Optimizing Portfolio Weights with Theory and Machine Learning

MASTERS OF financial insurance

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Abstract

If someone has spare money, should they place it in a risk-free account or invest in stocks for potentially higher returns? To address this question, I examine portfolio optimization strategies tailored to different risk tolerances. Backtesting results demonstrate that strategies customized to specific preferences can enhance returns, highlighting the advantages of combining financial theory with machine learning to improve investment decisions in uncertain markets.

Objectives

- Optimize Portfolio Weights Using Advanced Theoretical and Machine Learning Models
- Incorporate Various Real-World Investment Constraints and Conduct Thorough Backtesting
- Enhance Returns by Maximizing Expected Utility and Accounting for Risk Aversion Level

Methods

Define Objective Function → Apply Cts-Time Models to Derive Theoretical π → Estimate Key Parameters Σ, μ → Obtain ML Model → Incorporate Real-World Constraints → Conduct Backtesting & Analyze Results

Results

❖ Minimizing Portfolio Variance

Objective Function: $\min_{\pi_t}(\boldsymbol{\pi}_t^T \boldsymbol{\Sigma}_t \boldsymbol{\pi}_t)$ subject to $\mathbf{1}_m^T \boldsymbol{\pi}_t = 1$

$$\boldsymbol{\pi}_t = \frac{\boldsymbol{\Sigma}_t^{-1}\mathbf{1}_m}{\mathbf{1}_m^{-1}\boldsymbol{\Sigma}_t^{-1}\mathbf{1}_m}$$

2022.07 – 2023.07	No Short Selling	Allow Short Selling
No Rebalance	1.66%	4.61%
Monthly Rebalance	1.97%	2.02%
2023.07 – 2024.07	**No Short Selling**	**Allow Short Selling**
No Rebalance	1.99%	2.26%
Monthly Rebalance	1.71%	4.33%

Average Monthly Return Comparisons

❖ Linear-Quadratic Preference

Maximize the Expected Utility of Wealth, adjusted for the Risk Aversion Level. Objective Function:

$$\max_{\pi_t} \int_0^T \left[W_t \left(r + \boldsymbol{\pi}_t^T(\boldsymbol{\mu}_t - r\mathbf{1}_m) \right) - \frac{\gamma}{2} W_t \boldsymbol{\pi}_t^T \boldsymbol{\Sigma}_t \boldsymbol{\pi}_t \right] dt$$

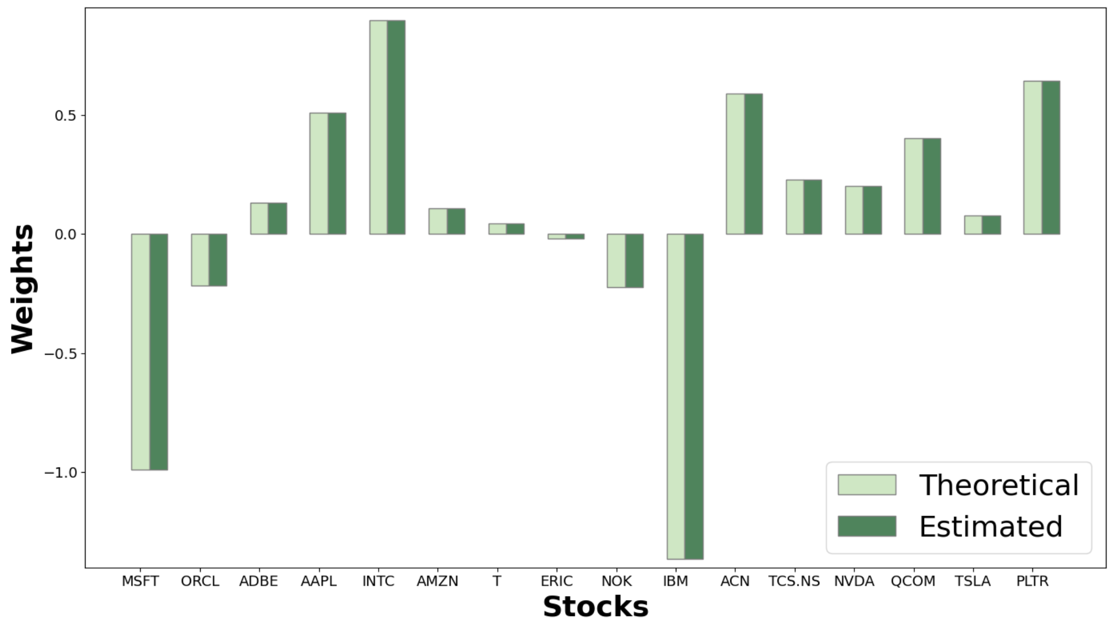
$$\boldsymbol{\pi}_t = \frac{1}{\gamma}\boldsymbol{\Sigma}_t^{-1}(\boldsymbol{\mu}_t - r\mathbf{1}_m)$$

