MXB103 Project Group 69: BUNGEE!

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1 Introduction

As part of Brisbane's "New World City" transformation, the Brisbane City Council is investigating a proposal to allow bungee jumping off the Story Bridge. This report addresses several key questions about the proposal.

In Section 2 of this report, the proposal and the key questions relating to it are discussed in more detail. In Section 3, the model ... In Section 4, etc.

2 The proposal

The proposal calls for a platform to be installed at the very top of the Story Bridge, from which the bungee jumps will take place.

Firstly, we have been asked to implement a second order or higher numerical method to model the bungee jump. The code that we have chosen will obtain the numerical solution for the model and the bungee jumper's position will be plotted against time.

Secondly, we are then asked to observe the results of our model and determine whether it agrees with the bungee jump company's suggestion that a standard jump will consist of 10 "bounces" which should take approximately 60 seconds.

Thirdly, the maximum speed of the bungee jumper and when it occurs in relation to the overall jump need to be determined. It asks us to graphically plot the jumper's velocity against time.

Fourthly, in order to solve the following question, numerical differentiation must be used to find the acceleration predicted by our model. After that, we have to plot the jumper's acceleration against time.

Fifthly, we are then asked to compute the integral to determine how far the jumper actually travels in the 60 second jump. To compute the integral numerical integration will be used.

Sixthly, a constituent of the proposal is to install a camera on the bridge deck at height D from the water. An accurate value for t needs to be computed such that y(t) = H - D.

Lastly, the bungee company then asks us to determine how close the jumper is to touch the water. An investigation into how the bungee rope could be altered to produce an accurate experience of a water touch for a jumper who weighs 80kg, while still maintaing 10 bounces in 60 seconds.

These questions that are asked will be answered in order as they are all related to each other. This way answers will be accurate. This approach will work well as it ensures that the questions are answered accordingly.

The next section will discuss the model in more detail, relating to the equation and its origin.

3 The model

The equation of motion for bungee jumping is

$$\frac{dv}{dt} = g - C|v|v - max(0, K(y - L))$$

Being aware of the forces (gravity (mg), drag(-c|v|v) and tension (-max(0,k(y-L))) that are regularly acting on the jumper, the mathematical model of performing bungee jumping can be derived.

The sum of the forces that are acting on the jumper equals the product of the jumper's mass and acceleration. The ODE which governs bungee jumping is

$$\frac{m}{dv}dt = mg - c|v|v - max(0, k(y - L))$$

The drag force acts in the opposite direction to motion which always slows the jumper down. The value is given by -c|v|v, c is the drag coefficient and the velocity of the jumper is v. The length of the unstretched rope is L, the tension when the rope is taut is k(y-L), where the "spring constant" k measures the bungee ropes elasticity. The tension force is - max(0,k(y-L)). When y > L the maximum function makes sure that the tension "switches on". And to make sure that it acts upwards the minus sign is placed in front.

The equation of motion however, is a simplified version by dividing through m. Where C = c/m and K = k/m.

3.1 Assumptions and limitations

There are a few assumptions and limitations of the given model written in this project. First of all, it is assumed that it is valid that the parameters of the model of the factors of the bridge are given by the bungee jump company. Secondly, it is assumed for the Story Bridge that the point in which jumping platform is set up is capable of sustaining the weight of platform and staffs on it. In terms of limitations of the given model, the only case of 80 kilograms jumper is considered as the result generated from the model. What is more, there is a possibility that the expected result generated by the model is different from the actual outcome of the jumping as the given model is a numerical solution.

There exist some limitations in these solutions after the case study conducted. First of all, this investigation ignores the case of rising tide. Therefore, the occurrence of 'water touch' would not exist in reality for a certain time. Moreover, this analysis ignores the case of ships moving under the bridge. Hence it is expected that the safety of the bungee jump is significantly low. On top of that, due to the weight of jumper and variables such as drag coefficient will not always be the same as given parameters, the actual situation of jumping may be variable according to the current situation.

