Decoupling Homophily and Reciprocity with Latent Space Network Models

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Dynamic Network Data

From	To	Date	Time
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- Point process models
- Statistical network models

Observed data: $\{(u, v, \mathcal{H}_{uv})\}_{u,v \in V}$

- $V = \{1, \ldots, n\}$: a fixed set of vertices throughout time period $[0, T)$.
- $\mathcal{H}_{uv} = \{t_i^{uv}\}_{i=1}^{n_{uv}}$: set of all time-points at which u sent v a message.
- $n_{uv} \geq 0$: total number of messages from u to v.

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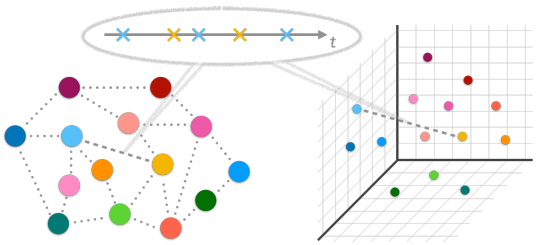
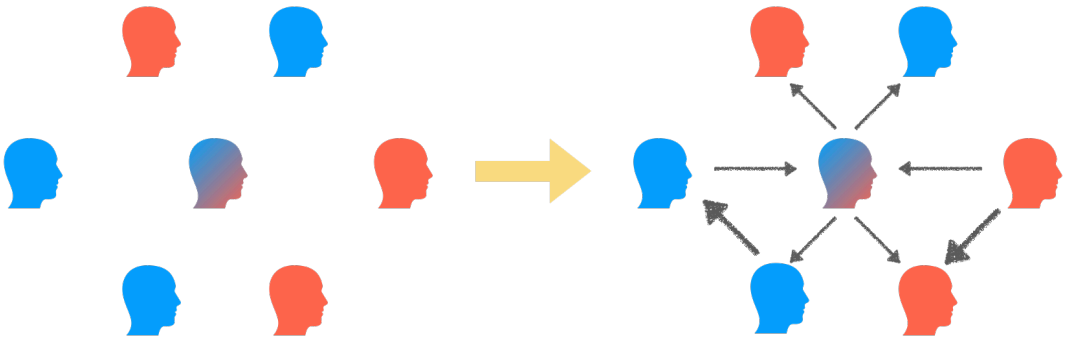
This work: Latent space point process models of dynamic networks:



- Model $\{t_i^{uv}\}_{i=1}^{n_{uv}}$ as realizations of a point process $N_{uv}(t)$, $t \in [0, T)$.
- Entire network $\Rightarrow n^2 - n$ processes (no self-loops). *Not independent!*
- Dependencies via latent space model.
- Network embedding & link prediction.

Homophily

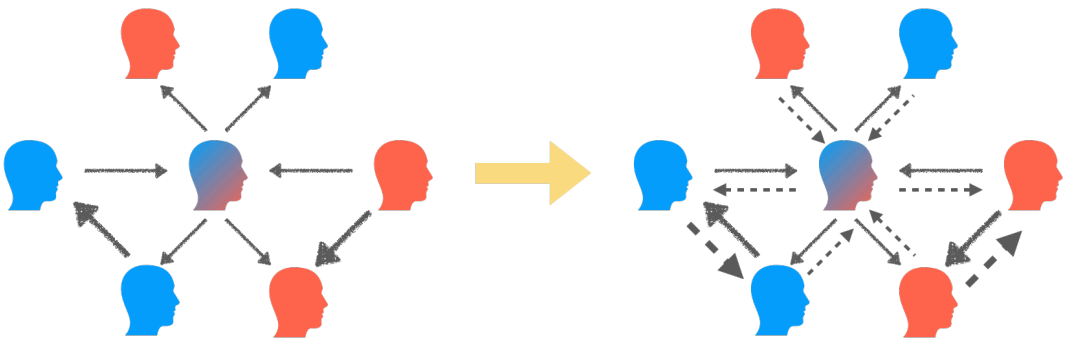
Communication occurs at a higher rate between individuals with similar features.



