QAOA Depth–Noise Tradeoff for Weighted MaxCut

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Overview

We study the depth–noise tradeoff in the Quantum Approximate Optimization Algorithm (QAOA) on weighted MaxCut instances. For each topology (`er`, `regular`, `complete`, `ba`) and weight model (`rademacher`, `uniform`, `gaussian`), we sweep the circuit depth p, optimize angles (γ, β) with COBYLA under an ideal simulator, and evaluate the same angles on a noisy simulator. The performance metric is the Approximation Ratio (AR), defined as the expected cut value divided by the brute-force optimum for the instance.

Methods

Graphs. Small graphs were generated per topology with sizes approximately ba=10, complete=9, er=10, regular=10. Weights follow three distributions; when signed weights are present, AR can be negative. **Optimization.** For each $p \in \{1, 2, 3, 4, 5, 6, 7$, we warm-started from the solution at depth $p - 1$ and ran COBYLA. **Evaluation.** Ideal expectations are computed from statevectors; noisy expectations use a sampling simulator with a lightweight depolarizing + readout noise model ("synthetic" backend) and transpiled circuits. We report AR, circuit depth, and two-qubit gate counts.

Results Summary

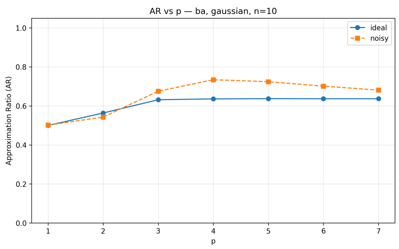
Table 1 reports, for each topology/weight pair, the depth that maximizes AR in the ideal and noisy settings, the corresponding AR (mean±std across seeds), and the average two-qubit gate counts at those depths. The last column shows the noisy AR improvement relative to $p = 1$.

AR vs depth

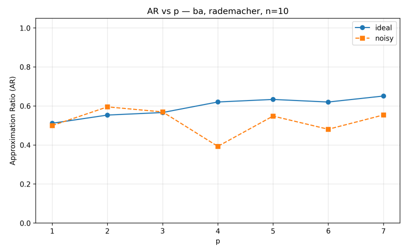
Figures 1 show AR vs depth with mean±std across seeds for each topology/weight pair.

Topo	Weights	$p^\star_{\text{ideal}}$	AR_{ideal}	2q (ideal)	$p^\star_{\text{noisy}}$	AR_{noisy}	2q (noisy)	$\Delta\text{AR}_{\text{noisy}}$ (vs p=1)
ba	gaussian	5	0.637	45	4	0.734	36	0.233
ba	rademacher	7	0.651	63	2	0.595	18	0.096
ba	uniform	7	0.717	63	2	0.574	18	0.071
complete	gaussian	7	0.722	252	1	0.692	36	0.000
complete	rademacher	7	1.000	252	1	0.976	36	0.000
complete	uniform	7	0.800	252	4	0.759	144	0.012
er	gaussian	7	0.704	175	3	0.667	75	0.016
er	rademacher	7	0.813	175	3	0.828	75	0.056
er	uniform	7	0.758	175	6	0.730	150	0.057
regular	gaussian	7	0.721	105	7	0.652	105	0.062
regular	rademacher	7	0.939	105	5	0.841	75	0.267
regular	uniform	7	0.982	105	6	0.818	90	0.113

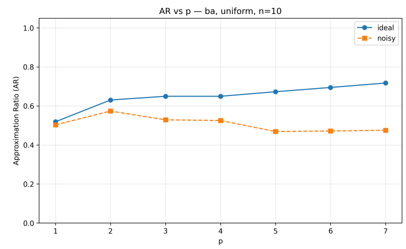
Table 1: Summary of best-performing QAOA runs across graph topologies and edge-weight distributions. For each configuration, the table reports the depth p^* yielding the highest approximation ratio (AR) under both ideal and noisy simulators, along with corresponding two-qubit (2q) gate counts and improvement $\Delta\text{AR}_{\text{noisy}}$ relative to $p = 1$. These values represent mean performance aggregated over random seeds.



