

A CONCISE INTRO TO GPT

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CONTENT

- NLP brief
- GPT brief
- GPT experiments

BASIC ROUTINE OF NLP

- Tokenization: I drink water \rightarrow ['I', 'drink', 'water']
- Embedding: 'I' \rightarrow vector
- Positional Encoding: encoding positional info
- Encoding: seq2seq, self attention...
- Decoding: same as encoding
- Output: I eat food

ATTENTION UNIT

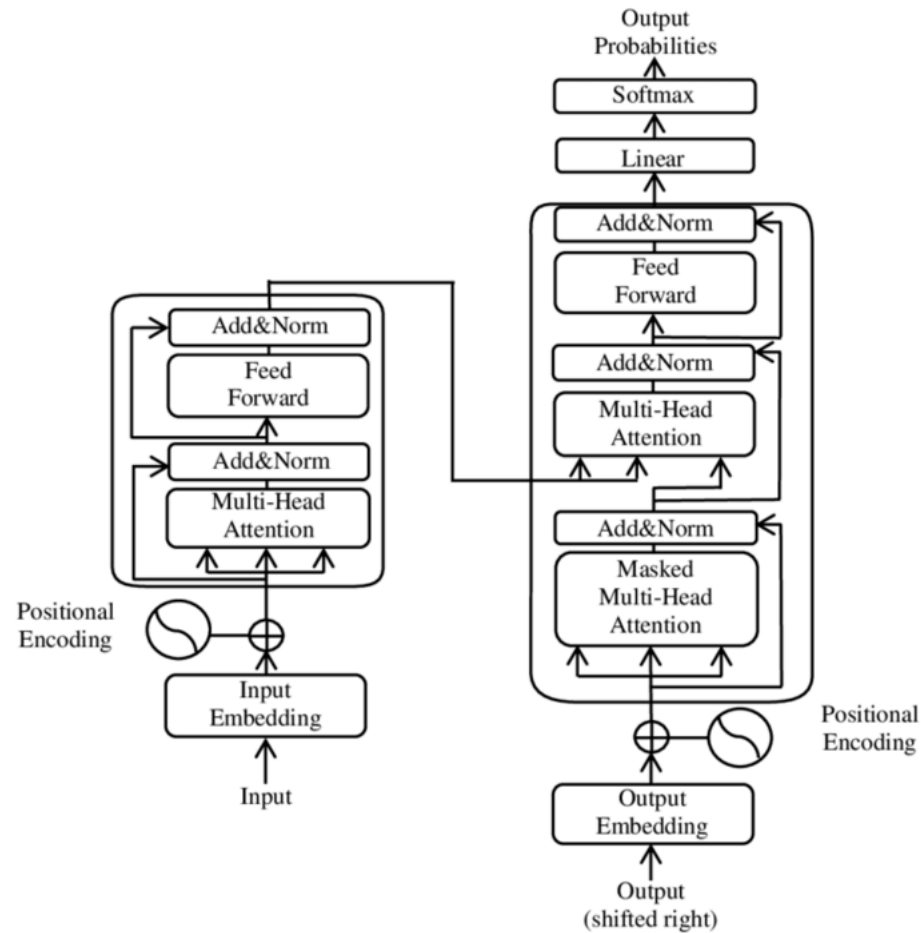
$$\text{Attention}(Q, K, V) = \text{softmax} \left(\frac{QK^T}{\sqrt{d_k}} \right) V$$

- Weights need to be learned: W_Q : query, W_K : key, W_V : value
- $q_i = x_i W_Q$, $k_i = x_i W_K$, $v_i = x_i W_V$
- Q, K makes the attention non-symmetric
- Connection with feedforward NN

[HTTPS://EN.WIKIPEDIA.ORG/WIKI/TRANSFORMER_\(MACHINE_LEARNING_MODEL\)](https://en.wikipedia.org/wiki/Transformer_(machine_learning_model))

[HTTPS://WWW.YOUTUBE.COM/WATCH?V=XLNMGVIQHPA&T=1987S](https://www.youtube.com/watch?v=XLNMGVIQHPA&T=1987S)

TRANSFORMER



[HTTPS://EN.WIKIPEDIA.ORG/WIKI/TRANSFORMER_\(MACHINE_LEARNING_MODEL\)](https://en.wikipedia.org/wiki/Transformer_(machine_learning_model))

- Generative Pre-trained Transformer



what is GPT?



