New Definition

Monday, July 11, 2022 9:26 AM

⇒ Dep Graph:

$V = \{ x^i \mid x^i \in LV(c) \}$

$E = \left\{ (x^i, y^j, w) \;\middle|\; \begin{array}{l} w = \max \{ |Diff(\tau_1, \tau_2, y^j)| \mid \forall \tau_1, \tau_2, \text{ s.t. } Dep(x^i, \tau_1, \tau_2, y^j, c) \} \\ \wedge\ w \geq 1 \wedge x^i, y^j \in LV(c) \end{array} \right\}$

functional

$\Rightarrow \left\{ \begin{array}{l} w : \tau_0 \to \mathbb{N}. \wedge x^i, y^j \in LV(C) \wedge \\ w(\tau_0) = \max \{ |Diff(\tau_1, \tau_2, y^j)| \mid \forall \tau_1, \tau_2 \text{ s.t. } Dep(x^i, y^j, \tau_1, \tau_2, \tau_0, c) \} \end{array} \right\}$

$$Diff(\tau_1, \tau_2, x^i) \triangleq \{ k \mid (k = 0, \dots, l_{min}-1 \wedge Seq(\tau_1)x^i[k] \neq Seq(\tau_2, x^i)[k]) \}$$

$\wedge\ (Seq(\tau_{max}, x^i)[k-1] \neq Seq(\tau_{max}, x^i)[k] \wedge k = l_{min}, \dots, l_{max})$

$\wedge\ l_{min} = \min(|Seq(\tau_1, x^i)|, |Seq(\tau_2, x^i)|)$

$l_{max} = \max(\cdots)$

τ_{max} = longer Sequence. τ $\}$

$$\Rightarrow Seq(\tau, x^i) \triangleq \begin{cases} seq(\tau', x^i) :: v & \tau = \tau' :: (x, i, v, \cdot) \\ seq(\tau', x^i) :: \alpha. & \tau = \tau' :: (x, i, v, \alpha) \\ [] & \tau = [] \\ seq(\tau_1') & \text{o.w.} \end{cases}$$

$Dep(x^i, \tau_1, \tau_2, y^j, c) \triangleq$

$\exists\ \epsilon_1', \tau_0, \tau_0'$

$\epsilon_1 = (x, i, v, -, -)$. $\quad \epsilon_2 = (y, j, -, -, -)$.

$\epsilon_1 \neq_{event} \epsilon_1'$

$\langle c, \tau_0 \rangle \to \langle c_1, \tau_0' :: \epsilon_1 \rangle \to \langle skip, \tau_0' :: \epsilon_1 ++ \tau_1 \rangle$

$\wedge \quad \langle c_1, \tau_0' :: \epsilon_1' \rangle \xrightarrow{*} \langle skip, \tau_0' :: \epsilon_1 ++ \tau_1 \rangle$

$Diff(\tau_1, \tau_2, y^j) \neq \emptyset$.

$Dep(x^i, y^j, \tau_1, \tau_2, \tau_0, c) \triangleq$.

$\Rightarrow$. $\exists\ \tau_0', \epsilon_1'$. $\epsilon_1 = (x^i, i, -, -)$, $\epsilon_2 = (y, j, -, -)$. $\epsilon_1 \neq \epsilon_1'$

$\langle C, \tau_0 \rangle \to \langle C_1, \tau_0' :: \epsilon_1 \rangle \xrightarrow{*} \langle skip, \tau_0' :: \epsilon_1 ++ \tau_1 \rangle$

$\langle C_1, \tau_0' :: \epsilon_1' \rangle \xrightarrow{*} \langle skip, \tau_0' :: \epsilon_1' ++ \tau_2 \rangle$

$\wedge \quad Diff(\tau_1, \tau_2, y^j) \neq \emptyset$.

