

1. TODO

2. (a) When the learning rate was set to 0.01, there was a stable decline in the function, when the learning rate was raised to 0.1, it became scattered. This is due to the function jumping around, trying to find the local minimum, but the learning rate is too high, so it's jumping more than it should.

- (b) Learning rate set to 0.01:

x start	y start	x min	y min	min value
0.1	0.1	0.24380496936478335	-0.23792582148617658	-1.8200785415471565
1	1	1.2180703009052047	0.7128119503387537	0.5932693743258357
-0.5	-0.5	-0.731377460409601	-0.23785536290147238	-1.332481062330978
-1	-1	-1.2180703009052047	-0.7128119503387537	0.5932693743258357

Learning rate set to 0.1:

x start	y start	x min	y min	min value
0.1	0.1	0.23624085417279922	0.2292203653889578	-1.6452216178404533
1	1	0.5638448167590557	-0.03503546573154889	-1.6997441668889812
-0.5	-0.5	-0.01859951030800544	0.4009863778325866	-1.3964666430345036
-1	-1	-0.5638448167590557	0.03503546573154889	-1.6997441668889812

3. (a)  $\text{cost}(\text{accept}) (+1) = 0 \times \text{correct} + c_a \times \text{incorrect} = c_a \times \text{incorrect} = (1 - g(x))c_a$   
 $\text{cost}(\text{reject}) (-1) = c_r \times \text{correct} + 0 \times \text{incorrect} = c_r \times \text{correct} = g(x)c_r$

- (b) We accept when  $g(x) \geq k$ , so

$$\begin{aligned}
 (1 - g(x))c_a &\geq g(x)c_r \\
 c_a - c_a g(x) &\geq g(x)c_r \\
 -c_a g(x) - g(x)c_r &\geq -c_a \\
 g(x)(-c_a - c_r) &\geq -c_a \\
 g(x) &\geq \frac{-c_a}{-c_a - c_r} \\
 g(x) &\geq \frac{c_a}{c_a + c_r}
 \end{aligned}$$

If we let

$$k = \frac{c_a}{c_a + c_r}$$

Then we have fulfilled our accepting condition  $g(x) \geq k$

- (c) Supermarket:  $c_a = 1$  and  $c_r = 10$

$$k = \frac{1}{1 + 10} = 0.09090909$$

CIA:  $c_a = 1000$  and  $c_r = 1$

$$k = \frac{1000}{1000 + 1} = 0.999000999$$

Intuitively, the CIA would have a higher threshold because the cost of accepting a person's fingerprints when it should've been rejecting is high. Whereas the supermarket, if you accept false accept, it is not a big deal and thus have low cost.