GPT stands for "Generative Pre-trained Transformer". It is a series of deep learning models developed by OpenAI for natural language processing tasks such as language modeling, text completion, and text generation. GPT models are based on the transformer architecture and are pre-trained on large amounts of text data using unsupervised learning methods, which allows them to capture the statistical regularities of natural language. The pre-trained models can then be fine-tuned on specific downstream tasks with relatively little labeled data. The GPT series includes GPT, GPT-2, GPT-3, and the more recent GPT-Neo and GPT-4, which have increasingly larger model sizes and capabilities.

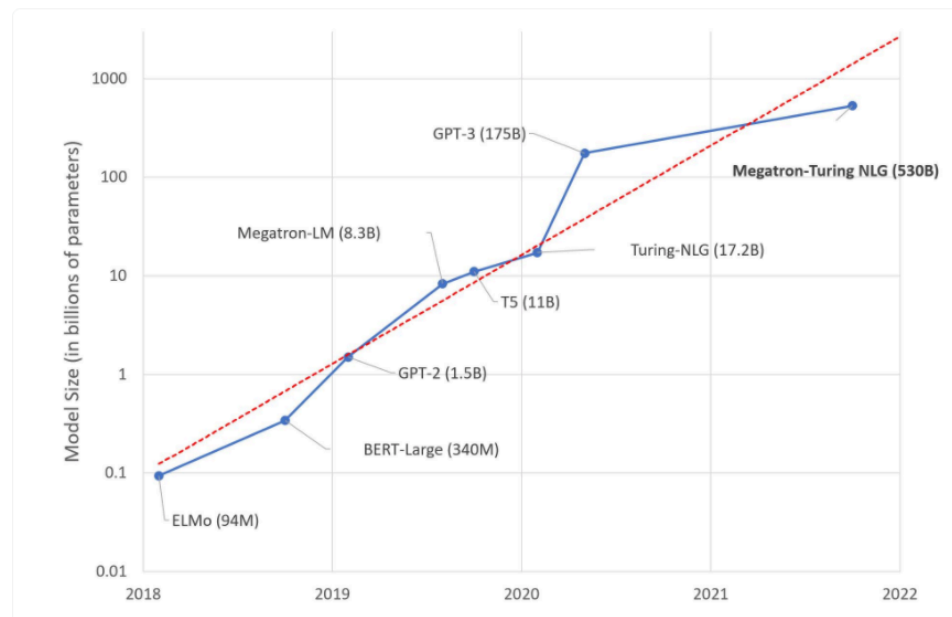


GPT

- Pretrained on large corpus
- Fine-tuned to downstream task

GPT-3 training data^{[1]:9}

Dataset	# tokens	Proportion within training
Common Crawl	410 billion	60%
WebText2	19 billion	22%
Books1	12 billion	8%
Books2	55 billion	8%
Wikipedia	3 billion	3%



[HTTPS://EN.WIKIPEDIA.ORG/WIKI/GPT-3](https://en.wikipedia.org/wiki/GPT-3)

[HTTPS://HUGGINGFACE.CO/BLOG/LARGE-LANGUAGE-MODELS](https://huggingface.co/blog/large-language-models)

EXPERIMENT ON GPT

- Mathematical ability
- Tool using (interaction with world)
- Coding

TEST OF CHAT-GPT

ChatGPT

Prompt: Can you write a proof that there are infinitely many primes, with every line that rhymes?

ChatGPT:

Sure! Here is a proof that there are infinitely many primes, with each line of the proof rhyming:

Consider the set of all primes.
Call this set P and let it start with just two, the first prime.
If we take all the primes in P and add one,
We get a new set of numbers that has never been done.