⇒ Example :

```
multipleRoundsSingle(k)
[j ← 0]^0; [z ← query(0)]^1; [p ← 0]^2;
if ([k = 0]^3, [y ← query(z)]^4, [skip]^5);
while [j ≠ k]^6 do
([p ← query(χ[y] + p)]^7; [j ← j + 1]^8
if ([j ≠ k − 2]^9, [p ← 0]^10, [skip]^10));
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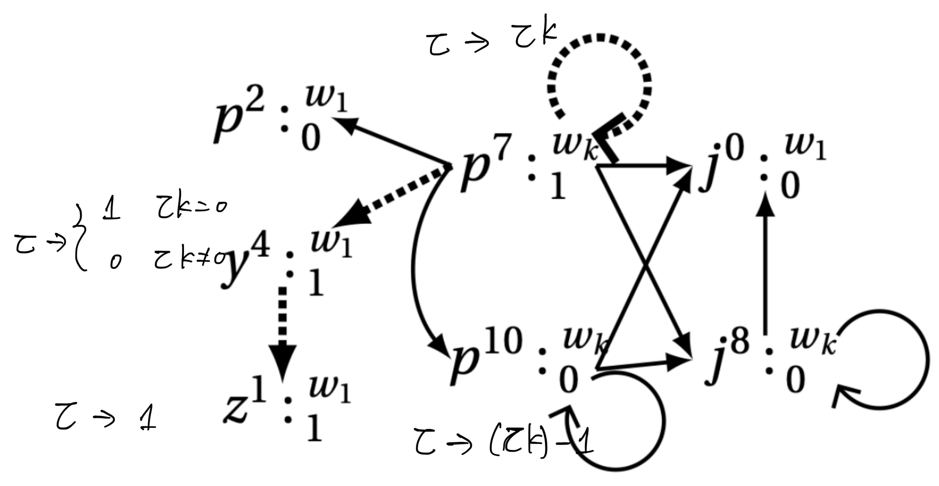
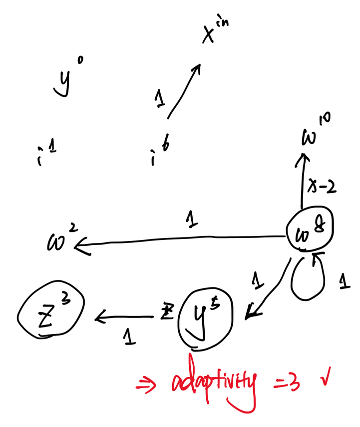




$\Rightarrow Dep(w^8, \tau_1, \tau_2, w^8, c)$, with max $|Diff(\tau_1, \tau_2, w^8)|$

$\forall x \in \mathbb{N}$.
$x = 4$

$\tau_1 = [(w^8, q_3(0,0), V_1), (w^{10}, 0), (i, 1),$
$(w^8, q_3(0,0), V_1), (w^{10}, 0)(i, 2),$
$(w^8, q_3(0,0), V_1), (i, x-1),$
$(w^8, q_3(0, V_1), V_2), (i, x)]$.

$\tau_2 = [(w^8, q_3(0,0), V_1'),$
$(w^8, q_3(0,0), V_1)$
$(w^8, q_3(0,0), V_1'), (i, x-1)$
$(w^8, q_3(0, V_1'), V_2), (i, x)]$.

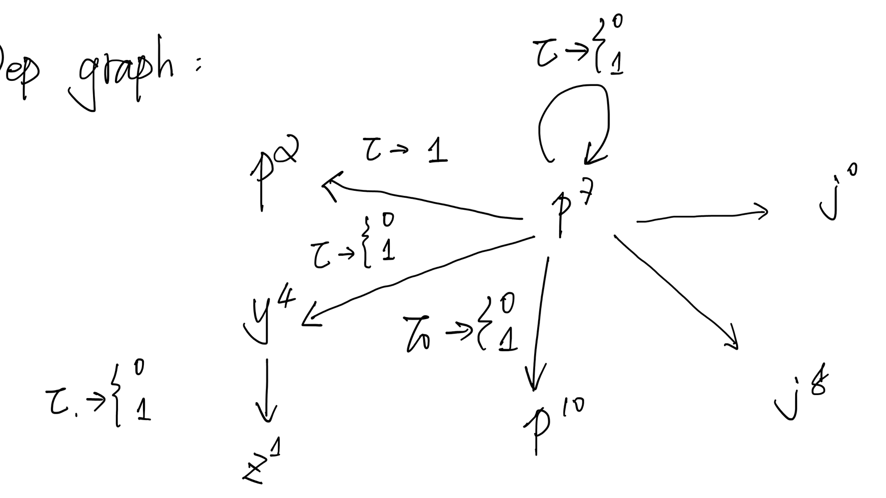
However.

$\max \{ |Diff(\tau_1, \tau_2, w^8)| \mid \forall \tau_1, \tau_2. \ Dep(w^{10}, \tau_1, \tau_2, w^8, c) \}$
$= 1$.

$\Rightarrow |Diff(\tau_1, \tau_2, w^8)| = 1$.

$\Rightarrow$ $[j \leftarrow k]^0$; $[z \leftarrow query(0)]^1$; $[p \leftarrow 0]^2$;
if $([k = 0]^3, [y \leftarrow query(z)]^4, [skip]^5)$;
while $[j \geq 0]^6$ do
$([p \leftarrow query(y + p)]^7$;
$[j \leftarrow j - 1]^8$;
if $([j \neq 1]^9, [p \leftarrow 0]^{10}, [skip]^{11})$;)

New Dep graph :
⇒



edge : (y^4, z^1) $\quad \tau_2 = \tau_1[\quad \cdots \quad (y, 4, V_2', V_1')]$

$\tau_0 = [(k, in, 0, \cdot)]$. $\quad \tau_1 = [(p, 2, 0, \cdot), (k=0, 3, true, \cdot), (y, 4, V_1, V_1)]$

$Dep(y^4, z^1, \tau_0, \tau_1, \tau_2, c)$ $\quad \epsilon_1 = (z, 1, V_1, 0)$ $\quad \epsilon_1' = (z, 1, V_1', 0)$

$Diff(\tau_1, \tau_2, y^4) = \{ (V_1', V_1) \}$.

$\Rightarrow (z^1, y^4, w_1 : \tau_0 \to \begin{cases} 1 & \rho(\tau_0)k = 0 \\ 0 & \rho(\tau_0)k \neq 0 \end{cases} \}.)$

