbol{\epsilon}_{ij}^{[n]}, \boldsymbol{\Lambda}^{[n]}, \alpha^{2[n-1]}]$
- $\alpha^{2[n]} \leftarrow [\alpha^2 | \{\boldsymbol{p}_{ij}\}, \boldsymbol{\epsilon}_{ij}^{[n]}, \boldsymbol{\Lambda}^{[n]}, \beta_j^{2,[n]}]$

当算法收敛时$\alpha^2, \beta^2, \Lambda$ **可以估计!**

合成孔径非同步测量(18)

非同步测量的空间基构造

基于广义谐波多项式（GHPs）基展开的解的近似

$$\begin{cases}\Delta p + k_\omega^2 p = 0 & \text{in} \quad D \\ \text{Boundary Condition} & \text{on } \partial D\end{cases} \qquad p \approx \sum_{l=0}^{L}\sum_{m=-l}^{l} \alpha_{l,m}^{L} j_l(k_\omega r) Y_{l,m}(\theta,\phi),$$

球贝塞尔函数　　球谐函数

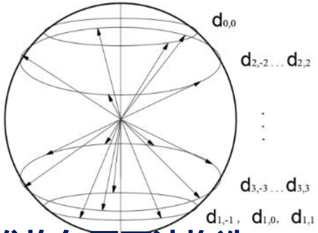
基于平面波（准均匀和不均匀）基展开的解的近似

$$\boxed{\theta_{l+2} - \theta_l = \frac{\pi}{q+2}}$$

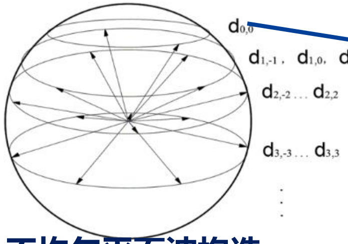
$$p(\boldsymbol{x}) \approx \sum_{k=1}^{K_p} \alpha_k e^{ik_\omega \boldsymbol{x} \cdot \boldsymbol{d}_k}$$

平面波的不同方向

$$\boxed{\boldsymbol{d}_{l,m} = (\sin\theta_l \cos\varphi_{l,m}, \sin\theta_l \sin\varphi_{l,m}, \cos\theta_l)}$$



