EXPLORING

PARRONDO'S PARADOX

IN FINANCIAL INSTRUMENTS

Poster 10

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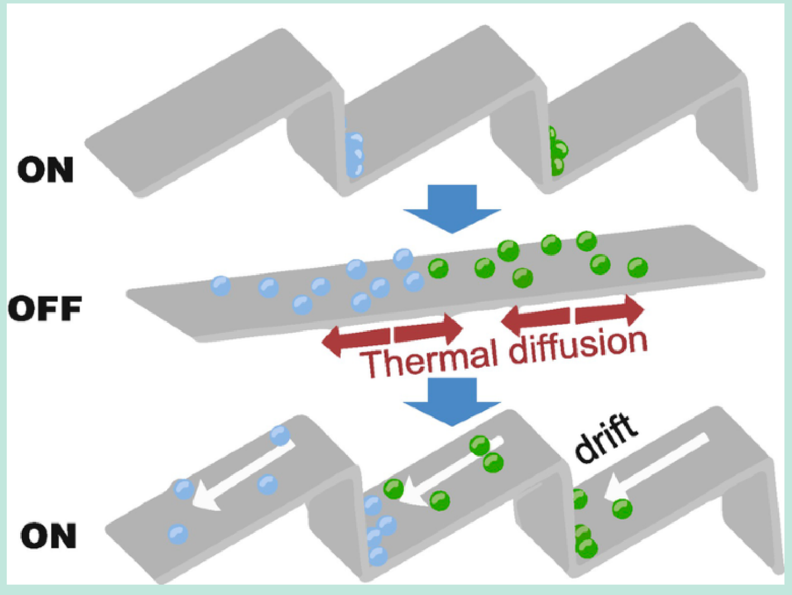
Introduction

Parrondo's Paradox is a paradox within the realm of game theory and is the combination of a pair of losing strategies to produce a winning strategy. It was devised by its creator, Juan Parrondo, in connection with his analysis of the Brownian ratchet, a thought experiment about a machine that can purposely extract energy from random heat motions popularised by Richard Feynman [1]. Previously, there have been attempts to see if Parrondo's Paradox exist in the stock market.

This project aims to build on top of that and explore if Parrondo's Paradox exists in other financial instruments apart from stocks.



