Assignment 6

Jiawen KE

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1

Bellman equation:

$$
\begin{aligned}
V(k) &= \max U(c) + \beta V(k') \\
\text{s.t.} \quad k' &= k^\alpha l^{1-\alpha} - c - \delta k \\
\text{where} \quad U(c) &= \frac{c^{1-\gamma}}{1-\gamma} - \frac{l^{1+\xi}}{1+\xi}
\end{aligned}
$$

Hence,

$$
V(k) = \max \quad \left[\frac{(k^\alpha l^{1-\alpha} - \delta k - k')^{1-\gamma}}{1-\gamma} - \frac{l^{1+\xi}}{1+\xi}\right] + \beta V(k') \tag{1}
$$

2

Algorithm to solve for $(a_0, a_1, a_2, a_3, a_4) = \mathbf{a}$:

- Solve the optimal l^*

$$
\begin{aligned}
l^{\xi+\alpha} &= (zk^\alpha l^{1-\alpha} - \delta k - k')^{-\gamma}(1-\alpha)zk^\alpha \\
\text{Since} \quad z = 1, \quad l^* &= (1-\alpha)^{\frac{1}{\xi+\alpha}}(k^\alpha l^{1-\alpha} - \delta k - k')^{\frac{-\gamma}{\xi+\alpha}} k^{\frac{\alpha}{\xi+\alpha}}
\end{aligned} \tag{2}
$$

- Solve the optimal k_{t+1}:

$$
(k^\alpha l^{1-\alpha} - \delta k - k')^{-\gamma} = \beta(a_1 + 2a_2 k' + 3a_3 k'^2 + 4a_4 k'^3) \tag{3}
$$

- Combine equations (2) and (3), we can get the optimal l^* and k'^*
- Plug the l^* and k'^* into equation (1), for each $k = k_i$, we will have $V(k_i) = V_i$. We pick up arbitrary five k_i, and hence we have five (k_i, V_i).
- At the same time, for each $k = k_i$, we also have:

$$V(k_i) = V_i = a_0 + a_1 k_i + a_2 k_i^2 + a_3 k_i^3 + a_4 k_i^4, i = 1, ..., 5$$

- Then we insert the initial guess of $(a_0, ..., a_4)$ as $(a_0, ..., a_4)^{(0)}$ and five groups of (k_i, V_i) to start the iteration until $(a_0, ..., a_4)^{(t)} = (a_0, ..., a_4)^{(t-1)}$.

3

Please refer to the code.

$$\mathbf{a} = \begin{bmatrix} 11.6935 \\ 1.7231 \\ -3.1268 \\ 3.7162 \\ -1.9150 \end{bmatrix}$$

4

4.1

Bellman Equation:

$$\begin{aligned} V(k, z_s) &= \max z_s U(c) + \beta E\left[\pi_{s,s} V(k', z_s) + \pi_{s,-s} V(k', z_{-s})\right] \\ \text{s.t.} \quad k' &= z_s k^\alpha l^{1-\alpha} - c - \delta k \end{aligned} \tag{4}$$

Where: $z_L = 0.8, z_H = 1.2$

Markov shifter with a transition matrix:

$$\Pi = \begin{bmatrix} \pi_{LL} & \pi_{LH} \\ \pi_{HL} & \pi_{HH} \end{bmatrix} = \begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{bmatrix}$$

4.2

$$\text{Optimal } \mathbf{a} = \begin{bmatrix} 14.4245 & 14.4267 \\ 1.7211 & 1.7302 \\ -3.1116 & -3.1379 \\ 3.6912 & 3.7273 \\ -1.9002 & -1.9200 \end{bmatrix}$$