Assignment 6

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1

Bellman equation:

$$\begin{aligned} V(k_t) &= \max U(c_t) + \beta V(k_{t+1}) \\ \text{s.t.} \quad k_{t+1} &= k_t^\alpha l_t^{1-\alpha} - c_t - \delta k_t \\ U(c_t) &= \frac{c_t^{1-\gamma}}{1-\gamma} - \frac{l_t^{1+\xi}}{1+\xi} \end{aligned}$$

2

Algorithm to solve for $(a_0, a_1, a_2, a_3, a_4) = \mathbf{a}$:

- Grid the space of k_j into $\mathbf{J}$ points
- Solve the optimal l_t^* in each period t
- Initial guess for the vector $\mathbf{a} = \mathbf{0}$, hence $v_j^{(0)} = 0$
- Update $v_j^{(1)} = max_{k=1,\ldots,j} u(w_j - w_k) + \beta v_k^{(0)}$
- Iterate until $v_j^{(t)} = v_j^{(t+1)}$
- Use 5 points of k_j and OLS to approximate and fit $\mathbf{a}$

3

Solve for the optimal l_t^* using $fsolve$ for each t:

$$(k_t^\alpha l_t^{1-\alpha} - \delta k_t - k_{t+1})^{-\gamma}(1-\alpha)k_t^\alpha = l_t^{\xi+\alpha}$$
$$\text{Rearrange, we have:} \quad A * B * l_t^{1-\alpha} - l_t^C - D * A = 0$$
$$\begin{aligned} \text{Where} \quad A &= ((1-\alpha)z_t k_t^\alpha)^{-1/\gamma} \\ B &= z_t k_t \\ C &= (\xi+\alpha) * (-1/\gamma) \\ D &= \delta k_t + k_{t+1} \\ z_t &= 1 \end{aligned}$$

- $\mathbf{a} = (0, 2, 0, 0, 0)$

4

4.1

Bellman Equation:

$$V(k_t) = \max z_t U(c_t) + \beta E_t V(k_{t+1})$$
$$\text{s.t.} \quad k_{t+1} = z_t k_t^\alpha l_t^{1-\alpha} - c_t - \delta k_t$$

4.2