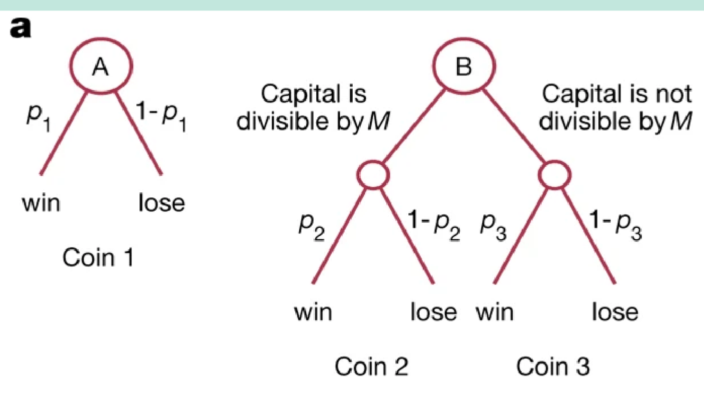
A Visual Example

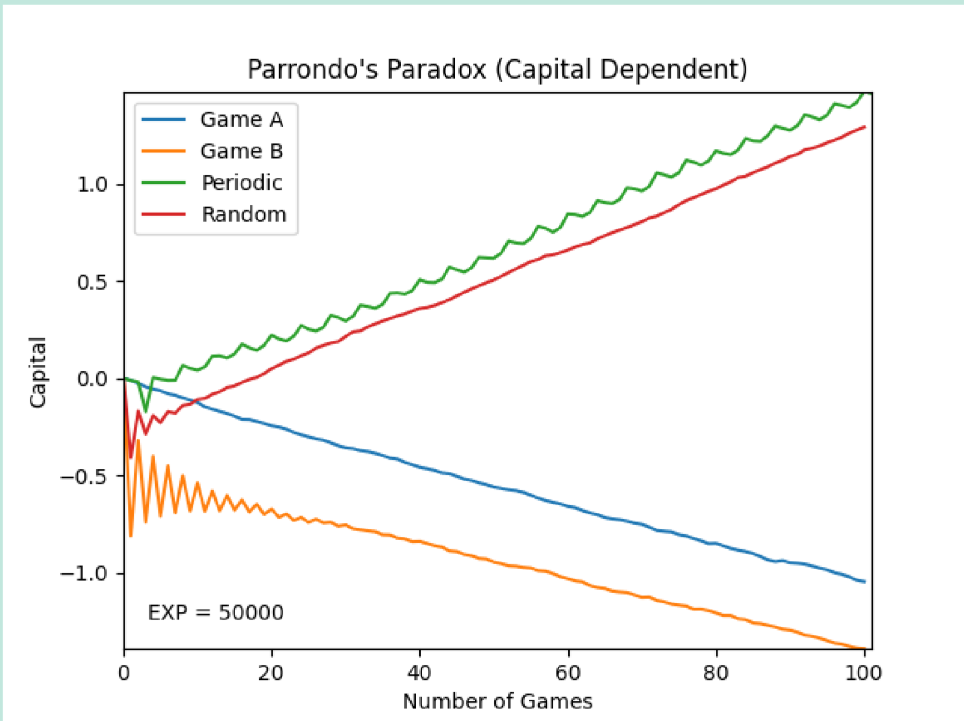
We can visualise Parrondo's Paradox by using a flashing Brownian ratchet. A flashing Brownian ratchet is a process that alternates between two regimes, a one-dimensional Brownian motion and a Brownian ratchet which produces directed motion.

Simulating Parrondo's Paradox

The goal of this simulation is to realise Parrondo's Paradox in its most basic form. To do this, we simulated two losing games, Game A and B, using biased coins. A player will start off with zero capital and will gain 1 capital whenever he wins a game and lose 1 capital whenever he loses. In Game A the player always flips a coin (p1) that is slightly biased to lose while in Game B the player flips another losing coin (p2) but if the capital is divisible by 3, they will instead flip a winning coin (p3). In particular, p1 = 1/2-e, p2 = 1/10-e and p3 = 3/4-e where "e" is the losing bias. In this case, e is 0.005. [2]

In the diagram, we can see that, after averaging 50,000 simulations, Game A and B are losing games on their own. To produce the effect of Parrondo's Paradox, we will introduce periodic and random switching to alternate between these two games. As a result, we can see that over time, the switching mechanism causes these two losing games to produce a winning one overall.





Parrondo's Paradox in the Stock Market

Previously, our switching mechanisms were periodic or random when choosing whether to play Game A or Game B. However, due to the context of the stock market where transaction fees exist, we will be changing the switching mechanism or algorithm to be time-based. Such an algorithm has been used in a previous paper called "Switching between two losing stocks may enable paradoxical win: an empirical analysis" by our professors here in SUTD. [3]

The time-based algorithm is mainly determined by two variables called the Periodic Cycle, T, and Strategy change time, t1. Given an initial capital and staying within the time scale of the stocks, if the remainder number of days passed, t, divided by the Periodic Cycle, T, is 0, all our capital will be used to purchase stock A. However, if the remainder equals the Strategy change time, t1, all our capital will be sold and used to purchase stock B instead. We also simulate that every transaction takes a fee of 0.1% of the total capital transferred. [3]

Below is the pseudo code for this switching algorithm and the graphs around show our reproduction of their results. For the graphs in this segment, the historical data dates from 1 Jan 2018 to 1 Jan 2019 for CAT/KSH and CTSH/CERN and 26 Nov 2021 to 26 Nov 2022 for PLUG/LAZR and WBD/PENN..