edge : (p^7, p^7)

$\Rightarrow \tau_0 = [(k, in, 3, \cdot)]$.

$\tau_1 = [(p, 7, V_1, 0), (j, 8, 2, \cdot), (j \neq 1, 9, true, \cdot), (p, 10, 0, \cdot),$
$(j \geq 0, true), (p, 7, V_1', 0), (j, 1) \quad (j \neq 1, false),$
$(j \geq 0, true), (p, 7, V_1'', V_1'), (j, 0) \quad (j \neq 1, true) \quad (p, 10, 0)$.
$(j \geq 0, true) \quad (p, 7, V_1''', 0), (j, -1) \quad (j \neq 1, true) \quad (p, 10, 0)$,
$(j \geq 0, false)]$.

$\Rightarrow$

$\tau_2 = [(p, 7, V_1, 0), (j, 8, 2, \cdot) \quad (j \neq 1, true) \quad (p = 0)$
$(p, 7, V_2, 0) \ (j, 1) \quad (false)$
$(p, 7, V_2', V_2) \ (j, 0) \quad (true) \quad (p, 0)$
$(p, 7, V_2'', 0) \quad (j, -1) \ (true) \quad (p, 0)$

$\Rightarrow Seq(\tau_1, \tau_2, p^7) = \left\langle \begin{array}{l} V_1' \cdot 0 \\ V_2 \cdot 0 \end{array} \right. \quad \Rightarrow |Diff| = 1$.

if

$\tau_0 = [(k, in, 1, 0 \cdot)] \quad \tau_0 = [(k, in, 0, \cdot)]$. $|Diff| = 0$.

$\Rightarrow \tau \to \begin{cases} 1 & \tau_0 k \geq 2. \\ 0 & \tau_0 k \leq 1. \end{cases}$

edge (p^7, p^{10}) :

$\tau_0\ k = [(k, in, 3, \cdot)]$.

$\Rightarrow \tau_2 = [\cdots \quad (p, 10, 1, \cdot)$,
$\cdots (p, 7, V_1', 1)(j, 1)$
$(p, 7, V_1'', V_1')(j, 0) \quad (p, 0)$
$(p, 7, V_1''', 0) \quad (j, -1) \quad (p, 0)]$.

$\Rightarrow Seq(\tau_1, \tau_2, p^7) = \left\langle \begin{array}{l} 0 \cdot V_1' \cdot 0 \\ 1 \cdot V_1' \cdot 0 \end{array} \right. \Rightarrow |Diff| = 1$.

$\wedge \quad \tau_0 k = 0 \Rightarrow |Diff| = 0$

$\Rightarrow \tau \to \begin{cases} 1 & \tau_0 k \geq 1 \\ 0 & \tau_0 k = 0. \end{cases}$

$\Rightarrow$ adaptivity : $\begin{cases} 2 & \tau_0 k = 0 \\ 0 & \tau_0 k = 1 \\ 1 & \tau_0 k \geq 2 \end{cases}$

⇒ the over-approximation for dependency depth between variables inside loop. caused from weight, & Dependency based on different traces.