

# Geometric phase effects in the excited state dynamics of N-dimensional linear vibronic coupling model

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## Introduction

- Molecular wave function in the adiabatic representation

$$\Psi(r, R, t) = \sum_j \chi_{n,j}(R, t) \phi_{e,j}(r; R)$$

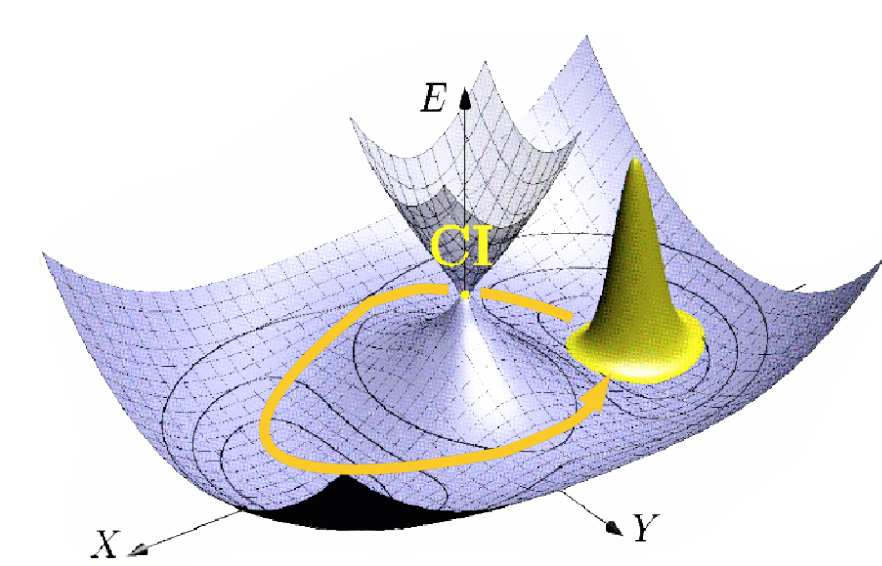
$$H_e \phi_{e,j} = E_j(R) \phi_{e,j}(r; R)$$

- If electronic surfaces,  $E_j(R)$ , intersect conically, geometric phase (GP) makes the electronic functions double-valued.

- To compensate for that:<sup>1</sup>

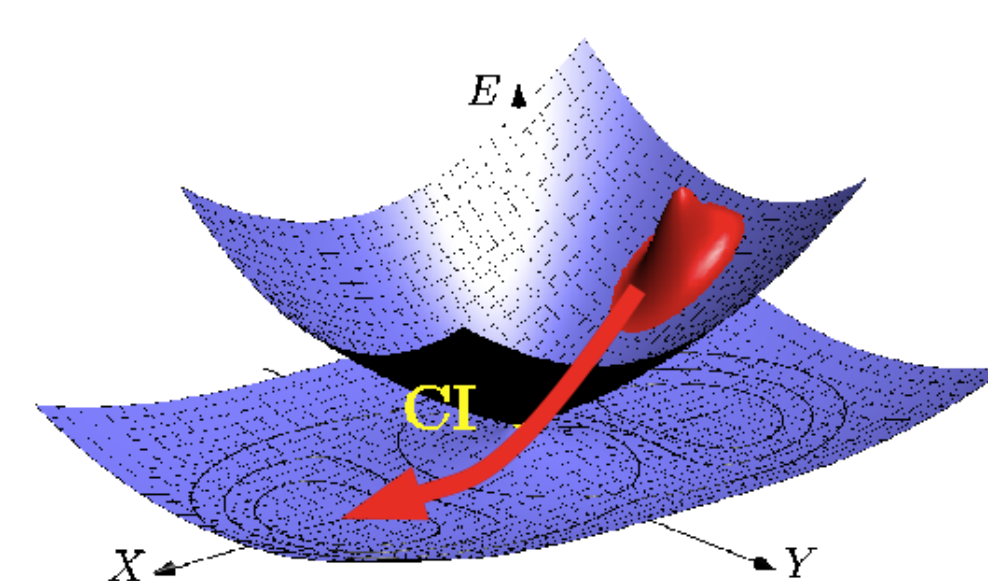
$$\phi_{e,j}(r; R) e^{-i\theta(R)}$$

$$\chi_{n,j}(R, t) e^{i\theta(R)}$$



Questions:

- When is GP important?
- What is the impact of extra nuclear degrees of freedom (DOF)?



Motivation:

To develop a qualitative picture of GP effects in excited state dynamics of large system.

## Modeling the system

N-dimensional LVC Hamiltonian for nuclear DOF

$$H_{ND} = \sum_j^N (p_j^2 + \omega_j^2 q_j^2) \mathbf{1}_2 + \begin{bmatrix} -\kappa_j q_j & c_j q_j \\ c_j q_j & \kappa_j q_j \end{bmatrix} + \begin{bmatrix} -\delta & 0 \\ 0 & \delta \end{bmatrix}$$

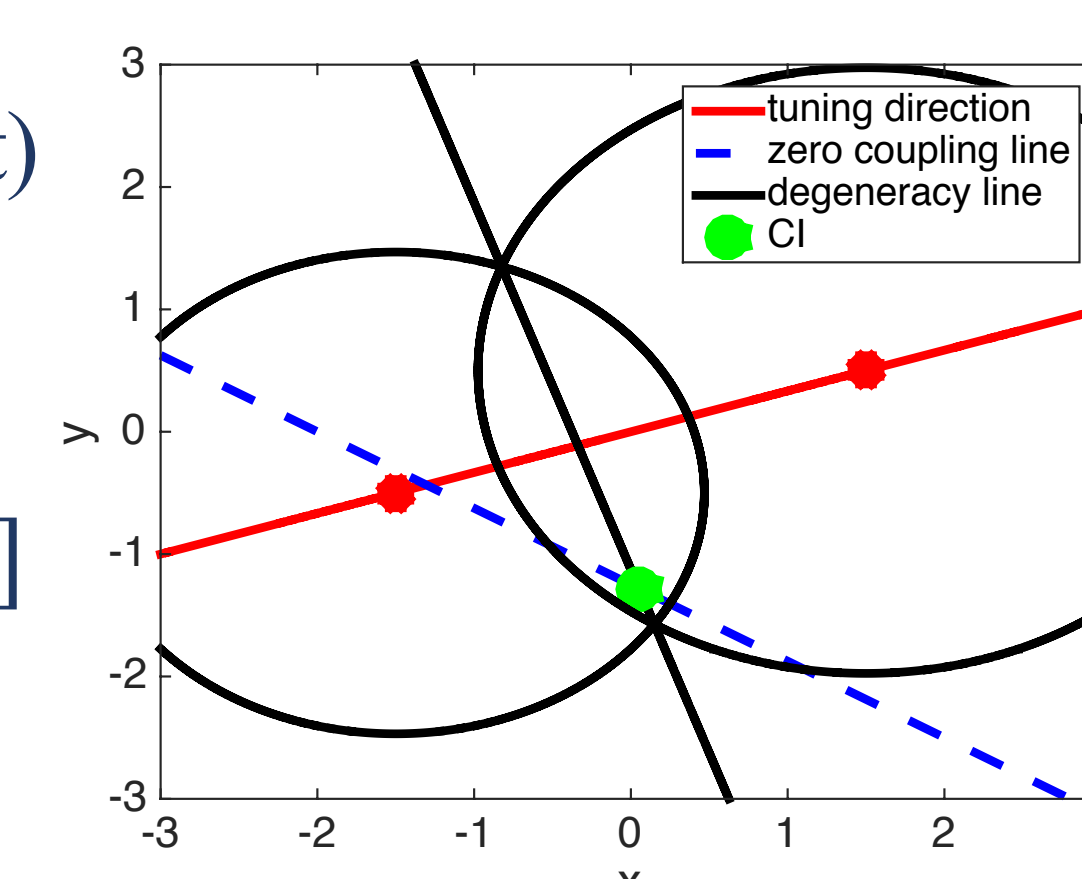
The short-time dynamics can be examined by an effective 2D system<sup>2</sup>. Extra DOF modify the 2D parameters.

Diabatic representation (GP is implicit)

$$H = -\frac{1}{2} \nabla^2 \mathbf{1}_2 + \begin{bmatrix} V_{11} & V_{12} \\ V_{12} & V_{22} \end{bmatrix}$$

$$V_{11,22} = \frac{1}{2} [\omega_x^2 (x \mp x_0)^2 + \omega_y^2 y^2 \mp \Delta]$$

$$V_{12} = c_x x + c_y y + \Delta_{12}$$



Adiabatic representation (no GP if single-value real wave functions are used):

$$H_{adi} = -\frac{1}{2} \nabla^2 \mathbf{1}_2 + \begin{bmatrix} W_- & 0 \\ 0 & W_+ \end{bmatrix} + \begin{bmatrix} \tau_{11} & \tau_{12} \\ -\tau_{12} & \tau_{22} \end{bmatrix}$$

$$W_{\pm} = \frac{1}{2} (V_{11} + V_{22}) \pm \frac{1}{2} \sqrt{(V_{11} - V_{22})^2 + 4V_{12}^2}$$

$$\tau_{ij} = -\langle \phi_i | \nabla \phi_j \rangle \nabla - \frac{1}{2} \langle \phi_i | \nabla^2 \phi_j \rangle$$

## GP effects for transfer

Expand the nuclear wave function:

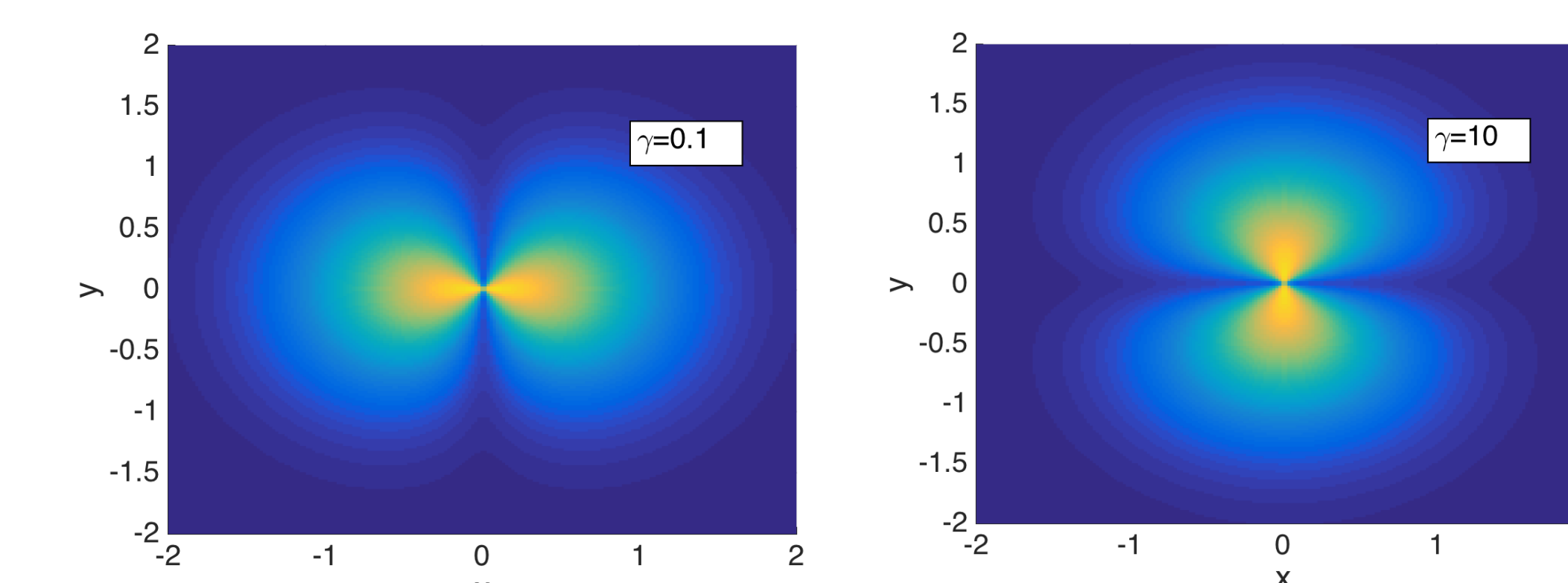
$$\chi = \chi_{nontran} + \chi_{tran}$$

Nontransferable part:

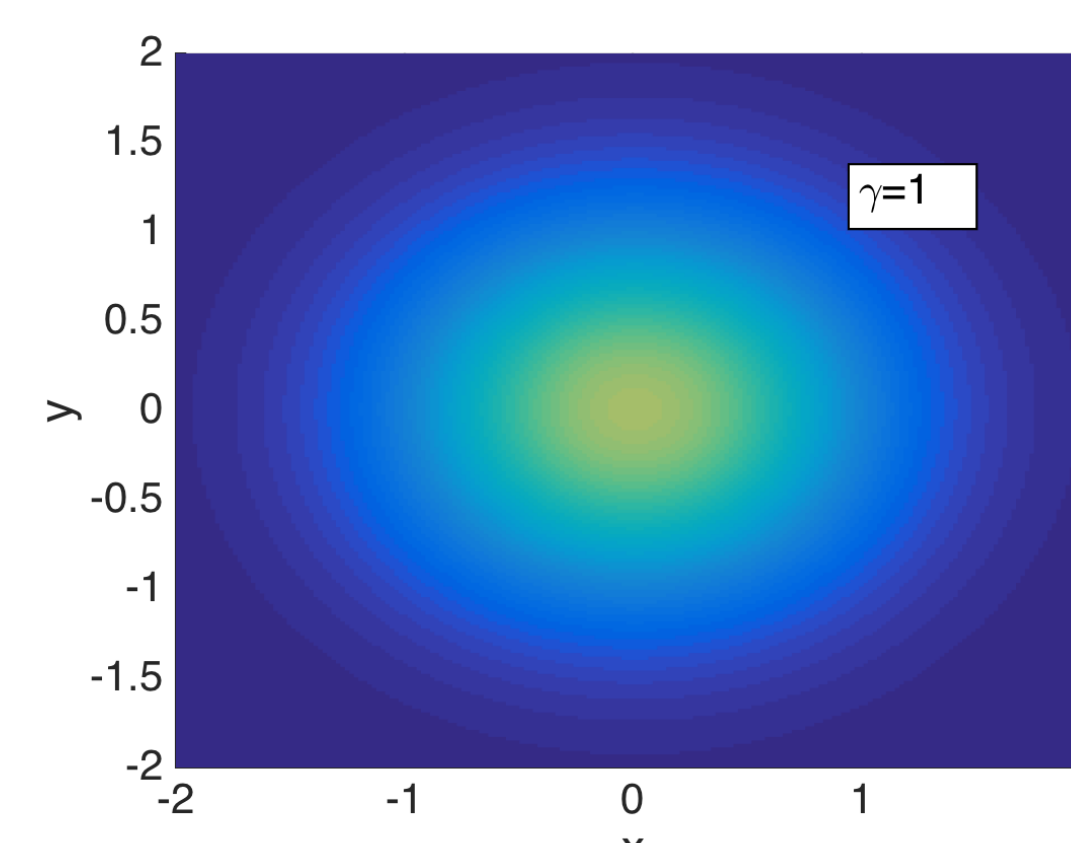
$$\tau_{12} \chi_{nontran} = 0 \times \chi_{nontran}$$

Exact solution:

$$\chi_{nontran}(r, \theta) = R \sqrt{\gamma \sin^2 \theta + \gamma^{-1} \cos^2 \theta}, \quad \gamma = \frac{c_y}{\omega_x^2 x_0}$$



In Mexican hat model (i.e.  $\gamma = 1$ ):  $\chi_{nontran} = R$

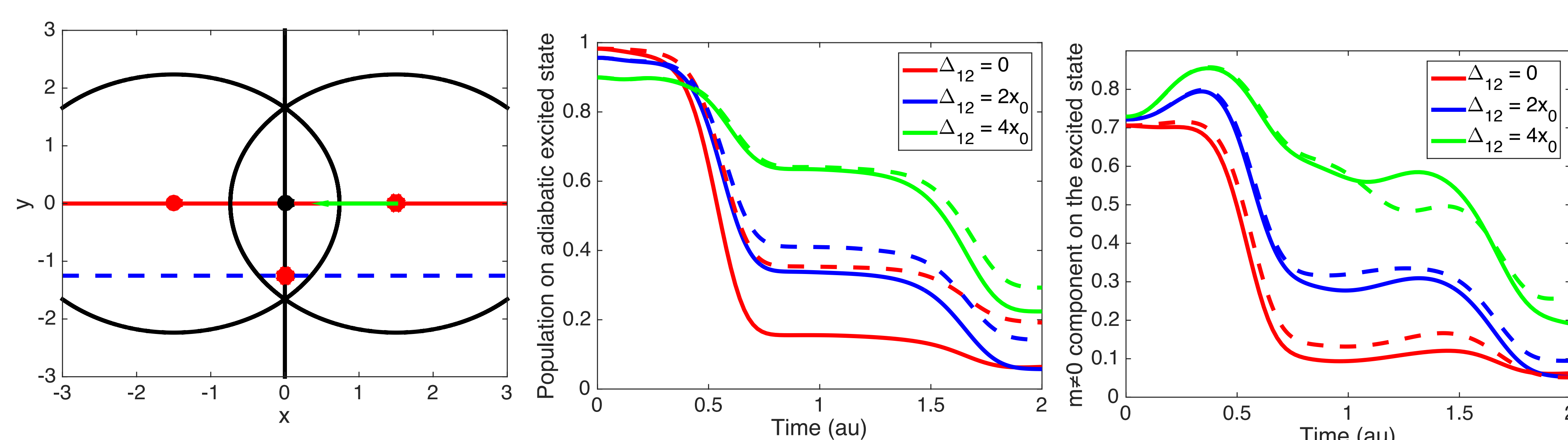


It is the angular-momentum free ( $m=0$ ) component of a wave packet<sup>3</sup>. Good approximation to  $\chi_{nontran}$  when  $\gamma \sim 1$ .

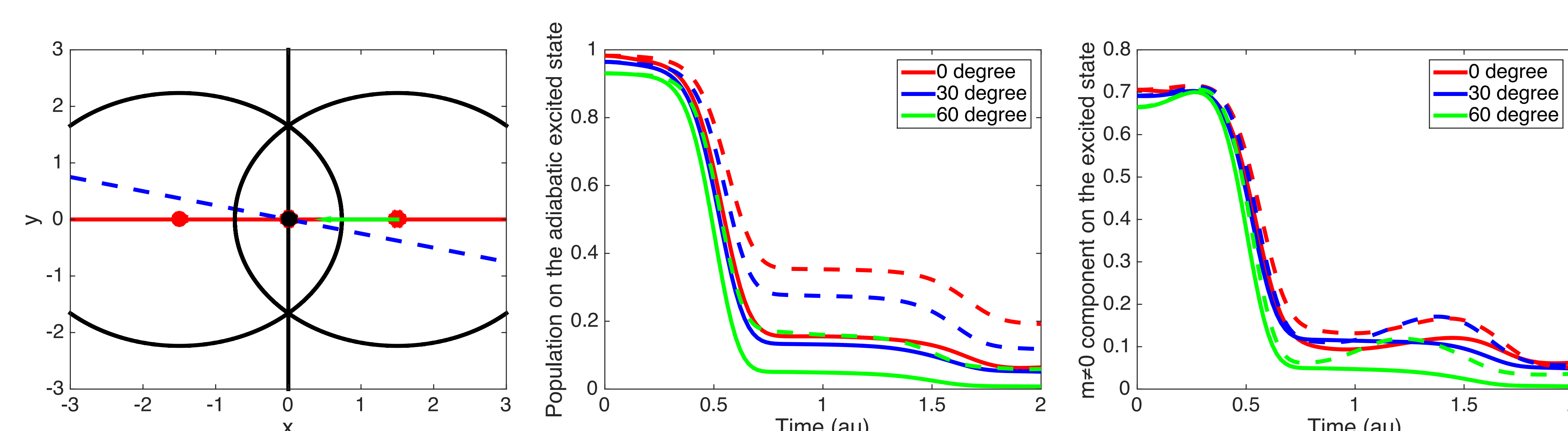
## How geometry affects the nontransferable weight ( $m=0$ ) and the importance of GP

Population plots: solid - with GP, dashed - without GP.

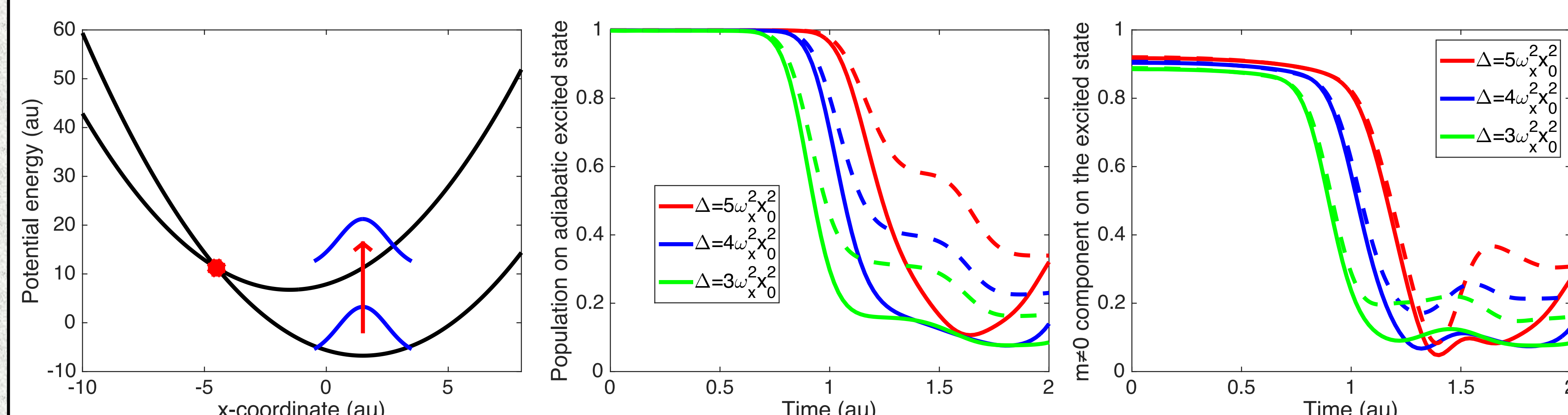
Constant coupling ( $\Delta_{12}$ ): a shift of CI along y-axis



Orthogonality of coupling: breaking down of the wave packet's symmetry



Difference in potential minima ( $\Delta$ ): a change in kinetic energy



## References

- C. A. Mead and D. G. Truhlar, J. Chem. Phys. **70**, 2284 (1979)
- L. Joubert-Doriol, I. G. Ryabinkin, and A. F. Izmaylov, J. Chem. Phys. **139**, 234103 (2013)
- I. G. Ryabinkin, L. Joubert-Doriol, and A. F. Izmaylov, J. Chem. Phys. **140**, 214116 (2014)

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