

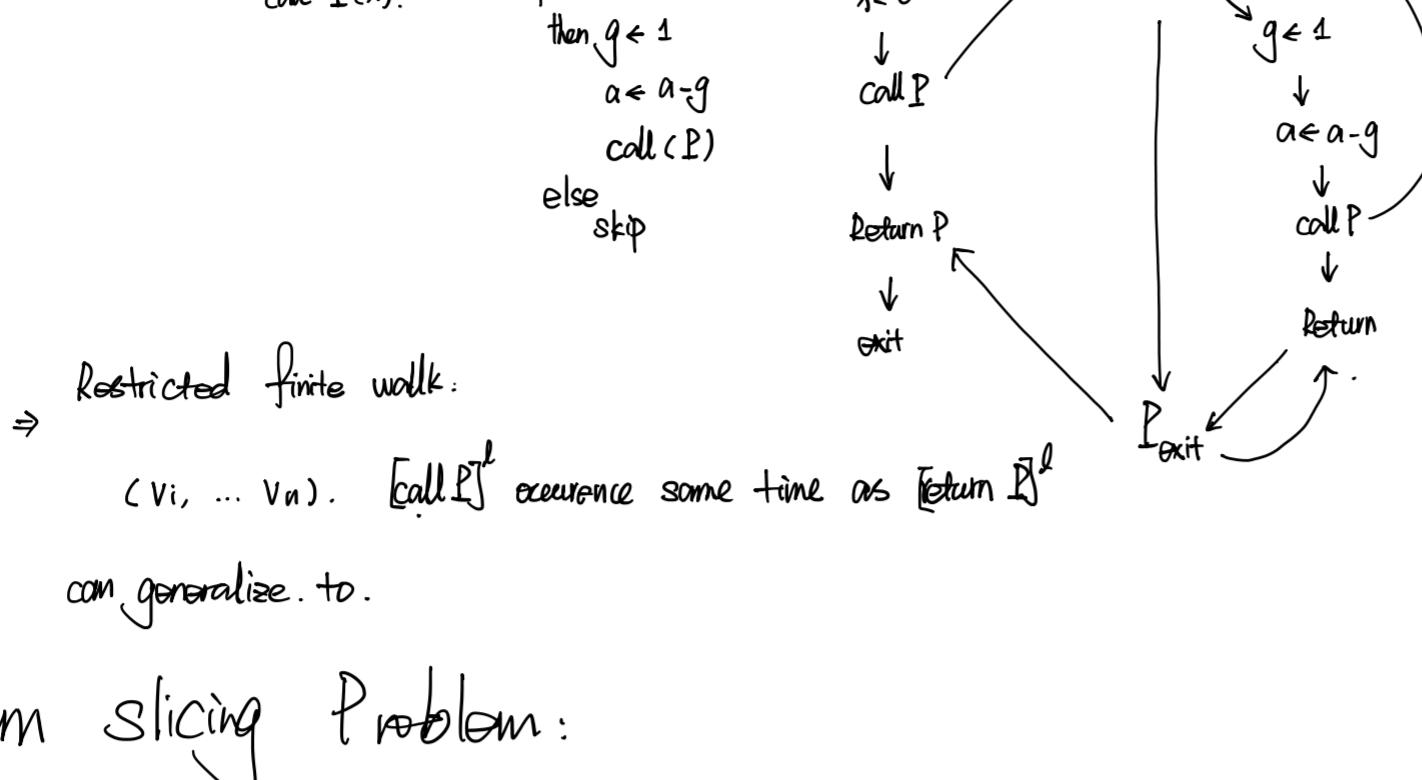
CFL Reduce to walk

Monday, July 11, 2022 9:58 PM

CFL - Reachability : labels of edges composing into a matched word
 label matching can be reduced to restrict the occurrence times equality
 \Rightarrow higher efficient, & symbolic occurrence time.

for example:

①. interprocedure call P can be reduced to valid walk by



⇒ Restricted finite walk:

(v_1, \dots, v_n). [$\text{call } P$] occurrence same time as [$\text{return } P$]

can generalize to.

