ECON 7115
Assignment 4

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1 Spatial Trade Model

1.1 Trade Share

$$
\begin{aligned}
\lambda_{lm} &= \frac{X_{lm}}{X_m} \\
&= \frac{P_{lm}}{P_m} \\
&= \frac{\left[\frac{w_l}{\bar{A}_l * L_l^{\alpha}} * \tau_{lm} * (1 + t_{lm})\right]^{1-\sigma}}{P_m^{1-\sigma}}
\end{aligned}
$$

1.2 Price Index of Region m

$$
P_m = \left[\sum_{i=1}^{N} \sum_{l' \in \mathcal{L}_i} \left[\frac{w_l'}{\bar{A}_l' * L_l'^{\alpha}} * \tau_{l'm} * (1 + t_{l'm})\right]^{1-\sigma}\right]^{1/1-\sigma}
$$

1.3 Labor Allocation

$$
\frac{L_l}{\bar{L}_n} = \frac{(\bar{B}_l * \frac{X_l/L_l}{P_l})^{1/\mu_n}}{\sum_{l \in \mathcal{L}_n} (\bar{B}_l * \frac{X_l/L_l}{P_l})^{1/\mu_n}}
$$

1.4 Total Expenditure and Eq'm Wage

$$
X_l = w_l * L_l + \frac{L_l}{\bar{L}_n} \sum_{l' \in \mathcal{L}_n} \sum_{i=1}^{N} \sum_{m \in \mathcal{L}_i} \frac{t_{ml'}}{1 + t_{ml'}} * \lambda_{ml'} * X_{l'}
$$

$$L_l = \frac{1}{w_l} * \sum_{i=1}^{N} \sum_{m \in \mathcal{L}_i} \frac{1}{1+t_{in}} \lambda_{lm} * X_m$$

1.5 Country 1 Maximizes its National Welfare

If arbitrary country n wants to max its national welfare, the question becomes:

$$\max \quad U_n = \bar{B}_l L_l^{-\mu_n} \frac{X_l/L_l}{P_l}$$

$$\text{Since} \quad \sum_{l \in \mathcal{L}_n} U_l = \bar{L_n},$$

$$\text{Equivalently,} \quad \max \quad U_n = \left(\sum_{l \in \mathcal{L}_n} \left(\bar{B}_l L_l^{-\mu_n} \frac{X_l/L_l}{P_l} \right)^{1/\mu_n} \right)^{\mu_n}$$

If country 1 wants to max its national welfare, the question becomes:

$$\max \quad U_1 = \left(\sum_{l \in \mathcal{L}_1} \left(\bar{B}_l L_l^{-\mu_1} \frac{X_l/L_l}{P_l} \right)^{1/\mu_1} \right)^{\mu_1}$$

$$\text{s.t.} \quad \lambda_{lm} = \frac{\left[\frac{w_l}{\bar{A}_l \cdot L_l^{\alpha}} \cdot \tau_{lm} \cdot (1+t_{lm}) \right]^{1-\sigma}}{P_m^{1-\sigma}} \tag{1}$$

$$P_m = \left[\sum_{i=1}^{N} \sum_{l' \in \mathcal{L}_i} \left[\frac{w_{l'}}{\bar{A}_{l'} \cdot L_{l'}^{\alpha}} \cdot \tau_{l'm} \cdot (1+t_{l'm}) \right]^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{2}$$

$$\frac{L_l}{\bar{L_n}} = \frac{\left(\bar{B}_l \cdot \frac{X_l/L_l}{P_l} \right)^{1/\mu_n}}{\sum_{l \in \mathcal{L}_n} \left(\bar{B}_l \cdot \frac{X_l/L_l}{P_l} \right)^{1/\mu_n}} \tag{3}$$

$$X_l = w_l \cdot L_l + \frac{L_l}{\bar{L_n}} \sum_{l' \in \mathcal{L}_n} \sum_{i=1}^{N} \sum_{m \in \mathcal{L}_i} \frac{t_{ml'}}{1+t_{ml'}} \cdot \lambda_{ml'} \cdot X_{l'} \tag{4}$$

$$L_l = \frac{1}{w_l} \cdot \sum_{i=1}^{N} \sum_{m \in \mathcal{L}_i} \frac{1}{1+t_{in}} \cdot \lambda_{lm} \cdot X_m \tag{5}$$

1.6 Exact-hat Algebra

First, denote the following:

- Z': the value of Z after the shock
- $\hat{Z} = Z'/Z$

Using the exact-hat algebra, the eq'm system becomes:

$$\hat{\lambda_{lm}} = \frac{\left[\frac{\hat{w}_l}{\hat{\bar{A}}_l \cdot \hat{L_l^{\alpha}}} \cdot \hat{\tau_{lm}} \cdot (1 + \hat{t_{lm}})\right]^{1-\sigma}}{\sum_{l' \in \mathcal{L}_n} \sum_{m=1}^{N} \lambda_{lm} \left[\frac{\hat{w}_l}{\hat{\bar{A}}_l \cdot \hat{L_l^{\alpha}}} \cdot \hat{\tau_{lm}} \cdot (1 + \hat{t_{lm}})\right]^{1-\sigma}} \tag{6}$$

$$\frac{\hat{L}_l}{\hat{\bar{L}}_n} = \frac{\left(\hat{\bar{B}}_l \cdot \frac{\hat{X}_l/\hat{L}_l}{\hat{P}_l}\right)^{1/\mu_n}}{\sum_{l' \in \mathcal{L}_n} \frac{L_l'}{\bar{L}_n} \left(\hat{\bar{B}}_l \cdot \frac{\hat{X}_l/\hat{L}_l}{\hat{P}_l}\right)^{1/\mu_n}} \tag{7}$$

$$\hat{X}_l X_l = \hat{w}_l w_l \cdot \hat{L}_l L_l + \frac{\hat{L}_l}{\hat{\bar{L}}_n} \sum_{l' \in \mathcal{L}_n} \sum_{i=1}^{N} \sum_{m \in \mathcal{L}_i} \frac{\hat{t_{ml'}} t_{ml'}}{(1 + \hat{t}_{ml'})(1 + t_{ml'})} \cdot \hat{\lambda_{ml'}} \lambda_{ml'} \cdot \hat{X}_{l'} X_{l'} \tag{8}$$

$$\hat{L}_l L_l = \frac{1}{\hat{w}_l w_l} \cdot \sum_{i=1}^{N} \sum_{m \in \mathcal{L}_i} \frac{1}{(1\hat{+}_{in})(1 + t_{in})} \cdot \hat{\lambda_{lm}} \lambda_{lm} \cdot \hat{X_m} X_m \tag{9}$$

The parameters we need to conduct the exact-hat algebra:

- From data: $\lambda_{lm}, X_{lm}, w_l$
- exogenous parameters: $\bar{A}_l, \bar{B}_l, \mu_n, \alpha, t, \tau, \sigma$

1.7 Optimal Tariff of Country 1 using Exact-hat Algebra

$$\max \quad U_1 = \left(\sum_{l \in \mathcal{L}_1} \left(\bar{B}_l L_l^{-\mu_1} \frac{X_l/L_l}{P_l} \right)^{1/\mu_1} \right)^{\mu_1}$$

s.t. the equilibrium system of under the exact-hat algebra in 1.6.

2

2.1 Equilibrium outcomes under zero tariffs

Each column in the matrix represents the following results: Welfare(U), Wage(w), Trade Volume (X), and Labor Allocation(L). Each row demonstrates regions 1-4, respectively.

$$\begin{bmatrix} 1.4690 & 0.2539 & 0.1396 & 0.5498 \\ 1.4690 & 0.2461 & 0.1108 & 0.4502 \\ 1.4690 & 0.2531 & 0.1396 & 0.5498 \\ 1.4690 & 0.2461 & 0.1108 & 0.4502 \end{bmatrix}$$

2.2 Unilaterally Optimal Tariffs for Country 1

When $Tar_2 = 0$, the optimal tariffs for country 1 is

$$Tar_1 = 0.3$$

For the computation, please refer to the code.

2.3 Nash Tariffs

The Nash Tariffs:

$$NashTar_1 = NashTar_2 = 0.2749$$

For the computation, please refer to the code.

2.4 When $\mu_1 = 0.4$

2.4.1 Zero Tariffs

Each column in the matrix represents the following results: Welfare(U), Wage(w), Trade Volume (X), and Labor Allocation(L). Each row demonstrates regions 1-4, respectively.

$$\begin{bmatrix} 1.3707 & 0.2535 & 0.1427 & 0.5630 \\ 1.3707 & 0.2464 & 0.1077 & 0.4370 \\ 1.4699 & 0.2540 & 0.1397 & 0.5502 \\ 1.4699 & 0.2461 & 0.1107 & 0.4498 \end{bmatrix}$$

2.4.2 Unilateral Tariffs for Country 1

When $Tar_2 = 0$, the optimal tariffs for country 1 is

$$Tar_1 = 0.3$$

For the computation, please refer to the code.

2.4.3 Nash Tariffs

The Nash Tariffs:

$$NashTar_1 = 0.2750$$

$$NashTar_2 = 0.2725$$

For the computation, please refer to the code.

2.5 Discussions

When $\mu_1 = \mu_2 = 0.5$, the equilibrium outcomes of the two countries are the same. Specifically,

- The welfare are the same
- The economic outcomes of the coastal and inner regions are identical
- The labor are basically allocated in the coastal regions
- The unilateral optimal tariff of country 1 is 0.3; and the both nash tariffs are 0.2749

When $\mu_1 = 0.4 < \mu_2 = 0.5$, which means the amenity of country 1 decreases,

- The economic status of country 1 is better in the coastal region. The gap between the coastal and inner region is enlarged
- The welfare of country 1 decreases and the inequality between the two countries increases
- The unilateral optimal tariff of country 1 does not change, but the nash tariffs are asymmetric

Hence, the deterioration of the living amenity would negatively affect the economy of a country.