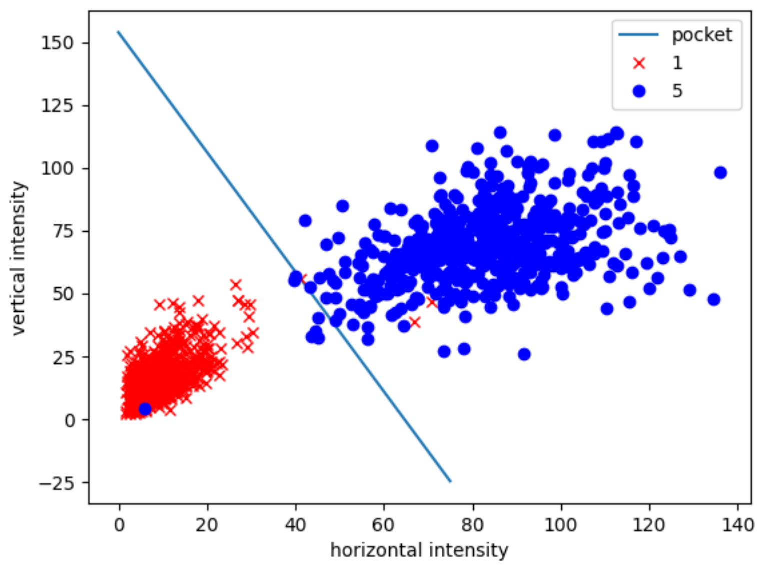
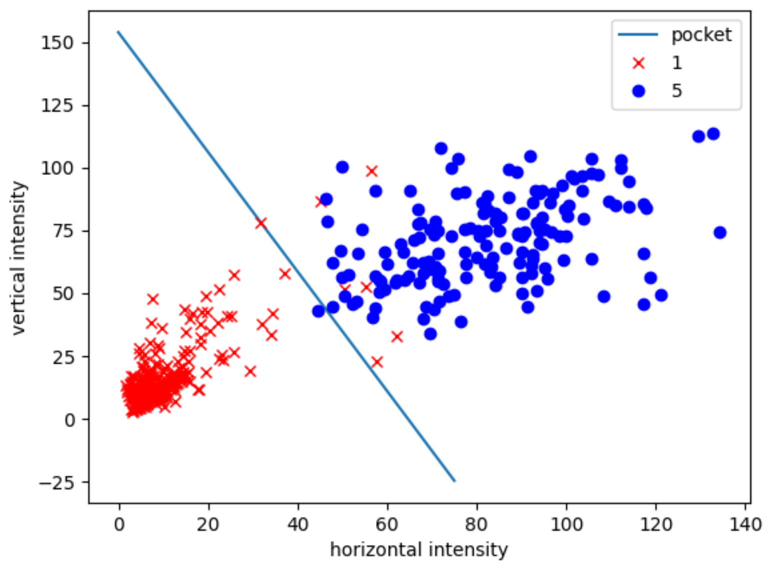
1. (a) linear regression with pocket

training:



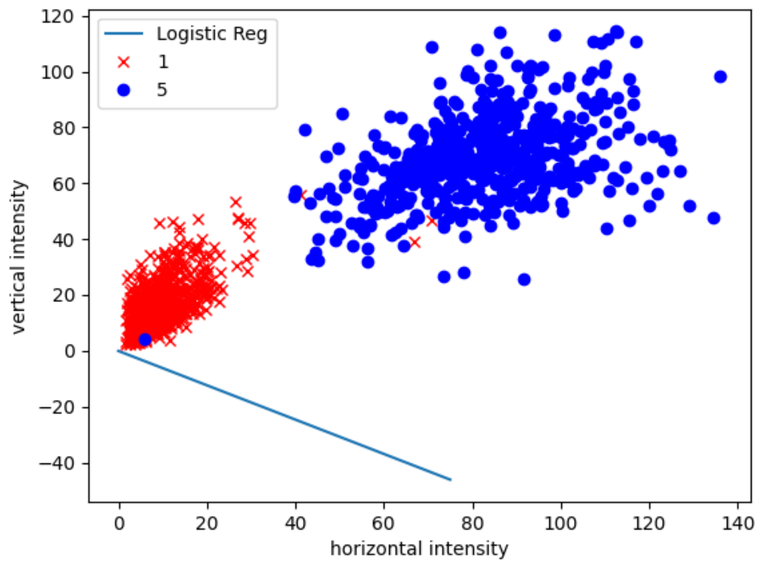
test:



