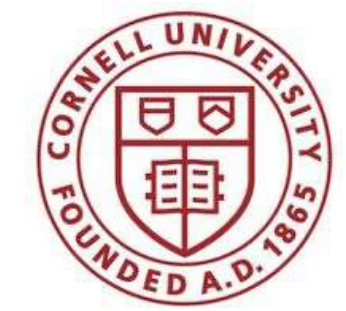# Ring Formation in Protoplanetary Discs Driven by an Eccentric Instability

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Abstract

Protoplanetary discs may spontaneously generate multiple concentric gas rings without embedded planets through an eccentric instability due to cooling.

We show that various background states may trap a slowly precessing, one-armed spiral mode that can become unstable when the disc cools fast enough. The spiral naturally evolves into ellipses as the instability saturates. The angular momentum required to excite this spiral comes at the expense of a corresponding radial mass transport that generically produces multiple rings.

Our fiducial long-term hydrodynamics simulation exhibits four long-lived, axisymmetric gas rings. We verify the instability evolution and ring formation mechanism from first principles with our linear theory, which shows remarkable agreement with the simulation results.

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Background

Eccentric modes are the gas complex eccentricity profiles $E(r)$ that evolve coherently across their host discs.
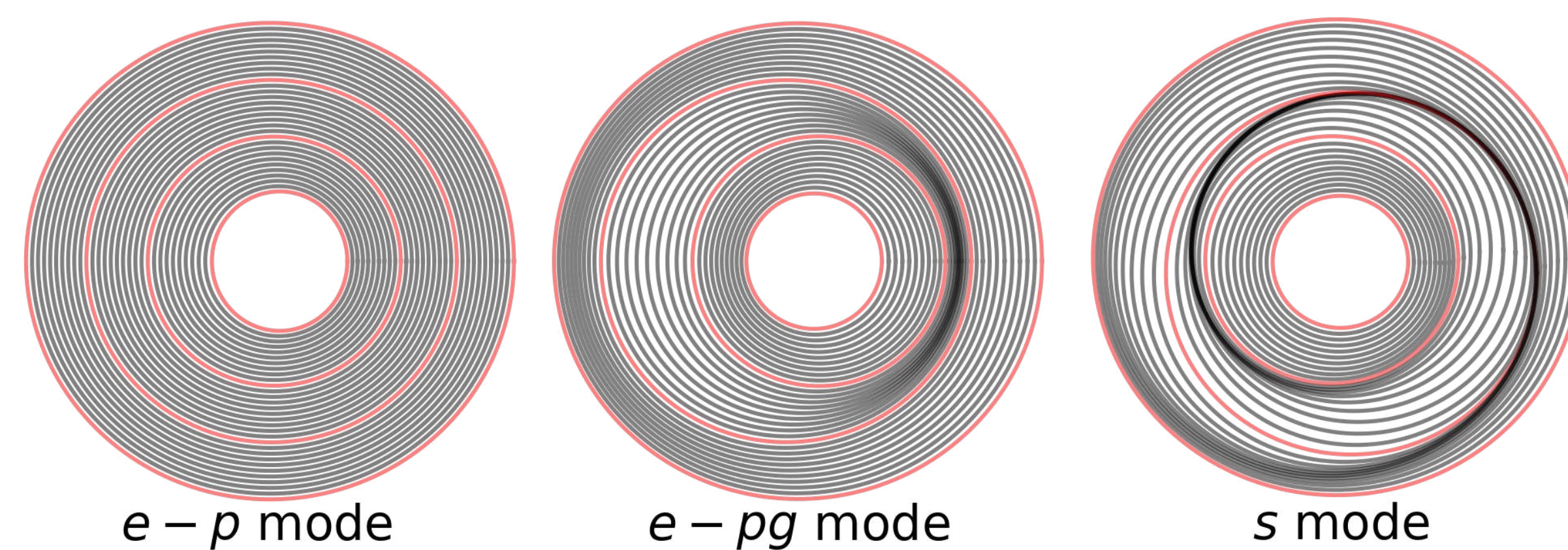
Figure 1. Three different kinds of eccentric modes (see Lee *et al.* 2019). Black: gas trajectories at different locations in a disc. Red: same as black but with larger radial spacing.

e − p mode   *e − pg* mode   *s* mode

S mode instability appears as a growing one-arm spiral in locally isothermal discs (Lin 2015).

