Network Externality Analysis and Application in Cryptocurrency

The final project report of EE 495

Jiaqi Zang, Shujie Cao *JZG0851,SCO3415*

December 1, 2023

1 Introduction and Motivation

The digital currency Bitcoin caught the interest of the mainstream media only from 2012. Unlike cash, these currencies are purely digital and used primarily online. While digital currencies are widely varied, the decentralized digital currencies that are based primarily on cryptography, called cryptocurrencies, have gotten the most attention. As a brand new market, cryptocurrencies seem impossible to predict. For example, the Classic Luna, which was once among the top 4 popular cryptocurrencies in the market, dropped 99.7 percent in under a week. The Classic Luna's price is lower than 0.000115$ by now. Everything seems so dramatic, but can we choose to buy any less risky cryptocurrencies? Can we use game theory to attain any useful ideas? And this leads to our project theme.
First, according to Kristoufek in [1], the factors driving cryptocurrency's price are investment attractiveness and the supply-demand relationship. Political and economic crises often lead to increased public acceptance of cryptocurrencies. People will become more reliant on cryptocurrencies during times of financial turmoil due to their ability to trade without banks as intermediaries and the prospect of high-yielding future investments. That is to say, the more popular a currency is, the more useful it is, and the easier it attracts new users. Therefore, one might expect that the most popular cryptocurrency would grow even more popular, eventually dominating the whole market. And this is a classic case showing the network externality. By definition from [2], network externality is the fact that the value of the service to a user depends on the number of other users in the 'network'. So in the following part of this report, we will start by learning theorems related to network externality in section 2, and verify the theorems in the cryptocurrency market in section 3. Also in section 4, we further develop the theorem in a newly proposed model, which generates some meaningful and interesting results.

2 Existing work

Katz and Shapiro ([3] [4] [5] [6])analyzed a lot centering the competition, and compatibility under different circumstances, making their works the milestones of

Network Externalities. The purposed problems ([7] [8]) due to network effects are abundant, but the most arresting one is a claim that markets may adopt an inferior product or network in the place of some superior alternative. Thus if network effects are a typical characteristic of modern technologies, the theory suggests that markets may be inadequate for managing the fruits of such technologies.
Our project concentrates on one of Katz's work [4], which is the origin of "First-mover wins" theorem.

2.1 Katz's Model

In Katz's model, there are n homogeneous firms and a continuum of consumers. The firms are divided into groups, where only products from the same group are compatible. All users of such a group form a network, and the utility of each user depends on the size of the network.

A key fact is that consumers will base their purchase decisions on expected network sizes before the actual network sizes are known. The procedure is the following: first, consumers form expectations about the size of the networks. For simplicity, they assume that all consumers have identical expectations of network sizes. Second, the firms play an output game based on consumers' expectations. Then prices are then generated in a Cournot manner such that all the products can be sold.

2.1.1 Consumers

Formally, consumers have a basic willingness r where r is uniformly distributed on $(-\infty, A]$ for some $A > 0$. Each consumer either buys one unit of product of one brand or buys nothing. The utility for buying a product of firm i is $r + v(y_i)$ where y_i is the network size, and v is a function representing the network externality. v satisfies the following conditions:

1. v is a continuously twice differentiable function on $[0, \infty)$,
2. v is increasing and convex,
3. $v(0) = 0$, $\lim_{x\to\infty} v'(x) = 0$.

However, upon making the decision, each consumer can only use $r + v(y_i^e)$ to evaluate the utility where y_i^e is the expected network size. He will buy the product that he thinks will maximize $r + v(y_i^e) - p_i$, where p_i is the price of firm i. Since all products will be sold, they must be equally attractive to the consumers. Thus, $\Phi = p_i - v(y_i^e)$, the hedonic price of the brand i, should be the same for all i. Only consumers with a basic willingness larger than the hedonic price will enter the market. Thus, if the firms together sell $z = \sum_i x_i$ units, so the prices must satisfies that $A - \Phi = z$. As a result, firm i will set their price to be $p_i = A + v(y_i^e) - z$. Here, for simplicity, the fixed cost is set to be 0. And the variable cost (cost per unit) can also be set to 0 since if it is a constant c, we can just adjust the A into $A - c$ and get equivalent results.