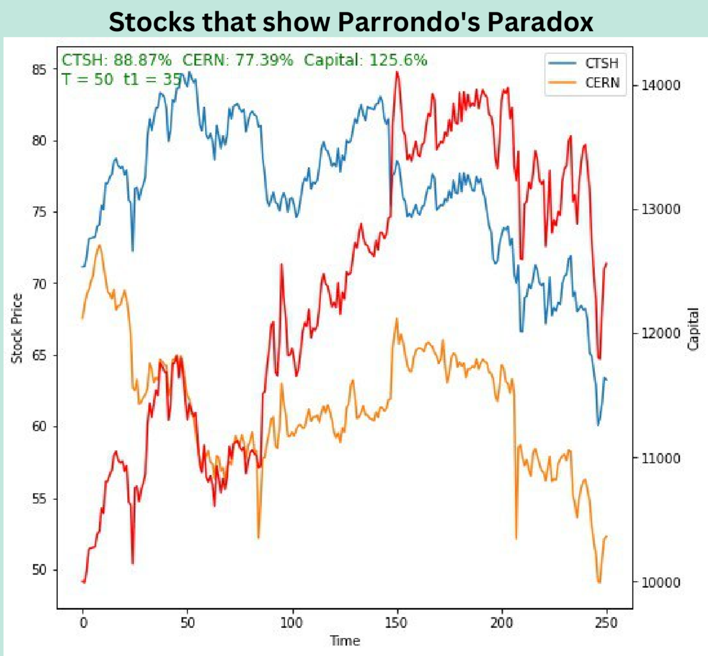
Algorithm 1 Time-based switching scheme in stock market.

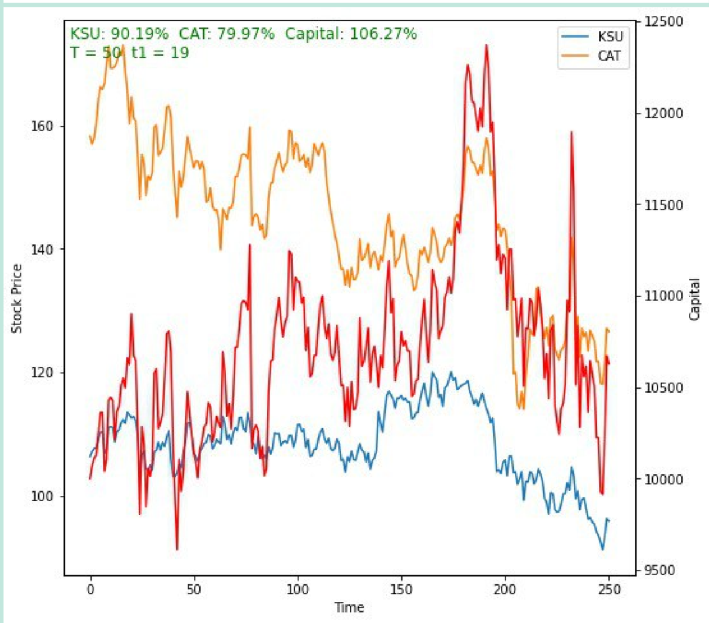
Input: Initial capital, C_0; Periodic cycle, T; Strategy change time, t_1; Time scale L;

Output: Total capital, C_L;

```
for time t = 1 to L do
    if mod(t, T) = 0 then
        Sell Stock B: C_t = C_{t-1} + (1 - α) N_t^B P_t^B, N_t^B = 0;
        Buy Stock A: N_t^A = (1 - α) C_t / P_t^A, C_t = C_t - N_t^A P_t^A / (1 - α);
    end if
    if mod(t, T) = t_1 then
        Sell Stock A: C_t = C_{t-1} + (1 - α) N_t^A P_t^A, N_t^A = 0;
        Buy Stock B: N_t^B = (1 - α) C_t / P_t^B, C_t = C_t - N_t^B P^B / (1 - α);
    end if
end for
```

Stocks that show Parrondo's Paradox





Stocks that do not show Parrondo's Paradox

3x Leveraged Inverse ETFs

IETFs , also known as Exchange Traded Funds, is a type of pooled investment security that holds multiple investment assets, traded on the exchange similar to a stock. For example, an leveraged ETF may be based on the S&P 500 index (SPXL). An inverse ETF, on the other hand, is set up so that its price rises (or falls) when the price of its target asset falls (or rises). This means the inverse ETF performs inversely to the asset it's tracking.

Leverage involves borrowing in order to amplify the returns of an investment. This means that potential gains and losses can be increased. In the case of this study, we are using 3x leverage. This means that the amplification of the profit or loss in our inverse ETFs are multiplied by 3 times. On the right, you can see the effects of using 3x leverage compared to without it.

