

1. Exercise 1.8 in LFD

binomial distribution tells us

$$p_x = \binom{n}{x} p^x q^{n-x}$$

$$n = 10$$

$$x = 0 \text{ and } x = 1$$

$$p = 0.9$$

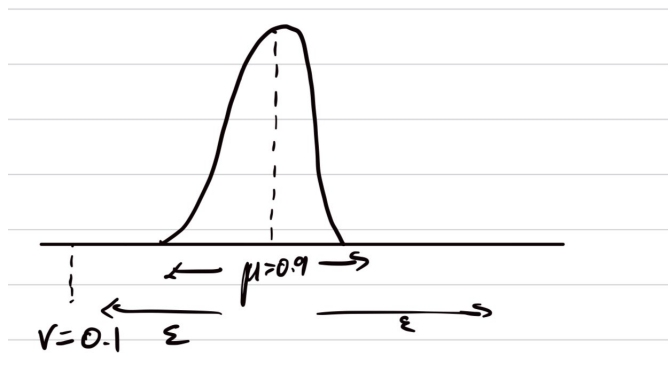
$$q = 1 - p = 0.1$$

$$p_0 = \binom{10}{0} 0.9^0 0.1^{10} = 0.1^{10} = 1 \times 10^{-10}$$

$$p_1 = \binom{10}{1} 0.9^1 0.1^9 = 10 \times 0.9 \times 0.1^9 = 9 \times 10^{-9}$$

$$p = p_0 + p_1 = 9.1 \times 10^{-9}$$

2. Exercise 1.9 in LFD



we want  $0.1 = \nu$  to  
happen, so  $\epsilon = |0.9 - 0.1| = 0.8$

Using  $P[|\nu - \mu| \geq \epsilon] \leq 2e^{-2\epsilon^2 N}$ , we can find the following bounds:

Using  $\epsilon = 0.8$  and  $N = 10$ , we can derive  $2e^{-2\epsilon^2 N} = 2e^{2(0.8)^2 \times 10}$ , which equals  $5.52 \times 10^{-6}$ .

Since this is a bound, it is reasonable for it to be greater than our answer in exercise 1.8

3. Exercise 1.10 in LFD

(a)  $\mu = 0.5$

(b) **FINISH HERE**

(c) **FINISH HERE**

(d)  $c_1$  and  $c_{rand}$  obey the Hoeffding bound,  $c_{min}$  does not because  $c_1$  and  $c_{rand}$  were selected without looking at the data, while  $c_{min}$  looks at the data before selecting.  $c_{min}$  represents the "unlucky" choice

(e) **FINISH HERE**

4. Exercise 1.11 in LFD

(a) **FINISH HERE**

(b) **FINISH HERE**

(c) **FINISH HERE**

(d) **FINISH HERE**

5. Exercise 1.12 in LFD

(a) We don't know anything about the sample, so we can't make any assumptions of how well our  $g$  can approximate  $f$ , in addition, the problem says "guarantee", which will never happen.

(b) No, in order to have a high probability that our  $g$  approximates  $f$  well out of sample, we need our  $E_{out} \approx E_{in} \approx 0$ . With 4000, which is a small data point, we can say  $E_{in} \approx 0$ , but we can't say anything about  $E_{out} \approx E_{in}$  since that requires  $N$  to be large.

(c) **This is the best choice**, more likely than not we will declare that we have failed, we don't have enough data to say anything about outside the  $N = 4000$  data points.

6. Problem 1.3 in LFD

(a)  $w^*$  separates the data, so  $x_n = y_n$ , let  $A_n = x_n = y_n$  and  $p = \min_{1 \leq n \leq N} A_n^2 w^*$ ,  $A_n^2$  will always be a positive number, so  $p > 0$

(b) Assume

$$w^T(t)w^* \geq w^T(t-1)w^* + p$$

$$w^T(t) = w^T(t-1) + y_*x_* \text{ update rule}$$

$$(w^T(t-1) + y_*x_*)w^* \geq w^T(t-1)w^* + p$$

$$w^T(t-1)w^* + y_*x_*w^* \geq w^T(t-1)w^* + p$$

Here, we see that  $w^T(t-1)w^*$  are the same on both side of the inequality, so we need to prove that  $y_*x_*w^* \geq p$ , but we already know that  $p \leq y_n(w^{*T}x_n)$  from part (a), so the statement is true ■

prove  $w^T(t)w^* \geq tp$  by induction:

base case:  $t = 0$ ,  $w(0) = 0$ , so  $0 \geq 0$  is true

induction step: assume  $k = t$  and create our induction hypothesis  $w^T(k)w^* \geq kp$

Prove  $w^T(k+1)w^* \geq (k+1)p$

$$(w^T(k) + y_*x_*)w^* \geq (k+1)p \text{ update rule}$$

$$w^T(k)w^* + y_*x_*w^* \geq kp + p$$

Our induction hypothesis says  $w^T(k)w^* \geq kp$ , and we know that  $y_*x_*w^* \geq p$  from part (a), so the statement is true ■

(c) with the update rule, we get

$$\|w(t-1) + x(t-1)y(t-1)\|^2 \leq \|w(t-1)\|^2 + \|x(t-1)\|^2$$

We can solve LHS, so

$$\begin{aligned} \|[w(t-1) + x(t-1)y(t-1)]^2\| &= \|w(t-1)^2 + 2w(t-1)x(t-1)y(t-1) + x(t-1)^2y(t-1)^2\| \\ y(t-1)^2 &= 1 \text{ since } y \in \{-1, 1\} \\ &= \|w(t-1)^2 + 2w(t-1)x(t-1)y(t-1) + x(t-1)^2\| \\ &= \|w(t-1)^2 + x(t-1)^2\| \text{ is in both sides of the inequality} \end{aligned}$$

On the LHS, we have  $2w(t-1)x(t-1)y(t-1)$ , which, when  $x(t-1)$  is misclassified by  $w(t-1)$ , becomes  $< 0$ , so

$$\|w(t-1)^2 + x(t-1)^2 + \text{negative number}\| < \|w(t-1)^2 + x(t-1)^2\|$$

We know  $\|w(t-1)^2 + x(t-1)^2\| \leq \|w(t-1)\|^2 + \|x(t-1)\|^2$  by subadditivity, so  $\|w(t)\|^2 \leq \|w(t-1)\|^2 + \|x(t-1)\|^2$  ■