If all of the networks are incompatible, then $y_i^e = x_i^e$, where x_i^e is the number of customers that consumers expect firm i to have. So the firm i earns profits $\pi_i = x_i(A - z + v(x_i^e))$. When all products are compatible, $y_i^e = z^e$, the firm's profits are $\pi_i = x_i(A - z + v(z^e))$.

2.1.2 Fulfilled Expectations Cournot Equilibrium

As a standard procedure, differentiating the firm's profits, the first-order condition $d\pi_i/dx_i = 0$ implies the equilibrium output must satisfy

$$x_i^* = A + v\left(y_i^e\right) - \sum_{j=1}^{n} x_j^* \tag{1}$$

, where the right side of equation equals p_i. Solving equation 1, the outcome is the standard linear demand Cournot equilibrium

$$x_i^* = \left\{ A + nv\left(y_i^e\right) - \sum_{j \neq i} v\left(y_j^e\right) \right\} / (n+1) \quad \text{for } i = 1, 2, \ldots, n. \tag{2}$$

Authors focus on fulfilled expectations Cournot equilibrium (FECE) where the expected sales are equal to actual sales in the Cournot equilibrium.

Now as for the welfare. Since the firm's output equals the firm's received price, the firm's profits in equilibrium are $\pi_i = (x_i^*)^2$. When the market output is z, consumers expects to have a surplus of $r + z - A$, so the overall consumers' expected surplus is

$$S(z) = \int_{A-z}^{A} (\rho + z - A) d\rho = z^2/2 \tag{3}$$

Finally, the social welfare is

$$\sum_{i=1}^{n} x_i^2 + z^2/2. \tag{4}$$

2.1.3 Characterization of Equilibria

When all products are mutually compatible, there is unique FECE. It is symmetric and the aggregate level of output is given implicitly by

$$z^c = (n/(n+1))\left(A + v\left(z^c\right)\right) \tag{5}$$

, where z^c deonotes the fulfilled expectations equilibrium value of total output. When each brand is incompatible with all the others, there exists a uique symmetric equilibrium in which $x_i = z^I/n$, and aggregate sales are given implicitly

$$((n+1)/n) z^I = A + v\left(z^I/n\right) \tag{6}$$

When the network externality is strong: $v'(0) \geq 1$(This condition is added by the authors of this report, which will be explained later). There might be other type of FECEs.

A case similar to the symmetric case is the partially symmetric case. In such a case, some of the firms make no output at all and all active firms make equal output. Suppose there are k active firms with total output z, then the output of each active firm is z/k. z is given by

$$((k+1)/k)z = A + v(z/k) \tag{7}$$

In addition, to ensure that the inactive firms have no incentive to enter, it suffices to require $v(A/k) \geq A/k$.

There are also cases where multiple firms have different nonzero outputs. These FECEs are difficult to characterize in general.

2.2 A complement specification for Katz's result

One may wonder when asymmetric equilibriums will occur. While this was not discussed by the author, it is not surprising that the effect of consumer expectation can only determine the result when the network externality is strong. We discovered the following criterion.

Proposition 1 *For n incompatible firms, asymmetric FECE with positive output for each firm can only appear when $v'(0) \geq 1$.*

The proof is lengthy and we only sketch it here due to the limitation of length. We may define an n-dimentional function Φ. $\Phi_i(x_1, ..., x_n) = A - z - x_i + v(x_i)$. A FECE is a zero of Φ. The derivative of Φ is an $n \times n$ matrix with diagonal elements > 1 and all other elements 1. Such conditions will ensure that it has full rank. Note that the convex combination of such matrices satisfies the same condition. If there are two different zero of Φ, apply the fundamental theorem of calculus along the segment connecting them and we get $\Delta\Phi = 0 = Br$ where B is the average of $D\Phi$ along the segment and r is the vector representing the segment. Since A is of full rank, this yields a contradiction.