3.2 Parameters

```
H = 74;                % Height of jump point (m)
D = 31;                % Deck height (m)
c = 0.9;               % Drag coefficient (kg/m)
m = 80;                % Mass of the jumper (kg)
L = 25;                % Length of bungee cord (m)
k = 90;                % Spring constant of bungee cord (N/m)
g = 9.8;               % Gravitational acceleration (m/s^2)
C = c/m;               % Scaled drag coefficient
K = k/m;               % Scaled spring constant
```

4 The numerical method

Solving the bungee jumping equation will require just a slight modification of numerical methods for solving ODEs that we have developed in our lectures. Because the equation of motion for bungee jumping involves two unknowns, v and y, a second equation is needed to relate the the two. The relationship is quite simple such that the jumper's velocity v is the derivative of the jumpers position y. This problem can be considered as two ODEs.

$$\frac{dy}{dt} = v$$

$$\frac{dv}{dt} = g - C|v|v - max(0, K(y - L))$$

Any numerical method can now be applied to these two equations.

# 4.1 Parameters

```
T = 60;                % Final time in simulation (s)
n = 10000;             % Number of subintervals (you decide how many you
 need)
```

4.2 Solution

The ordinary differential equations are solved using the Classical Fourth Order Runge-Kutta method.

```
[t, y, v, h] = rk4_bungee(T, n, g, C, K, L);

figure
plot(t, y);
xlabel('time (s)');
ylabel('distance fallen (m)');
title('Figure 1: Distance between Jumper and Platform.');
```

Figure 1: Distance between Jumper and Platform.
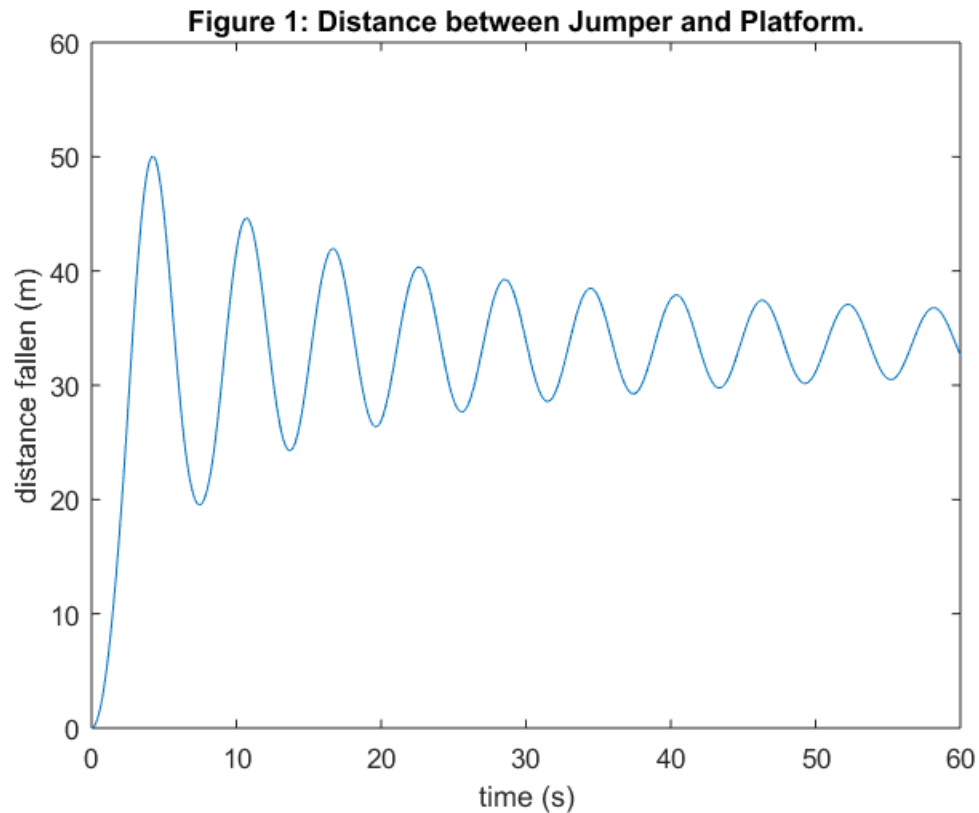
5 Analysis

In this section, the model predictions are analysed with respect to the key questions being asked about the proposal.

