Assignment 4

Jiawen KE

March 2025

1

1.1 Trade Share

$$\begin{aligned}\lambda_{lm} &= \frac{X_{lm}}{X_m} \\ &= \frac{P_{lm}}{P_m} \\ &= \frac{[\frac{w_l}{\bar{A}_l * L_l^{\alpha}} * \tau_{lm} * (1 + t_{lm})]^{1-\sigma}}{P_m^{1-\sigma}}\end{aligned}$$

1.2 Price Index of Region m

$$P_m = [\sum_{i=1}^{N} \sum_{l' \in \mathcal{L}_i} [\frac{w_l'}{\bar{A}_l' * L_l'^{\alpha}} * \tau_{l'm} * (1 + t_{l'm})]^{1-\sigma}]^{1/1-\sigma}$$

1.3 Labor Allocation

$$\frac{L_l}{\bar{L_n}} = \frac{(\bar{B}_l * \frac{X_l/L_l}{P_l})^{1/\mu_n}}{\sum_{l \in \mathcal{L}_n} (\bar{B}_l * \frac{X_l/L_l}{P_l})^{1/\mu_n}}$$

1.4 Total Expenditure and Eq'm Wage

$$X_l = w_l * L_l + \frac{L_l}{\bar{L_n}} \sum_{l' \in \mathcal{L}_n} \sum_{i=1}^{N} \sum_{m \in \mathcal{L}_i} \frac{t_{ml'}}{1 + t_{ml'}} * \lambda_{ml'} * X_{l'}$$

$$L_l = \frac{1}{w_l} * \sum_{i=1}^{N} \sum_{m \in \mathcal{L}_i} \frac{1}{1 + t_{in}} \lambda_{lm} * X_m$$

1.5 Country 1 Maximizes its National Welfare

If arbitrary country n wants to max its national welfare, the question becomes:

$$\begin{aligned} &\max \quad U_n = \bar{B}_l L_l^{-\mu_n} \frac{X_l/L_l}{P_l} \\ \text{Since} \quad &\sum_{l\in\mathcal{L}_n} U_l = \bar{L_n}, \\ \text{Equivalently,} \quad &\max \quad U_n = \left(\sum_{l\in\mathcal{L}_n} \left(\bar{B}_l L_l^{-\mu_n} \frac{X_l/L_l}{P_l} \right)^{1/\mu_n} \right)^{\mu_n} \end{aligned}$$

If country 1 wants to max its national welfare, the question becomes:

$$\max \quad U_1 = \left(\sum_{l\in\mathcal{L}_1} \left(\bar{B}_l L_l^{-\mu_1} \frac{X_l/L_l}{P_l} \right)^{1/\mu_1} \right)^{\mu_1}$$

$$\text{s.t.} \quad \lambda_{lm} = \frac{\left[\frac{w_l}{\bar{A}_l \cdot L_l^{\alpha}} \cdot \tau_{lm} \cdot (1+t_{lm}) \right]^{1-\sigma}}{P_m^{1-\sigma}} \tag{1}$$

$$P_m = \left[\sum_{i=1}^{N} \sum_{l'\in\mathcal{L}_i} \left[\frac{w_{l'}}{\bar{A}_{l'} \cdot L_{l'}^{\alpha}} \cdot \tau_{l'm} \cdot (1+t_{l'm}) \right]^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{2}$$

$$\frac{L_l}{\bar{L_n}} = \frac{\left(\bar{B}_l \cdot \frac{X_l/L_l}{P_l} \right)^{1/\mu_n}}{\sum_{l\in\mathcal{L}_n} \left(\bar{B}_l \cdot \frac{X_l/L_l}{P_l} \right)^{1/\mu_n}} \tag{3}$$

$$X_l = w_l \cdot L_l + \frac{L_l}{\bar{L_n}} \sum_{l'\in\mathcal{L}_n} \sum_{i=1}^{N} \sum_{m\in\mathcal{L}_i} \frac{t_{ml'}}{1+t_{ml'}} \cdot \lambda_{ml'} \cdot X_{l'} \tag{4}$$

$$L_l = \frac{1}{w_l} \cdot \sum_{i=1}^{N} \sum_{m\in\mathcal{L}_i} \frac{1}{1+t_{in}} \cdot \lambda_{lm} \cdot X_m \tag{5}$$

1.6 Exact-hat Algebra

First, denote the following:

- Z': the value of Z after the shock
- $\hat{Z} = Z'/Z$

Using the exact-hat algebra, the eq'm system becomes:

$$\hat{\lambda_{lm}} = \frac{\left[\frac{\hat{w}_l}{\hat{\bar{A}}_l \cdot \hat{L_l^\alpha}} \cdot \hat{\tau_{lm}} \cdot (1+\hat{t_{lm}})\right]^{1-\sigma}}{\sum_{l' \in \mathcal{L}_n} \sum_{m=1}^{N} \lambda_{lm} \left[\frac{\hat{w}_l}{\hat{\bar{A}}_l \cdot \hat{L_l^\alpha}} \cdot \hat{\tau_{lm}} \cdot (1+\hat{t_{lm}})\right]^{1-\sigma}} \tag{6}$$

$$\frac{\hat{L}_l}{\hat{\bar{L}}_n} = \frac{\left(\hat{\bar{B}}_l \cdot \frac{\hat{X}_l/\hat{L}_l}{\hat{P}_l}\right)^{1/\mu_n}}{\sum_{l' \in \mathcal{L}_n} \frac{L_l'}{\bar{L}_n} \left(\hat{\bar{B}}_l \cdot \frac{\hat{X}_l/\hat{L}_l}{\hat{P}_l}\right)^{1/\mu_n}} \tag{7}$$

$$\hat{X}_l X_l = \hat{w}_l w_l \cdot \hat{L}_l L_l + \frac{\hat{L}_l}{\hat{\bar{L}}_n} \sum_{l' \in \mathcal{L}_n} \sum_{i=1}^{N} \sum_{m \in \mathcal{L}_i} \frac{\hat{t_{ml'}} t_{ml'}}{(1+\hat{t}_{ml'})(1+t_{ml'})} \cdot \hat{\lambda_{ml'}} \lambda_{ml'} \cdot \hat{X}_{l'} X_{l'} \tag{8}$$

$$\hat{L}_l L_l = \frac{1}{\hat{w}_l w_l} \cdot \sum_{i=1}^{N} \sum_{m \in \mathcal{L}_i} \frac{1}{(1\hat{+}_{in})(1+t_{in})} \cdot \hat{\lambda_{lm}} \lambda_{lm} \cdot \hat{X_m} X_m \tag{9}$$

The parameters we need to conduct the exact-hat algebra:

- From data: $\lambda_{lm}, X_{lm}, w_l$
- exogenous parameters: $\bar{A}_l, \bar{B}_l, \mu_n, \alpha, t, \tau, \sigma$

1.7 Optimal Tariff of Country 1 using Exact-hat Algebra

$$\max \quad U_1 = \left(\sum_{l \in \mathcal{L}_1} \left(\bar{B}_l L_l^{-\mu_1} \frac{X_l/L_l}{P_l}\right)^{1/\mu_1}\right)^{\mu_1}$$

s.t. the equilibrium system of under the exact-hat algebra in 1.6.