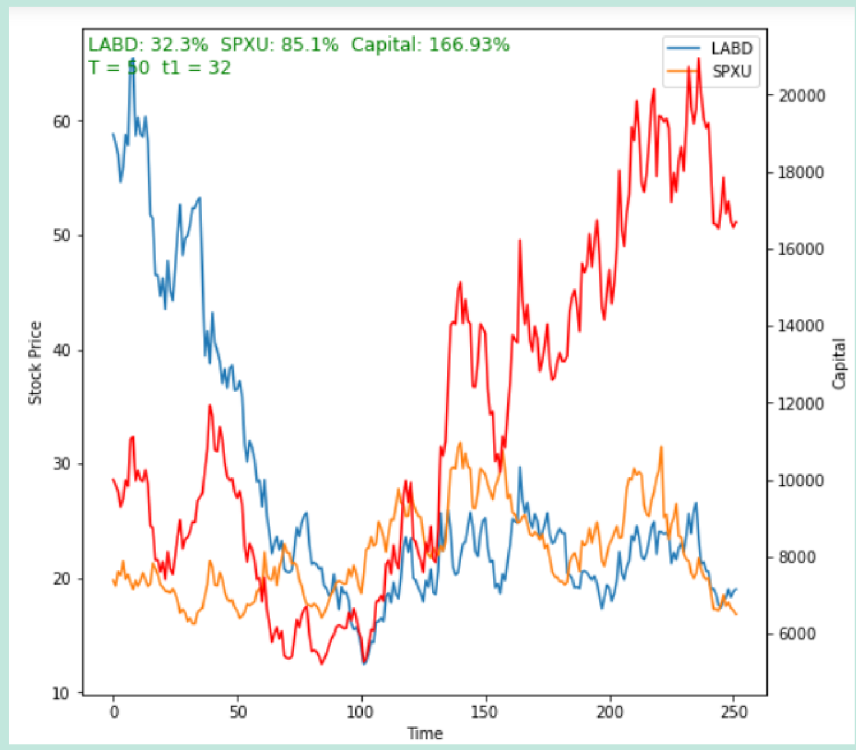
In this project, we will be using 3x leveraged inverse ETFs as our financial instrument of choice. The rationale for such decision is that ETFs are that they are less susceptible to exhibit drastic changes towards sudden market movement, inverse ETFs have downward movement in the long term, and 3x leverages would increase the amplitude the stochastic nature of the instrument.

SPY (1x) vs SPXL (3x)

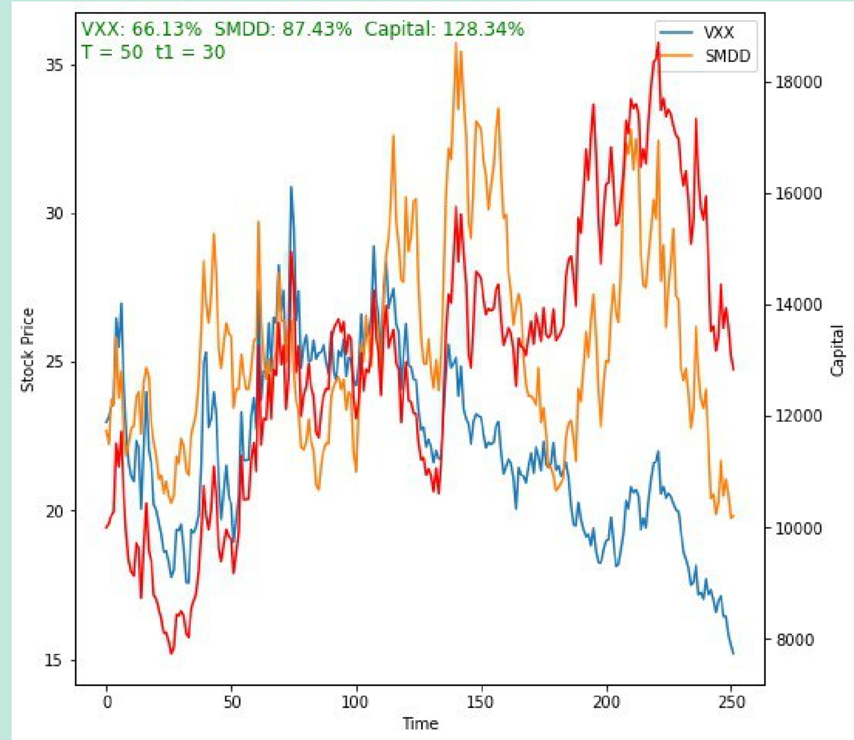
Parrondo's Paradox in 3x Leveraged Inverse ETFs