Latent space models of (static) networks: $p_{uv} \propto e^{-\|\mathbf{z}_u - \mathbf{z}_v\|_2^2}$, $\mathbf{z}_u, \mathbf{z}_v \in \mathbb{R}^d$. (Hoff *et al.*, 2002)

Reciprocity

Individuals tend to reciprocate communications from other individuals.

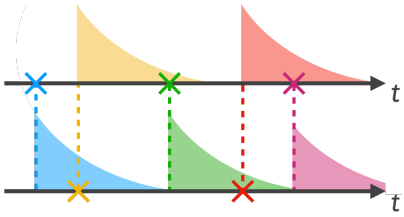


Modeling reciprocating relationships with Hawkes processes. (Blundell *et al.*, 2012)

Model the pair of processes $N_{uv}(t)$ and $N_{vu}(t)$ as a bivariate Hawkes process:

$$\lambda_{uv}(t|\mathcal{H}_{uv}, \mathcal{H}_{vu}) = \gamma_{uv} + \sum_{k:\, t_k^{vu} < t} \phi_{uv}(t - t_k^{vu})$$

Event history: $\mathcal{H}_{uv}(t) = \{t_k^{uv} : t_k^{uv} < t\}$.



(Hawkes, 1971)

Weighted combination of basis kernels:

$$\underbrace{\phi_{uv}(\cdot)}_{\text{Triggering kernel}} = \sum_{b=1}^{B} \xi_b^{uv} \phi_b(\cdot)$$

e.g., $\phi_b(t) = e^{-t/\tau}$, $e^{-t/\tau} \sin^2\left(\frac{\pi t}{\tau}\right)$.

Hawkes Process (HP) Model

$$\lambda_{uv}(t) = \gamma + \sum_{k:\, t_k^{vu} < t} \sum_{b=1}^{B} \xi_b \, \phi_b(t - t_k^{vu})$$

$$N_{uv}(\cdot) \sim \text{HawkesProcess}(\lambda_{uv}(\cdot))$$

(*cf.* Blundell *et al.*, 2012)

Latent Space Point Process Models

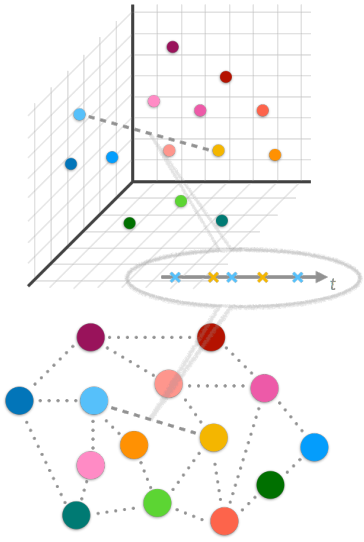
- Poisson-rate Latent Space (PLS) Model
- Hawkes Dual Latent Space (DLS) Model
- Hawkes Base-rate Latent Space (BLS) Model
- Hawkes Reciprocal Latent Space (RLS) Model

Poisson-rate Latent Space (PLS) Model

$$\mathbf{z}_v \sim \mathcal{N}(0, \sigma^2 \, \mathrm{I}_{d\times d}) \qquad \forall v \in V$$

$$\lambda_{uv}(t) = \gamma \, e^{-\|\mathbf{z}_u - \mathbf{z}_v\|_2^2} \qquad \forall u \neq v$$

$$N_{uv}(\cdot) \sim \text{PoissonProcess}(\lambda_{uv}(\cdot)) \qquad \forall u \neq v$$



Hawkes Dual Latent Space (DLS) Model

$$\mathbf{z}_v \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I}_{d\times d}) \qquad \forall v \in V$$

