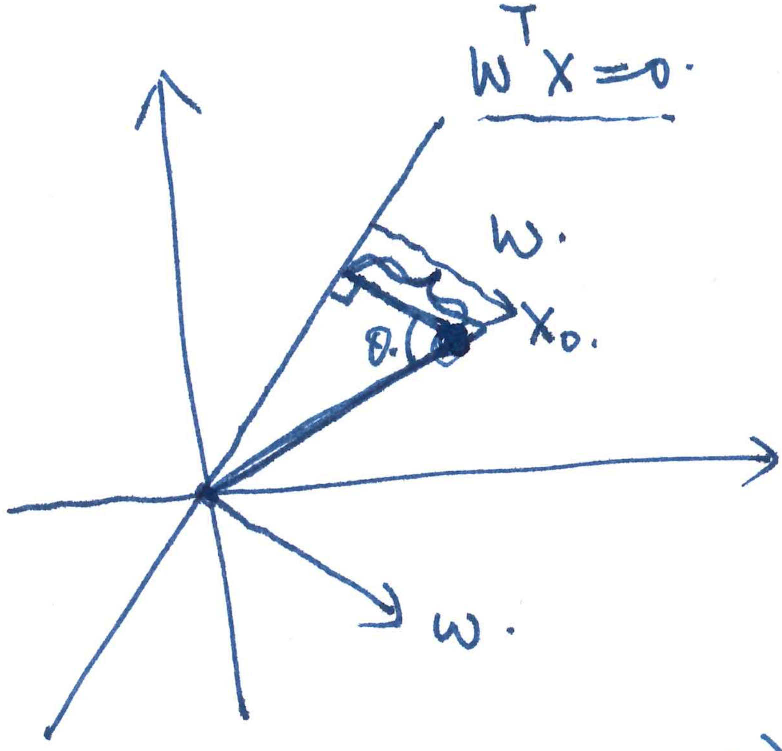
$$\text{signed distance} = \frac{y \cdot w^T x}{\|w\|_2}$$



$$\frac{\|x_0\| \cdot \cos\theta}{}$$

$$y \cdot \frac{w^T x_0}{\|w\|_2} = \frac{(\|w\|_2 \|x_0\|_2 \cos\theta)}{\|w\|_2} y$$

$$= \|x_0\|_2 \cdot \cos\theta \cdot y$$

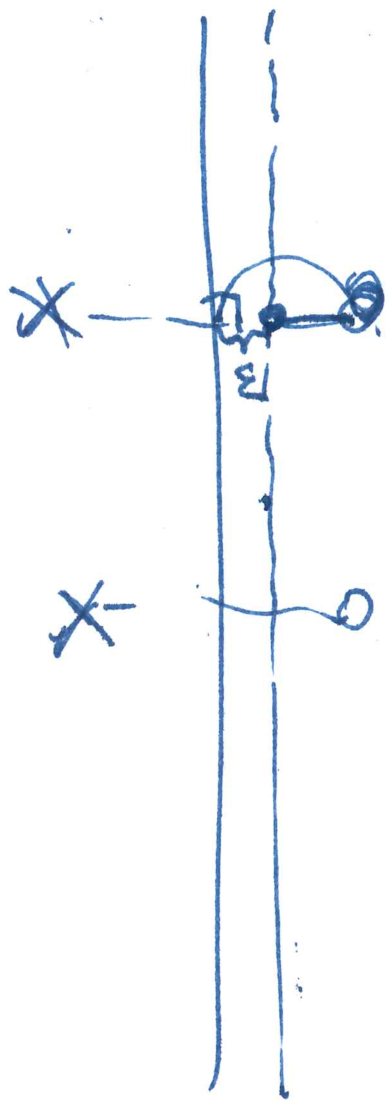
$$\max_{w \in \mathbb{R}^d} \frac{\min_{i \in [n]} \frac{y^{(i)} \cdot w^T x^{(i)}}{\|w\|_2}}{}$$

$$\Leftrightarrow \max_{w \in \mathbb{R}^d} \frac{\min_{i \in [n]} y^{(i)} \cdot w^T x^{(i)}}{\|w\|_2}$$

$$\Leftrightarrow \max_{\substack{w \in \mathbb{R}^d \\ c > 0}} \frac{\boxed{\min_{i \in [n]} y^{(i)} (c \cdot w) \cdot x^{(i)}}}{c \|w\|_2}$$

$$\Leftrightarrow \max_{w \in \mathbb{R}^d} \frac{1}{\|w\|_2}$$

$$\text{s.t.} \quad y^{(i)} \cdot w^T x^{(i)} \geq 1. \ \forall i \in [n].$$



soft-margin SVM:

$$\min_{\substack{w\in\mathbb{R}^d \\ \cancel{\xi\in\mathbb{R}^n}}} \sum_{i=1}^{n} \xi_i + \frac{\lambda}{2}\|w\|_2^2.$$

s.t. $\forall i\in[n].\quad y^{(i)}\cdot w^T x^{(i)} \geq 1-\xi_i.$

$\forall i\in[n].\quad \xi_i \geq 0.$

$\Rightarrow \forall i\in[n].\quad \xi_i \geq 1-y^{(i)}\cdot w^T x^{(i)}.$

$\xi_i \geq 0.$

$$\Rightarrow \xi_i^{*} = \max\{0,\ 1-y^{(i)}\cdot w^T x^{(i)}\}.$$

$$\Leftrightarrow \min_{w\in\mathbb{R}^d} \sum_{i=1}^{n} \max\{0,\ 1-y^{(i)}\cdot w^T x^{(i)}\} + \frac{\lambda}{2}\|w\|_2^2$$

Define $l_{\text{hinge}}(t) = \max\{0,\ 1-t\}.$

$$\Leftrightarrow \min_{w\in\mathbb{R}^d} \sum_{i=1}^{n} l_{\text{hinge}}(y^{(i)}\cdot w^T x^{(i)}) + \frac{\lambda}{2}\|w\|_2^2.$$