(a) ba, gaussian

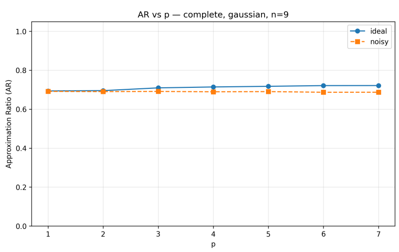


(b) ba, rademacher

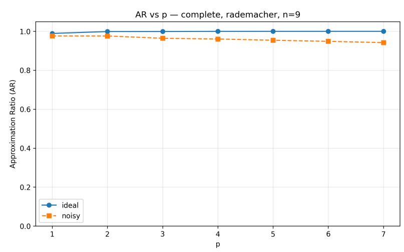


(c) ba, uniform

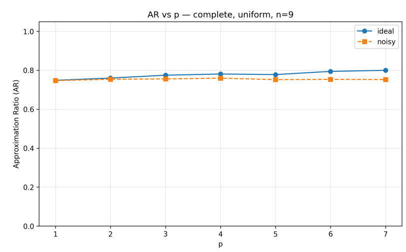
Figure 1: Mean±std AR vs depth p aggregated across random seeds. Solid: ideal simulator; dashed markers: noisy simulator.



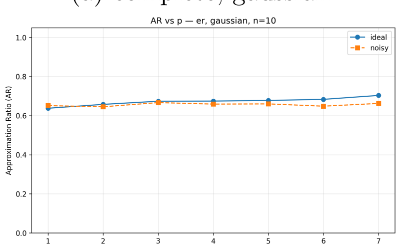
(a) complete, gaussian



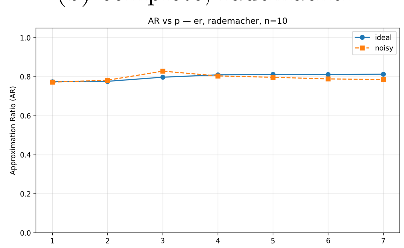
(b) complete, rademacher



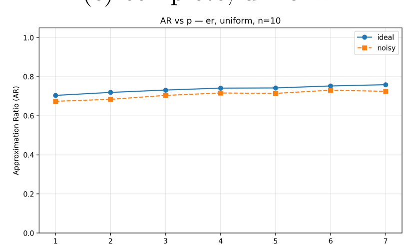
(c) complete, uniform



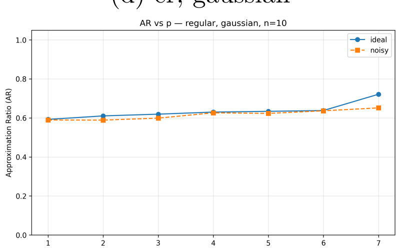
(d) er, gaussian



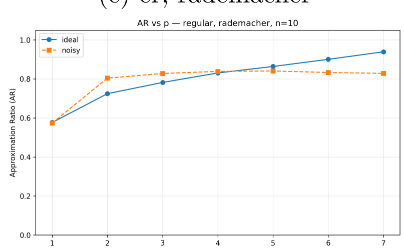
(e) er, rademacher



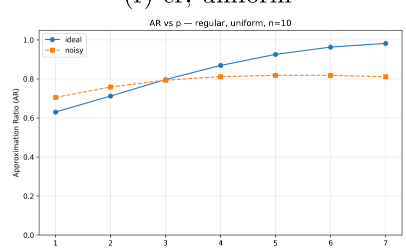
(f) er, uniform



(g) regular, gaussian



(h) regular, rademacher



(i) regular, uniform

Noisy AR vs two-qubit gates

Figure 3 scatters the mean noisy AR against the mean two-qubit count across all instances and depths. The downward trend as two-qubit count grows illustrates a typical onset of noise-limited performance even when ideal AR keeps improving with depth.