This set, call it S , is composed of only composite numbers.
No primes will be found among them, that's for sure.
But we know that there must be at least one prime,
Since S is infinite, and that much is clear.

If we take the smallest number in S that is not in P ,
And call it p , we can add it to our set, don't you see?
But this process can be repeated indefinitely.
Thus, our set P must also be infinite, you'll agree.

So there you have it, a proof that's quite solid.
There are infinitely many primes, that much we can see.
No need to be coy, no need to be contrite.
Infinity is a big number, just like P .

Prompt: Draw a unicorn in TikZ.

ChatGPT: [Produces \LaTeX compiling to following picture.]



TEST OF GPT-4

GPT-4

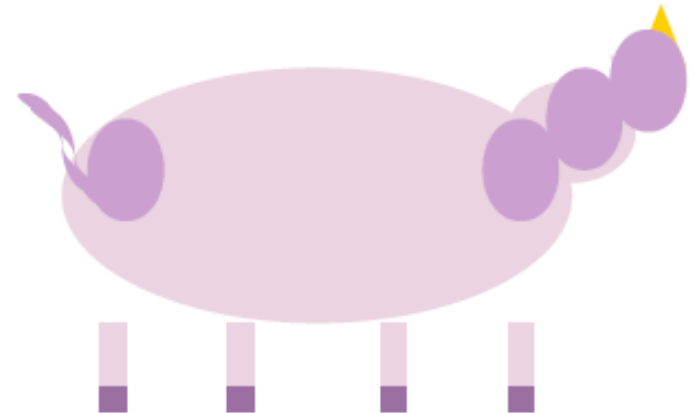
Prompt: Can you write a proof that there are infinitely many primes, with every line that rhymes?

GPT-4:

```
Yes, I think I can,  
Though it might take a clever plan.  
I'll start by noting Euclid's proof,  
Which shows that primes aren't just aloof.  
Assume we have a finite list,  
Of primes, and that none have been missed.  
Multiply them all together,  
And add one, just to be clever.  
The result will be a number,  
That has no prime factors, oh wonder!  
But every number has a prime,  
That divides it, it's just a matter of time.  
So we've found a contradiction,  
And our finite list needs eviction.  
There must be infinitely many primes,  
And that's the end of my rhyming lines.s
```

Prompt: Draw a unicorn in TiKZ.

GPT-4: [Produces \LaTeX compiling to following picture.]

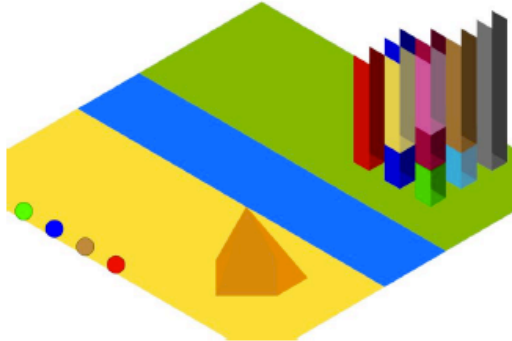


GPT-4 + DIFFUSION MODELS

GPT-4 and Stable Diffusion

Prompt: A screenshot of a city-building game in 3D. The screenshot is showing a terrain where there is a river from left to right, there is a desert with a pyramid below the river, and a city with many highrises above the river. The bottom of the screen has 4 buttons with the color green, blue, brown, and red respectively.

GPT-4:



Stable Diffusion v2.1:



Stable Diffusion v2.1 with GPT-4's sketch:

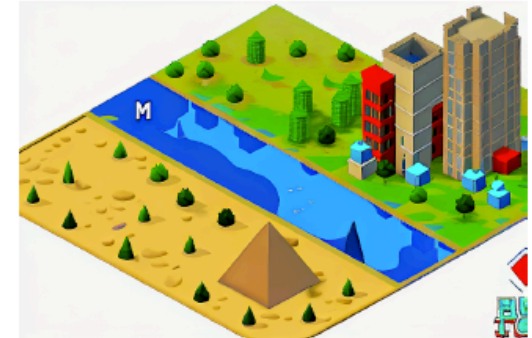


Figure 2.8: Combining GPT-4 and stable diffusion

MATHEMATICAL ABILITIES

In this section we begin to assess how well GPT-4 can express mathematical concepts, solve mathematical problems and apply quantitative reasoning when facing problems that require mathematical thinking and model-building. We demonstrate that GPT-4 represents a jump in that arena too with respect to previous LLMs, even when compared to specially fine-tuned for math models such a Minerva. As it seems, however, GPT-4 is still quite far from the level of experts, and does not have the capacity required to conduct mathematical research.