Average Monthly Return Comparisons

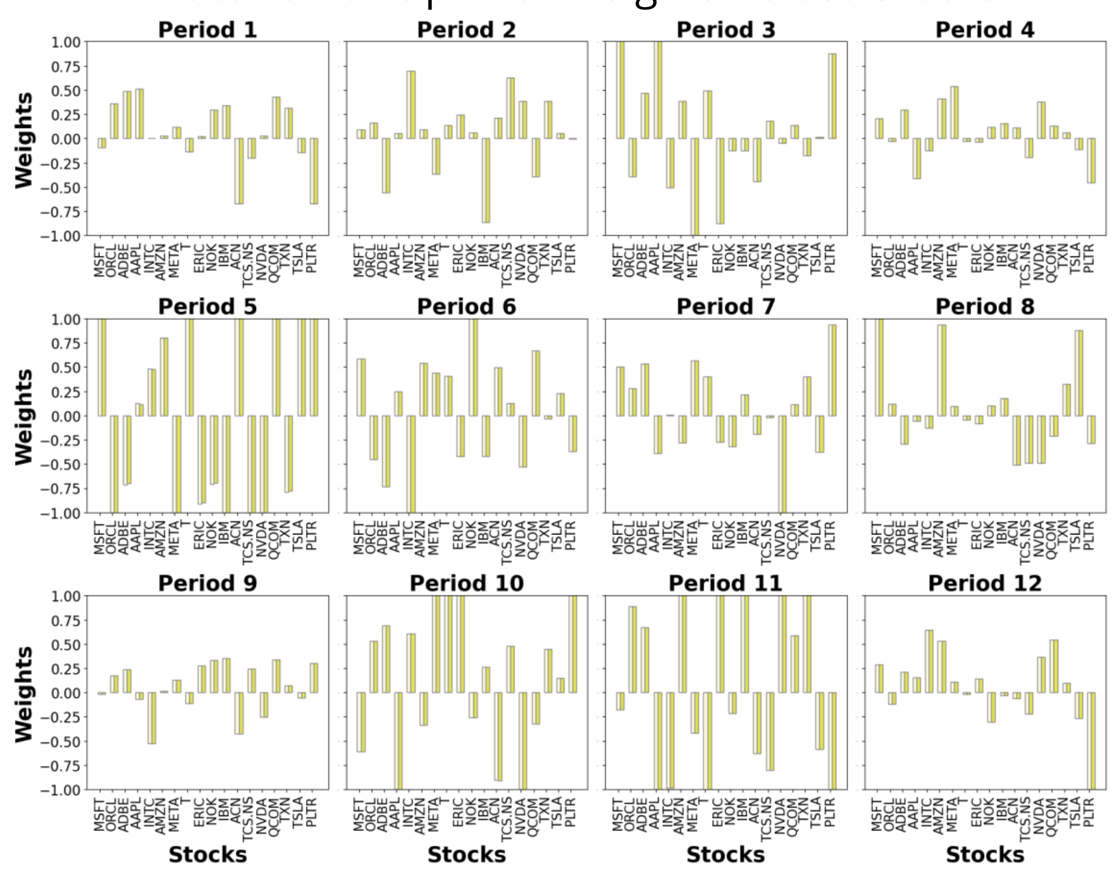
2022.07 – 2023.07	No Short Selling	Allow Short Selling
No Rebalance	3.61%	4.46%
Monthly Rebalance	4.30%	6.42%
2023.07 – 2024.07	**No Short Selling**	**Allow Short Selling**
No Rebalance	4.16%	4.79%
Monthly Rebalance	3.10%	5.51%