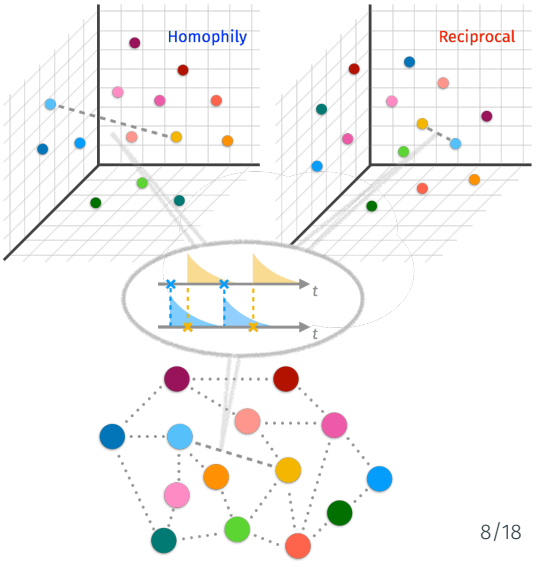
$$\boldsymbol{\mu}_v \sim \mathcal{N}(\mathbf{0}, \sigma_\mu^2 \mathbf{I}_{d\times d}) \qquad \forall v \in V$$

$$\boldsymbol{\varepsilon}_v^{(b)} \sim \mathcal{N}(\mathbf{0}, \sigma_\varepsilon^2 \mathbf{I}_{d\times d}) \qquad \forall v \in V,\ b = 1, \ldots, B$$

$$\mathbf{x}_v^{(b)} \sim \boldsymbol{\mu}_v + \boldsymbol{\varepsilon}_v^{(b)} \qquad \forall v \in V,\ b = 1, \ldots, B$$

$$\lambda_{uv}(t) = \underbrace{\gamma\, e^{-\|\mathbf{z}_u - \mathbf{z}_v\|_2^2}}_{\text{Homophily base-rate}} + \underbrace{\sum_{k:\, t_k^{vu} < t} \sum_{b=1}^{B} \beta\, e^{-\|\mathbf{x}_u^{(b)} - \mathbf{x}_v^{(b)}\|_2^2}\, \phi_b(t - t_k^{vu})}_{\text{Reciprocal component}}$$

$$N_{uv}(\cdot) \sim \text{HawkesProcess}(\lambda_{uv}(\cdot)) \qquad \forall u \neq v$$



Base-rate Latent Space (BLS) Model

$$\mathbf{z}_v \sim \mathcal{N}(0, \sigma^2 \, \mathrm{I}_{d \times d})$$

$$\lambda_{uv}(t) = \underbrace{\gamma \, e^{-\|\mathbf{z}_u - \mathbf{z}_v\|_2^2}}_{\text{Homophily base-rate}} + \sum_{k:\, t_k^{vu} < t} \sum_{b=1}^{B} \xi_b \, \phi_b(t - t_k^{vu})$$

$$N_{uv}(\cdot) \sim \text{HawkesProcess}(\lambda_{uv}(\cdot))$$

Reciprocal Latent Space (RLS) Model

$$\mathbf{x}_v \sim \mathcal{N}(0, \sigma^2 \, \mathrm{I}_{d \times d})$$

$$\lambda_{uv}(t) = \gamma + \underbrace{\sum_{k:\, t_k^{vu} < t} \sum_{b=1}^{B} \xi_b \, e^{-\|\mathbf{x}_u - \mathbf{x}_v\|_2^2} \, \phi_b(t - t_k^{vu})}_{\text{Reciprocal component}}$$

$$N_{uv}(\cdot) \sim \text{HawkesProcess}(\lambda_{uv}(\cdot))$$

(*cf.* Linderman and Adams, 2014)