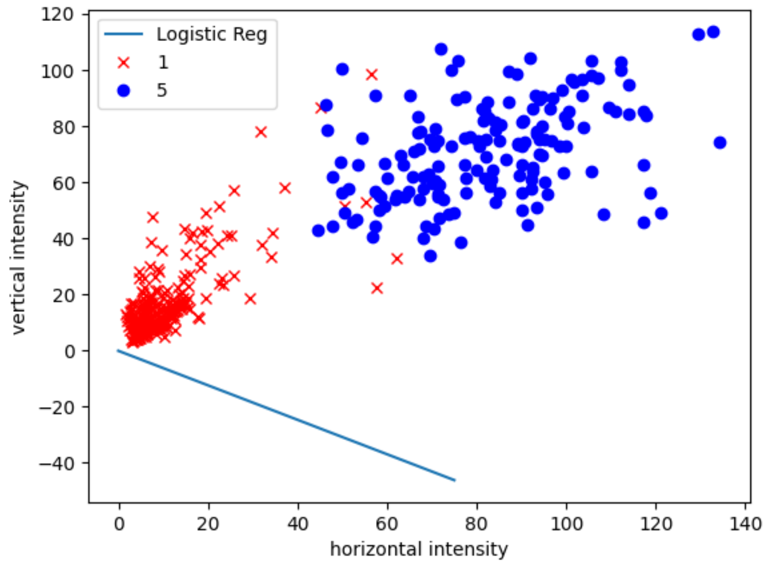
NOTE: my logistic regression functions didn't work :(, I followed the algorithm on page 95 of the book. I still included it to show progress, the attempted code file is submitted as well.

logistic regression with gd

training:



test:



(b) linear regression with pocket
$E_{in} = 0.0058$, $E_{test} = 0.0165$

logistic regression with gradient descent

$E_{in} = 0.3562$, $E_{test} = 0.3774$

(c) Since my algorithm for logistic regression did not work, I will assume linear regression w/ pocket is the method with the minimum E_{in}. With approximation & generalization, we have derived the following formula:

$$E_{out}(g) \leq E_{in}(g) + \sqrt{\frac{8}{N} \ln \frac{4m_H(N)}{\delta}}$$

where our error bar

$$\Omega = \sqrt{\frac{8}{N} \ln \frac{4m_H(N)}{\delta}}$$

We can plug in $N = 1561$, $\delta = 0.05$, $d_{vc} = 3$, $m_H(1561) = (2(1561))^3 + 1$, and $E_in = 0.0058$, then we derive the equation:

$$\begin{aligned} E_{out}(g) &\leq 0.0058 + \sqrt{\frac{8}{1561} \ln \frac{4((2(1561))^3 + 1)}{0.05}} \\ &\leq 0.388 \end{aligned}$$

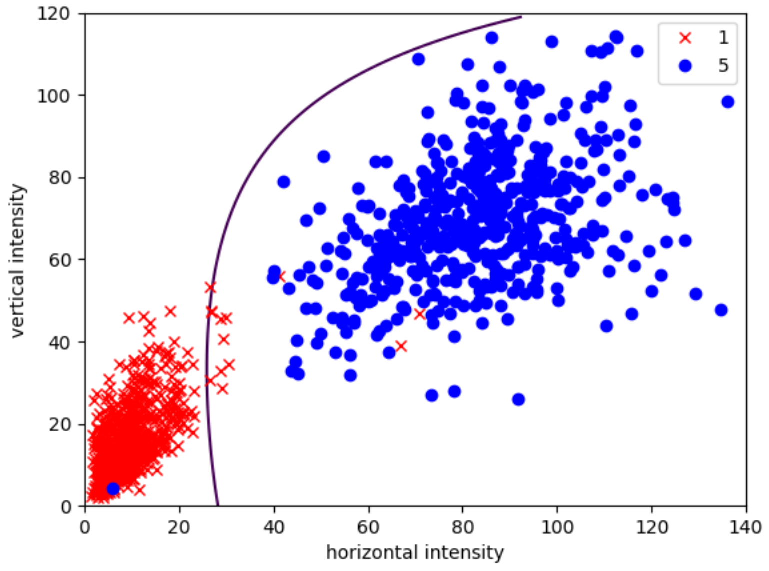
(d) For our testing data, we can use (2.1)

$$E_{out}(g) \leq E_{test}(g) + \sqrt{\frac{1}{2N} \ln \frac{2M}{\delta}}$$

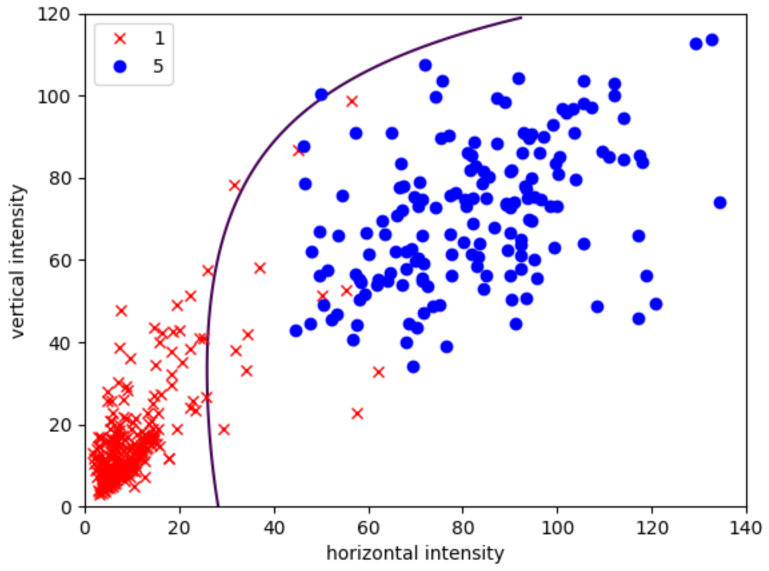
Where $N = 424$, $\delta = 0.05$, $M = 1$

$$\begin{aligned} E_{out}(g) &\leq 0.0118 + \sqrt{\frac{1}{2(424)} \ln \frac{2}{0.05}} \\ &\leq 0.0824 \end{aligned}$$