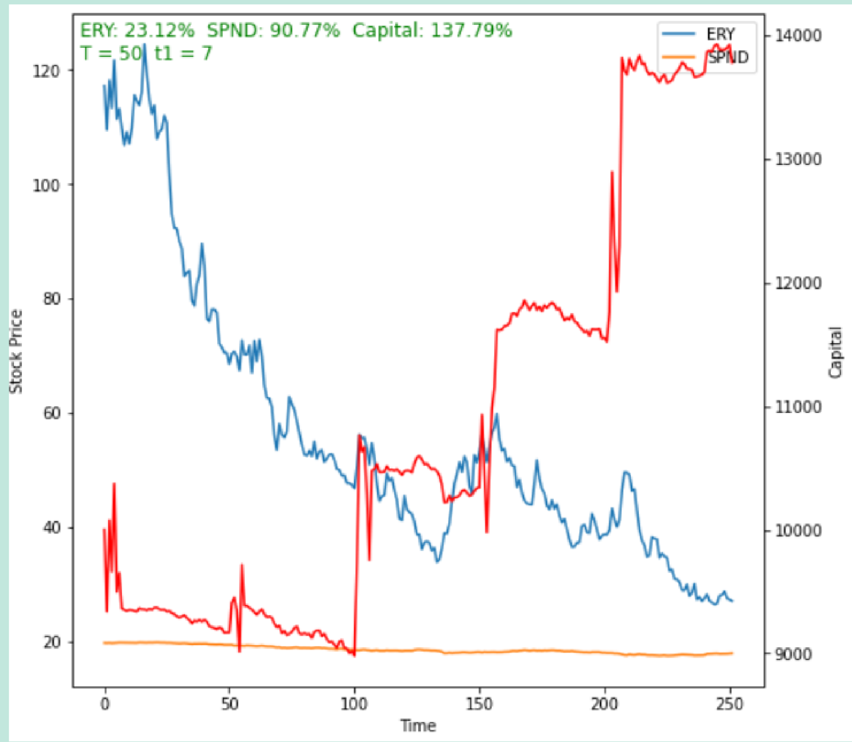
Using the same algorithm previously mentioned [3]. We will now apply them to 3x Leveraged Inverse ETFs. For the historical data, it dates from 26 Nov 2021 to 26 Nov 2022. except LABD/SPXU which dates from 13 Sept 2020 to 13 Sept 2021

LABD & SPXU



VXX & SMDD





ERY & SBND

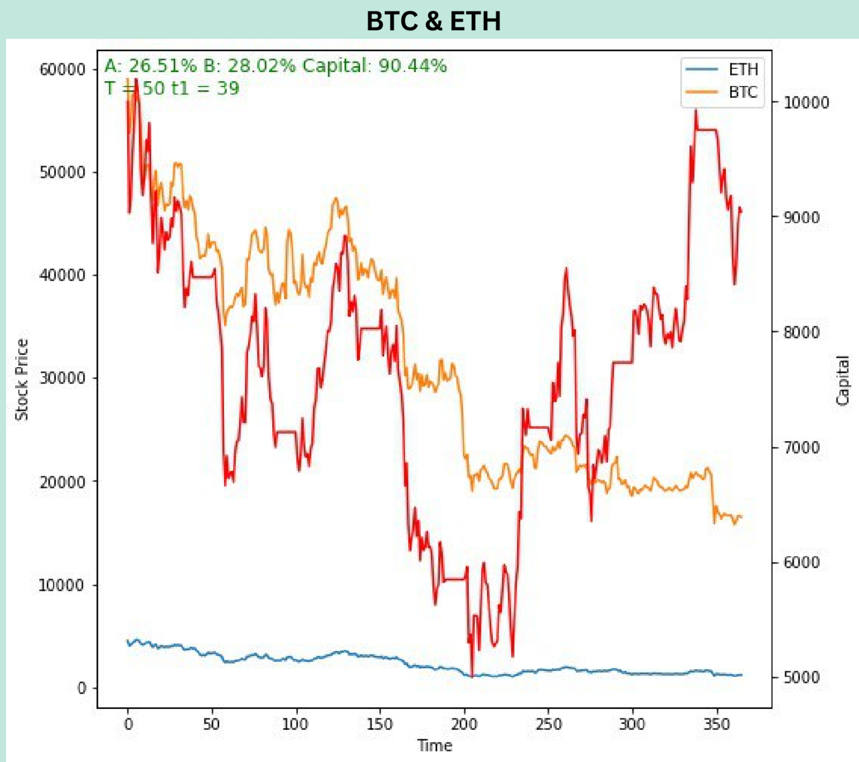
With respect to the resultant capital generated from the ETF pairings , we can observe that returns are very pleasant. The pairings has exhibited strong Parrondo's Paradox. The returns have all been observably greater than the stock pairings we have displayed, with all 3 pairings providing more than 1.25 times return.

