Physics 2A Spring 2020
Discussion 10

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1. **Warm-ups**[1]

(a) A penny-farthing bicycle has a front wheel that is twice as large (in radius) as the back wheel and is moving on the ground with constant speed. Assuming the wheels have the same shape and mass, what is true about the relative rotational energies of the wheels?

A. Front wheel has 1/4 times the energy

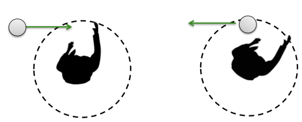
B. Front wheel has 1/2 times the energy

C. Front wheel has the same energy

D. Front wheel has twice the energy

E. Front wheel has 4 times the energy

Solution: C. $KE = \frac{1}{2}I\omega^2$. The moment of inertia of the front wheel is 4 times larger, but its angular velocity is half as that of the back wheel, so overall the kinetic energy is the same for both wheels.



(b) A person rotates clockwise on a chair and a ball bounces off his hand as shown. If the collision were elastic, how fast would the person in the chair rotate after the collision compared to the inelastic case (catches the ball)?

A. Not rotate

B. Rotate faster

C. Rotate slower

[1]From Prof. Burgasser's TPS worksheet

Solution: B. Collision is elastic, so there is no energy loss and should be more energy available for rotation as compared to the inelastic case.

2. **Angular momentum**[2]
   A bug of mass $m = 0.020$ kg is at rest on the edge of a solid cylindrical disk ($M = 0.10$ kg, $R = 0.10$ m) rotating in a horizontal plane around the vertical axis through its center. The disk is rotating at $\omega = 10.0$ rad/s. The bug crawls to the center of the disk.

   (a) What is the new angular velocity of the disk?

   Solution: The initial moment of inertia is $I_0 = mR^2 + \frac{1}{2}MR^2$, and the angular momentum is $L = I_0\omega$. After the bug crawls to the center, the new moment of inertia is $I_1 = \frac{1}{2}MR^2$. Since the angular momentum is conserved, the new angular velocity is $\omega_1 = I_0\omega/I_1 = (2m + M)\omega/M = 14$ rad/s.

   (b) What is the change in the kinetic energy of the system?

   Solution: $\Delta KE = \frac{1}{2}I_1\omega_1^2 - \frac{1}{2}I_0\omega^2 = ((2m+M)^2/2M - m - M/2)R^2\omega^2/2 =$ 0.014 J.

   (c) If the bug crawls back to the outer edge of the disk, what is the angular velocity of the disk then?

   Solution: The moment of inertia becomes I_0 again, and since L is conserved, the angular velocity returns to being 10.0 rad/s.

   (d) What is the new kinetic energy of the system?

   Solution: $KE = \frac{1}{2}I_0\omega^2 = 0.035$ J. This is the original value.

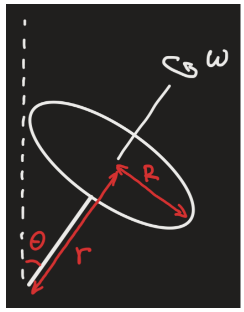
   (e) What is the cause of the increase and decrease of kinetic energy?

   Solution: The change in kinetic energy comes from the work done by the bug as it crawls on the disk.

3. **Precession of a top**
   A top spins with its tip on the ground and is spinning with negligible frictional resistance. Assume the top consists of a solid disk of mass 0.3 kg and radius 5.0 cm, and a stick going through the center of the disk. The disk is spinning at 20 rev/s. The top's center of mass is 5.0 cm from the pivot. When the rotational axis is tilted by an angle $\theta = 10°$, the top starts precessing. What is the period of precession of the top?

   Solution: Consider the following diagram

[2] OpenStax *University Physics*. Chapter 11 problem 59



where r is the distance between the pivot and the center of mass of the top. The weight of the top causes a torque $\tau = rmg\sin\theta$ into the page. Assume the precession frequency is much smaller than the angular frequency of the top. Since $\tau = \frac{dL}{dt}$ and the change in angular momentum is $dL = L\sin\theta d\phi$ where ϕ is the angle by which the top precesses, we have

$$\omega_p = \frac{d\phi}{dt} = \frac{\tau}{L\sin\theta} = \frac{rmg}{L} = \frac{rmg}{I\omega} = \frac{2rg}{R^2\omega} = 3.12 \text{ rad/s} \tag{1}$$

Note that this is independent of the angle θ. The period is $T_p = 2\pi/\omega_p = 2.0$ s.