Model Summary

Model	$\lambda_{uv}(t)$	#parameters
HP	$\gamma + \sum_{k:\, t_k^{vu} < t} \sum_{b=1}^{B} \xi_b \, \phi_b(t - t_k^{vu})$	$\mathcal{O}(B)$
PLS	$\gamma \, e^{-\|\mathbf{z}_u - \mathbf{z}_v\|_2^2}$	$\mathcal{O}(n \cdot d)$
BLS	$\gamma \, e^{-\|\mathbf{z}_u - \mathbf{z}_v\|_2^2} + \sum_{k:\, t_k^{vu} < t} \sum_{b=1}^{B} \xi_b \, \phi_b(t - t_k^{vu})$	$\mathcal{O}(n \cdot d)$
RLS	$\gamma + \sum_{k:\, t_k^{vu} < t} \sum_{b=1}^{B} \xi_b \, e^{-\|\mathbf{x}_u - \mathbf{x}_v\|_2^2} \, \phi_b(t - t_k^{vu})$	$\mathcal{O}(n \cdot d)$
DLS	$\gamma \, e^{-\|\mathbf{z}_u - \mathbf{z}_v\|_2^2} + \sum_{k:\, t_k^{vu} < t} \sum_{b=1}^{B} \beta \, e^{-\|\mathbf{x}_u^{(b)} - \mathbf{x}_v^{(b)}\|_2^2} \, \phi_b(t - t_k^{vu})$	$\mathcal{O}(n \cdot d \cdot B)$

HP Hawkes Process Model (*cf.* Blundell *et al.*, 2012)

PLS Poisson-rate Latent Space Model

RLS Reciprocal Latent Space Model (*cf.* Linderman and Adams, 2014)

BLS Base-rate Latent Space Model

DLS Dual Latent Space Model

Perform maximum a posteriori (MAP) inference for all model parameters.

Full log-likelihood of observed communications $\left\{(u, v, \{t_i^{uv}\}_{i=1}^{n_{uv}})\right\}_{u,v \in V}$:

$$\log \mathcal{L} = \sum_{\substack{u,v=1 \\ u \neq v}}^{n} \left\{ -\Lambda_{uv}(0, T) + \sum_{i=1}^{n_{uv}} \log \lambda_{uv}(t_i^{uv}) \right\}$$

and gradients available in closed form.

Cache data statistics: $\forall u, v \in V, b = 1, \ldots, B$,

$$\Delta_{b,T}^{vu} \triangleq \sum_{k=1}^{n_{vu}} [\Phi_b(T - t_k^{vu}) - \Phi_b(0)] \qquad \delta_{b,i}^{uv} \triangleq \sum_{k:\, t_k^{vu} < t_i^{uv}} \phi_b(t_i^{uv} - t_k^{vu})$$

Optimization via L-BFGS-B (Byrd et al., 1995).

Experimental Evaluation

Dataset Description

Enron Core network of the Enron email dataset during 01/2000–04/2002; 155 Enron executives, 9,646 email messages over 453 days.

Email Email communications within Purdue University during 07/2011—02/2012. Filtered out mailing-lists, extracted 100 largest-degree nodes; 34,438 emails over 237 days.

Facebook Facebook wall messages among Purdue students during 03/2007–03/2008. Extracted 100 largest-degree nodes; 18,865 wall messages over 385 days.

Experiment Setup

For each dataset, sort the messages according to their time-stamps.
Training set: first 70% messages. Test set: remaining 30% messages.

$B = 4$ Hawkes basis kernels: $\phi_1(t) = e^{-\frac{t}{1/24}}$, $\phi_2(t) = e^{-t}$, $\phi_3(t) = e^{-t/7}$, $\phi_4(t) = e^{-t/7} \sin^2\left(\frac{\pi t}{7}\right)$.

Randomly sample 100 time-points t_i during the test period.

For each pair of nodes, the predicted probability that at least a link will appear in the $[t_i, t_i + \delta)$ time window is $p_{uv} = 1 - \exp\{-\int_{t_i}^{t_i+\delta} \lambda_{uv}(s)\, ds\}$. Set $\delta = 14$ (days).