②. Program slicing Problem:

Example Application Q:

program slicing Problem:

Backward / Forward slicing:

⇒ by define as a single-target $L(\underline{\text{slice}})$ -problem:

where L defined as:

unbalanced-right := unbalanced-right matched
 $|$ unbalanced-right); $1 \leq i \leq \text{callsite}$. $\Rightarrow \checkmark$
 $|_{\Sigma}$

slice := unbalanced-right realizable.

⇒ by restricting inequality between occurrence times of vertices.

③. Data flow

standard Data reachability definition analysis is a special case of valid walk.

by set the vertex kill x . weight 0

vertex gen x weight 1.

⇒ valid walk = feasible path.

④. JFDS: (Interprocedural, Finite Distributive, Subset problems)

⇒ 1. supergraph G^*

2. D : domain of data-flow facts, $\rightarrow \Delta \in D$. the initial data fact.

Each program point $\Delta \in D$ is associated to a member $\Delta \in \Delta^D$.

3. Assign of distributive dataflow functions: $\Delta^D \rightarrow \Delta^D$, to edges of G^* .

⇒ solve it as realizable-path reachability problem.

⇒ exploded supergraph: G^* . for each node in G^* :

$$\Rightarrow G^*(n) = \{ V_{G^*(n)}, E_{G^*(n)} \}$$

$d \in V_{G^*(n)}$: $d \in D$: a dataflow fact.

$$e = (d_1, d_2), d_1 \in V_{G^*(n_1)}$$

$$d_2 \in V_{G^*(n_2)}$$

dataflow facts d_1 in node n_1 if d_1 is true in n_1 .

↓

... d_2 in node n_2 if the fact transformation function from n_1 to n_2

$f(d_1) = d_2$ is true in n_2 .

⇒ Formally: G^* :

$$V = \bigcup_n \text{node } n \in G^*, \exists \text{ a node } \langle n, \Delta \rangle \text{ in } G^*$$

$$\bigcup_n \text{node } n \in G^*, d \in D, \exists \text{ a node } \langle n, d \rangle \text{ in } G^*$$

$$\exists \text{ edge } e = (\langle m, \Delta \rangle, \langle n, d \rangle) \wedge d \in f(\phi)$$

$$\vee \text{ edge } e = (\langle m, d_1 \rangle, \langle n, d_2 \rangle) \wedge d_1, d_2 \in D, d_2 \in f(d_1) \wedge d_2 \notin f(\phi)$$

$$\vee \text{ edge } e = (\langle m, \Delta \rangle, \langle n, \Delta \rangle).$$

Example :

data flow facts: $\Delta \cup \{x, y\}$: $\bullet \exists x$: x is a possible uninitialized variable.

$\langle 1, x \rangle$: the node for start point in program where data-flow fact x is true.

fact transformation function $f: \lambda S. \{x, y\}$

$$\therefore x \in f(\Delta) = \{x, y\} \Rightarrow \text{edges: } (\langle 1, x \rangle, \langle 2, x \rangle), (\langle 1, y \rangle, \langle 2, y \rangle)$$

⇒ composition of edges. \Rightarrow pasting nodes.

CFL-II

Friday, June 3, 2022 3:40 PM

⇒ Dyck-language : Balanced "[" "]" → kleene Closure.

⇒ IFDS, ↳ Interprocedural finite, distributive subset problem.
Sharir-Pnueli framework.

IDE ↳ Interprocedural Distributive Environment.

CFL → chain program

→ dataflow fact is set of environment.

f: environment transition function.

variable propagation. → through chain.

⇒ interconvertable to Set Constraints Problem. ↳

David Melski & Thomas Reps. Dec 1997.