3 Cryptocurrency Market Validation

One of the most famous theorems driven from the previous paper is that the Network externality favors the first mover, known as"first-mover advantage", which means a firm's ability to be better off than its competitors as a result of being first to market in a new product category. As for the cryptocurrency market, Bitcoin is a typical first-mover. And there is a voice like"There can only be one. The Network effect is simply too strong. Bitcoin has orders of magnitudes more adoption, acceptance and use compared to any other cryptocurrency on the market. The game is over and Bitcoin won." But there is also the opposite voice"substantially better product will almost always find its place in a market where the cost to move from one option to the other is cheap and easy".

In a nutshell, the network externality and substitution effect are two balancing factors, driving the contest to two opposite ends. So, we have made some experiments to testify which factor is more powerful.

	r(Eth)	r(Tether)	r(BNB)	r(Solana)	r(LTC)
May 23 2021 - May 30 2022					
r(BTC)	-0.93	0.146	-0.502	-1.047	-0.349
R^2	0.69	0.37	0.59	0.42	0.29

3.1 Experiment Setup

When analyzing assets including currencies, researchers and market analysts are typically interested in returns (i.e., percentage changes) than in price levels. And changes in prices, i.e. returns, can directly capture changes in the strength of the network effects of different cryptocurrencies, since the attractiveness of the platform increases when its network effects become stronger.
Here, the return of altcoin is defined as the following,

$$r_t(\text{ altcoin }) = \frac{p_t\left(\frac{\text{USD}}{\text{altcoin}}\right) - p_{t-1}\left(\frac{\text{USD}}{\text{altcoin}}\right)}{p_{t-1}\left(\frac{\text{USD}}{\text{altcoin}}\right)} \tag{8}$$

Based on returns, we run regressions of $r_t(\text{ altcoin })$ and $r_t(\text{ BTC })$, to see the relationship between their prices' change.
The data is crawled from the coinbase with API. And we choose the first 5 most popular cryptocurrencies other than Bitcoin: Ethereum(Eth), Tether, Binance Coin(BNB), Solana(SOL), Litecoin(LTC). The data is from May 23 2022 to May 30,2022. And application programming interface (MatLab) is employed.

3.2 Experiment Result

As can be seen easily from the table, the constants are all negative except for the Tether. This means that good news for BTC are usually bad news for the altcoins.

Also, R^2 demonstrates the overall fit of the regression equation and expresses the overall relationship between the dependent variable and all independent variables. As in the table, the R^2 of Eth is almost 0.7 which is extremely high, illustrating that 70 percent of Eth's price's variation can be explained by the change of the price of BTC.

The experiment shows that the first mover in the cryptocurrency market: BTC is getting more shares and it can be explained that people expect BTC to be popular. It can be seen that the google search intensity of BTC is much higher than the other altcoins. So, when it comes to investments in the cryptocurrency market, BTC is a safer choice compared to other altcoins.

4 Newly Proposed Model

In Katz's model, they just forced the resulting equilibrium to meet the expectation of the consumer expectations. There is no theoretical background ensuring that such cases will happen. We propose a new model to avoid the mysterious role of consumer expectation.

In the following discussion, we assume that there are two homogeneous firms. The settings and notations remain the same except that the consumers always evaluate their utility by the ACTUAL network size. The utility function for a consumer of type r is

$$U_r = r + v(y), \tag{9}$$

where y is the actual network size. Here we make a slight change of the assumptions on v. Namely,

1. v is a continuously twice differentiable function on $[0, \infty)$,
2. v is increasing and convex.
3. $v(0) = 0$, $v'(0) < 1$.

Here we require that $v'(0) < 1$ to prevent the size of the network from going to infinity and to ensure that the best response of the firms is unique. This condition means that the network externality should not be unrealistically strong.