For each t_i, compute ROC-AUC over all node-pairs. Report mean and SD across t_i's.

Model	Enron	Email	Facebook
HP	0.750 (0.070)	0.881 (0.088)	0.931 (0.095)
PLS	0.681 (0.041)	0.843 (0.087)	0.874 (0.078)
BLS	0.738 (0.065)	0.868 (0.095)	0.927 (0.096)
RLS	0.750 (0.070)	0.881 (0.088)	0.931 (0.095)
DLS	**0.928 (0.018)**	**0.971 (0.006)**	**0.979 (0.008)**

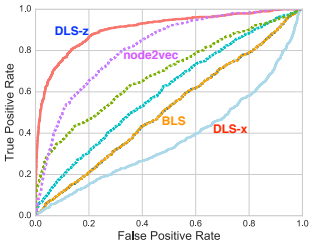
Given the learned latent feature vectors $\{z_v\}_{v \in V}$, compute the predicted probability that an edge exists in the test graph via $p_{uv} \propto e^{-\|z_u - z_v\|_2^2}$.

Compute (static) link prediction ROC-AUC for all pairs of nodes.

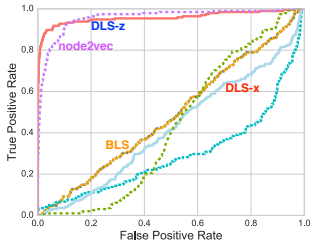
Static graph embeddings: spectral clustering and node2vec (Grover and Leskovec, 2016).

Model	Enron	Email	Facebook
PLS	0.512	0.483	0.505
BLS	0.512	0.483	0.505
RLS	0.601	0.295	0.445
DLS	**0.906**	**0.958**	0.947
Spectral	0.687	0.428	0.452
node2vec	0.829	**0.958**	**0.956**

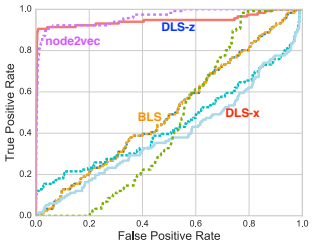
Homophily and Reciprocal Latent Spaces



Enron

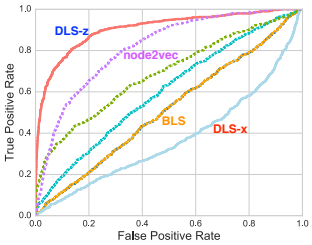


Email

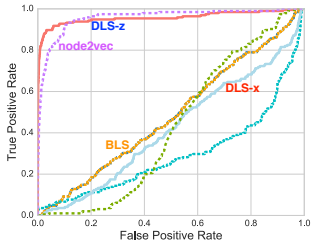


Facebook

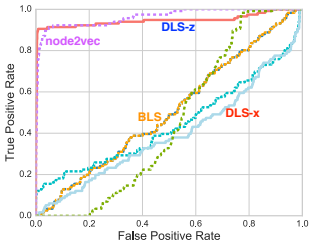
Homophily and Reciprocal Latent Spaces



Enron



Email



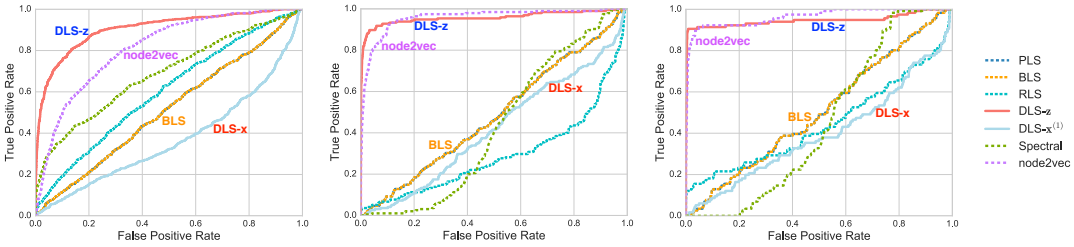
Facebook

(DLS) $$\lambda_{uv}(t) = \gamma\, e^{-\|\mathbf{z}_u - \mathbf{z}_v\|_2^2} + \sum_{k:\, t_k^{vu} < t} \sum_{b=1}^{B} \beta\, e^{-\|\mathbf{x}_u^{(b)} - \mathbf{x}_v^{(b)}\|_2^2}\, \phi_b(t - t_k^{vu})$$