<https://research.cs.wisc.edu/wpis/papers/pepm97.pdf>

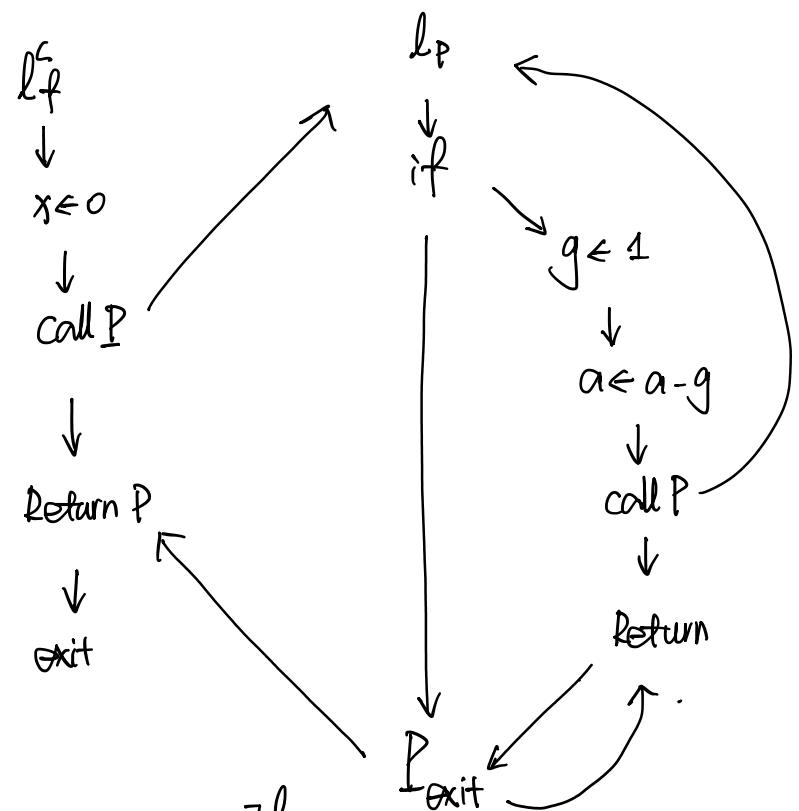
CFL-III-InterProcedure

Monday, July 11, 2022 9:06 AM

CFL - Reachability : labels of edges composing into a matched word

Example: $f : x \leftarrow 0$
call $P(x)$.

$P(x) : (a)$
if ($a > 0$)
then $g \leftarrow 1$
 $a \leftarrow a - g$
call (P)
else skip



Restricted finite walk:

⇒

(v_i, \dots, v_n) . $[call P]^l$ occurrence same time as $[return P]^l$

can generalize to.

Context Free language reachability

Wednesday, May 25, 2022 4:04 PM

convert program-analysis \Rightarrow CFL-reachability

\Rightarrow ordinary graph-reachability.

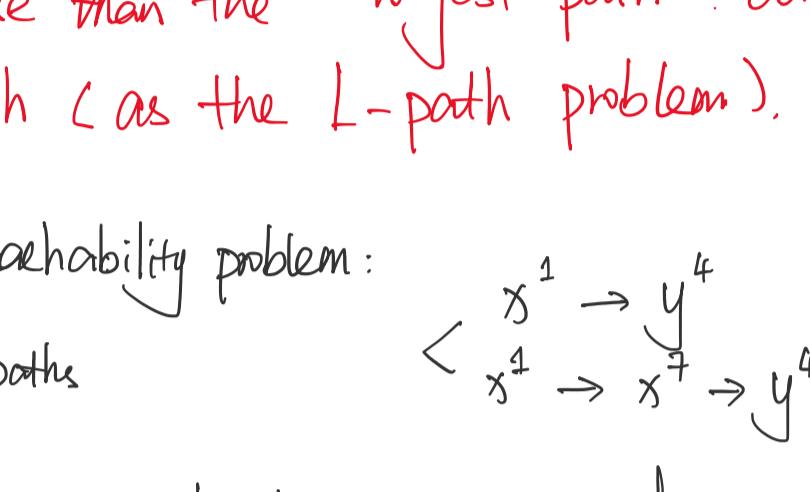
1. Generalization of graph-reachability.

with restrictions as labels on edge, (restricting visiting orders of edges?)

excluded infeasible path (i.e. paths on graph which don't correspond to actual program executions.)

\Rightarrow compare to longest walk definition.