5.1 Timing and bounces

The bungee jump company suggests that the standard jump will consist of 10 "bounces" which should take approximately 60 seconds. Our model, Figure 1, does agree with the timing.
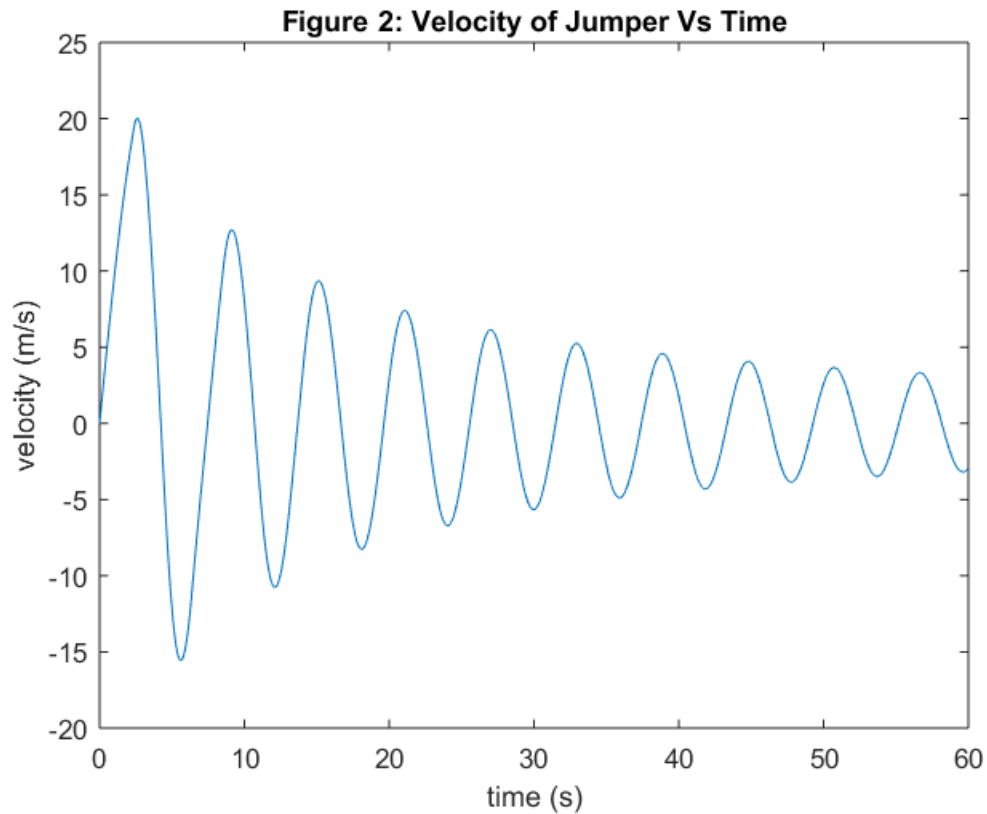
5.2 Maximum speed experienced by the jumper

The "thrill factor" of bungee jumping is partly determined by the maximum speed experienced by the jumper.

We can find the maximum velocity by:

```
[t, y, v, h] = rk4_bungee(T, n, g, C, K, L);

figure
plot(t,v);
xlabel('time (s)');
ylabel('velocity (m/s)');
title('Figure 2: Velocity of Jumper Vs Time');
```

```
% According to the graph, it seems that the maximum velocity achieved
 by
% the jumper is approximately 20 m/s.
```

Figure 2: Velocity of Jumper Vs Time

5.3 Maximum acceleration experienced by the jumper

Another factor for thrill-seekers is the maximum acceleration experienced by the jumper. More acceleration equals bigger thrills, however too much acceleration may be dangerous. Using numerical differentiation, the acceleration can be predicted by it.