The homophily latent space $\{\mathbf{z}_v\}_{v \in V}$ outperforms reciprocal latent spaces $\{\{\mathbf{x}_v^{(b)}\}_{v \in V}\}_{b=1}^{B}$.

However, BLS ("DLS - $\{\{\mathbf{x}_v^{(b)}\}_{v \in V}\}_{b=1}^{B}$") also performs poorly in link prediction.

Homophily and Reciprocal Latent Spaces



Enron Email Facebook

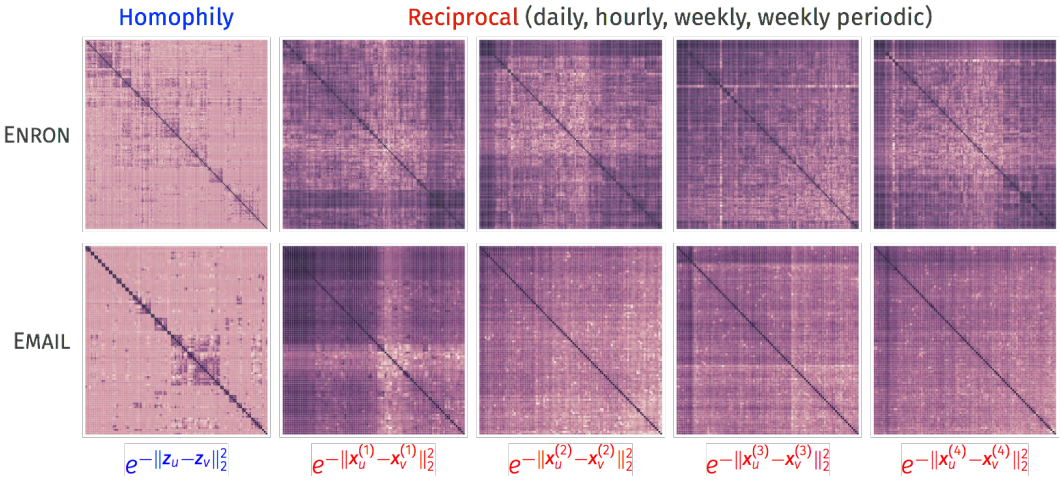
(DLS) $$\lambda_{uv}(t) = \gamma\, e^{-\|z_u - z_v\|_2^2} + \sum_{k:\, t_k^{vu} < t} \sum_{b=1}^{B} \beta\, e^{-\|x_u^{(b)} - x_v^{(b)}\|_2^2}\, \phi_b(t - t_k^{vu})$$

The homophily latent space $\{z_v\}_{v \in V}$ outperforms reciprocal latent spaces $\{\{x_v^{(b)}\}_{v \in V}\}_{b=1}^{B}$.

However, BLS ("DLS - $\{\{x_v^{(b)}\}_{v \in V}\}_{b=1}^{B}$") also performs poorly in link prediction.

Explanation: The reciprocal latent spaces achieve a denoising effect that "explains away" communications primarily driven by reciprocity.

