Adaptivity analysis

AExpr.	a	::=	$n \mid x \mid a_1 + a_2 \mid a_1 - a_2 \mid a_1 * a_2$
BExpr	b	::=	$v \mid a_1 < a_2 \mid a_1 = a_2 \mid \neg b \mid b_1 \wedge b_2 \mid b_1 \vee b_2$
Command	c	::=	$Skip \mid c_1; c_2 \mid$ `if` b `then` c_1 `else` $c_2 \mid$ `while`(b) c
			$\mid x \leftarrow a \mid x \leftarrow^l \delta^l(a)$
Label	l	$\in$	$\mathbb{Z}$
Trace	T	::=	$\{[(x_1, l_1), \ldots, (x_i, l_i)], \ldots, [(y_1, l_1), \ldots, (y_i, l_i)]\}$
Environment	θ	::=	$x_1 \mapsto (n_1, T_1), \ldots, x_n \mapsto (v_n, T_n)$
Node	N	::=	$Empty \mid D_1(x, T) \mid D_2(x, T) \mid IFT(T_b, N)$
			$\mid IFF(T_b, N) \mid W(T_b, N) \mid N_1; N_2$
Trace	T	$\in$	$Set < List < Var \times Label >>$

$\boxed{\theta, a \to^{T} n}$

$$\frac{\theta(x) = (n, T)}{\theta, x \to^{T} n}\ \textsf{var} \qquad \frac{}{\theta, n \to^{()} n}\ \textsf{const} \qquad \frac{\theta, a_1 \to^{T_1} n_1 \qquad \theta, a_2 \to^{T_2} n_2}{\theta, a_1 + a_2 \to^{T_1 \cup T_2} n_1 + n_2}\ \textsf{sum}$$

$\boxed{\theta, b \to^{T} v}$

$$\frac{\theta, a_1 \to^{T_1} n_1 \qquad \theta, a_2 \to^{T_2} n_2}{\theta, a_1 + a_2 \to^{T_1 \cup T_2} n_1 < n_2}\ \textsf{les} \qquad \frac{\theta, b_1 \to^{T_1} v_1 \qquad \theta, b_2 \to^{T_2} v_2}{\theta, b_1 \wedge b_2 \to^{T_1 \cup T_2} v_1 \wedge v_2}\ \textsf{land}$$

$\boxed{\theta, c \to^{N} \theta'}$

$$\frac{}{\theta, Skip \to \theta}\ \textsf{skip} \qquad \frac{\theta, c_1 \to^{N_1} \theta_1 \qquad \theta_1, c_2 \to^{N_2} \theta_2}{\theta, c_1; c_2 \to^{N_1;N_2} \theta_2}\ \textsf{Seq}$$

$$\frac{\theta, a \to^{T} n}{\theta, x \leftarrow a \to^{D_1(x,T)} \theta[x \to (n, T)]}\ \textsf{Assign}$$

$$\frac{\theta, a \to^{T} n \qquad \delta(n) = n_1}{\theta, x \leftarrow^{l} \delta^{l}(a) \to^{D_2(x, T + (x,l))} \theta[x \to (n_1, T + (x, l))]}\ \textsf{FAssign}$$

$$\frac{\theta, b \to^{T_b} false \qquad \theta, c_2 \to^{N_2} \theta_2}{\theta, \texttt{if}\ b\ \texttt{then}\ c_1\ \texttt{else}\ c_2 \to^{IFF(T_b, N_2)} \theta_2}\ \textsf{IFF}$$

$$\frac{\theta, b \to^{T_b} true \qquad \theta, c_1 \to^{N_1} \theta_1}{\theta, \texttt{if}\ b\ \texttt{then}\ c_1\ \texttt{else}\ c_2 \to^{IFT(T_b, N_1)} \theta_1}\ \textsf{IFT}$$

$$\frac{\theta, b \to^{T_b} false}{\theta, \texttt{while}(b)\ c \to^{WF(T_b)} \theta}\ \textsf{WHILEF}$$

$$\frac{\theta, b \to^{T_b} true \qquad \theta, c \to^{N_1} \theta_1 \qquad \theta_1, \texttt{while}(b)\ c \to^{N_2} \theta_2}{\theta, \texttt{while}(b)\ c \to^{WT(T_b, N_1, N_2)} \theta_2}\ \textsf{WHILET}$$

$$\begin{array}{lll} \{\} + (x, l) & \triangleq & \{(x, l)\} \\ \{[(x_1, l_1), \ldots, (x_i, l_i)], \ldots, [(y_1, l'_1), \ldots, (y_i, l'_i)]\} + (x, l) & \triangleq & \\ \{[(x_1, l_1), \ldots, (x_i, l_i), (x, l)], \ldots, [(y_1, l'_1), \ldots, (y_i, l'_i), (x, l)]\} & & \end{array}$$

Figure 1: Big-step semantics