Therefore the maximum accleration can be determined by

```
[a, ~] = maximum_acceleration_bungee(v,h,n);

figure
plot(t,abs(a));
xlabel('Time(Seconds)');
ylabel('Acceleration of Jumper(m/s/s)');
title('Figure 3: Jumpers Acceleration against Time');
```

Figure 3: Jumpers Acceleration against Time
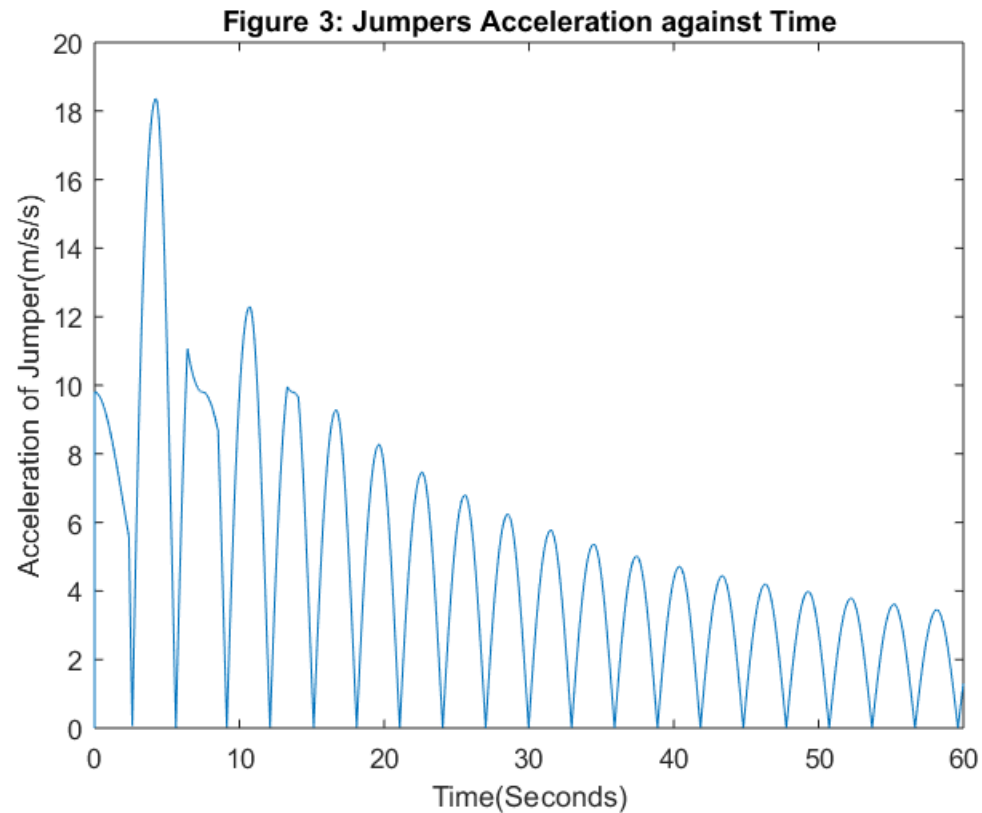
According to the graph, the maximum acceleration experienced by the jumper occurs at approximately 4 seconds. Also, the jumper will be able to experience a maximum accleration of 18.3366m/s/s. What is more, the claim of "up to 2g" acceleration is not supported by the model.

5.4 Distance travelled by the jumper

It is of interest to know how far the jumper actually travels in the 60 second jump. Using numerical integration, the answer to this question can be computed to determine how far the jumper travels.