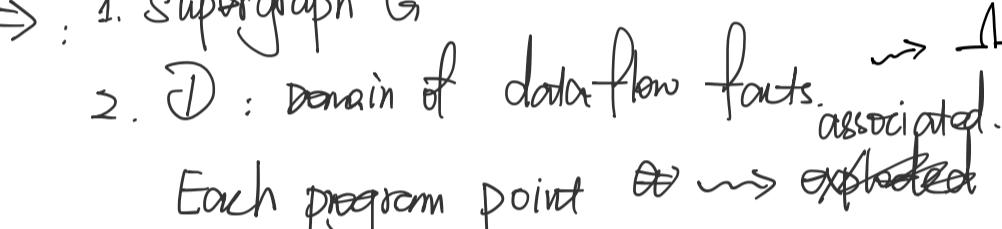
Example: while without off dep. recursive val dep \checkmark .



Adaptivity (able to generalize to general resource consumption) computed by longest walk.

more precise than the "longest path" computed from the Best/Most Accurate CFL-graph (as the L-path problem).

CFL-reachability problem:



both are corresponding to actual execution. can't be excluded.

\Rightarrow solve as all-pairs L-path problem $\Rightarrow 1 + k! + k$ is the longest path. \hookrightarrow over-approximated.

\Rightarrow solve as longest finite walk $\Rightarrow 1 + k! + 1$ is the actual (Resource consumption) Adaptivity.

Example Application on interprocedural dataflow-analysis problem:

①. IFDS: (Interprocedural, Finite, Distributive, Subset problems)

\Rightarrow 1. Supergraph G^*

2. D : domain of dataflow facts $\hookrightarrow \perp \in D$. the initial datafact.

Each program point \hookrightarrow \hookrightarrow associated to a member $\hookrightarrow \in \omega^D$.

3. Assign of distributive dataflow functions: $\omega^D \rightarrow \omega^D$, to edges of G^* .

\Rightarrow solve it as realizable-path reachability problem.

\Rightarrow exploded supergraph: G^* . for each node in G^* :

$$\Rightarrow G^*(n) = \{ V_{G^*(n)}, E_{G^*(n)} \}$$

$d \in V_{G^*(n)}$: $d \in D$: a dataflow fact.

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dataflow facts d_1 in node n_1 . if d_1 is true in n_1 .

\downarrow

\dots d_2 in node n_2 . then $f(d_1) = d_2$ is true in n_2 .

\downarrow if: the fact transformation function from n_1 to n_2 .

\Rightarrow Formally: G^* :

$$V = \bigcup_n \{ \text{node } n \in G^*, \exists \text{ a node } \langle n, \perp \rangle \text{ in } G^* \}$$

$$\bigcup_n \{ n \in G^*, d \in D, \exists \text{ a node } \langle n, d \rangle \text{ in } G^* \}$$

$$E \ni e = \text{edge } e = (\langle m, \perp \rangle, \langle n, d \rangle) \ni d \in f(\perp)$$

$$\vee \text{edge } e = (\langle m, d_1 \rangle, \langle n, d_2 \rangle) \ni d_1, d_2 \in D, d_1 \in f(d_2) \wedge d_2 \notin f(\perp)$$

$$\vee \text{edge } e = (\langle m, \perp \rangle, \langle n, \perp \rangle).$$

Example:

dataflow facts: $\perp \cup \{x, g\}$. \perp : x is a possible uninitialized variable.

$\langle 1, x \rangle$: the node for start point in program where dataflow fact x is true.

fact transformation function f : $\lambda s. \perp, g \mapsto$

$$\perp, x, g \mapsto \text{edges: } \langle \perp, x \rangle, \langle \perp, y \rangle, \langle \perp, \perp \rangle$$

$$\langle 1, \perp \rangle \xrightarrow{\perp} \langle 2, x \rangle \xrightarrow{\perp} \langle 3, g \rangle$$