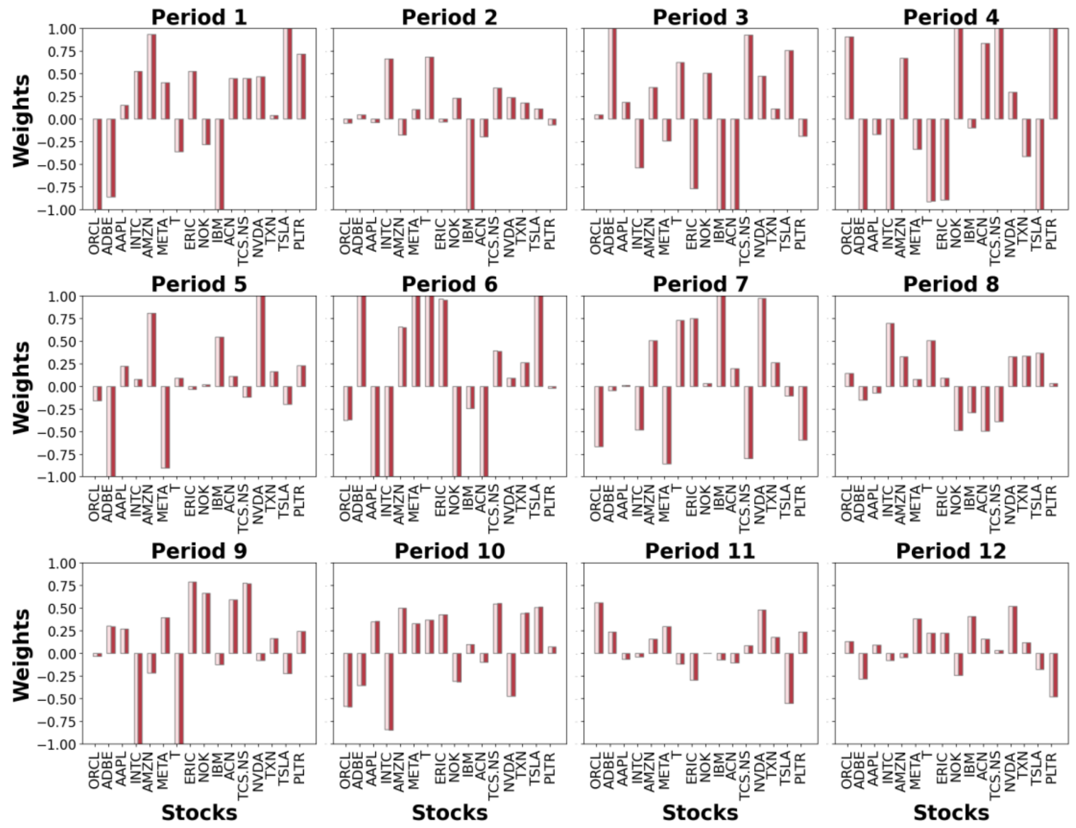
Real-World Factors and Assumptions

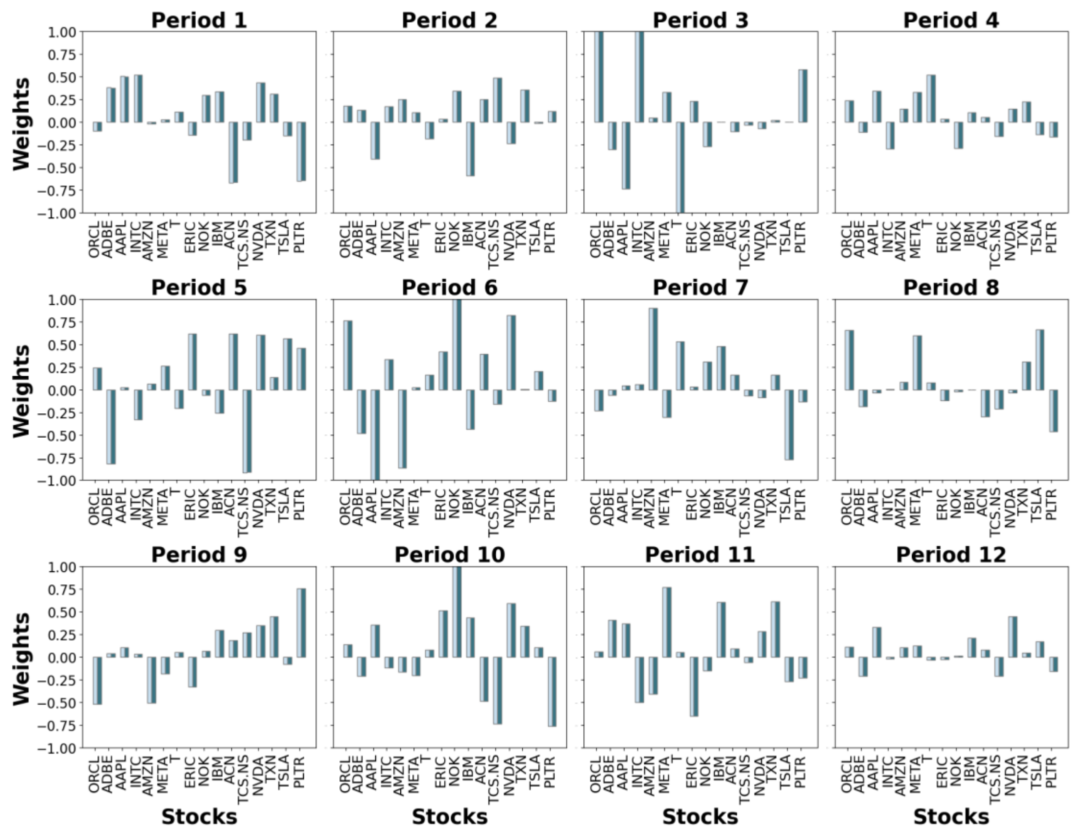
Transaction Fees: 2%		Risk-Free Rate r	5% for 2023-2024
Tax on Gains: 1.5%			3.75% for 2022-2023
Risk Aversion Level γ: 3 (Moderately Risk-Aversion)			
Data Frequency	Daily Returns to Estimate the Covariance Matrix $\boldsymbol{\Sigma}$		
	Monthly Returns to Estimate Expectations $\boldsymbol{\mu}$		



Allocation of Optimal Weights Across Stocks







Conclusion

1. **Short Selling Benefits**: it can lead to higher returns by providing more opportunities to capitalize on market movement
2. **Rebalancing Considerations**: while rebalancing incurs transaction costs, it is crucial to adjust portfolios in response to significant market changes to optimize returns
3. **Variance Minimization Limitations**: a strategy focused on minimizing variance is conservative and may yield lower returns, especially in volatile markets
4. **Integrating Theory and Machine Learning**: combining theoretical models with machine learning techniques can better adapt to specific market conditions and investor preferences, enhancing portfolio performance

References

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[2] Goldberg, L. R., Papanicolaou, A., & Shkolnik, A. (2022). The Dispersion Bias. *SIAM Journal on Financial Mathematics* **13**(2), 521–550.