(e) (a) linear regression w/ pocket:
training:



test:



As mentioned above, algorithms for logistic regression didn't work despite following the code in page 95 :(, attempt is shown in code, so no plot was created for logistic regression

(b) for linear regression w/ pocket: $E_{in} = 0.0032$, $E_{test} = 0.0212$

(c) We can reuse the formula from above:

$$E_{out}(g) \leq E_{in}(g) + \sqrt{\frac{8}{N} \ln \frac{4m_H(N)}{\delta}}$$

We can plug in $N = 1561$, $\delta = 0.05$, $d_{vc} = 10$, $m_H(1561) = (2(1561))^{10} + 1$, and $E_in = 0.0058$, then we derive the equation:

$$\begin{aligned} E_{out}(g) &\leq 0.0058 + \sqrt{\frac{8}{1561} \ln \frac{4((2(1561))^{10} + 1)}{0.05}} \\ &\leq 0.665 \end{aligned}$$

(d) We can reuse the formula from above:

$$E_{out}(g) \leq E_{test}(g) + \sqrt{\frac{1}{2N} \ln \frac{2M}{\delta}}$$

Where $N = 424$, $\delta = 0.05$, $M = 1$

$$\begin{aligned} E_{out}(g) &\leq 0.0212 + \sqrt{\frac{1}{2(424)} \ln \frac{2}{0.05}} \\ &\leq 0.08716 \end{aligned}$$

(f) Linear model without the 3rd order polynomial transform. It may seem counter-intuitive that, despite a lower E_{in} and E_{test}, we don't choose the one with 3rd order transform. As discussed in lecture, when a problem gets more complex, our solution becomes simpler, this means a lower d_{vc}, we can't assume the size of the dataset that the customer will provide, but if we assume that it is large, then we want the simplest curve possible, which we obtain from the linear model without the 3rd order polynomial transform.

2. (a) When the learning rate was set to 0.01, there was a stable decline in the function, when the learning rate was raised to 0.1, it became scattered. This is due to the function jumping around, trying to find the local minimum, but the learning rate is too high, so it's jumping more than it should.

(b) Learning rate set to 0.01:

x start	y start	x min	y min	min value
0.1	0.1	0.24380496936478335	-0.23792582148617658	-1.8200785415471565
1	1	1.2180703009052047	0.7128119503387537	0.5932693743258357
-0.5	-0.5	-0.731377460409601	-0.23785536290147238	-1.332481062330978
-1	-1	-1.2180703009052047	-0.7128119503387537	0.5932693743258357

Learning rate set to 0.1:

x start	y start	x min	y min	min value
0.1	0.1	0.23624085417279922	0.2292203653889578	-1.6452216178404533
1	1	0.5638448167590557	-0.03503546573154889	-1.6997441668889812
-0.5	-0.5	-0.01859951030800544	0.4009863778325866	-1.3964666430345036
-1	-1	-0.5638448167590557	0.03503546573154889	-1.6997441668889812

3. (a) cost(accept) (+1) $= 0 \times correct + c_a \times incorrect = c_a \times incorrect = (1 - g(x))c_a$
cost(reject) (-1) $= c_r \times correct + 0 \times incorrect = c_r \times correct = g(x)c_r$

(b) We accept when $g(x) \geq k$, so

$$\begin{aligned}(1 - g(x))c_a &\geq g(x)c_r \\ c_a - c_a g(x) &\geq g(x)c_r \\ -c_a g(x) - g(x)c_r &\geq -c_a \\ g(x)(-c_a - c_r) &\geq -c_a \\ g(x) &\geq \frac{-c_a}{-c_a - c_r} \\ g(x) &\geq \frac{c_a}{c_a + c_r}\end{aligned}$$

If we let

$$k = \frac{c_a}{c_a + c_r}$$

Then we have fulfilled our accepting condition $g(x) \geq k$

(c) Supermarket: $c_a = 1$ and $c_r = 10$

$$k = \frac{1}{1 + 10} = 0.09090909$$

CIA: $c_a = 1000$ and $c_r = 1$

$$k = \frac{1000}{1000 + 1} = 0.999000999$$

Intuitively, the CIA would have a higher threshhold because the cost of accepting a person's fingerprints when it should've been rejecting is high. Whereas the supermarket, if you accept false accept, it is not a big deal and thus have low cost.