ECON7115: Assignment 4

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1 Spatial Trade Model

1.1 Trade Share

$$\lambda_{lm} = \frac{X_{lm}}{X_m} = \frac{P_{lm}}{P_m} = \frac{\left[\frac{w_l}{\bar{A}_l * L_l^{\alpha}} * \tau_{lm} * (1 + t_{lm})\right]^{1-\sigma}}{P_m^{1-\sigma}}$$

1.2 Price Index of Region m

$$P_m = \left[\sum_{i=1}^{N} \sum_{l' \in \mathcal{L}_i} \left[\frac{w_l'}{\bar{A}_l' * L_l'^{\alpha}} * \tau_{l'm} * (1 + t_{l'm})\right]^{1-\sigma}\right]^{1/1-\sigma}$$

1.3 Labor Allocation

$$\frac{L_l}{\bar{L_n}} = \frac{(\bar{B}_l * \frac{X_l/L_l}{P_l})^{1/\mu_n}}{\sum_{l \in \mathcal{L}_n} (\bar{B}_l * \frac{X_l/L_l}{P_l})^{1/\mu_n}}$$

1.4 Total Expenditure and Eq'm Wage

$$X_l = w_l * L_l + \frac{L_l}{\bar{L_n}} \sum_{l' \in \mathcal{L}_n} \sum_{i=1}^{N} \sum_{m \in \mathcal{L}_i} \frac{t_{ml'}}{1 + t_{ml'}} * \lambda_{ml'} * X_{l'}$$

$$L_l = \frac{1}{w_l} * \sum_{i=1}^{N} \sum_{m \in \mathcal{L}_i} \frac{1}{1 + t_{in}} \lambda_{lm} * X_m$$

1.5 Country 1 Maximizes its National Welfare

If arbitrary country n wants to max its national welfare, the question becomes:

$$\max \quad U_n = \bar{B}_l L_l^{-\mu_n} \frac{X_l/L_l}{P_l}$$

$$\text{Since} \quad \sum_{l \in \mathcal{L}_n} U_l = \bar{L_n},$$

$$\text{Equivalently,} \quad \max \quad U_n = \left(\sum_{l \in \mathcal{L}_n} \left(\bar{B}_l L_l^{-\mu_n} \frac{X_l/L_l}{P_l} \right)^{1/\mu_n} \right)^{\mu_n}$$

If country 1 wants to max its national welfare, the question becomes:

$$\max \quad U_1 = \left(\sum_{l \in \mathcal{L}_1} \left(\bar{B}_l L_l^{-\mu_1} \frac{X_l/L_l}{P_l} \right)^{1/\mu_1} \right)^{\mu_1}$$

$$\text{s.t.} \quad \lambda_{lm} = \frac{\left[\frac{w_l}{\bar{A}_l \cdot L_l^{\alpha}} \cdot \tau_{lm} \cdot (1 + t_{lm}) \right]^{1-\sigma}}{P_m^{1-\sigma}} \tag{1}$$

$$P_m = \left[\sum_{i=1}^{N} \sum_{l' \in \mathcal{L}_i} \left[\frac{w_{l'}}{\bar{A}_{l'} \cdot L_{l'}^{\alpha}} \cdot \tau_{l'm} \cdot (1 + t_{l'm}) \right]^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{2}$$

$$\frac{L_l}{\bar{L_n}} = \frac{\left(\bar{B}_l \cdot \frac{X_l/L_l}{P_l} \right)^{1/\mu_n}}{\sum_{l \in \mathcal{L}_n} \left(\bar{B}_l \cdot \frac{X_l/L_l}{P_l} \right)^{1/\mu_n}} \tag{3}$$

$$X_l = w_l \cdot L_l + \frac{L_l}{\bar{L_n}} \sum_{l' \in \mathcal{L}_n} \sum_{i=1}^{N} \sum_{m \in \mathcal{L}_i} \frac{t_{ml'}}{1 + t_{ml'}} \cdot \lambda_{ml'} \cdot X_{l'} \tag{4}$$

$$L_l = \frac{1}{w_l} \cdot \sum_{i=1}^{N} \sum_{m \in \mathcal{L}_i} \frac{1}{1 + t_{in}} \cdot \lambda_{lm} \cdot X_m \tag{5}$$

1.6 Exact-hat Algebra

First, denote the following:

- Z': the value of Z after the shock
- $\hat{Z} = Z'/Z$

Using the exact-hat algebra, the eq'm system becomes:

$$\hat{\lambda_{lm}} = \frac{\left[\frac{\hat{w}_l}{\hat{\bar{A}}_l \cdot \hat{L_l^{\alpha}}} \cdot \hat{\tau_{lm}} \cdot (1+\hat{t_{lm}})\right]^{1-\sigma}}{\sum_{l' \in \mathcal{L}_n} \sum_{m=1}^{N} \lambda_{lm} \left[\frac{\hat{w}_l}{\hat{\bar{A}}_l \cdot \hat{L_l^{\alpha}}} \cdot \hat{\tau_{lm}} \cdot (1+\hat{t_{lm}})\right]^{1-\sigma}} \tag{6}$$

$$\frac{\hat{L}_l}{\hat{\bar{L}}_n} = \frac{\left(\hat{\bar{B}}_l \cdot \frac{\hat{X}_l/\hat{L}_l}{\hat{P}_l}\right)^{1/\mu_n}}{\sum_{l' \in \mathcal{L}_n} \frac{L_l'}{\bar{L}_n} \left(\hat{\bar{B}}_l \cdot \frac{\hat{X}_l/\hat{L}_l}{\hat{P}_l}\right)^{1/\mu_n}} \tag{7}$$

