

Q1 (20 points; 2 per part) DPV Problem 0.1 (parts a through j): In each of the following situations, indicate whether $f = O(g)$, or $f = \Omega(g)$, or both (in each case $f = \theta(g)$).

	$f(n)$	$g(n)$	answer
(a)	$n - 100$	$n - 200$	$f = \theta(g)$
(b)	$n^{\frac{1}{2}}$	$n^{\frac{2}{3}}$	$f = O(g)$
(c)	$100n + \log n$	$n + (\log n)^2$	$f = \theta(g)$
(d)	$n \log n$	$10n \log 10n$	$f = \theta(g)$
(e)	$\log 2n$	$\log 3n$	$f = \theta(g)$
(f)	$10 \log n$	$\log(n^2)$	$f = \theta(g)$
(g)	$n^{1.01}$	$n \log^2 n$	$f = \Omega(g)$
(h)	$\frac{n^2}{\log n}$	$n(\log n)^2$	$f = \Omega(g)$
(i)	$n^{0.1}$	$(\log n)^{10}$	$f = \Omega(g)$
(j)	$(\log n)^{\log n}$	$\frac{n}{\log n}$	answer

Q2 (10 points) Consider the following pseudo-code which takes the integer $n \geq 0$ as input:

```

Function bar(n)
    Print '*';
    if n == 0 then
        Return;
    end
    for i = 0 to n - 1 do
        bar(i);
    end

```

Let $T(n)$ be the number of times the above function prints a star (*) when called with input $n \geq 0$. What is $T(n)$ exactly, in terms of only n (and not values like $T(n - 1)$ or $T(n - 2)$)? Prove your statement

Answer = n^2

The function $\text{bar}(n)$ calls itself recursively n times. Inside of each call, it calls itself n more times, until it finally reaches the base case, where $n = 0$, terminating the recursion. Each of the calls are independent from each other, therefore reaching the $T(n^2)$ runtime.

Q3 (30 points) Let $f(n)$ and $g(n)$ be asymptotically nonnegative functions. Using the basic definition of θ -notation, prove that $\max(f(n), g(n)) = \theta(f(n) + g(n))$

Q4 (10 points; 5 per part)

- (a) is $2^{2n} = O(2^n)$?
No

(b) why?

n^a dominates n^b if $a > b$. In this instance, no matter the value for n , $2n$ will always dominate n (unless negative, but that doesn't apply here). In addition, 2^{2n} can be rewritten as $(2^2)^n$, therefore it is in $O(4^n)$.