With regards to LABD/SPXU and ERY/SBND, they have very differing trendlines with regards to their individual ETF movement as compared to VXX/SMDD that has similar trendlines. The resulting profit for the differing stock pairs is significantly greater than the stock pair with similar ETF movement. VXX/SMDD pairing tracks a generic market whereas the other pairings, they each track different market sectors compared to their counterpart, thus providing a very different individual trendline. The differing trendline generates a significantly higher profit as observed.

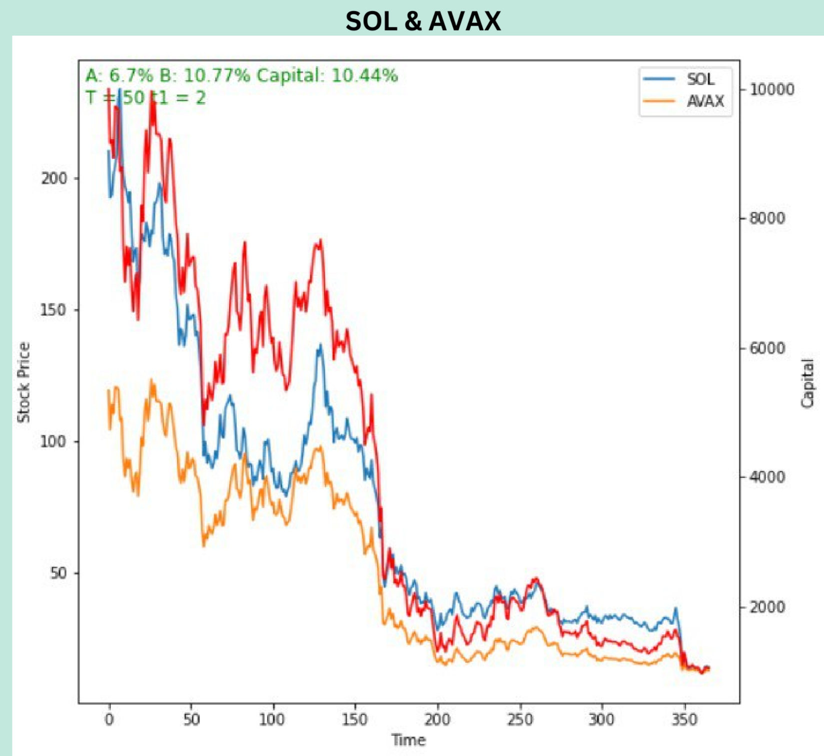
Parrondo's Paradox in Cryptocurrency

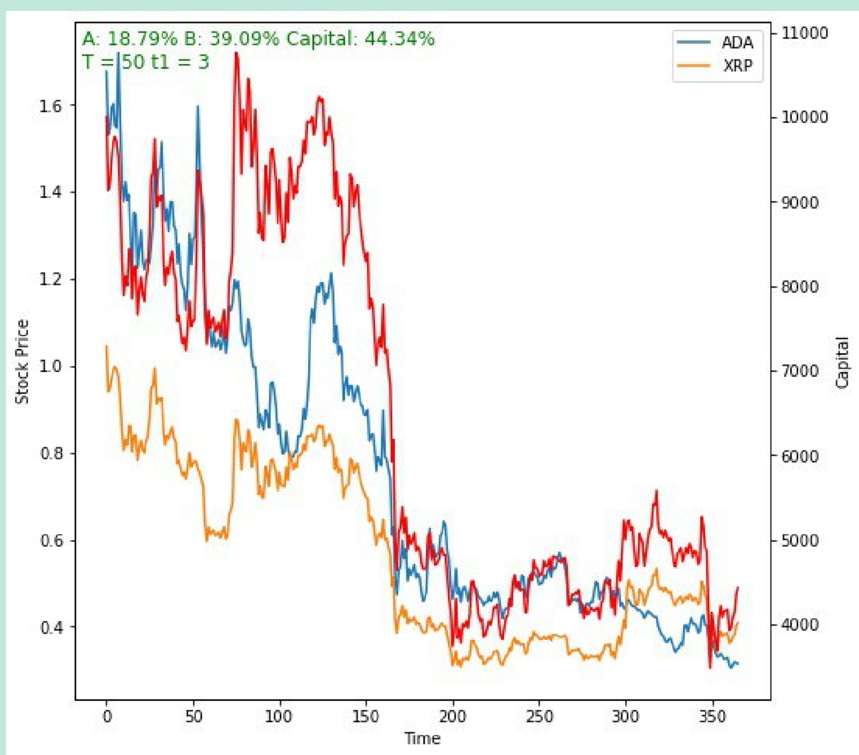
Apart from traditional finance, we wanted to explore if Parrondo's Paradox exists in the decentralised financial space. Cryptocurrency is very new in the financial space compared to other financial instruments and is considered a high risk investment tool. It is also known for its manipulation by high capital players and its aggressive price movement compared to traditional stocks.

BTC & ETH



SOL & AVAX





ADA & XRP

From these graphs, we can observe that all of these cryptocurrency pairs do not produce Parrondo's Paradox. In terms of performance, we can see that both ADA/XRP and SOL/AVAX pairings produce a capital movement that is similar to the price movements of the cryptocurrencies that made them. However, the BTC/ETH pairing produces a capital movement that performs better than both BTC or ETH alone.

Additionally, we have noticed that the performance of the capital movement produced by switching between two cryptocurrencies is linked to how similar the price movement is between them. Therefore, we would like to hypothesise that the more similar the price movement is of two cryptocurrencies, the less likely they will exhibit Parrondo's Paradox.

Future Improvements

