Physics 2A Spring 2020
Discussion 4

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1. **Warm-ups**[1]

   (a) An object moving in a circle at constant speed has 0 net force acting on it

   A. True

   B. False

   Solution: False. There is a centripetal force $F = mv^2/r$.

   (b) A bee and a train collide. What is true about this collision?

   A. The bee experiences a greater force

   B. The train experiences a greater force

   C. The bee experiences a greater acceleration

   D. The train experiences a greater acceleration

   E. More than one answer is true

   Solution: C. The two experience the same force but the bee has a much smaller mass so it experiences a much greater acceleration.

2. **Newton's laws**

   This exercise aims at clearing some common confusions about Newton's laws of motion. Before we start, let us review the three laws:

   1. If not acted upon by a force, an object either stays at rest or moves at a constant velocity.[2]
   2. The sum of the forces $\mathbf{F}$ on an object is equal to its mass m times its acceleration $\mathbf{a}$: $\mathbf{F} = m\mathbf{a}$.[3]
   3. When one object exerts a force on a second object, the second object also exerts an equal and opposite force on the first object at the same time.

[1] From Prof. Burgasser's TPS worksheet

[2] In an inertial frame of reference

[3] In an inertial frame of reference

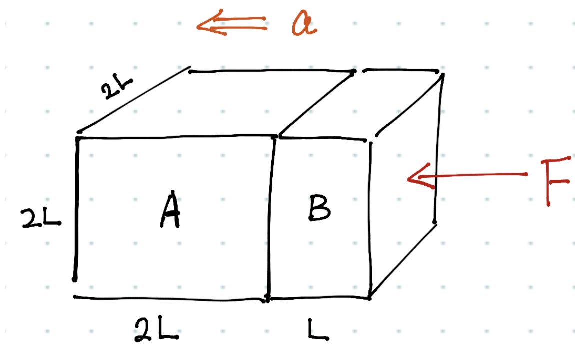


Figure 1: Two boxes A (mass m_A) and B (mass $m_B = m_A/2$) are placed next to each other. A constant force $\mathbf{F}$ is applied from the right as shown, giving the boxes an acceleration of $\mathbf{a}$ pointing to the left. The two boxes remain stuck together.

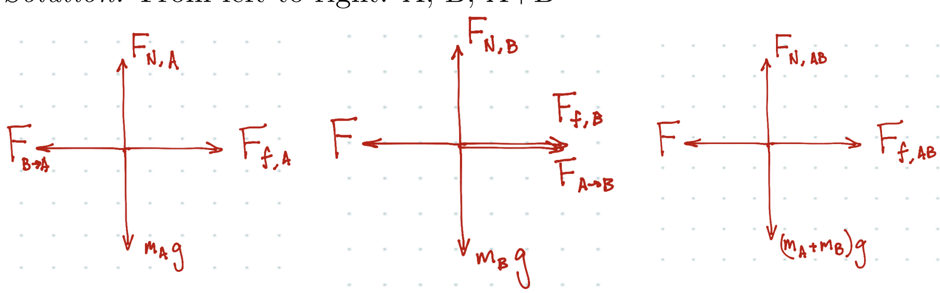
For the following questions, define up as the $+y$ direction and left as the $+x$ direction.

(a) Relate the total mass to $\mathbf{F}$ using the second law.

Solution: $\mathbf{F} = (m_A + m_B)\mathbf{a} = \frac{3}{2}m_A\mathbf{a}$

(b) Draw the free body diagrams for box A, box B, and the combination of boxes A and B.

Solution: From left to right: A, B, A+B



The key to drawing the free body diagram of an object is to include all forces acting *on* the object, and not the forces it exerts on others. This is a common source of confusion about Newton's third law.

(c) Based on your diagrams, what are the normal forces on A, B, and A+B?

Solution: The only forces in the vertical direction are the weight and the normal force, which should add up to zero since there is no vertical acceleration: $\mathbf{F}_N + \mathbf{F}_w = 0$. Therefore, the normal forces are:

- $\mathbf{F}_{N,A} = -\mathbf{F}_{w,A} = m_A\mathbf{g}$
- $\mathbf{F}_{N,B} = -\mathbf{F}_{w,B} = m_B\mathbf{g} = \frac{1}{2}m_A\mathbf{g}$
- $\mathbf{F}_{N,AB} = -\mathbf{F}_{w,AB} = \frac{3}{2}m_A\mathbf{g}$

(d) The surface has a coefficient of kinetic friction μ. For each object the magnitude of the frictional force $\mathbf{F}_f$ acting on the object is given by $F_f = \mu F_N$. The direction of friction is opposite the direction of motion. Write down the friction on each of the objects (A, B, and A+B).