Results and Analysis

Table 1 summarizes the best-performing QAOA configurations across all graph topologies and weight initializations. In the noiseless simulator, deeper circuits generally improve the approximation ratio (AR), with optimal depths p^*_{ideal} often reaching the upper end of the search range ($p = 5{-}7$). This aligns with theoretical expectations: additional QAOA layers enable finer control of the cost landscape and allow closer approach to the true MaxCut optimum.

However, under the noisy simulator, the trend shifts markedly. The optimal depth p^*_{noisy} is typically smaller—between 2 and 4 for most topologies—reflecting the growing impact of gate errors and decoherence at large circuit depths. Even when two-qubit counts increase substantially, the AR improvement under noise saturates or declines, showing the familiar trade-off between expressivity and noise robustness.

Across topologies, `regular` and `complete` graphs maintain consistently high AR_{noisy}, suggesting that dense connectivity allows QAOA to retain structure even when gate errors accumulate. By contrast, the `ba` (Barabási–Albert) graphs exhibit more noise sensitivity, likely because their heterogeneous degree distribution amplifies the effect of individual gate errors on hub nodes.

The edge-weight distribution also plays a role. Rademacher weights (± 1) produce the highest ideal AR values, consistent with balanced positive and negative interactions yielding a clearer optimization landscape. Gaussian and uniform weights, while smoother, result in smaller $\Delta\text{AR}_{\text{noisy}}$ gains beyond $p = 1$, indicating that the additional QAOA layers provide diminishing returns under hardware noise.

Finally, the last column, $\Delta\text{AR}_{\text{noisy}}$, quantifies the effective benefit of circuit depth in realistic conditions. Values near zero (e.g., for `complete, rademacher`) indicate that noise completely offsets the advantage of deeper circuits, while moderate positive values (e.g., for `regular, rademacher`) reveal cases where increased depth still yields measurable improvement. Together, these results emphasize that optimal depth selection must account for both topology and noise level, not just idealized convergence.

Discussion

Increasing p generally improves the *ideal* AR, but it also increases transpiled depth and two-qubit gates, so the *noisy* AR tends to saturate or decline beyond a problem-dependent $p^{\star}$. The gap between ideal and noisy curves quantifies sensitivity to noise. The two-qubit count provides a convenient predictor of the onset of noise-limited performance.

Acknowledgements

We acknowledge IBM's public QAOA tutorial materials that informed our implementation approach, and the assistance of an AI (GPT-5 Thinking) for coding and drafting plots.

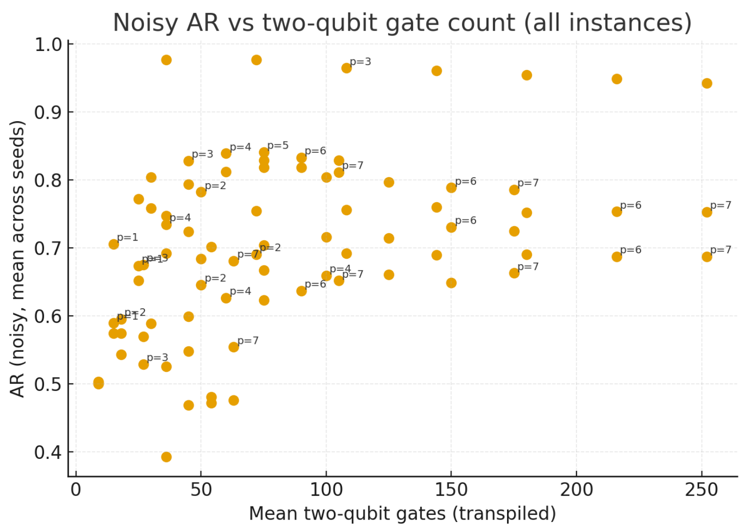


Figure 3: Mean noisy AR vs mean two-qubit gate count across all (topology, weight, p).