Motivating Example

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$E = \left\{ (x^i, w, y^j) \;\middle|\; \begin{array}{l} x^i, y^j \in LVar \\ w = \max\{ \xi(\tau) \mid \forall \tau.\ Dep(x^i, y^j, \tau, c) \} \end{array} \right\}$

example: input x:

```
y=0;
i=0 ;
w=0;
z <- q1;
if x = 0
{ y <- q2(z); i=x-1}
else
{skip};
while i \neq x
{w <- q3(y,w); if i neq x-2 then w=0 else skip ; i=i+1}
```

a trace τ_1. $\varepsilon_1 \overset{=}{\&} w.2$. $\varepsilon_2 \overset{=}{\&} w.2$.

change $\varepsilon_1 \overset{=}{\&} w.2'$ $\Rightarrow \varepsilon_2' = \varepsilon\, w'.2$

then another trace: $\varepsilon_1 = w.2$, $\varepsilon_2 = w.2$, $\varepsilon_3 = w.2$.

change ↓ ||| ↓change

$\varepsilon_1' = w.2.v'$. $\varepsilon_2 = w_*.2$. $\varepsilon_3' = w^*, 2, v_3'$.

$\forall$ trace. s.t. ~~$\varepsilon_1, \tau, \varepsilon$~~ $Dep(w^2, \tau, w^2)$, # of occurrence time of 2 $\geq$ # of event where value of w^2 is changed.

then count: $\forall$ trace τ s.t. $Dep(w^2, \tau, w^2)$.

$|Diff(\tau_1, \tau_2, w^2)|$

$\Rightarrow$ weight on edge (x^i, y^j, w).

$w = \max\{ |Diff(\tau_1, \tau_2, y^j)| \mid \forall \tau_1, \tau_2.\ s.t.\ Dep(x^i, \tau_1, \tau_2, y^j) \}$.

$\Rightarrow$ 1. Dep on 2 traces. redefine.

$\Rightarrow$ 2. Control Dep. ??

$\Rightarrow$ add iteration id.

inorder to know the value changed in the 2 traces are in the same iteration.

then,

$\rightsquigarrow$ possible unsound ??

if cases $\forall t$. observe the same value, but in different iterations? $\Rightarrow$

$\Downarrow$

then intuitively no-dependency. it is good.

$\Rightarrow$ 3. $Diff(\tau_1, \tau_2, x^i)$

$\Rightarrow$ ~~$\forall \tau \varepsilon$~~

$seq(\tau_1)\, x^i \triangleq \begin{cases} \tau_1 = \tau_1' :: (x, i, v, \cdot) \rightarrow seq(\tau_1') :: v \\ \tau_1 = \tau_1' :: (x, i, v, \alpha) \rightarrow seq(\tau_1') :: \alpha. \\ \tau_1 = [] \rightarrow [] \\ o.w. \rightarrow seq(\tau_1') \end{cases}$

without loop iteration id.

for $seq(\tau_2)\, x^i$. where $Dep(y^j, x^i, \tau_1, \tau_2)$.

cases of $seq(\tau_1) \neq seq(\tau_2)$:

①. $len_1 \neq len_2$. $\Rightarrow Diff(seq(\tau_1), seq(\tau_2), x^i) = \{k \mid seq(\tau_1)[k] \neq Seq(\tau_2)[k]\} \cup \{len_1, len_1+1, \ldots len_2\}$.

②. $len_1 = len_2 \Rightarrow Diff(seq(\tau_1), seq(\tau_2), x^i) = \{k \mid seq(\tau_1)[k] \neq Seq(\tau_2)[k]\}$

$\Rightarrow$ potential issue: the different observed value is in different iterations.

case: if query/variable is affected by moving to different iteration, but it always executed the same time with same value.

then it doesn't matter which iteration it is in, there isn't dependence / adaptivity.

$\Rightarrow$ for case ②. where they have same len, it doesn't loss adaptivity if I observe the same value in the same location of $seq(\tau_1)$ and $seq(\tau_2)$ ~~to and~~ where they actually comes from different loop iteration.

i.e. it is sound that only count the diff of same location.

$\Rightarrow$. for case ①. $len_1 \neq len_2$.

in the 2 seqs from $[0, \min(l_1, l_2)]$, it is still sound by just observe the different value in same location,

for the same reason as case ①.

in the longer seq from $[\min(l_1, l_2), \max(l_1, l_2)]$.

case
```
i = 0
while i < x do:
    i = i+1
    y = q(a).
```

$\Rightarrow$ when $x = 100$. still safe.

$(y, 1, v_1)$ $(y, 1, v_2)$ $(y, 1, v_3)$. $(y, 2, 0 \ldots)$

$(y, 1, v_1')$. $(y, 1, v_2')$ $(y, 1, v_3')$ $(y, 2, 0) \ldots$

$\Rightarrow$ $G_{y^*}(inf, 5)$.

in order to get different length, only way is by changing value affect guards.

if in the ~~long~~ seq from $\min(l_1, l_2)$ to $\max(l_1, l_2)$.

$(x, in, 1)$ $(y, 1, v_1)$

$(x, in, 100)$, $(y, 1, v_1)$, $(y, 1, v_2) \ldots$

$\Rightarrow$ all the value are the same. then adaptivity is actually 1.

2 approches:

$\rightsquigarrow$

$\Rightarrow$ if they are different, then either guard variable is used in evaluating y. or y itself is used (through a chain) is used in evaluating y.

```
i=0
while i < x.
  w = q(w)
  if i < 3 , w=0, skip
  i=i+1.
```

$x = 2$ $(w, 0)$, $(w, 1, 0)$

$x = 100$. $(w, 1, 0)$ $(w, 1, 0)$ $(w, 1, q(1))$ $(w, 1, q(q(1))) \ldots$

Dep : infinite.