Solution:

- $F_{f,A} = \mu F_{N,A} = \mu m_A g$
- $F_{f,B} = \mu F_{N,B} = \frac{1}{2}\mu m_A g$
- $F_{f,AB} = \mu F_{N,AB} = \frac{3}{2}\mu m_A g$

Note that these are magnitudes of forces. Later when you add them with other horizontal forces make sure to attach a minus sign to be consistent with our definition of directions.

(e) For each object, write down all of the horizontal forces acting on it, and check if they satisfy Newton's second law. That is, do they produce the same acceleration for all three objects, as we should expect?

Solution:

- $m_A a = F_{B\to A} - F_{f,A} = F_{B\to A} - \mu m_A g$
- $m_B a = \frac{1}{2} m_A a = F - F_{f,B} - F_{A\to B} = F - \frac{1}{2}\mu m_A g - F_{B\to A}$
- $(m_A + m_B)a = \frac{3}{2} m_A a = F - F_{f,AB} = F - \frac{3}{2}\mu m_A g$

In the second line we used Newton's third law ($F_{A\to B} = F_{B\to A}$). We can see that the first two equations add up to the third equation, so they are consistent.

(f) Are the three equations you wrote down consistent with each other? If they are not, what could you have missed?

Solution: If your equations are not consistent, most likely you have missed the $F_{A\to B}$ in the free body diagram of B or the $F_{B\to A}$ in the FBD of A.

3. **Two boxes on an inclined plane**

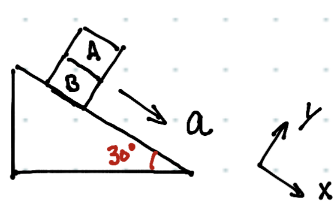
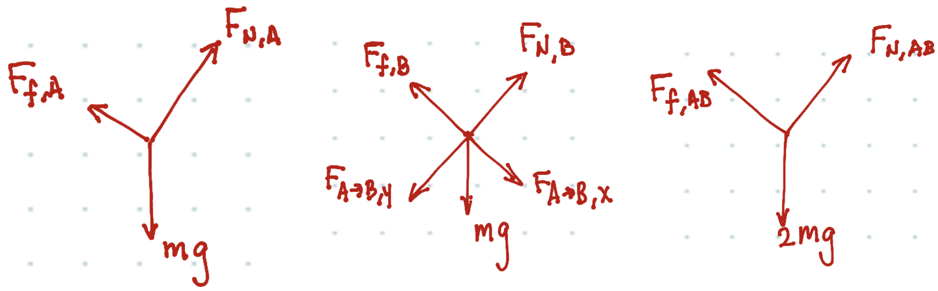


Figure 2: Two boxes of the same mass m are placed on a slope with an angle $\theta = 30°$ from the ground as shown. The two boxes slide down the incline with acceleration a and box A is just on the brink of sliding off of B, but the two remain stuck together. Both the static friction coefficient between the boxes and the kinetic friction coefficient between the slope and box B are μ.

(a) Draw the free body diagrams of A, B, and A+B.

Solution: From left to right: A, B, A+B



Again, we draw only the forces acting *on* each object. Notice that in the diagram for B, I did not include the weight of A as a force acting on B because the weight of A is a force exerted by the Earth on A, and therefore its reaction force is exerted by A on the Earth, not B.

(b) Check that they all produce the same relationship $g\sin\theta - \mu g\cos\theta = a$.

Solution: For A we can write down the following:

- $mg\cos\theta = F_{N,A}$
- $mg\sin\theta - F_{f,A} = mg\sin\theta - \mu F_{N,A} = ma$

which simplify to $g\sin\theta - \mu g\cos\theta = a$. For A+B the equations are similar and we just replace m with $2m$. For B we have

- $mg\cos\theta + F_{A\to B,y} = mg\cos\theta + F_{N,A} = F_{N,B}$
- $mg\sin\theta + F_{A\to B,x} - F_{f,B} = mg\sin\theta + F_{f,A} - \mu F_{N,B} = ma$

Plug the first equation into the second and again we recover the desired result.