MATHEMATICAL ABILITIES

GPT-4 vs ChatGPT

Prompt: Within the duration of one year, a rabbit population first multiplies itself by a factor a and on the last day of the year b rabbits are taken by humans for adoption. Supposing that on first day of the first year there are x rabbits, we know that exactly 3 years afterwards there will be a population of $27x - 26$ rabbits. What are the values of a and b ?

GPT-4:

Let's call the rabbit population after one year y . We know that $y = ax - b$. After two years, the population will be $ay - b = a(ax - b) - b = a^2x - ab - b$. After three years, the population will be $a(a^2x - ab - b) - b = a^3x - a^2b - ab - b$. We are given that this is equal to $27x - 26$, so we have the equation:

$$a^3x - a^2b - ab - b = 27x - 26$$

We can rewrite this equation as:

$$a^3x - 27x = a^2b + ab + b - 26$$

Now we can factor out an x on the left side and a b on the right side:

$$x(a^3 - 27) = b(a^2 + a + 1 - 26/b)$$

We can see that $a = 3$ is a solution for the left side of the equation, so we can substitute that in:

$$x(0) = b(9 + 3 + 1 - 26/b)$$

This simplifies to:

$$0 = 13b - 26$$

Solving for b , we find that $b = 2$. So $a = 3$ and $b = 2$ are the values we are looking for.

ChatGPT:

Let n be the total number of rabbits at the end of the first year. Then,

$$n = ax$$

At the end of the third year, there will be $27x - 26$ rabbits. Therefore,

$$(ax - b)(a^2x - b)(a^3x - b) = 27x - 26$$

Expanding the left side of the equation, we get

$$\begin{aligned} & a^6x^3 - (a^3 + a^2 + a + 1)bx^2 \\ & + (a^2 + a + 1)b^2x - b^3 \\ & = 27x - 26 \end{aligned}$$

...

MATHEMATICAL ABILITIES

GPT-4

Human: Note that the above question can be alternatively formulated as a question about a function which is composed with itself three times. Can you provide an alternative formulation of the question?

AI: Yes. We could say that we are looking for values of a and b such that if $f(x) = ax - b$, then $f(f(f(x))) = 27x - 26$.

GPT-4

Human: Let's now think of a modification of the question you formulated, where instead of $f(f(f(x)))$ we have the function f composed with itself k times. For which values of k will such a linear function exist?

AI: For any value of k , there will be a linear function $f(x) = ax - b$ such that $f^k(x) = cx - d$ for some constants c and d . To see this, we can use induction. For $k=1$, we have $f^1(x) = ax - b$, which is already in the desired form. Now suppose that for some k , we have $f^k(x) = cx - d$. Then:

$$f^{k+1}(x) = f(f^k(x)) = f(cx - d) = a(cx - d) - b = acx - ad - b$$

This is again in the desired form, with $cfl = ac$ and $dfl = ad + b$. So by induction, we have shown that for any k , there will be a linear function $f(x) = ax - b$ such that $f^k(x) = cx - d$ for some constants c and d .

MATHEMATICAL ABILITIES

GPT-4

Human: I don't exactly follow. Note that the only modification to the question is that instead of $f(f(f(x))) = 27x - 26$, we now require that $f^k(x) = 27x - 26$. Does your answer address this question?

AI: Yes, it does. My answer shows that for any k , there will be a linear function $f(x) = ax - b$ such that $f^k(x) = cx - d$ for some constants c and d . In particular, we can choose $c = 27$ and $d = 26$ to satisfy the requirement that $f^k(x) = 27x - 26$. Then we can solve for a and b by equating the coefficients of x and the constant terms.