$$\hat{X}_l X_l = \hat{w}_l w_l \cdot \hat{L}_l L_l + \frac{\hat{L}_l}{\hat{\bar{L}}_n} \sum_{l' \in \mathcal{L}_n} \sum_{i=1}^{N} \sum_{m \in \mathcal{L}_i} \frac{\hat{t_{ml'}} t_{ml'}}{(1+\hat{t}_{ml'})(1+t_{ml'})} \cdot \hat{\lambda_{ml'}} \lambda_{ml'} \cdot \hat{X}_{l'} X_{l'} \tag{8}$$

$$\hat{L}_l L_l = \frac{1}{\hat{w}_l w_l} \cdot \sum_{i=1}^{N} \sum_{m \in \mathcal{L}_i} \frac{1}{(1\hat{+}_{in})(1+t_{in})} \cdot \hat{\lambda_{lm}} \lambda_{lm} \cdot \hat{X_m} X_m \tag{9}$$

The parameters we need to conduct the exact-hat algebra:

- From data: $\lambda_{lm}, X_{lm}, w_l$
- exogenous parameters: $\bar{A}_l, \bar{B}_l, \mu_n, \alpha, t, \tau, \sigma$

1.7 Optimal Tariff of Country 1 using Exact-hat Algebra

$$\max \quad U_1 = \left(\sum_{l \in \mathcal{L}_1} \left(\bar{B}_l L_l^{-\mu_1} \frac{X_l/L_l}{P_l}\right)^{1/\mu_1}\right)^{\mu_1}$$

s.t. the equilibrium system of under the exact-hat algebra in 1.6.

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2.1 Equilibrium outcomes under zero tariffs

Each column in the matrix represents the following results: Welfare(U), Wage(w), Trade Volume (X), Price Index(P), and Labor Allocation(L). Each row demonstrates regions 1-4, respectively.

$$\begin{bmatrix} 1.1653 & 0.2853 & 0.3329 & 0.3384 & 0.8932 \\ 1.1653 & 0.2973 & 0.3074 & 0.3587 & 0.8147 \\ 1.1490 & 0.2041 & 0.2152 & 0.2440 & 0.8384 \\ 1.1490 & 0.2134 & 0.2206 & 0.2576 & 0.8220 \end{bmatrix}$$

2.2 Unilaterally Optimal Tariffs for Country 1

When $Tar_2 = 0$, the optimal tariffs for country 1 is

$$Tar_1 = 0.51273$$

For the computation, please refer to the code.

2.3 Nash Tariffs

The Nash Tariffs:

$$NashTar_1 = NashTar_2 = 0.5192$$

For the computation, please refer to the code.

2.4 When $\mu_1 = 0.4$

2.4.1 Zero Tariffs

Each column in the matrix represents the following results: Welfare(U), Wage(w), Trade Volume (X), Price Index(P), and Labor Allocation(L). Each row demonstrates regions 1-4, respectively.

$$\begin{bmatrix} 1.1279 & 0.2395 & 0.3541 & 0.3016 & 1.0291 \\ 1.1279 & 0.2608 & 0.3418 & 0.3447 & 0.9121 \\ 1.1455 & 0.2392 & 0.3394 & 0.3019 & 0.9876 \\ 1.1455 & 0.2606 & 0.3362 & 0.3450 & 0.8978 \end{bmatrix}$$

2.4.2 Unilateral Tariffs for Country 1

When $Tar_2 = 0$, the optimal tariffs for country 1 is

$$Tar_1 = 0.3892$$

For the computation, please refer to the code.

2.4.3 Nash Tariffs

The Nash Tariffs:

$$NashTar_1 = 0.4010$$

$$NashTar_2 = 0.5033$$

For the computation, please refer to the code.

2.4.4 Discussions

When μ_1 decreases, that is, the amenity B_1 increases. My observations from the computational results:

- When both tariffs are zero,
    1. Both countries experience welfare declines.
    2. Wages decrease in country 1 but increase in country 2.
    3. Trade volumes increase for both countries, with country 2 showing a more substantial increase.
    4. The price index falls in country 1 while rising in country 2.
    5. Labor forces concentrate in coastal regions.
- Country 1's optimal unilateral tariff decreases.
- Nash equilibrium tariffs decrease for both countries.

Under zero tariffs, an increase in country 1's amenity generates distinct impacts across both countries. In country 1, enhanced amenity attracts labor force inflows, leading to downward pressure on wages and a reduction in the price index. This catalyzes increased trade activity. However, the ultimate decline in country 1's welfare reveals the substantial economic costs associated with this amenity improvement.

For country 2, the situation presents different challenges. Unable to stem the outflow of labor to country 1, it faces labor shortages that drive up both wages and the price index. The rising demand from country 1, potentially coupled with industrial restructuring there, necessitates increased international trade. Consequently, country 2 experiences a marked surge in trade volumes. Yet, this expansion in international trade proves insufficient to offset the economic losses from labor force depletion, resulting in diminished overall welfare.

The evolving economic landscape fosters an environment where bilateral cooperation becomes imperative. This necessity for cooperation is reflected in the observed reductions in both country 1's optimal unilateral tariff and the Nash equilibrium tariffs. These declining tariffs signal a shift toward more integrated economic relationships between the two countries.

Some possible economic mechanisms here:

- Labor mobility responding to amenity differentials
- Price adjustments in both goods and labor markets
- Trade pattern changes compensating for resource reallocation
- Strategic trade policy adaptation to new economic conditions

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