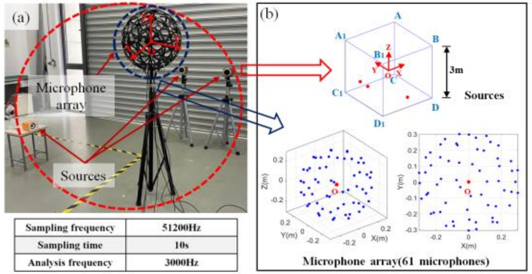
准均匀平面波构造

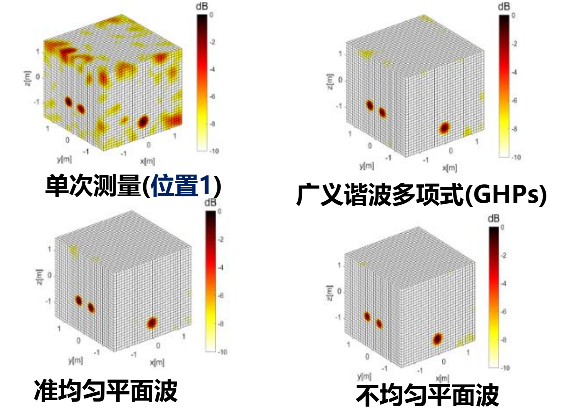


不均匀平面波构造

初始 $\frac{\pi}{q+2}$

$$\boxed{\theta_{l+1} - \theta_l = \frac{\pi}{q+2}}$$

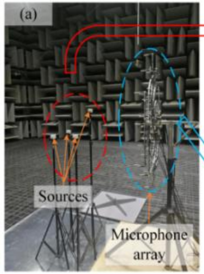




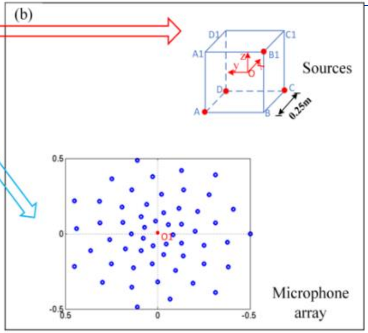
3000Hz下不同基函数的非同步测量三维成像图

Dingyu Hu etc. Spatial basis interpretation for implementing the acoustic imaging of non-synchronous measurements, Applied Acoustics, 2021

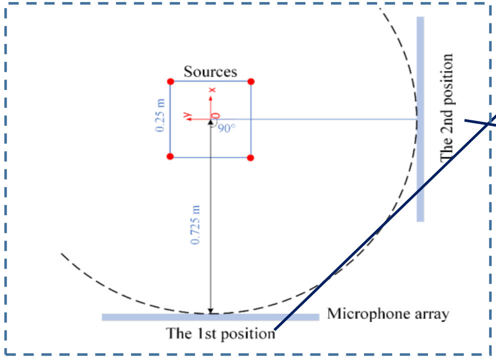
合成孔径非同步测量(19)



Sampling frequency	50kHz
Sampling time	30s
Experimental environment	anechoic chamber

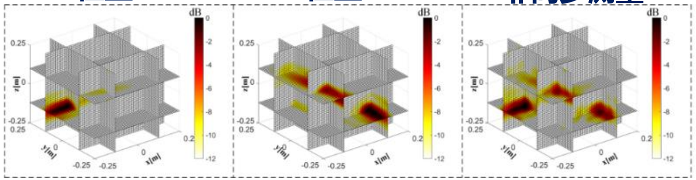


声源及传声器阵列位置示意图



传声器阵列移动方法的俯视图

位置1　位置2　非同步测量



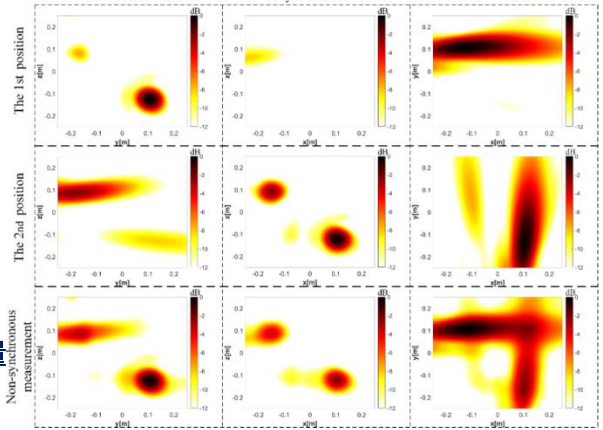
单个平面传声器阵列与非同步测量之间的3D波束形成结果比较

x=-0.13m　y=-0.13m　z=-0.13m

位置1

位置2

非同步测量



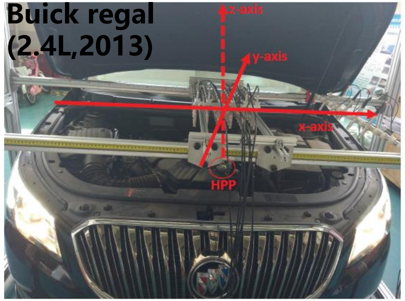
单个平面传声器阵列与非同步测量之间的3D波束形成切片结果比较

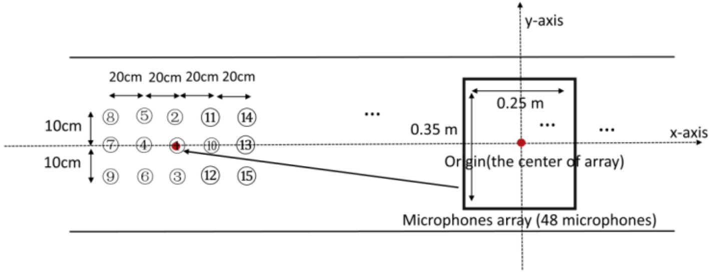
非同步测量可以显著提高平面阵列法线方向上的空间分辨率，实现3D声源定位

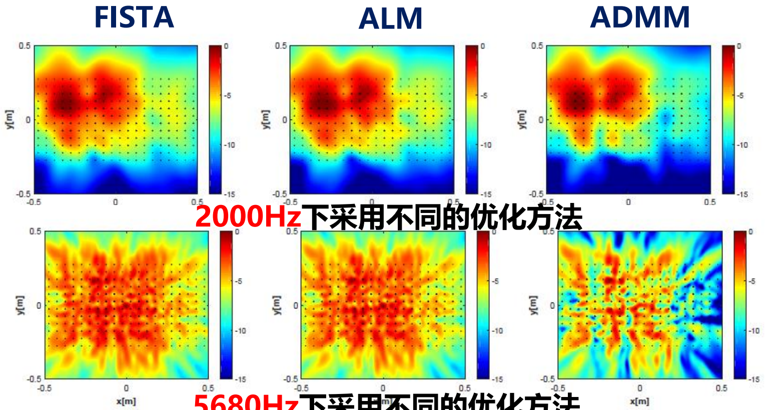
Liang Yu etc. Achieving 3D Beamforming by Non-Synchronous Microphone Array Measurements, Sensors, 2020

应用(1)

工业应用: 合成孔径非同步测量（汽车发动机）







2000Hz下采用不同的优化方法

5680Hz下采用不同的优化方法

Table 1
Matrix completion error (MCE) and computation time (CT) of non-synchronous beamforming with three algorithms (FISTA, ALM and ADMM).

	SNR = 10 dB	SNR = 20 dB	SNR = 30 dB	SNR = 60 dB
(MCE,CT)	4 non-synchronous measurements			
FISTA	(12.94%, 0.077s)	(12.16%, 0.064s)	(12.27%, 0.060s)	(12.27%, 0.057s)
ADMM	(21.70%, 0.028s)	(20.34%, 0.024 s)	(19.94%, 0.022s)	(20.17%, 0.020s)
ALM	(79.64%, 0.011s)	(79.72%, 0.009s)	(79.65%, 0.009s)	(79.87%, 0.008s)
(MCE,CT)	9 non-synchronous measurements			
FISTA	(10.92%, 0.61s)	(8.75%, 0.51s)	(8.47%, 0.48s)	(8.41%, 0.49s)
ADMM	(11.57%, 0.37s)	(9.23%, 0.33s)	(8.97%, 0.30s)	(8.89%, 0.28s)
ALM	(91.08%, 0.05s)	(91.03%, 0.04s)	(90.92%, 0.04s)	(90.96%, 0.04s)
(MCE,CT)	16 non-synchronous measurements			
FISTA	(6.65%, 2.20s)	(3.16%, 2.13s)	(2.51%, 2.11s)	(2.41%, 2.11s)
ADMM	(8.98%, 1.37s)	(6.67%, 1.35s)	(6.54%, 1.34s)	(6.40%, 1.37s)
ALM	(94.86%, 0.18s)	(94.83%, 0.17s)	(94.95%, 0.17s)	(94.82%, 0.17s)
(MCE,CT)	25 non-synchronous measurements			
FISTA	(9.22%, 7.77s)	(6.37%, 7.81s)	(6.06%, 7.86s)	(6.03%, 8.11s)
ADMM	(15.55%, 5.13s)	(14.20%, 5.17s)	(13.83%, 6.08s)	(13.91%, 4.71s)
ALM	(96.78%, 0.54s)	(96.69%, 0.54s)	(96.71%, 0.55s)	(96.71%, 0.54s)