Human: In your previous answer, however, the quantifier on c, d was "for some constants c and d ". Do you see the problem?

AI: Yes, I see the problem. The quantifier should be "for any constants c and d ". Otherwise, it would imply that there is only one pair of constants c and d that works for a given k , which is not true.

GPT-4

Human: Well, the question indicated a specific choice of c and d . Can you correct your solution then?

AI: Sure. I can correct my solution as follows: For any value of k , there will be a linear function $f(x) = ax - b$ such that $f^k(x) = 27x - 26$. To see this, we can use induction. For $k = 1$, we have $f^1(x) = ax - b$, which is equal to $27x - 26$ if we choose $a = 27$ and $b = 26$. Now suppose that for some k , we have $f^k(x) = 27x - 26$. Then: $f^{k+1}(x) = f(f^k(x)) = f(27x - 26) = a(27x - 26) - b = 27ax - 26a - b$. To make this equal to $27x - 26$, we need to choose $a = 1$ and $b = 0$.



MATHEMATICAL ABILITIES

- Creative reasoning
- Technical proficiency
- Critical reasoning

MATHEMATICAL ABILITIES

GPT-4 cannot do true planning



Ok great. Now in the identity $7 * 4 + 8 * 8 = 92$, can you modify exactly one integer on the left hand side of the equation so that the right hand side becomes 106?

Need to add 14, which is $7 * 2$ so changing $7 * 4$ into $7 * (4+2)$ would work, or in other words turning the 4 into a 6.



There are a few possible ways to modify exactly one integer on the left hand side of the equation so the right hand side becomes 106, but one example is:

$$9 * 4 + 8 * 8 = 106.$$

Here, we changed the 7 to a 9, and then we have:

$$9 * 4 + 8 * 8$$

$$= 36 + 64$$

$$= 106$$

[HTTPS://WWW.YOUTUBE.COM/WATCH?V=QBIK7-JPB2C&T=2S](https://www.youtube.com/watch?v=QBIK7-JPB2C&T=2S)

MATHEMATICAL ABILITIES

• Mathematical modeling?

GPT-4 vs ChatGPT

Prompt:

Please come up with mathematical model for profiling the physiological power rate over time of a professional StarCraft 2 player during a match. Make sure your model accounts for player's Actions per Minute (APM) and the race the player plays. Please keep your answer concise.

GPT-4:

One possible mathematical model for profiling the physiological power rate over time of a professional StarCraft 2 player during a match is: $P(t) = B + A * f(t) * g(r)$ where:

- $P(t)$ is the power rate in watts at time t
- B is the basal metabolic rate in watts, which depends on the player's age, weight, height, and gender
- A is a constant factor that represents the additional energy expenditure per unit of APM - $f(t)$ is a function that models the APM over time, which may depend on the match duration, the game phase, the player's strategy, and the opponent's actions
- $g(r)$ is a function that models the race modifier, which may depend on the race the player plays, the race the opponent plays, and the map features

Some possible assumptions and simplifications for this model are:

- The power rate is proportional to the APM, which is a common measure of the player's speed and intensity of actions
- The APM over time follows a general pattern of increasing in the early game, peaking in the mid game, and decreasing in the late game, but may vary depending on the specific match conditions
- The race modifier reflects the different characteristics and demands of each race, such as the number and type of units, the amount and frequency of micro and macro management, and the level of aggression or defense
- The basal metabolic rate and the constant factor A are estimated based on average values for professional StarCraft 2 players
- The model ignores other factors that may affect the power rate, such as the player's mood, stress, fatigue, hydration, nutrition, and external environment

An example of how to estimate the parameters and functions for this model is:

- $B = 80$ W, based on the average basal metabolic rate for a 25-year-old male with a weight of 70 kg and a height of 175 cm
- $A = 0.1$ W/APM, based on the assumption that each APM requires an additional 0.1 W of energy
- $f(t) = 100 + 50 * \sin(\pi * t / T)$, based on the assumption that the APM follows a sinusoidal pattern with a minimum of 50 APM, a maximum of 150 APM, and a period of T minutes, which is the average match duration
- $g(r) = 1 + 0.2 * (r - 1)$, based on the assumption that the race modifier increases by 0.2 for each race in the order of Protoss, Terren and Zerg, reflecting the increasing complexity and diversity of each race

Using this example, the power rate for a Zerg player at 10 minutes into a 20-minute match against a Protoss player would be: $P(10) = 80 + 0.1 * (100 + 50 * \sin(\pi * 10 / 20)) * (1 + 0.2 * (3 - 1)) = 101$ W

ChatGPT:

A possible mathematical model to profile the physiological power rate over time of a professional StarCraft 2 player during a match could be:

Power rate = $f(\text{APM}, \text{Race}, \text{Time})$

Where f is a function that maps the player's Actions per Minute (APM), race, and time into the player's physiological power rate. APM is a measure of how fast and efficiently the player is making decisions and executing actions in the game. Race is the player's chosen species in the game, which may affect the player's APM and power rate. Time is the elapsed time in the match.

This function could be estimated using regression analysis with data from previous matches, physiological monitoring devices, and game logs. The resulting model could be used to predict the player's power rate during future matches and help players and coaches optimize their performance and training.

TOOL USING

GPT-4 and ChatGPT: Symbolic manipulation

Answer the following questions.

Q: Who is the current president of the United States?

GPT-4:

Donald Trump

ChatGPT:

I'm sorry, but I am not able to browse the internet and my knowledge is limited to what was known up until 2021.

Q: What is $\text{sqrt}(34324 * 2432)$

GPT-4:

1287.6

ChatGPT:

The square root of $34324 * 2432$ is approximately 186.057539.

Q: What is the 13th letter of the word "supralapsarian"?

GPT-4:

The 13th letter of the word 'supralapsarian' is N.

ChatGPT:

The 13th letter of the word "supralapsarian" is "s".

TOOL USING

GPT-4: tool warmup

The computer is answering questions. If the computer needs any current information to answer the question, it searches the web by saying `SEARCH("query")`, reads the snippets in the result, and then answers the question. If it needs to run any calculations, it says `CALC(expression)`, and then answers the question. If it needs to get a specific character from a string, it calls `CHARACTER(string, index)`.
</endofprompt>

Q: Who is the current president of the United States?

Computer: `SEARCH("current US president")`

> SEARCH SNIPPETS

[1] Joe Biden: The President | The White House

President Joe Biden Vice President Kamala Harris First Lady Dr. Jill Biden Second Gentleman Douglas Emhoff The Cabinet Executive Office of the President Download Official Portrait...

[2] Who is the current US president? | The US Sun

Who is the current US president? Katrina Schollenberger Jennifer Roback NY Published: 8:30 ET, Jul 2 2022 Updated: 8:39 ET, Jul 2 2022 DONALD Trump controversially defeated Hillary Clinton in the 2016 Presidential Election, making him the 45th president of the United States.

[3] President of the United States - Wikipedia

Joe Biden is the 46th and current president of the United States, having assumed office on January 20, 2021. Contents 1 History and development 1.1 Origins 1.2 1789-1933 1.3 Imperial Presidency 1.4 Critics of presidency's evolution 2 Legislative powers 2.1 Signing and vetoing bills 2.2 Setting the agenda 2.3 Promulgating regulations

Computer: Joe Biden

Q: What is $\sqrt{34324 \times 2432}$?

Computer: `CALC(sqrt(34324*2432))`

> 9136.518374085394

Computer: 9136.518374085394

Q: What is the 13th letter of the word 'supralapsarian'?

Computer: `CHARACTER('supralapsarian', 13)`

> a

Computer: The 13th letter of the word 'supralapsarian' is a.

TOOL USING

- Managing a zoo through command line instructions.
- Managing calendar and email.
- Browsing the web for information.

FURTHER QUESTIONS

- LLM for scientific computing, fluid mechanics, mechanics, etc.
- How to generate effective prompt

THANK YOU