Econ7115: Structural Models and Numerical Methods in Economics
Assignment W1

Due 23 April 2025

Zi Wang
HKBU

Spring 2025

---

1. Consider the following equilibrium system of the Armington model:

   - $$w_i L_i = \sum_{n=1}^{N} \frac{1}{1+t_{in}} \lambda_{in} X_n, \quad \lambda_{in} = \frac{\left(\frac{w_i \kappa_{in}}{A_i}\right)^{1-\sigma}}{\sum_{k=1}^{N} \left(\frac{w_k \kappa_{kn}}{A_k}\right)^{1-\sigma}}, \quad \kappa_{in} = \tau_{in}\left(1+t_{in}\right). \quad (1)$$

   - $$X_n = w_n L_n + \sum_{i=1}^{N} \frac{t_{in}}{1+t_{in}} \lambda_{in} X_n. \quad (2)$$

   (a) Suppose that $t_{in} = 0$ for all (i, n).

   - Derive the equilibrium system for $(w_i)_{i=1}^{N}$.
   - Derive the Jacabian matrix of the equilibrium system above.
   - Consider the following parameterization: $N = 3$, $\sigma = 4$, $A = [3; 1; 1]$, $L = [1; 2; 5]$, and $\tau_{in} = 2$ for all $i \neq n$.
     - Please solve for the equilibrium outcomes $(w_i)_{i=1}^{3}$ using the *Newton's method*, with the normalization $\sum_{i=1}^{3} w_i = 1$. Compare your solution with that using fsolve in the Matlab (Note: if you do not use Matlab, you can compare your answer with that derived by any packaged nonlinear solver)
     - Consider reducing τ_{in} for all $i \neq n$ from 2 to 1.2. Please derive the welfare in each country with respect to these changes in τ_{in}.

   (b) Suppose that $t_{in} = 0.05$ for all $i \neq n$. Consider again the following parameterization: $N = 3$, $\sigma = 4$, $A = [3; 1; 1]$, $L = [1; 2; 5]$, and $\tau_{in} = 2$ for all $i \neq n$.

   - Please derive the first-order effect of $\log(1 + t_{in})$ on $\log \lambda_{in}$ for all $i \neq n$ in this general equilibrium system. Compare these GE effects with the reduced-form partial elasticity, $1 - \sigma$.

- Suppose that $t_{in} = 0.25$ for all $i \neq n$. Please derive the first-order effect of $\log(1 + t_{in})$ on $\log \lambda_{in}$ for all $i \neq n$ in this general equilibrium system.

2. Consider an extension of the Armington model discussed in class: production uses both labor and intermediates, *i.e.* the unit cost of producing in country i can be expressed as

$$c_i = \frac{w_i^{\beta} P_i^{1-\beta}}{A_i}, \quad \beta \in (0, 1], \tag{3}$$

where β is the value-added share and P_i is the price index of the final consumption goods in country i.

- Please derive the equilibrium system of this extension.
- Please derive the linear system that can be used to compute the Jacobian matrix of the equilibrium system above.