作业14参考答案

13.

$$\begin{aligned}A &= \{1,2,7,8\}\\ B &= \{0,1,2,3,4,5,6,7\}\\ C &= \{0,3,6,9,12,15,18\}\\ D &= \{1,2,4,8,16,32\}\end{aligned}$$

(2) $A \cap (B \cap (C \cap D)) = \varnothing$

(4) $(B - A) \cup D = \{0,1,2,3,4,5,6,8,16,32\}$

14. $\{3,4,\{3,4\}\}$

15. $P(\varnothing) = \{\varnothing\}$

$PP(\varnothing) = \{\varnothing, \{\varnothing\}\}$

$PPP(\varnothing) = \{\varnothing, \{\varnothing\}, \{\{\varnothing\}\}, \{\varnothing, \{\varnothing\}\}\}$

(1) $\bigcup\{PPP(\varnothing), PP(\varnothing), P(\varnothing), \varnothing\} = \{\varnothing, \{\varnothing\}, \{\{\varnothing\}\}, \{\varnothing, \{\varnothing\}\}\}$

16. $P(A) = \{\varnothing, \{\{\varnothing\}\}, \{\{\{\varnothing\}\}\}, \{\{\varnothing\}\}\}\}$

$\bigcup P(A) = \{\{\varnothing\}, \{\varnothing\}\}$

17.

(6)

① 设 $A \cap B = \varnothing$，对任意的 x

$x \in A \Rightarrow x \in A - \varnothing \Rightarrow x \in A - A \cap B \Rightarrow x \in -(A \cap B) \Rightarrow x \in -B$

所以，$A \subseteq -B$.

② 设 $A \subseteq -B$，对任意的 x

$x \in B \Rightarrow x \notin -B \Rightarrow x \notin A \Rightarrow x \in -A$

所以，$B \subseteq -A$.

③ 设 $B \subseteq -A$，对任意的 x

$x \in A \Rightarrow x \notin -A \Rightarrow x \notin B \Rightarrow x \in A \cap B$

所以，$A \cap B = \varnothing$.

从而，$A \cap B = \varnothing \Leftrightarrow A \subseteq -B \Leftrightarrow B \subseteq -A$ 得证.

18.

$$
\begin{aligned}
\text{因为}\ & A \oplus A = \varnothing \\
\text{同时有}\ & A \oplus A = A \oplus (A \oplus B) \\
& = (A \oplus A) \oplus B \qquad\qquad \text{(对称差的结合律)} \\
& = \varnothing \oplus B = B
\end{aligned}
$$

因此$B = \varnothing$

19. 充要条件为$A \subseteq B \cap C$

$$
\begin{aligned}
& (A - B) \cup (A - C) = \varnothing \\
& \Leftrightarrow A - B = \varnothing \wedge A - C = \varnothing \\
& \Leftrightarrow A \subseteq B \wedge A \subseteq C \\
& \Leftrightarrow A \subseteq B \cap C
\end{aligned}
$$

22.

方法一：

（1）设 A 是传递集合，由定理 9.5.14 得$\bigcup A$ 也是传递集合.

对任意的 x，$x \in \bigcup A \Rightarrow x \subseteq \bigcup A \Rightarrow x \in A$

所以，$\bigcup A \subseteq A$.

（2）设$\bigcup A \subseteq A$，对任意的 x 和 y

$x \in y \wedge y \in \bigcup A \Rightarrow x \in y \wedge (y \subseteq A \vee y \in A) \Rightarrow x \in A$

所以，A 是传递集合.

从而，集合 A 是传递集合当且仅当$\bigcup A \subseteq A$ 得证.

方法二：

A 是传递集合

$\Leftrightarrow (\forall x)(\forall y)(x \in y \wedge y \in A \rightarrow x \in A)$

$\Leftrightarrow (\forall x)((\exists y)(x \in y \wedge y \in A \rightarrow x \in A))$

$\Leftrightarrow (\forall x)(x \in \bigcup A \rightarrow x \in A)$

$\Leftrightarrow \bigcup A \subseteq A$