Physics 2A Spring 2020
Discussion 7

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1. **Warm-ups**[1]

   (a) A stone is launched upward into the air. In addition to the force of gravity, the stone is subject to drag force. The average speed of the stone as it moves upward is

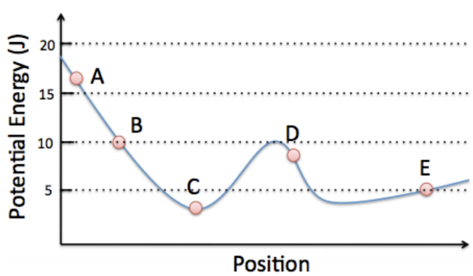
   A. larger than

   B. equal to

   C. smaller than

   the the average speed of the stone as it moves downward.

   Solution: A. Energy is dissipated by the drag force as the stone travels up and then down, so during the second half it will have less total energy and therefore less kinetic energy available at each fixed height.



   (b) In the potential energy diagram above, at which point will an object experience a force toward the left?

   Solution: E. An object always wants to move to a lower energy configuration. At E the slope is positive, so the object will experience a force that pushes it to the left.

2. **Dissipative work**[2]

   A box slides on a frictionless surface with a total energy of 50 J. It hits a spring and compresses the spring a distance of 25 cm from equilibrium. If the same box with the same initial energy slides on a rough surface, it only compresses the

[1]From Prof. Burgasser's TPS worksheet

[2]OpenStax *University Physics*. Chapter 8 problem 89

spring a distance of 15 cm, how much energy must have been lost by sliding on the rough surface?

Solution: We can find the spring constant from the frictionless case: 50 J $= E_{\text{initial}} = \frac{1}{2}k(\Delta x)^2 = \frac{1}{2}k(0.25\text{ m})^2$ which gives $k = 1600\text{ kg/s}^2$. With friction, the total energy left when the spring is fully compressed is $\frac{1}{2}k(0.15\text{ m})^2 = 18$ J, so the energy lost to friction is 32 J.

3. **Impulse and average force**

   An automatic rifle is firing 4.0 g bullets into a wall at a rate of exactly 800 bullets per minute. The bullets impact the wall moving with a speed of 750 m/s at a 90° angle to the wall, and remain embedded in the wall. What is the time-averaged force exerted on the wall by the bullets?

   Solution: Each bullet goes from an initial state of $p_i = mv_i = 3$ kg·m/s to a final state of $p_f = 0$ kg·m/s, so the impulse due to each bullet is $\Delta p = 3$ kg·m/s. From the firing rate we can calculate the time interval between bullets: $\Delta t = 1/(800\text{ min}^{-1}) = 0.075$ s. The average force is then $\langle F \rangle = \frac{\Delta p}{\Delta t} = 40$ N.

4. **Momentum conservation in center of mass frame**

   Sometimes when we are dealing with problems involving the motion of two objects, the equations end up hard to solve, and it is often convenient to first convert the problem into the center of mass frame (CM). The idea is that in CM the equations are easier to solve (because the motion of the center of mass is effectively a 1-body problem), and once we solve the problem in CM, we can easily convert the result back to the lab frame. To illustrate this procedure, consider the following problem: two carts with masses m_1 and m_2 on a straight track collide head on elastically. If the initial velocities of the two carts are v_1 and v_2 respectively, what are their final velocities in terms of the quantities given above?

   (a) Write down the equations for momentum and energy conservation in the lab frame. You may find these equations not easy to solve.

   Solution: momentum conservation: $m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$; energy conservation: $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$

   (b) Convert the problem into center of mass coordinates: $\vec{x}_{cm} = \frac{m_1\vec{x}_1 + m_2\vec{x}_2}{m_1+m_2}$ and $\vec{x}_r = \vec{x}_1 - \vec{x}_2$, then write down the equations for momentum and energy conservation in CM in the same form as in the last part. What are the "masses" associated with the new coordinates?

   Solution: using the given definitions we can write the lab coordinates in terms of the CM coordinates: $\vec{x}_1 = \vec{x}_{cm} + \frac{m_2}{M}\vec{x}_r$ and $\vec{x}_2 = \vec{x}_{cm} - \frac{m_1}{M}\vec{x}_r$. Since the masses are constants, we can just replace the $\vec{x}$ with $\vec{v}$ to get the respective velocities. We can then plug these into the equations in (a). After some simplification, we should find

$$M\vec{v}_{cm} = M\vec{v}'_{cm} \tag{1}$$

$$\frac{1}{2}Mv_{cm}^2 + \frac{1}{2}\mu v_r^2 = \frac{1}{2}Mv_{cm}'^2 + \frac{1}{2}\mu v_r'^2 \tag{2}$$

where $M = m_1 + m_2$ is the total mass, and $\mu = \frac{m_1 m_2}{m_1+m_2}$ is the reduced mass.

(c) Now solve for the final velocity of the center of mass $\vec{v}_{cm}$ and the final relative velocity $\vec{v}_r$.

Solution: the center of mass velocity is $\vec{v}'_{cm} = \vec{v}_{cm} = \frac{m_1\vec{v}_1+m_2\vec{v}_2}{M}$, and the relative velocity satisfies $v_r'^2 = v_r^2$. Note that this procedure is valid up to this point for 2d and 3d problems as well. Since the collision is elastic and we are dealing with a 1d problem, we can safely assume that the relative velocity has switched sign, so $\vec{v}'_r = -\vec{v}_r = \vec{v}_2 - \vec{v}_1$. This assumption is not necessarily true for 2d or 3d.

(d) Convert the results back to the lab frame coordinates using the transformations we found in part (b). Do they satisfy the equations you wrote down in (a)?

Solution:

$$\vec{v}'_1 = \vec{v}'_{cm} + \frac{m_2}{M}\vec{v}'_r = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{M} + \frac{m_2}{M}(\vec{v}_2 - \vec{v}_1) = \frac{(m_1 - m_2)\vec{v}_1 + 2m_2\vec{v}_2}{M} \tag{3}$$

$$\vec{v}'_2 = \vec{v}'_{cm} - \frac{m_1}{M}\vec{v}'_r = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{M} - \frac{m_1}{M}(\vec{v}_2 - \vec{v}_1) = \frac{2m_1\vec{v}_1 + (m_2 - m_1)\vec{v}_2}{M} \tag{4}$$

These indeed satisfy the equations we wrote down in part (a).