Thus, the game is simplified to a modified version of the Cournot competition. And we will characterize the Nash equilibria of this game. If the output of firm i is x_i, then the price of firm i is

$$p_i = A + v(y_i) - z, \tag{10}$$

where y_i are the network sizes and z is the total output. Thus, the profit of firm i is

$$\pi_i = x_i(A + v(y_i) - z). \tag{11}$$

After careful consideration and computation, we can rule out the monopoly case as long as $v'(0) < 1/2$. For the case that both firms have nonzero output, the best response x_i^* of firm i is characterized by $\partial\pi_i/\partial x_i = 0$ fixing the opponent's strategy. Thus we have

$$A + v(y_i^*) - z^* = (1 - v'(y_i^*))x_i^*. \tag{12}$$

For simplicity, we omit the superscript * in the following computation. Note that $A+v(y_i)-z$ is decreasing in x_i and $(1-v'(y_i))x_i$ is increasing in x_i, the best response is uniquely determined.

4.1 Characterization of the equilibrium

The Nash equilibrium satisfies the best response equation for both firms. For the compatible case, it is obvious that there is only one NE. The equations imply that the NE is symmetric with the total output satisfying that

$$A + v(z) - z = (1 - v'(z))z/2. \tag{13}$$

For the incompatible case, the best response of firm i is characterized by

$$x_{-i} = A + v(x_i) - (2 - v'(x_i))x_i. \tag{14}$$

For general v, the solution may be asymmetric and there is nothing much to conclude. But if we assume that the externality is moderately weak, i.e., $v'(0) < 1/2$, we get $\partial x_{-i}/\partial x_i < -1$. Thus, the equilibrium is unique and symmetric since the reaction equations are also symmetric.

Actually, for the case with n firms, the equilibrium is also unique and symmetric as long as we assume $v'(0) < 1/2$. The proof is a bit lengthy and we omit it here since the case with 2 firms has already revealed the essence of this problem.

From now on we keep the assumption that $v'(0) < 1/2$. At the equilibrium, the response is $(z/2, z/2)$ where the total output z satisfies

$$A + v(z/2) - z = (1 - v'(z/2))z/2. \tag{15}$$

Compare the equilibrium equations. Since the term $-v'$ is negative, we know the output z is higher than that in the original model for both compatible and incompatible cases. Also, it is quite obvious that the output of the compatible case is higher than that of the incompatible case.

4.2 The welfare

The main purpose of this section is to compare the welfare of the two models under the same A and v. We use z_0 and z_1 to denote the corresponding equilibrium total output in this subsection. The method is to represent the welfare by a function of z.

In the new model, the consumer of the lowest type that enters the market still gets a marginal surplus of 0. Thus the total consumer surplus is $z^2/2$. Since $z_1 > z_0$, we know

$$z_1^2/2 > z_0^2/2. \tag{16}$$

The consumer surplus is higher in the new model.

The welfare of society is the total utility of the consumers and is independent of the price of the product. Thus it can be written as

$$\int_{A-z}^{A} v(y) + r dr, \tag{17}$$

where $y = z$ or $z/2$ is the network size. Since $z_1 > z_0$, $v(y_1) \geq v(y_0)$ and $A - z_1 + v(y_1) = p > 0$. Thus,

$$\int_{A-z_1}^{A} v(y_1) + r dr \geq \int_{A-z_0}^{A} v(y_0) + r dr. \tag{18}$$

The welfare of society is higher in the new model.

The total profit of the firms are $\Pi(z) = z(A - z + v(y))$. For the compatible case, assume $z \in [z_0, z_1]$. Then $3z/2 \geq A + v(z)$, and thus

$$\Pi'(z) = A + v(z) - z(2 - v'(z)) \leq z(v'(z) - 1/2) < 0. \tag{19}$$

Similarly, $\Pi'(z) < 0$ on $[z_0, z_1]$ for the incompatible case. Thus $\Pi(z_1) < \Pi(z_0)$.

The profit is lower in the new model.

In conclusion, the private incentive to disclose the output is insufficient. Consequently, consumers should be encouraged to form organizations and negotiate with the firms to increase their surplus and social welfare.

4.3 High externality

The behavior of the equilibriums is generally messy when $v'(0) \geq 1/2$. We are unable to reach any general conclusions. However, we may give several examples by simulation where v is elaborately designed to show some insights of the game.

Example 1 $A = 1$, $v(x) = x/2$.