Hydro Simulations

We use LA-COMPASS (Li *et al.* 2005, 2009) to simulate 2d discs with **cooling** $\beta = t_{cool}\Omega_K$ (Gammie 2001), **self-gravity**, and **no planets**:
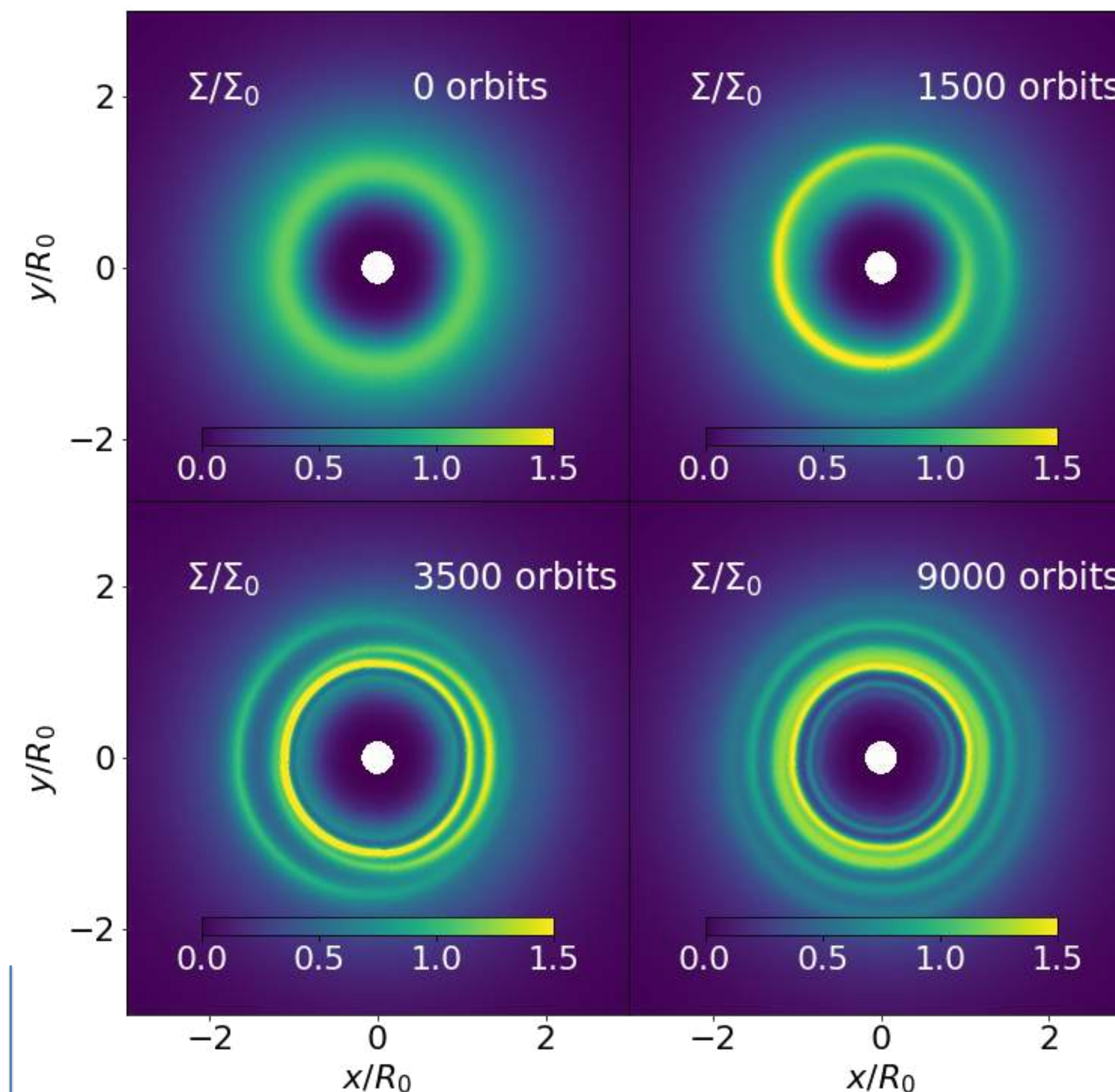
Figure 2. Simulation snapshots are taken at four different stages:
- **Initial:** inner hole; outer taper; no coherent substructures
- **Linear instability:** *s* mode; **rings and gaps form in this stage**
- **Saturation:** no spiral, but coherent ellipses can live (*e-pg* modes)
- **Long-term damping:** eccentric modes may be damped at the boundary.

Figure 3. Rings and gaps in the simulation:
- They are generated by the growing spiral.
- They appear at the early stage of the evolution and can be seen in $\langle\Sigma\rangle$.
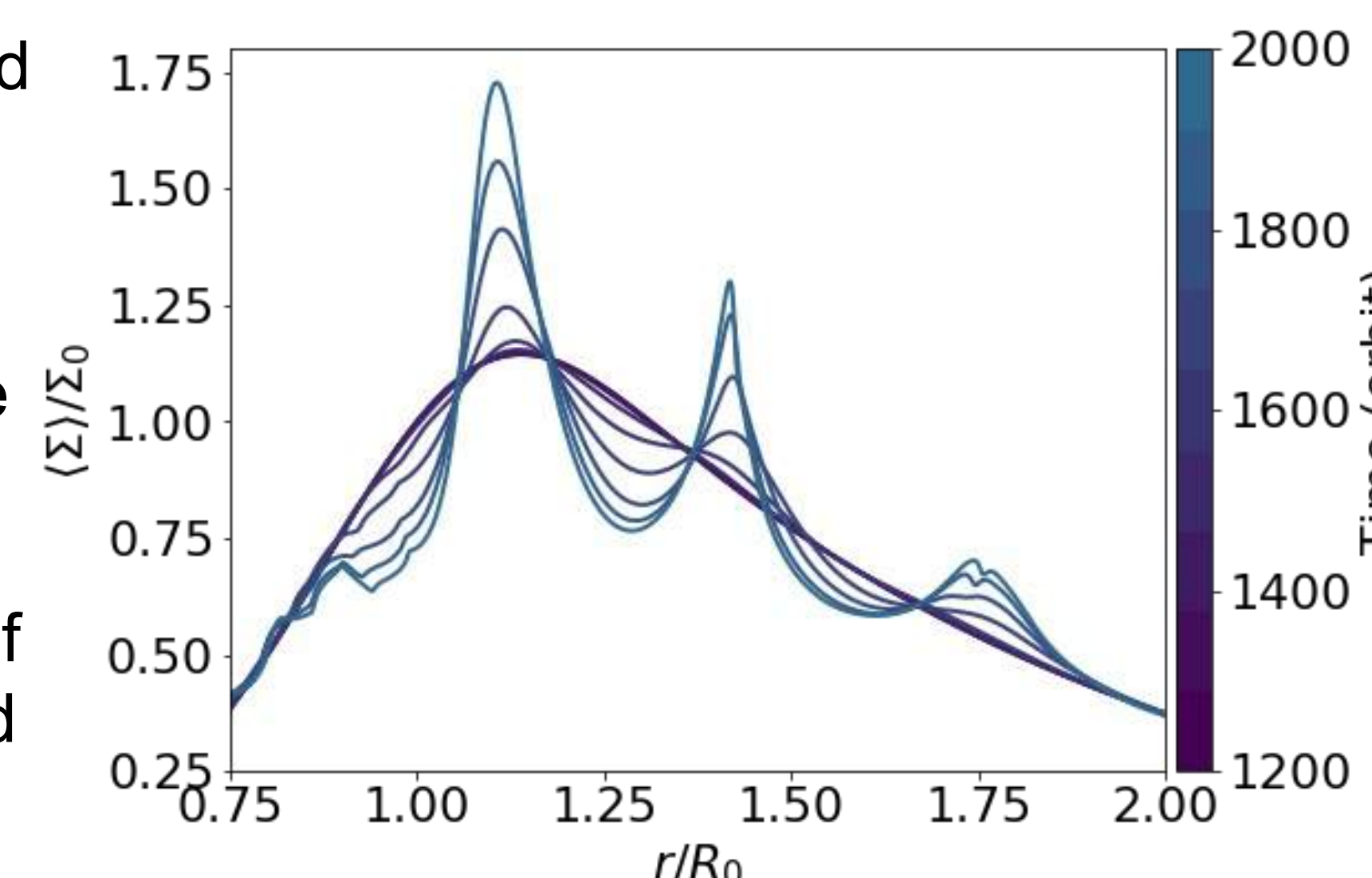
Linear Theory

Eigenmode equation for eccentricity:

$$2r^3\Omega_K\Sigma\omega = \frac{1}{1+\beta^2}\frac{d}{dr}\left\{r^3 P\left[c_{\rm iso}^2\frac{d}{dr}\left(\frac{E}{c_{\rm iso}^2}\right) + \gamma\beta^2\frac{dE}{dr}\right] + i\beta r^3 P\left[(\gamma-1)\frac{dE}{dr} + \frac{d\ln c_{\rm iso}^2}{dr}E\right]\right\} + r^2\frac{dP}{dr}E$$

$$\underbrace{-\Sigma r\frac{d}{dr}\left(r^2\frac{d\Phi_0}{dr}\right)E - \Sigma\frac{d}{dr}(r^2\Phi_1)}_{\text{disk self-gravity}}$$

Instability criteria: The disc needs to have
1. a $g(r) = \pi G\Sigma r/c_s^2$ large enough to trap a *s* mode,
2. and a cooling rate $\beta < g_{max}^{-1}$ to destabilize the *s* mode.