$$\langle 2, x \rangle \xrightarrow{x} \langle 3, g \rangle$$

$$\langle 3, g \rangle \xrightarrow{g} \langle 4, \perp \rangle$$

$$\langle 4, \perp \rangle \xrightarrow{\perp} \langle 5, \perp \rangle$$

$$\langle 5, \perp \rangle \xrightarrow{\perp} \langle 6, \perp \rangle$$

$$\langle 6, \perp \rangle \xrightarrow{\perp} \langle 7, \perp \rangle$$

$$\langle 7, \perp \rangle \xrightarrow{\perp} \langle 8, \perp \rangle$$

$$\langle 8, \perp \rangle \xrightarrow{\perp} \langle 9, \perp \rangle$$

$$\langle 9, \perp \rangle \xrightarrow{\perp} \langle 10, \perp \rangle$$

$$\langle 10, \perp \rangle \xrightarrow{\perp} \langle 11, \perp \rangle$$

$$\langle 11, \perp \rangle \xrightarrow{\perp} \langle 12, \perp \rangle$$

$$\langle 12, \perp \rangle \xrightarrow{\perp} \langle 13, \perp \rangle$$

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$$\langle 66, \perp \rangle \xrightarrow{\perp} \langle 67, \perp \rangle$$

$$\langle 67, \perp \rangle \xrightarrow{\perp} \langle 68, \perp \rangle$$

$$\langle 68, \perp \rangle \xrightarrow{\perp} \langle 69, \perp \rangle$$

$$\langle 69, \perp \rangle \xrightarrow{\perp} \langle 70, \perp \rangle$$

$$\langle 70, \perp \rangle \xrightarrow{\perp} \langle 71, \perp \rangle$$

$$\langle 71, \perp \rangle \xrightarrow{\perp} \langle 72, \perp \rangle$$

$$\langle 72, \perp \rangle \xrightarrow{\perp} \langle 73, \perp \rangle$$

Memory resources

Monday, July 11, 2022 9:03 AM

Shape Analysis :

possible "shapes" that heap-allocated structure can take:

⇒ example:

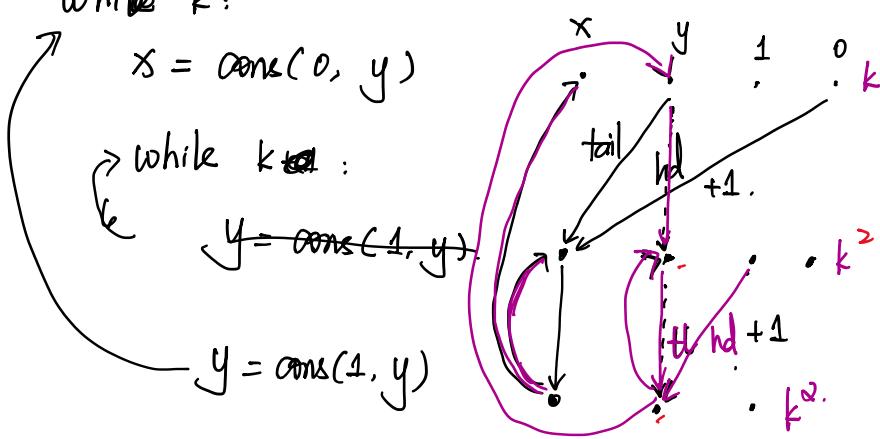
while k :

$x = \text{cons}(0, y)$

while $k \neq 0$:

$y = \text{cons}(1, y)$

$y = \text{cons}(1, y)$



$$\Rightarrow \text{cond}(x) = k^3 \text{ or } k + k^2.$$

actually
 $\text{cond}(x) = k^0$.

⇒ by reduce to single-target path problem:

longest weighted x -target path is:

$$y^0 \xrightarrow{k^2} y^3 \xrightarrow{k^2} y^1 \xrightarrow{k} x^2 \xrightarrow{k^2} x^3. \quad \square k^2.$$

$$\{k^2 + k^2 + k\}.$$

⇒ by longest finite walk

$$y^{k^2} \quad \text{longest } y\text{-target walk: } \square k^2.$$

$$\downarrow \quad \Rightarrow \quad \text{long } x\text{-target walk: } \square k^2 \rightarrow x. \\ x^k$$

$$\Rightarrow k^2 + 1.$$