Consider the incompatible case. The equilibrium equations are

$$x_{-i} = 1 - x_i. \tag{20}$$

Thus there is a continuum of asymmetric equilibriums.

Example 2 $A = 100, v(x) = \begin{cases} x & x \in [0, 200/3) \\ x - 90(x/100 - 2/3)^2 & x \in [200/3, 1100/9) \\ 1700/18 & x \in [1100/9, \infty). \end{cases}$

Consider the incompatible case. There are two discrete asymmetric equilibriums for this example, one of which is $(30, 100)$. The equilibrium reaction curves are plotted as Figure 1.

Even if the equilibrium is symmetric, the pattern of the welfare can be different from that in the low externality case.

Example 3 $A = 1, v(x) = 0.8x$.

Consider the compatible case. There is only one symmetric equilibrium in both models. From the formulas one may solve

$$z_0 = 10/7, p_0 = 5/7, \Pi(z_0) = 50/49, \tag{21}$$

$$z_1 = 10/3, p_1 = 1/3, \Pi(z_0) = 10/9. \tag{22}$$

Thus the profit of firms in the new model is actually higher in this example when network externality is extremely high. That is to say, both consumers and firms sides will gain benefits from making information transparent, which is beyond our expectation.

References

[1] L. Kristoufek, "Ladislav kristoufek- what are the main drivers of the bitcoin price," 07 2015.

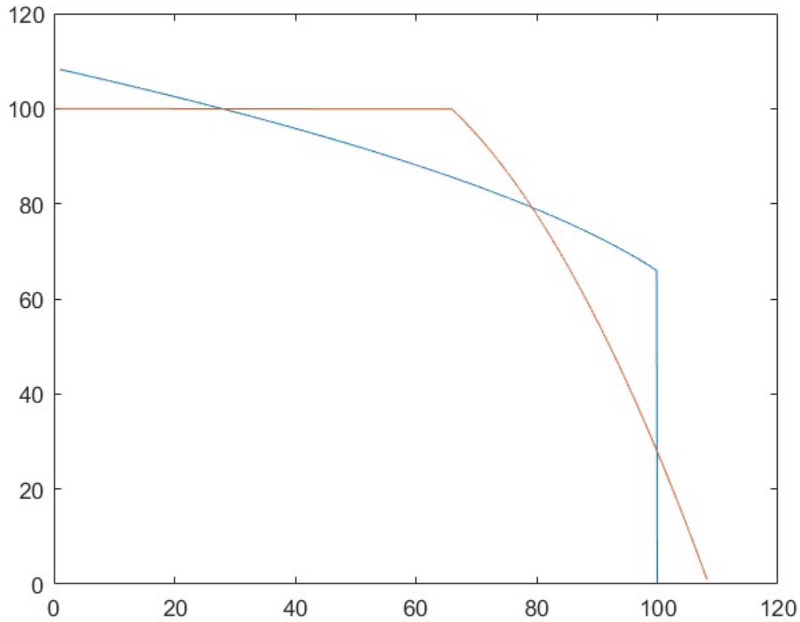

Figure 1: Equilibrium reation curves for x_1 and x_2

[2] R. A. Berry and R. Johari, *Economic Modeling in Networking: A Primer*, 2013.

[3] M. L. Katz and C. Shapiro, "On the licensing of innovations," *The RAND Journal of Economics*, pp. 504–520, 1985.

[4] ——, "Network externalities, competition, and compatibility," *The American economic review*, vol. 75, no. 3, pp. 424–440, 1985.

[5] ——, "Product introduction with network externalities," *The journal of industrial economics*, pp. 55–83, 1992.

[6] ——, "Technology adoption in the presence of network externalities," *Journal of political economy*, vol. 94, no. 4, pp. 822–841, 1986.

[7] J. Farrell and G. Saloner, "Standardization, compatibility, and innovation," *the RAND Journal of Economics*, pp. 70–83, 1985.

[8] S. J. Liebowitz and S. E. Margolis, "Network externality: An uncommon tragedy," *Journal of economic perspectives*, vol. 8, no. 2, pp. 133–150, 1994.