Eccentric modes and **the corresponding radial mass flux** can be solved for arbitrary T, Σ, β profiles with linear theories:

Ring forming spot (convergent flow)

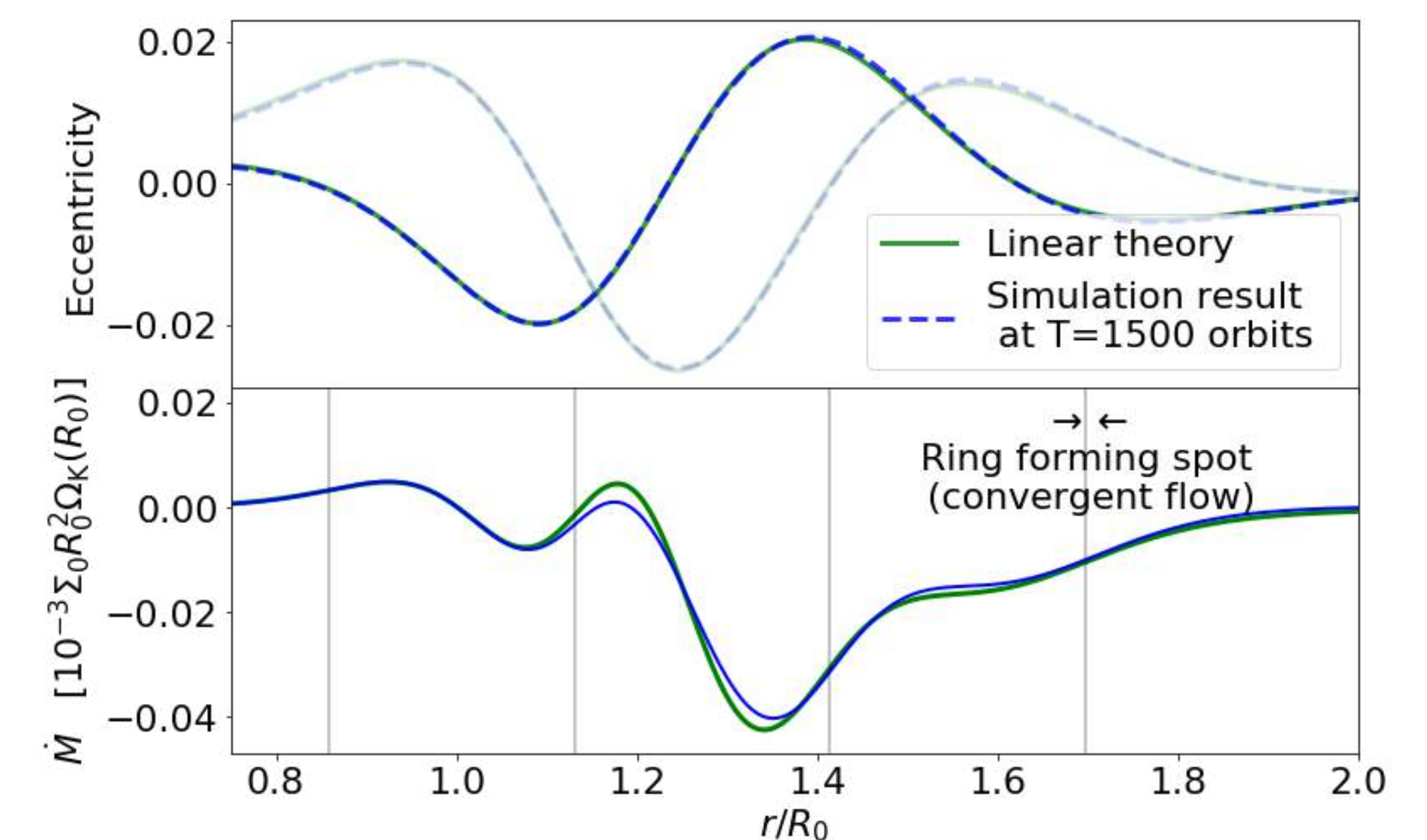Linear theory
Simulation result at T=1500 orbits

Figure 4. Comparison between the linear theory prediction and hydro simulation results at $T = 1500$ orbits:
- **Top:** Eccentricity profile $E(r)$. The real and imaginary parts are shown as opaque and transparent lines, respectively. The spiral growth rate is $\gamma_1 \approx 0.001\,\Omega_K|_{r=R_0}$ in both the linear and simulated results.
- **Bottom:** Radial mass flux $\dot{M} = -2\pi r\langle\Sigma v_r\rangle$. The vertical lines represent the ring forming spots in the simulation (Fig. 3).

Reference

Gammie 2001, ApJ, **553**, 174
Lee *et al.*, 2019, ApJ, **872**, 184
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Lin 2015, MNRAS, **448**, 4