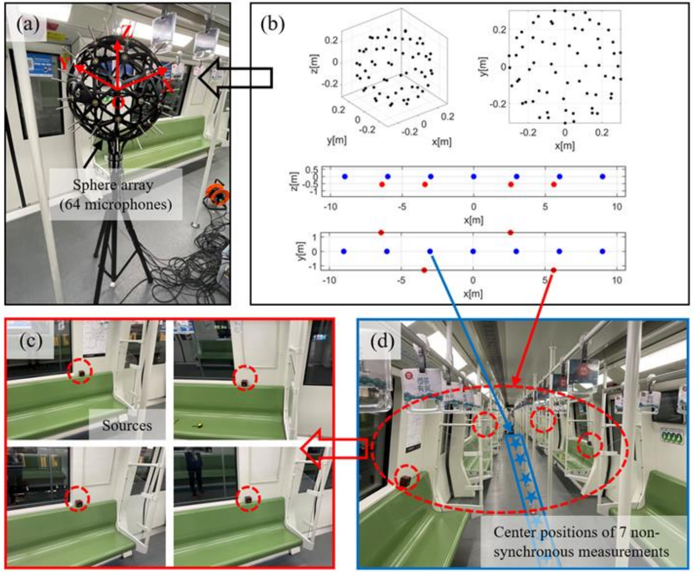
FISTA，ALM和ADMM的的矩阵补全误差（MCE）和计算时间（CT）

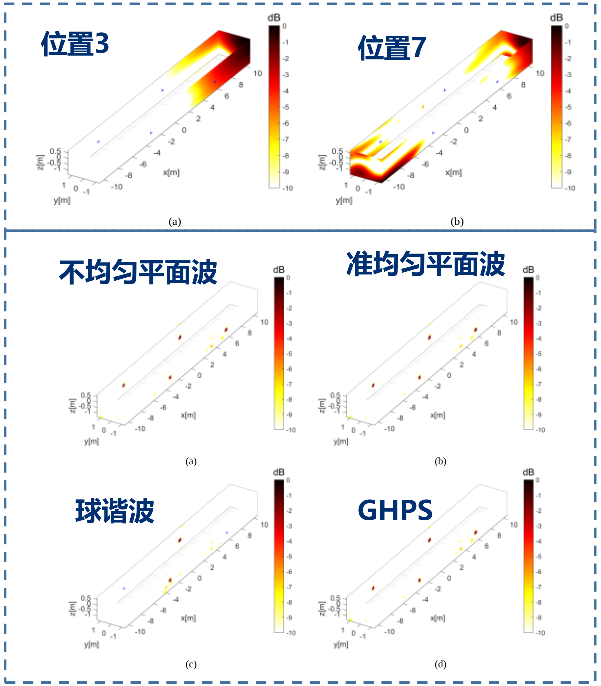
ALM和ADMM与FISTA的结果几乎相同，但运算快得多

Liang Yu etc. Fast iteration algorithms for implementing the acoustic beamforming of non-synchronous measurements, Mechanical Systems and Signal Processing, 2019

应用(2)

去混响和在列车内的应用





- 单次测量在混响车厢内**无法成像**
- 球阵列合成孔径非同步测量显著减少了高长宽比空间和混响干扰对声源位置的影响，**解决了大混响场中的高分辨率声成像问题。**

胡定玉.上海工程技术大学

结论

近场——合成孔径非同步测量

应用层面：

- 工业声成像应用
- 振动测量和运行模态分析
- 去混响和在列车内的应用
- 喷流噪声的可视化
- 服务机器人和手机合成孔径测量
- 全场景智能声学故障诊断与定位

算法层面：侵入式方法和非侵入式方法

期望最大化 (EM)

吉布斯采样 (MCMC)

互质位置的合成孔径非同步测量(CP-NSM)

快速迭代收缩阈值算法 (FISTA)

增广拉格朗日乘子法（ALM）

交替方向乘子法（ADMM）

理论层面：低阶建模+声场的谐波表示

Merci 谢谢！

Homepage

http://me.sjtu.edu.cn/teacher_directory2/yuliang.html