Physics 2A Spring 2020
Discussion 2

TA: Jiashu Han

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1. **Warm-ups**

(1) Approximately how fast does your car accelerate?

A. 0.3 m/s^2

B. 3 m/s^2

C. 30 m/s^2

D. 300 m/s^2

E. 3000 m/s^2

Solution: The typical speed of a car is 60 km/h (roughly 35 mph), or 17 m/s. Cars typically take about 10 seconds to accelerate from rest, so this gives us an acceleration of 1.7 m/s^2. The closest answer is (B).

(2) A train moves at a constant speed of 36 km/h along a straight track, which terminates 3 km ahead. This means the operator must bring the train to a complete stop before it reaches the end of the track and has to figure out the required (negative) acceleration. Assume he can only apply a constant acceleration to the train. Which of the following equations is *best* for this problem?

A. $v(t) = v(0) + at$

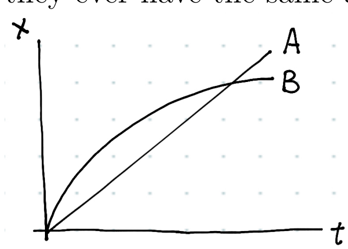
B. $v(t)^2 = v(0)^2 + 2a\Delta x$

C. $\Delta x = \frac{1}{2}(v(0) + v(t))t$

D. $x(t) = x(0) + v(0)t + \frac{1}{2}at^2$

Solution: We are given the initial and final velocity of the train, and the distance traveled. The quantity we are looking for is the acceleration. (B) relates all of these directly.

(3) The following diagram shows the positions of A and B as functions of time. Do the two objects ever have the same velocity? If so, at what time? Do they ever have the same acceleration?



Solution: The two objects have the same velocity when the two curves have the same slope (velocity). A's position is a linear function of time, meaning it has a constant velocity and therefore 0 acceleration. However, curve B is concave down, so it has a negative acceleration throughout and the two objects never have the same acceleration.

2. **2D kinematics**
A ball is launched in the horizontal direction with velocity 3 m/s off a cliff with a height of 500 m. How far is the ball from the cliff when it lands? (Assume $g = 9.8$ m/s^2 and no air resistance. The ground is flat.) Also find and sketch the trajectory of the ball.

Solution: We can break this problem into two 1D problems (since we can assume time is the same for both sub-problems). We are looking for the final horizontal position x of the ball. Since there is no acceleration in the x direction, $x = v_x t$ where $v_x = 3$ m/s. We use the vertical motion to find t: $\Delta y = v_y(0)t - \frac{1}{2}gt^2 = -\frac{1}{2}(9.8 \text{ m/s}^2)t^2 = -500$ m, which gives $t = 10.1$ s, so $x = 30.3$ m. To find the trajectory of the ball $y(x)$, we first write down $y(t) = y_0 - \frac{1}{2}gt^2$ and $x(t) = v_x t$. Invert the latter ($t = x/v_x$) and plug it into $y(t)$ to get $y(x) = y_0 - \frac{gx^2}{2v_x^2}$.

3. **Non-constant acceleration**
A bead is connected to the ceiling through a spring and oscillates up and down about its equilibrium position $x = 0$ with initial position x_0 and initial velocity v_0. Find the velocity v and position y of the bead as functions of time for the following scenarios:

(a) $a = A\cos(\omega t)$ where $A = 2$ cm/s^2 and $\omega = 2\pi$ s^{-1}.
Solution: Integrate $a(t)$ to get $v(t)$:

$$v(t) = v_0 + \int_0^t dt' \; A\cos(\omega t') = v_0 + \frac{A}{\omega}\sin\omega t \tag{1}$$

Integrate again to get $x(t)$:

$$x(t) - x_0 = \int_0^t dt' \; \left(v_0 + \frac{A}{\omega}\sin\omega t'\right) = \frac{A}{\omega^2} + v_0 t - \frac{A}{\omega^2}\cos(\omega t) \tag{2}$$

We can check that A/ω^2 indeed has the same unit as length.

(b) $a = a_0 e^{bv}$ where $a_0 = 1$ cm/s^2 and $b = 1$ s/cm (you can use $\int dx \ln x = -x + x\ln x + C$).
Solution: Use $a = dv/dt$ to write $dv/dt = a_0 e^{bv}$:

$$\frac{1}{a_0}\int_{v_0}^{v(t)} dv \; e^{-bv} = \frac{1}{a_0 b}\left(e^{-bv_0} - e^{-bv(t)}\right) = \int_0^t dt' = t \tag{3}$$

Solve the equation to get

$$v(t) = -\frac{1}{b}\ln\left(e^{-bv_0} - a_0 bt\right) \tag{4}$$

We can check that this indeed gives $v(0) = v_0$. Also, all terms inside the natural log are dimensionless. Further integrate this using the given integral and a change of variables:

$$\begin{aligned}
x(t) &= x_0 - \frac{1}{b}\int_0^t dt' \ln\left(e^{-bv_0} - a_0 bt'\right) \\
&= x_0 + \frac{1}{a_0 b^2}\int_{e^{-bv_0}}^{e^{-bv_0}-a_0 bt} dy \ln y \\
&= x_0 + \frac{1}{a_0 b^2}\left(a_0 bt + \left(e^{-bv_0} - a_0 bt\right)\ln\left(e^{-bv_0} - a_0 bt\right) + bv_0 e^{-bv_0}\right) \\
&= x_0 + \frac{t}{b} + \frac{1}{a_0 b^2}\left(e^{-bv_0} - a_0 bt\right)\ln\left(e^{-bv_0} - a_0 bt\right) + \frac{v_0}{a_0 b}e^{-bv_0} \qquad (5)
\end{aligned}$$

Indeed, all terms have units of length, and $x(0) = x_0$[1].

[1] In fact, I made quite a few mistakes when I wrote up the solution, and they are all identifiable by checking units.