Physics 2A Spring 2020
Discussion 9

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1. **Warm-ups**[1]

   (a) A dog is riding a bicycle. In which direction does the angular rate of his wheel point?

   A. To the left

   B. To the right

   C. Into the screen

   D. Out of the screen

   E. Toward the center of the wheel

   Solution: C. Use right hand rule.

   (b) The dog applies the brakes to slow down. In which direction does the torque on the tire act, which is caused by friction between the road and tire surfaces?

   A. Into the screen

   B. Down

   C. Toward the left

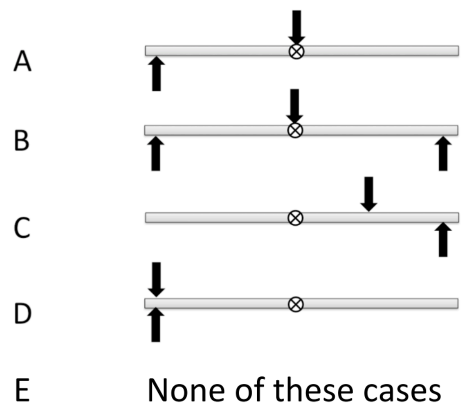
   D. Toward the right

   E. Friction applies no torque

   Solution: A. Use right hand rule.

   (c) A yardstick is acted on by forces acting at different locations. Each arrow represents a force of the same magnitude, while the symbol $\otimes$ indicates the center of mass. In which of these cases is the yardstick in dynamical equilibrium?

[1] From Prof. Burgasser's TPS worksheet



Solution: D. In order to be in dynamical equilibrium, all forces must add up to 0. The same must also be true for all torques.

2. **Moment of inertia**
Derive the moment of inertia of a spherical shell with outer radius R, mass m, and thickness l. Show that in the limit $l \to 0$ this gives us the moment of inertia of a solid sphere, and in the limit $l \to R$ this gives us the moment of inertia of a hollow sphere. What is the moment of inertia if the axis of rotation is at a distance d from the center of the shell? Finally, try deriving the moment of inertia of a hollow sphere without taking limits.

Solution: Assume the shell has uniform density, so $\rho = \frac{3m}{4\pi(R^3-(R-l)^3)}$. The moment of inertia is then

$$\begin{aligned} I &= \int_0^{2\pi} \int_0^{\pi} \int_{R-l}^{R} (r\sin\theta)^2 \rho r^2 \sin\theta dr d\theta d\phi \\ &= \frac{3m}{2(R^3-(R-l)^3)} \int_{-1}^{1} (1-\cos^2\theta) d(\cos\theta) \int_{R-l}^{R} r^4 dr \\ &= \frac{3m}{2(R^3-(R-l)^3)} \frac{4}{3}\frac{1}{5}(R^5-(R-l)^5) \\ &= \frac{2}{5} m \frac{R^5-(R-l)^5}{R^3-(R-l)^3} \end{aligned} \tag{1}$$

Clearly, when $l = R$, this gives us $I = \frac{2}{5}mR^2$. When $l = 0$, we expand the fraction

$$\frac{R^5-(R-l)^5}{R^3-(R-l)^3} = \frac{5R^4 - 10R^3l + 10R^2l^2 - 5Rl^3 + l^4}{3R^2 - 3Rl + l^2} \tag{2}$$

This is equal to $5R^2/3$, so $I = \frac{2}{3}mR^2$.

If the axis of rotation is at a distance d from the center of mass, we can just use parallel axis theorem

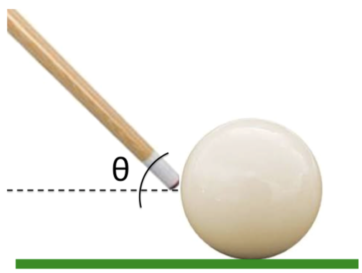
$$I = I_{cm} + md^2 \tag{3}$$

where I_{cm} is the moment of inertia we calculated above.

Alternatively, we can find the moment of inertia of a hollow sphere by treating it as a stack of rings. The surface mass density is $\sigma = \frac{m}{4\pi R^2}$ and the area element is $dA = 2\pi R dz$

$$I = \int_{-R}^{R} r^2(z)\sigma dA = \frac{m}{2R}\int_{-R}^{R}(R^2 - z^2)dz = \frac{2}{3}mR^2 \tag{4}$$

3. **Rotational dynamics**[2]



We strike a ball at its center from an angle $\theta = 30°$ with an average force $F = 25$ N for 30 ms. Assume the ball has uniform density and has a mass $m = 200$ g and radius $R = 3$ cm. The coefficient of kinetic friction between the table and the ball is $\mu_k = 0.2$. How far will the ball travel before it reverses direction?

Solution: We define x to be pointing to the right, and y to be pointing up. By the right hand rule, z points out of the page. The y component of the force applies a torque on the ball and results in the ball gaining angular momentum

$$\vec{L} = \langle\vec{\tau}\rangle\Delta t = \vec{r}\times\langle\vec{F}\rangle\Delta t = FR\Delta t\sin\theta\hat{z} \tag{5}$$

while the x component of the force gives the ball a linear momentum

$$\vec{p} = \langle\vec{F}\rangle\Delta t = F\Delta t\cos\theta\hat{x} \tag{6}$$

We can use these to find the initial velocity and initial angular velocity. The ball will be slowed down by friction ($\mu_k mg$) as it slips on the table. We can then use kinematics to figure out the distance the ball travels before it comes to a temporary stop (I found 2.7 m).

[2]From Homework 8