Therefore the distance travelled by the jumper is determined by

```
[t, y, ~, ~, x] = rk_bungee_integral(T, n, g, C, K, L);

figure
plot(t,x);
xlabel('Time(s)');
ylabel('Distance(m)');
title('Figure 4: Distance of jumper travelled');
```

Figure 4: Distance of jumper travelled
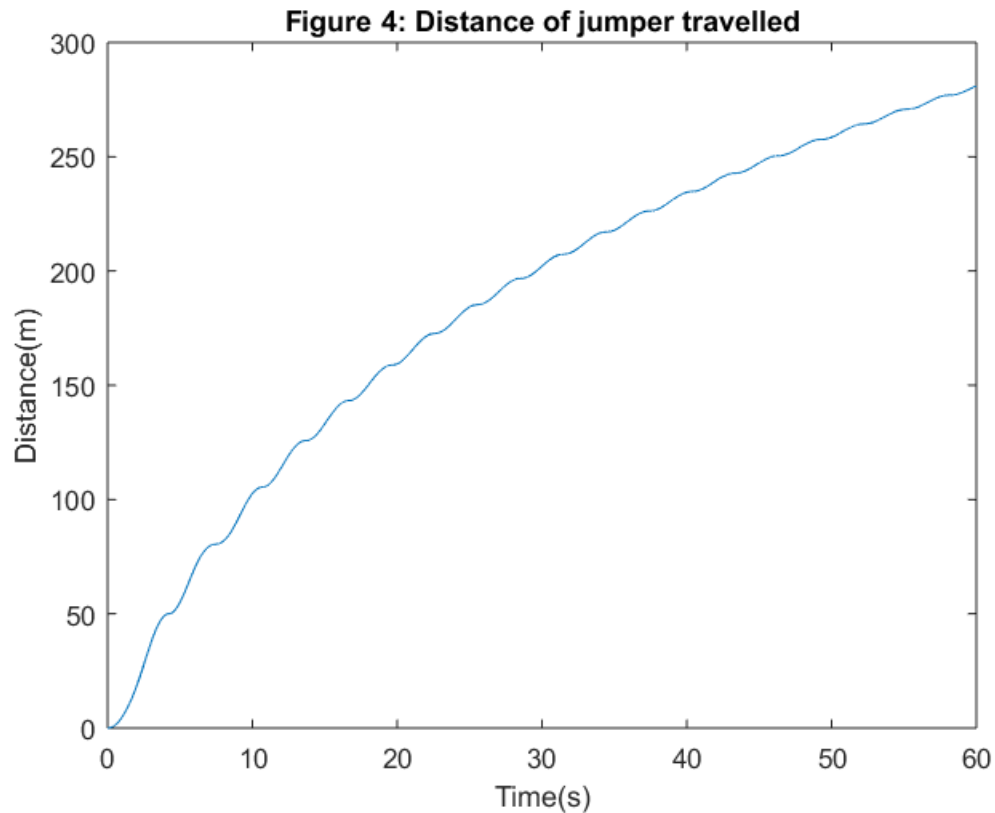
5.5 Automated camera system

Describe the question, and then answer it. In this case, you will fit an interpolating polynomial through the four points in your solution y that lie either side of the camera location. Then use that polynomial to solve for when the jumper passes the camera.

5.6 Water touch option

Describe the question, and then answer it. In this case, you will re-solve the equations with different parameters for L and k. Experiment to find which values work best for the water touch option, but include only the best combination that you found in the submitted code.

6 Conclusion

We are able to analyse different views of the bungee jump in order to meet the requirements of the bungee jump company with the Classical Fourth Runge-Kutta method which has been used to model the bungee jump. What the bungee company suggest is answered in order. The answers to question 1 and 2 use numerical solutions. Also, numerical differentiation and integration are used to solve the question 3 and 4.

The numerical solution and analysis for questions1, question2, question3, and question4 are well presented in this document. However, one of our team members (Saint) does not participate in this project, therefore, the numerical solution and analysis for question 5 and 6 are not presented at all. Due to the fact that the answers are based on the numerical model, further investigation and more accurate solution, such as analytical solution and real field experiment on the Story Bridge, are highly recommended if the bungee jump company are still interested in this project.

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