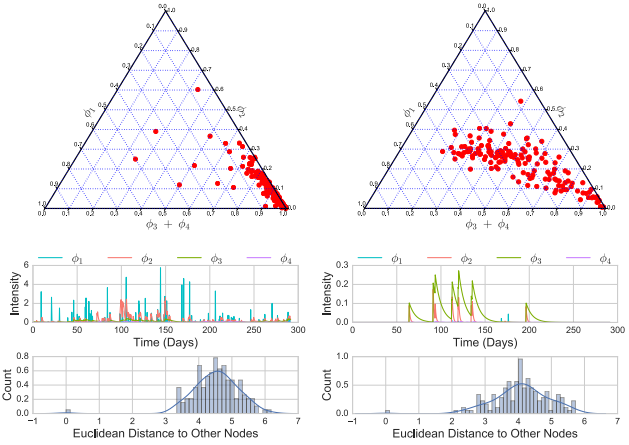
Visualization of Inferred Node-Similarity Matrices



For each pair of nodes u and v, compute relative similarity in the b-th kernel via

$$p_{uv}^{(b)} \triangleq \frac{e^{-\|x_u^{(b)} - x_v^{(b)}\|_2^2}}{\sum_{h=1}^{B} e^{-\|x_u^{(h)} - x_v^{(h)}\|_2^2}}$$

Embed pairs of nodes onto a probability simplex where each pair $u, v \in V$ is represented by a point $(p_{uv}^{(1)}, \ldots, p_{uv}^{(B)})^\top$.



(e) Enron Node #108 (f) Enron Node #92

Concluding Remarks

Model dynamic network data with latent space point process models.

Crucial to account for heterogeneity across both homophily and reciprocity:

- In dynamic link prediction, incorporating the homophily latent space produces a significant gain across all three real-world datasets.
- In static link prediction, the reciprocal latent spaces greatly improve the quality of the estimated homophily latent space.

Our finding is related to that of Rudolph et al. (2016): Modeling each observation conditioned on its context improves the quality of the learned embeddings.

Questions?

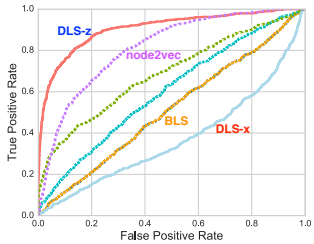
Thank you!

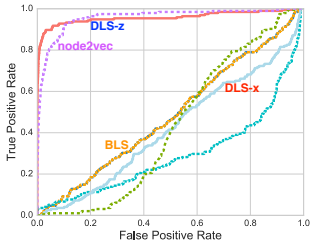
Predictive Log-Likelihood

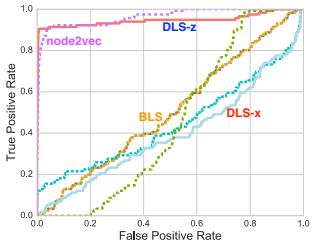
Learn each model on training set; compute predictive log-likelihood on test set.

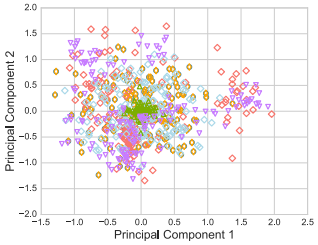
Model	Enron	Email	Facebook
HP	**-16226.155**	-2129.940	-7871.895
PLS	-37803.978	-112684.130	-66742.379
BLS	-21779.686	-9850.932	-12119.869
RLS	-16565.449	-2113.254	-7867.870
DLS	-16422.946	**185.264**	**-6421.609**

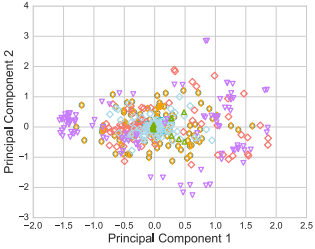
Network Embedding

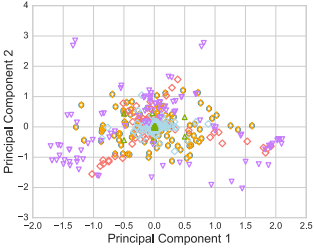


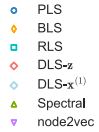












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