Adaboost

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Boosting

Overview

Bagging Reduce variance of a low bias, high variance estimator by ensembling many estimators trained in parallel.

Boosting Reduce the error rate of a high bias estimator by ensembling many estimators trained in sequential.

- A **weak/base learner** is a classifier that does slightly better than chance.
- Weak learners are like "rules of thumb":
  - "Viagra" $\implies$ spam
  - From a friend $\implies$ not spam
- **Key idea**:
  - Each weak learner focuses on different examples (*reweighted data*)
  - Weak learners have different contributions to the final prediction (*reweighted classifier*)

AdaBoost: Setting

- *Binary* classification: $\mathcal{Y} = \{-1, 1\}$
- Base hypothesis space $\mathcal{H} = \{h : \mathcal{X} \to \{-1, 1\}\}$.
- Typical base hypothesis spaces:
  - **Decision stumps** (tree with a single split)
  - Trees with few terminal nodes
  - Linear decision functions

Weighted Training Set

Each base learner is trained on weighted data.

- Training set $\mathcal{D} = ((x_1, y_1), \ldots, (x_n, y_n))$.
- Weights $(w_1, \ldots, w_n)$ associated with each example.
- **Weighted empirical risk**:

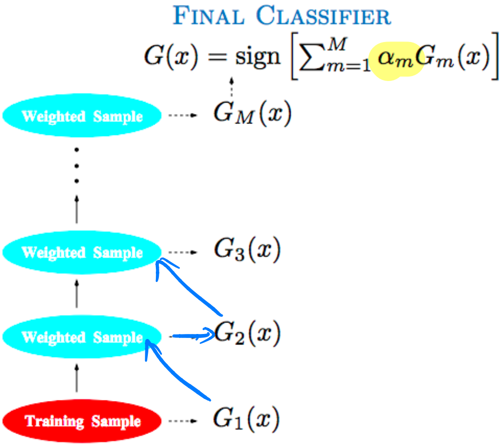
$$\hat{R}_n^w(f) \stackrel{\text{def}}{=} \underbrace{\frac{1}{W}}_{\text{normalizer}} \sum_{i=1}^{n} w_i \ell(f(x_i), y_i) \quad \text{where } W = \sum_{i=1}^{n} w_i$$

- Examples with larger weights have more influence on the loss.

AdaBoost - Rough Sketch

- Training set $\mathcal{D} = ((x_1, y_1), \ldots, (x_n, y_n))$.
- Start with equal weight on all training points $w_1 = \cdots = w_n = 1$.
- Repeat for $m = 1, \ldots, M$:
  - Find base classifier $G_m(x)$ that tries to fit weighted training data (but may not do that well)
  - Increase weight on the points $G_m(x)$ misclassifies
- So far, we've generated M classifiers: $G_1, \ldots, G_M : \mathcal{X} \to \{-1, 1\}$.

AdaBoost: Schematic



From ESL Figure 10.1

AdaBoost - Rough Sketch

- Training set $\mathcal{D} = \{(x_1, y_1), \ldots, (x_n, y_n)\}$.
- Start with equal weight on all training points $w_1 = \cdots = w_n = 1$.
- Repeat for $m = 1, \ldots, M$:
  - Base learner fits weighted training data and returns $G_m(x)$
  - Increase weight on the points $G_m(x)$ misclassifies
- Final prediction $G(x) = \operatorname{sign}\left[\sum_{m=1}^{M} \alpha_m G_m(x)\right]$. (recall $G_m(x) \in \{-1, 1\}$)
- What are desirable α_m's?
  - nonnegative
  - larger when G_m fits its weighted $\mathcal{D}$ well
  - smaller when G_m fits weighted $\mathcal{D}$ less well

Adaboost: Weighted Classification Error

- Weights of base learners depend on their performance. How to evaluate each base learner?
- In round m, base learner gets a weighted training set.
    - Returns a base classifier $G_m(x)$ that minimizes weighted $0-1$ error.
- The **weighted 0-1 error** of $G_m(x)$ is

$$\text{err}_m = \frac{1}{W}\sum_{i=1}^{n} w_i \mathbb{1}(y_i \neq G_m(x_i)) \quad \text{where } W = \sum_{i=1}^{n} w_i.$$

- Notice: $\text{err}_m \in [0,1]$.

AdaBoost: Classifier Weights

- The weight of classifier $G_m(x)$ is $\boxed{\alpha_m = \ln\left(\frac{1-\text{err}_m}{\text{err}_m}\right).}$

- Higher weighted error $\Longrightarrow$ lower weight
- When is $\alpha_m < 0$?

Adaboost: Example Reweighting

- We train G_m to minimize weighted error, and it achieves $_m$.
- Then $\alpha_m = \ln\left(\frac{1-\text{err}_m}{\text{err}_m}\right)$ is the weight of G_m in final ensemble.

We want the base learner to focus more on examples misclassified by the previous learner.

- Suppose w_i is weight of example i before training:
  - If G_m classfies x_i correctly, then w_i is unchanged.
  - Otherwise, w_i is increased as

$$\begin{aligned} w_i &\leftarrow w_i e^{\alpha_m} \\ &= w_i \left(\frac{1-\text{err}_m}{\text{err}_m}\right) \end{aligned}$$

  - For $\text{err}_m < 0.5$ (weak learner), this always increases the weight.

