DEEP LEARNING

HOMEWORK 1

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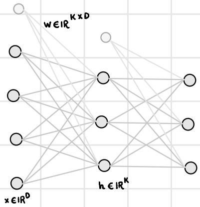
Question 3



1 $h = g(Wx) = (Wx)^2$ e queremos escrever $h = A_\theta\, \Phi(x)$, em que $\Phi(x)$ é uma transformação de x tal que $\Phi: \mathbb{R}^D \to \mathbb{R}^{\frac{D(D+1)}{2}}$ (independe de θ), e em que $A_\theta \in \mathbb{R}^{K \times \frac{D(D+1)}{2}}$

$$W = \begin{array}{cc} & \begin{array}{ccc} 1 & & D \end{array} \\ \begin{array}{c} 1 \\ \\ K \end{array} & \begin{bmatrix} | & & | \\ w_1 & \cdots & w_D \\ | & & | \end{bmatrix} \end{array} \qquad x = \begin{bmatrix} x_1 \\ \vdots \\ x_D \end{bmatrix} \qquad A_\theta = \begin{array}{cc} & \begin{array}{ccc} 1 & & \frac{D(D+1)}{2} \end{array} \\ \begin{array}{c} 1 \\ \\ K \end{array} & \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \end{array} \qquad \Phi(x) = \begin{array}{cc} \begin{bmatrix} \\ \\ \\ \end{bmatrix} & \begin{array}{c} 1 \\ \\ \frac{D(D+1)}{2} \end{array} \end{array}$$

$$h = \Big(\underbrace{\begin{bmatrix} & & \\ & & \end{bmatrix}}_{\substack{K \times D \\ W}} \underbrace{\begin{bmatrix} \\ \\ \end{bmatrix}}_{\substack{D \times 1 \\ x}} \Big)^2 = \left(\begin{bmatrix} w_1 x \\ \vdots \\ w_K x \end{bmatrix} \right)^2$$

square of the sum

$$(w_K x)^2 = \Big(\sum_{j=1}^{D} w_{Kj} x_j\Big)^2 = \sum_{j=1}^{D} (w_{Kj} x_j)^2 + 2 \sum_{j=1}^{D} \sum_{i=1}^{j-1} (w_{Kj} x_j)(w_{Ki} x_i) = \sum_{j=1}^{D} (w_{Kj} x_j)^2 + 2 \sum_{j=1}^{D} \sum_{i=1}^{j-1} (w_{Kj} w_{Ki})(x_j x_i)$$

$$h = \underbrace{\begin{bmatrix} & & \\ & & \end{bmatrix}}_{\substack{K \times \frac{D(D+1)}{2} \\ A_\theta}} \underbrace{\begin{bmatrix} \\ \\ \end{bmatrix}}_{\substack{\frac{D(D+1)}{2} \times 1 \\ \Phi(x)}} = \begin{bmatrix} a_1 \Phi(x) \\ \vdots \\ a_K \Phi(x) \end{bmatrix}$$

$$a_K\, \Phi(x) = \sum_{j=1}^{D} a_{Kj}\, \Phi(x_j) = \sum_{j=1}^{D} (w_{Kj} x_j)^2 + 2 \sum_{j=1}^{D} \sum_{i=1}^{j-1} (w_{Kj} w_{Ki})(x_j x_i)$$

$$= \sum_{j=1}^{D} w_{Kj}^2 x_j^2 + 2 \sum_{j=1}^{D} \sum_{i=1}^{j-1} (w_{Kj} w_{Ki})(x_j x_i)$$

! Cada j e i dos somatórios correspondem a um elemento de a_K e outro de $\Phi(x)$

Queremos definir a_K e $\Phi(x)$ tal que o seu produto interno dê o somatório → cada entrada é cada elemento somado x ordem

a_{Ki} e $\Phi(x_i)$ são tirados extraindo os mesmos índices do somatório