Case :
```
i = 0
while i < x :
    w = q(w)
    if i > 3, w=0, skip.
    i=i+1
```

$x=100$: $(w, 1, q(1))$, $(w, 1, q(1))$ $(w, 1, q(3))$, $(w, 1, 0)$ $(w, 1, 0) \ldots$

$\Downarrow$

two approaches:

① prove there must exist value dependency from x to y or from other variable to y.

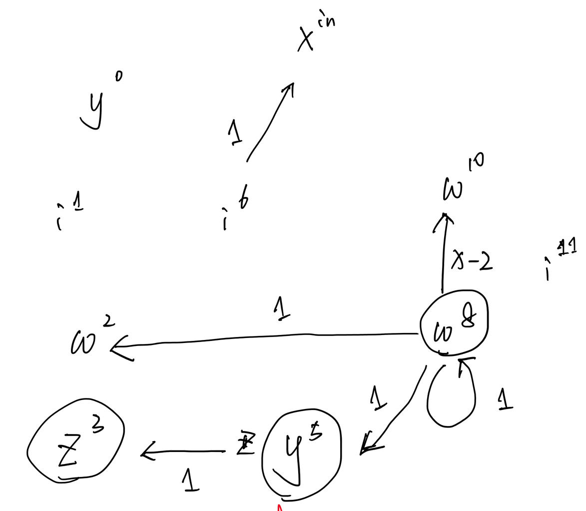
unclear to prove, because change x could change length and value in the same time.

② define $|Diff|$ as $\{seq(\tau_{max})[min : max]\}$. $\rightsquigarrow$ sound. set of unique values.

possible case. $(1, 0, 1, 0, 1, 0)$ I actually have no intuition does this adaptive or not.

$\Rightarrow$ if it is. the $|Diff|$ defined as $\{k \mid seq[k-1] \neq seq[k]\}$ can capture.

but "intuitive adaptivity" is unclear.



$\Rightarrow$ adaptivity $=3$ ✓

$\Rightarrow Dep(w^8, \tau_1, \tau_2, w^8, c)$, with max $|Diff(\tau_1, \tau_2, w^8)|$

$\forall x \in \mathbb{N}$.

$x = 4$

$\tau_1 = [(w^8, q_3(0,0), V_1), (w^{10}, 0), (i, 1),$
$(w^8, q_3(0,0), V_1), (w^{10}, 0)\, (i, 2),$
$(w^8, q_3(0,0), V_1), (i, x-1),$
$(w^8, q_3(0, V_1), V_2), (i, x)]$.

$\tau_2 = [(w^8, q_3(0,0), V_1'),$
$(w^8, q_3(0,0), V_1)$
$(w^8, q_3(0,0), V_1'), (i, x-1)$
$(w^8, q_3(0, V_1'), V_2), (i, x)]$.

However.

$\max\left\{ |Diff(\tau_1, \tau_2, w^8)| \;\middle|\; \begin{array}{l} \forall \tau_1, \tau_2. \\ Dep(w^{10}, \tau_1, \tau_2, w^8, c) \end{array} \right\}$

$= 1$.

$\Rightarrow |Diff(\tau_1, \tau_2, w^8)| = 1$.

$\Rightarrow$ Dep Graph:

$V(c) = \{x^i \mid x^i \in LV(c)\}$

$E(c) = \left\{ (x^i, y^j, w) \;\middle|\; \begin{array}{l} w = \max\{ |Diff(\tau_1, \tau_2, y^j)| \mid \forall \tau_1, \tau_2,\ s.t.\ Dep(x^i, \tau_1, \tau_2, y^j, c) \} \\ \wedge\ w \geq 1 \wedge x^i, y^j \in LV(c) \end{array} \right\}$

$Diff(\tau_1, \tau_2, x^i) \triangleq \{ k \mid (k = 0, \ldots, l_{min}-1 \wedge Seq(\tau_1)\, x^i [k] \neq Seq(\tau_2, x^i)[k])\}$

$\wedge\ (Seq(\tau_{max}, x^i)[k-1] \neq Seq(\tau_{max}, x^i)[k] \wedge k = l_{min}, \ldots, l_{max})$

$\wedge\ l_{min} = \min(|Seq(\tau_1, x^i)|, |Seq(\tau_2, x^i)|)$

$l_{max} = \max(\cdots)$

τ_{max} = longer Sequence. τ }

$\Rightarrow Seq(\tau, x^i) \triangleq \begin{cases} seq(\tau', x^i) :: v & \tau = \tau' :: (x, i, v, \cdot) \\ seq(\tau', x^i) :: \alpha. & \tau = \tau' :: (x, i, v, \alpha) \\ [] & \tau = [] \\ seq(\tau_1') & o.w. \end{cases}$

$Dep(x^i, \tau_1, \tau_2, y^j, c) \triangleq$

$\exists\ \varepsilon_1', \tau_0, \tau_0'$

$\varepsilon_1 = (x, i, v\_, \_)$. $\varepsilon_2 = (y, j, \_, \_)$.

$\varepsilon_1 \neq_{event} \varepsilon_1'$

$\langle c, \tau_0 \rangle \rightarrow \langle c_1, \tau_0' :: \varepsilon_1 \rangle \rightarrow \langle skip, \tau_0' :: \varepsilon_1 ++ \tau_1 \rangle$

$\wedge$ $\langle c_1, \tau_0' :: \varepsilon_1' \rangle \not\rightarrow \langle skip, \tau_0' :: \varepsilon_1 ++ \tau_1 \rangle$

$Diff(\tau_1, \tau_2, y^j) \neq \emptyset$.

$\Rightarrow$ example: