Assignment 4

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1.1 Trade Share

$$\begin{aligned}\lambda_{lm} &= \frac{X_{lm}}{X_m} \\ &= \frac{P_{lm}}{P_m} \\ &= \frac{[\frac{w_l}{\bar{A}_l * L_l^{\alpha}} * \tau_{lm} * (1 + t_{lm})]^{1-\sigma}}{P_m^{1-\sigma}}\end{aligned}$$

1.2 Price Index of Region m

$$P_m = [\sum_{i=1}^{N} \sum_{l' \in \mathcal{L}_i} [\frac{w_l'}{\bar{A}_l' * L_l'^{\alpha}} * \tau_{l'm} * (1 + t_{l'm})]^{1-\sigma}]^{1/1-\sigma}$$

1.3 Labor Allocation

$$\frac{L_l}{\bar{L_n}} = \frac{(\bar{B}_l * \frac{X_l/L_l}{P_l})^{1/\mu_n}}{\sum_{l \in \mathcal{L}_n} (\bar{B}_l * \frac{X_l/L_l}{P_l})^{1/\mu_n}}$$

1.4 Total Expenditure and Eq'm Wage

$$X_l = w_l * L_l + \frac{L_l}{\bar{L_n}} \sum_{l' \in \mathcal{L}_n} \sum_{i=1}^{N} \sum_{m \in \mathcal{L}_i} \frac{t_{ml'}}{1 + t_{ml'}} * \lambda_{ml'} * X_{l'}$$

$$L_l = \frac{1}{w_l} * \sum_{i=1}^{N} \sum_{m \in \mathcal{L}_i} \frac{1}{1 + t_{in}} \lambda_{lm} * X_m$$

1.5 Country 1 Maximizes its National Welfare

If arbitrary country n wants to max its national welfare, the question becomes:

$$\max \quad U_n = \bar{B}_l L_l^{-\mu_n} \frac{X_l/L_l}{P_l}$$

$$\text{Since} \quad \sum_{l \in \mathcal{L}_n} U_l = \bar{L_n},$$

$$\text{Equivalently,} \quad \max \quad U_n = \left(\sum_{l \in \mathcal{L}_n} \left(\bar{B}_l L_l^{-\mu_n} \frac{X_l/L_l}{P_l} \right)^{1/\mu_n} \right)^{\mu_n}$$

If country 1 wants to max its national welfare, the question becomes:

$$\max \quad U_1 = \left(\sum_{l \in \mathcal{L}_1} \left(\bar{B}_l L_l^{-\mu_1} \frac{X_l/L_l}{P_l} \right)^{1/\mu_1} \right)^{\mu_1}$$

$$\text{s.t.} \quad \lambda_{lm} = \frac{\left[\frac{w_l}{\bar{A}_l \cdot L_l^{\alpha}} \cdot \tau_{lm} \cdot (1 + t_{lm}) \right]^{1-\sigma}}{P_m^{1-\sigma}} \tag{1}$$

$$P_m = \left[\sum_{i=1}^{N} \sum_{l' \in \mathcal{L}_i} \left[\frac{w_{l'}}{\bar{A}_{l'} \cdot L_{l'}^{\alpha}} \cdot \tau_{l'm} \cdot (1 + t_{l'm}) \right]^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{2}$$

$$\frac{L_l}{\bar{L_n}} = \frac{\left(\bar{B}_l \cdot \frac{X_l/L_l}{P_l} \right)^{1/\mu_n}}{\sum_{l \in \mathcal{L}_n} \left(\bar{B}_l \cdot \frac{X_l/L_l}{P_l} \right)^{1/\mu_n}} \tag{3}$$

$$X_l = w_l \cdot L_l + \frac{L_l}{\bar{L_n}} \sum_{l' \in \mathcal{L}_n} \sum_{i=1}^{N} \sum_{m \in \mathcal{L}_i} \frac{t_{ml'}}{1 + t_{ml'}} \cdot \lambda_{ml'} \cdot X_{l'} \tag{4}$$

$$L_l = \frac{1}{w_l} \cdot \sum_{i=1}^{N} \sum_{m \in \mathcal{L}_i} \frac{1}{1 + t_{in}} \cdot \lambda_{lm} \cdot X_m \tag{5}$$

1.6 Exact-hat Algebra

First, denote the following:

- Z': the value of Z after the shock
- $\hat{Z} = Z'/Z$

Using the exact-hat algebra, the eq'm system becomes:

$$\hat{\lambda_{lm}} = \frac{\left[\frac{\hat{w_l}}{\hat{\bar{A}_l}\cdot\hat{L_l^{\alpha}}}\cdot\hat{\tau_{lm}}\cdot(1+\hat{t_{lm}})\right]^{1-\sigma}}{\sum_{l'\in\mathcal{L}_n}\sum_{m=1}^{N}\lambda_{lm}\left[\frac{\hat{w_l}}{\hat{\bar{A}_l}\cdot\hat{L_l^{\alpha}}}\cdot\hat{\tau_{lm}}\cdot(1+\hat{t_{lm}})\right]^{1-\sigma}} \tag{6}$$

$$\frac{\hat{L_l}}{\hat{L_n}} = \frac{\left(\hat{\bar{B}_l}\cdot\frac{\hat{X_l}/\hat{L_l}}{\hat{P_l}}\right)^{1/\mu_n}}{\sum_{l'\in\mathcal{L}_n}\frac{L_l'}{L_n}\left(\hat{\bar{B}_l}\cdot\frac{\hat{X_l}/\hat{L_l}}{\hat{P_l}}\right)^{1/\mu_n}} \tag{7}$$

$$\hat{X_l}X_l = \hat{w_l}w_l\cdot\hat{L_l}L_l + \frac{\hat{L_l}}{\hat{L_n}}\sum_{l'\in\mathcal{L}_n}\sum_{i=1}^{N}\sum_{m\in\mathcal{L}_i}\frac{\hat{t_{ml'}}t_{ml'}}{1+\hat{t}_{ml'}1+t_{ml'}}\cdot\hat{\lambda_{ml'}}\lambda_{ml'}\cdot\hat{X_{l'}}X_{l'} \tag{8}$$

$$L_l = \frac{1}{w_l}\cdot\sum_{i=1}^{N}\sum_{m\in\mathcal{L}_i}\frac{1}{1+t_{in}}\cdot\lambda_{lm}\cdot X_m \tag{9}$$

The parameters we need to conduct the exact-hat algebra:

- From data: $\lambda_{lm}, X_{lm}, w_l$
- exogenous parameters: $\bar{A}_l, \bar{B}_l, \mu_n, \alpha, t, \tau, \sigma$

1.7 Optimal Tariff of Country 1 using Exact-hat Algbra

$$\max \quad U_1 = \left(\sum_{l\in\mathcal{L}_1}\left(\bar{B}_l L_l^{-\mu_1}\frac{X_l/L_l}{P_l}\right)^{1/\mu_1}\right)^{\mu_1}$$

s.t. the equilibrium system of under the exact-hat algebra in 1.6.