Research can done into examining how exactly the similarity between the two base price movements effect the produced capital movement and whether Parrondo's Paradox will be evident or not. We would also like to mention how a different switching algorithm (perhaps a historical dependent one) may affect our results.

Additionally, a deep learning model can be implemented to pinpoint the corresponding strategy change time t1 for each pairing, through unsupervised learning from the stock's historical data. This allows a greater possibility in the algorithm's application on the pairing for real time use.

Conclusion

Throughout the project, we have realised that the amount of dissimilarity of the financial instruments matters a lot for Parrondo's Paradox to appear. This can be visualised in a flashing Brownian ratchet where the saw-tooth-like plane is too similar to the flat plane to cause a directed motion if the dissimilarity is too low (or the similarity is too high). Thus, the produced capital movement is similar to the general price movements of the pairing and will not result in the paradoxical win of Parrondo's Paradox.

This hypothesis is evident when we compare our results between the 3x Leveraged Inverse ETFs and Cryptocurrency. Since the market cap and diversification of cryptocurrencies are generally smaller compared to the ETFs, they are more prone to manipulation and will lead to their price movements being more similar from one another. This would result in a lower likelihood for their pairings to produce Parrondo's Paradox which can be seen from our results.

We like to remind readers that this project does aim to recommend using Parrondo's Paradox as an investment strategy in any shape or form.

REFERENCES

[1] Harmer, G. P., Abbott, D., Taylor, P. G. & Parrondo, J. M. R. in Proc. 2nd Int. Conf. Unsolved Problems of Noise and Fluctuations
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[3] Wen, T., Lai, J. W., & Cheong, K. H. (2022). Switching between two losing stocks may enable paradoxical win: An Empirical Analysis. Fractals.