

Econ7115: Structural Models and Numerical Methods in Economics

Assignment W6

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1. Consider the following growth model: Given $k_0 > 0$,

$$\begin{aligned} \max_{(c_t, l_t, k_{t+1})_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\gamma}}{1-\gamma} - \frac{l_t^{1+\xi}}{1+\xi} \right] \\ \text{s.t. } k_{t+1} = z_t k_t^{\alpha} l_t^{1-\alpha} - c_t - \delta k_t \end{aligned} \quad (1)$$

1. Suppose that $z_t = 1$ for all t . Please write the Bellman equation for this problem.
2. Let $V(k)$ be the value function w.r.t. state k . Suppose that we approximate $V(k)$ by the following polynomial:

$$V(k) = a_0 + a_1 k + a_2 k^2 + a_3 k^3 + a_4 k^4 \quad (2)$$

Please write down the value function iteration algorithm to solve for $(a_0, a_1, a_2, a_3, a_4)$.

3. Suppose that $\beta = 0.95$, $\alpha = 0.33$, $\gamma = 0.5$, $\xi = 0.2$, $\delta = 0.07$. Please solve $(a_0, a_1, a_2, a_3, a_4)$.
4. Suppose that $z_t \in (z_L, z_H)$ where $z_L = 0.8$ and $z_H = 1.2$. Let π_{ij} be the probability of shifting from z_i to z_j where $i, j \in (L, H)$. Suppose that $\pi_{LL} = 0.8$ and $\pi_{HH} = 0.6$. Please (i) write the Bellman equation for this problem; (ii) approximate the value function by a fourth degree polynomial; and (iii) solve this dynamic programming problem by value function iterations.