一.

(1)

$$\neg(\exists x)(\exists y)(P(x)\wedge P(y)\wedge Q(x)\wedge Q(y)\wedge R(x,y))$$
$$=(\forall x)(\forall y)\neg(P(x)\wedge P(y)\wedge Q(x)\wedge Q(y)\wedge R(x,y))$$
$$=(\forall x)(\forall y)(\neg(P(x)\wedge P(y)\wedge Q(x)\wedge Q(y))\vee\neg R(x,y))$$
$$=(\forall x)(\forall y)((P(x)\wedge P(y)\wedge Q(x)\wedge Q(y))\rightarrow\neg R(x,y))$$

(3)

$$(\forall x)(P(x)\vee q)\rightarrow(\exists x)(P(x)\wedge q)$$
$$=\neg(\forall x)(P(x)\vee q)\vee(\exists x)(P(x)\wedge q)$$
$$=(\exists x)\neg(P(x)\vee q)\vee(\exists x)(P(x)\wedge q)$$
$$=((\exists x)\neg P(x)\wedge\neg q)\vee(\exists x)(P(x)\wedge q)$$

(5)

$$(\forall x)(P(x)\rightarrow q)$$
$$=(\exists x)P(x)\rightarrow q$$

(7)

$$(\exists x)P(x)\rightarrow(\forall x)Q(x)$$
$$=\neg(\exists x)P(x)\vee(\forall x)Q(x)$$
$$=(\forall x)\neg P(x)\vee(\forall x)Q(x)$$
$$\Rightarrow(\forall x)(\neg P(x)\vee Q(x))$$
$$=(\forall x)(P(x)\rightarrow Q(x))$$

(9)

$$((\forall x)P(x)\wedge(\forall x)Q(x)\wedge(\exists x)R(x))\vee((\forall x)P(x)\wedge(\forall x)Q(x)\wedge(\exists x)S(x))$$
$$=((\forall x)(P(x)\wedge Q(x))\wedge(\exists x)R(x))\vee((\forall x)(P(x)\wedge Q(x))\wedge(\exists x)S(x))$$
$$=(\forall x)(P(x)\wedge Q(x))\wedge((\exists x)R(x)\vee(\exists x)S(x))$$
$$=(\forall x)(P(x)\wedge Q(x))\wedge(\exists x)(R(x)\vee S(x))$$

二 .

(2) 不是普遍有效

在{1,2}域上分析,若 $P(1)=Q(2)=\mathrm{F}$, $P(2)=Q(1)=\mathrm{T}$,该式为假.

(4) 不是普遍有效

在{1,2}域上分析,若 $P(1)=Q(1)=\mathrm{F}$, $P(2)=Q(2)=\mathrm{T}$,该式为假.

(6) 不是普遍有效

在{1,2}域上分析,若 $P(1)=Q(2)=\mathrm{F}$, $P(2)=Q(1)=\mathrm{T}$,该式为假.

(8) 不是普遍有效

在{1,2}域上分析,若 $P(1,2)=P(2,1)=\mathrm{F}$, $P(1,1)=P(2,2)=\mathrm{T}$,该式为假.

三 .

(1)

$(\forall x)(P(x)\to(\exists y)Q(x,y))$

$=(\forall x)(\neg P(x)\vee(\exists y)Q(x,y))$

$=(\forall x)(\exists y)(\neg P(x)\vee Q(x,y))$

(2)

$(\forall x)(\forall y)(\forall z)(P(x,y,z)\wedge((\exists u)Q(x,u)\to(\exists w)Q(y,w)))$

$=(\forall x)(\forall y)(\forall z)(P(x,y,z)\wedge(\neg(\exists u)Q(x,u)\vee(\exists w)Q(y,w)))$

$=(\forall x)(\forall y)(\forall z)(P(x,y,z)\wedge((\forall u)\neg Q(x,u)\vee(\exists w)Q(y,w)))$

$=(\forall x)(\forall y)(\forall z)(\forall u)(\exists w)(P(x,y,z)\wedge(\neg Q(x,u)\vee Q(y,w)))$

(3)

$(\exists x)P(x,y)\leftrightarrow(\forall z)Q(z)$

$=((\exists x)P(x,y)\wedge(\forall z)Q(z))\vee(\neg(\exists x)P(x,y)\wedge\neg(\forall z)Q(z))$

$=((\forall x)\neg P(x,y)\vee(\forall z)Q(z))\wedge((\exists x)P(x,y)\vee(\exists z)\neg Q(z))$

$=((\forall x)\neg P(x,y)\vee(\forall z)Q(z))\wedge((\exists u)P(u,y)\vee(\exists v)\neg Q(v))$

$=(\forall x)(\forall z)(\exists u)(\exists v)(\neg P(x,y)\vee Q(z))\wedge(P(u,y)\vee\neg Q(v))$

(4)

$(\neg(\exists x)P(x)\vee(\forall y)Q(y))\to(\forall z)R(z)$

$=\neg(\neg(\exists x)P(x)\vee(\forall y)Q(y))\vee(\forall z)R(z)$

$=(\neg\neg(\exists x)P(x)\wedge\neg(\forall y)Q(y))\vee(\forall z)R(z)$

$=((\exists x)P(x)\wedge(\exists y)\neg Q(y))\vee(\forall z)R(z)$

$=(\exists x)(\exists y)(\forall z)((P(x)\wedge\neg Q(y))\vee R(z))$

(5)

$(\forall x)(P(x)\to(\forall y)((P(y)\to(Q(x)\to Q(y)))\vee(\forall z)P(z)))$

$=(\forall x)(\neg P(x)\vee(\forall y)((\neg P(y)\vee(\neg Q(x)\vee Q(y)))\vee(\forall z)P(z)))$

$=(\forall x)(\forall y)(\forall z)(\neg P(x)\vee\neg P(y)\vee\neg Q(x)\vee Q(y)\vee P(z))$

(9)

$(\forall x)(P(x)\to(\exists y)Q(x,y))\vee(\forall z)R(z)$

$=(\forall x)(\neg P(x)\vee(\exists y)Q(x,y))\vee(\forall z)R(z)$

$=(\forall x)(\exists y)(\forall z)(\neg P(x)\vee Q(x,y)\vee R(z))$

Skolem 范式

$(\forall x)(\forall z)(\neg P(x)\vee Q(x,f(x))\vee R(z))$

(10)

$(\exists y)(\forall x)(\forall z)(\exists u)(\forall v)P(x,y,z,u,v)$

Skolem 范式

$(\forall x)(\forall z)(\forall v)P(x,a,z,f(x,z),v)$