

ECON7115: Assignment 4

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1 Spatial Trade Model

1.1 Trade Share

$$\begin{aligned}\lambda_{lm} &= \frac{X_{lm}}{X_m} \\ &= \frac{P_{lm}}{P_m} \\ &= \frac{\left[\frac{w_l}{\bar{A}_l * \bar{L}_l^\alpha} * \tau_{lm} * (1 + t_{lm}) \right]^{1-\sigma}}{P_m^{1-\sigma}}\end{aligned}$$

1.2 Price Index of Region m

$$P_m = \left[\sum_{i=1}^N \sum_{l' \in \mathcal{L}_i} \left[\frac{w'_l}{\bar{A}'_l * \bar{L}'_l^\alpha} * \tau_{l'm} * (1 + t_{l'm}) \right]^{1-\sigma} \right]^{1/1-\sigma}$$

1.3 Labor Allocation

$$\frac{L_l}{\bar{L}_n} = \frac{(\bar{B}_l * \frac{X_l/L_l}{\bar{P}_l})^{1/\mu_n}}{\sum_{l \in \mathcal{L}_n} (\bar{B}_l * \frac{X_l/L_l}{\bar{P}_l})^{1/\mu_n}}$$

1.4 Total Expenditure and Eq'm Wage

$$X_l = w_l * L_l + \frac{L_l}{\bar{L}_n} \sum_{l' \in \mathcal{L}_n} \sum_{i=1}^N \sum_{m \in \mathcal{L}_i} \frac{t_{ml'}}{1 + t_{ml'}} * \lambda_{ml'} * X_{l'}$$

$$L_l = \frac{1}{w_l} * \sum_{i=1}^N \sum_{m \in \mathcal{L}_i} \frac{1}{1 + t_{in}} \lambda_{lm} * X_m$$

1.5 Country 1 Maximizes its National Welfare

If arbitrary country n wants to max its national welfare, the question becomes:

$$\begin{aligned} \max \quad & U_n = \bar{B}_l L_l^{-\mu_n} \frac{X_l/L_l}{P_l} \\ \text{Since} \quad & \sum_{l \in \mathcal{L}_n} U_l = \bar{L}_n, \\ \text{Equivalently,} \quad & \max \quad U_n = \left(\sum_{l \in \mathcal{L}_n} \left(\bar{B}_l L_l^{-\mu_n} \frac{X_l/L_l}{P_l} \right)^{1/\mu_n} \right)^{\mu_n} \end{aligned}$$

If country 1 wants to max its national welfare, the question becomes:

$$\max \quad U_1 = \left(\sum_{l \in \mathcal{L}_1} \left(\bar{B}_l L_l^{-\mu_1} \frac{X_l/L_l}{P_l} \right)^{1/\mu_1} \right)^{\mu_1}$$

$$\text{s.t.} \quad \lambda_{lm} = \frac{\left[\frac{w_l}{\bar{A}_l \cdot L_l^\alpha} \cdot \tau_{lm} \cdot (1 + t_{lm}) \right]^{1-\sigma}}{P_m^{1-\sigma}} \quad (1)$$

$$P_m = \left[\sum_{i=1}^N \sum_{l' \in \mathcal{L}_i} \left[\frac{w_{l'}}{\bar{A}_{l'} \cdot L_{l'}^\alpha} \cdot \tau_{l'm} \cdot (1 + t_{l'm}) \right]^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad (2)$$

$$\frac{L_l}{\bar{L}_n} = \frac{\left(\bar{B}_l \cdot \frac{X_l/L_l}{P_l} \right)^{1/\mu_n}}{\sum_{l \in \mathcal{L}_n} \left(\bar{B}_l \cdot \frac{X_l/L_l}{P_l} \right)^{1/\mu_n}} \quad (3)$$

$$X_l = w_l \cdot L_l + \frac{L_l}{\bar{L}_n} \sum_{l' \in \mathcal{L}_n} \sum_{i=1}^N \sum_{m \in \mathcal{L}_i} \frac{t_{ml'}}{1 + t_{ml'}} \cdot \lambda_{ml'} \cdot X_{l'} \quad (4)$$

$$L_l = \frac{1}{w_l} \cdot \sum_{i=1}^N \sum_{m \in \mathcal{L}_i} \frac{1}{1 + t_{in}} \cdot \lambda_{lm} \cdot X_m \quad (5)$$

1.6 Exact-hat Algebra

First, denote the following:

- Z' : the value of Z after the shock
- $\hat{Z} = Z'/Z$

Using the exact-hat algebra, the eq'm system becomes:

$$\hat{\lambda}_{lm} = \frac{\left[\frac{\hat{w}_l}{\hat{\bar{A}}_l \cdot \hat{L}_l^\alpha} \cdot \tau_{lm} \cdot (1 + \hat{t}_{lm}) \right]^{1-\sigma}}{\sum_{l' \in \mathcal{L}_n} \sum_{m=1}^N \lambda_{lm} \left[\frac{\hat{w}_l}{\hat{\bar{A}}_l \cdot \hat{L}_l^\alpha} \cdot \tau_{lm} \cdot (1 + \hat{t}_{lm}) \right]^{1-\sigma}} \quad (6)$$

$$\frac{\hat{L}_l}{\hat{\bar{L}}_n} = \frac{\left(\hat{\bar{B}}_l \cdot \frac{\hat{X}_l / \hat{L}_l}{\hat{\bar{P}}_l} \right)^{1/\mu_n}}{\sum_{l' \in \mathcal{L}_n} \frac{L'_l}{\bar{L}_n} \left(\hat{\bar{B}}_l \cdot \frac{\hat{X}_l / \hat{L}_l}{\hat{\bar{P}}_l} \right)^{1/\mu_n}} \quad (7)$$

$$\hat{X}_l X_l = \hat{w}_l w_l \cdot \hat{L}_l L_l + \frac{\hat{L}_l}{\hat{\bar{L}}_n} \sum_{l' \in \mathcal{L}_n} \sum_{i=1}^N \sum_{m \in \mathcal{L}_i} \frac{\hat{t}_{ml'} t_{ml'}}{(1 + \hat{t}_{ml'})(1 + t_{ml'})} \cdot \hat{\lambda}_{ml'} \lambda_{ml'} \cdot \hat{X}_{l'} X_{l'} \quad (8)$$

$$\hat{L}_l L_l = \frac{1}{\hat{w}_l w_l} \cdot \sum_{i=1}^N \sum_{m \in \mathcal{L}_i} \frac{1}{(1 + \hat{t}_{in})(1 + t_{in})} \cdot \hat{\lambda}_{lm} \lambda_{lm} \cdot \hat{X}_m X_m \quad (9)$$

The parameters we need to conduct the exact-hat algebra:

- From data: $\lambda_{lm}, X_{lm}, w_l$
- exogenous parameters: $\bar{A}_l, \bar{B}_l, \mu_n, \alpha, t, \tau, \sigma$

1.7 Optimal Tariff of Country 1 using Exact-hat Algebra

$$\max U_1 = \left(\sum_{l \in \mathcal{L}_1} \left(\bar{B}_l L_l^{-\mu_1} \frac{X_l / L_l}{\bar{P}_l} \right)^{1/\mu_1} \right)^{\mu_1}$$

s.t. the equilibrium system of under the exact-hat algebra in 1.6.

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2.1 Equilibrium outcomes under zero tariffs

Each column in the matrix represents the following results: Welfare(U), Wage(w), Trade Volume (X), and Labor Allocation(L). Each row demonstrates regions 1-4, respectively.

$$\begin{bmatrix} 1.4690 & 0.2539 & 0.1396 & 0.5498 \\ 1.4690 & 0.2461 & 0.1108 & 0.4502 \\ 1.4690 & 0.2531 & 0.1396 & 0.5498 \\ 1.4690 & 0.2461 & 0.1108 & 0.4502 \end{bmatrix}$$

2.2 Unilaterally Optimal Tariffs for Country 1

When $Tar_2 = 0$, the optimal tariffs for country 1 is

$$Tar_1 = 0.3$$

For the computation, please refer to the code.

2.3 Nash Tariffs

The Nash Tariffs:

$$NashTar_1 = NashTar_2 = 0.2749$$

For the computation, please refer to the code.

2.4 When $\mu_1 = 0.4$

2.4.1 Zero Tariffs

Each column in the matrix represents the following results: Welfare(U), Wage(w), Trade Volume (X), and Labor Allocation(L). Each row demonstrates regions 1-4, respectively.

$$\begin{bmatrix} 1.3707 & 0.2535 & 0.1427 & 0.5630 \\ 1.3707 & 0.2464 & 0.1077 & 0.4370 \\ 1.4699 & 0.2540 & 0.1397 & 0.5502 \\ 1.4699 & 0.2461 & 0.1107 & 0.4498 \end{bmatrix}$$

2.4.2 Unilateral Tariffs for Country 1

When $Tar_2 = 0$, the optimal tariffs for country 1 is

$$Tar_1 = 0.3$$

For the computation, please refer to the code.

2.4.3 Nash Tariffs

The Nash Tariffs:

$$NashTar_1 = 0.2750$$

$$NashTar_2 = 0.2725$$

For the computation, please refer to the code.

2.5 Discussions

When $\mu_1 = \mu_2 = 0.5$, the equilibrium outcomes of the two countries are the same. Specifically,

- The welfare are the same
- The economic outcomes of the coastal and inner regions are identical
- The labor are basically allocated in the coastal regions
- The unilateral optimal tariff of country 1 is 0.3; and the both nash tariffs are 0.2749

When $\mu_1 = 0.4 < \mu_2 = 0.5$, which means the amenity of country 1 decreases,

- The economic status of country 1 is better in the coastal region. The gap between the coastal and inner region is enlarged
- The welfare of country 1 decreases and the inequality between the two countries increases
- The unilateral optimal tariff of country 1 does not change, but the nash tariffs are asymmetric

Hence, the deterioration of the living amenity would negatively affect the economy of a country.