If G_m is good
$\Rightarrow$ α_m is large
$\Rightarrow$ if x_i is misclassified
then its weight is
increased more
e^{α_m}

AdaBoost: Algorithm

Given training set $\mathcal{D} = \{(x_1, y_1), \ldots, (x_n, y_n)\}$.

1. Initialize observation weights $w_i = 1$, $i = 1, 2, \ldots, n$.
2. For $m = 1$ to M:
    1. Base learner fits weighted training data and returns $G_m(x)$
    2. Compute *weighted empirical 0-1 risk*:

$$\mathrm{err}_m = \frac{1}{W} \sum_{i=1}^{n} w_i 1(y_i \neq G_m(x_i)) \quad \text{where } W = \sum_{i=1}^{n} w_i.$$

    3. Compute *classifier weight*: $\alpha_m = \ln\left(\frac{1-\mathrm{err}_m}{\mathrm{err}_m}\right)$. if example is misclarified
    4. Update *example weight*: $w_i \leftarrow w_i \cdot \exp\left[\alpha_m 1(y_i \neq G_m(x_i))\right]$
3. Return *voted classifier*: $G(x) = \mathrm{sign}\left[\sum_{m=1}^{M} \alpha_m G_m(x)\right]$.

- After 1 round:

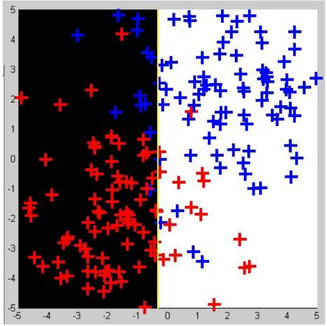


Figure: Plus size represents weight. Blackness represents score for red class.

KPM Figure 16.10

AdaBoost with Decision Stumps

- After 3 rounds:

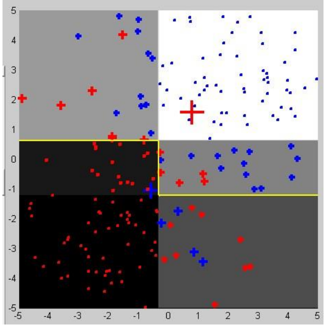


Figure: Plus size represents weight. Blackness represents score for red class.

KPM Figure 16.10

AdaBoost with Decision Stumps

- After 120 rounds:

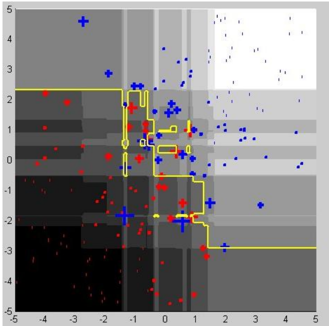
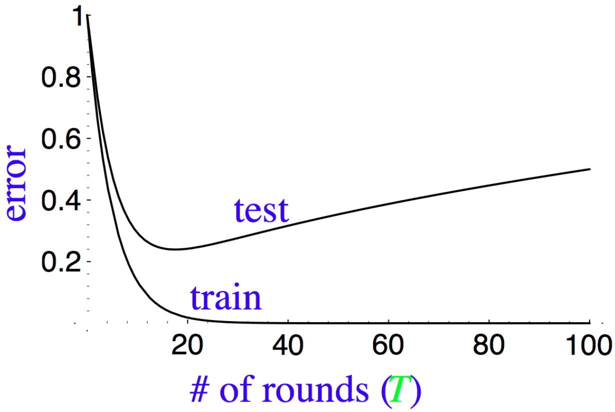


Figure: Plus size represents weight. Blackness represents score for red class.

KPM Figure 16.10

Typical Train / Test Learning Curves

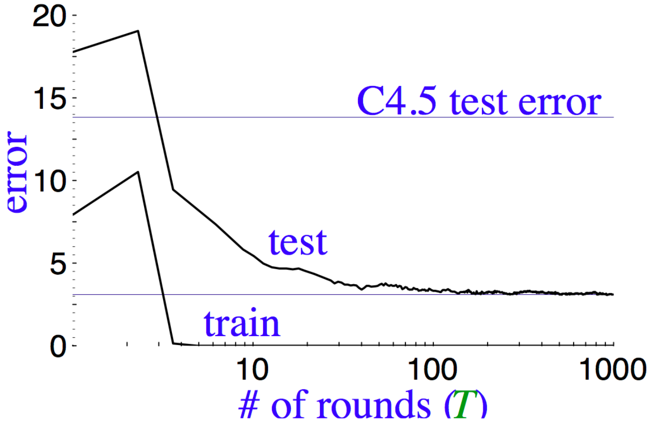
- Might expect too many rounds of boosting to overfit:



From Rob Schapire's NIPS 2007 Boosting tutorial.

Learning Curves for AdaBoost

- In typical performance, AdaBoost is surprisingly resistant to overfitting.
- Test continues to improve even after training error is zero!



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Summary

- Shallow decision tree + boosting
  - "best off-the-shelf classifier in the world"—Leo Brieman
  - Used in the first successful real-time face detector (Viola and Jones, 2001)
  - XGBoost: very popular in competitions
- Next week
  - What is the objective function of Adaboost?
  - Generalize to other loss functions.